Towards Autonomous Bound Constraint Handling: Study on an Adaptive Correction in Differential Evolution -Supplementary Material-

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I. Results for LSHADE coupled with individual BCHMs

The results presented in Table II report the final error and population variance (varPop) values obtained by averaging over 5 independent runs, considering the largest common number of iterations for LSHADE coupled with `'sat'`, `'beta'`, `'expC_B'`, `'vectT'`, `'vectB'` BCHMs and the adaptive operator variants.

Algorithm	f3		f4		f5		f16		f23	
	Error	varPop	Error	varPop	Error	varPop	Error	varPop	Error	varPop
LSHADE_vectT	4.61E-07	9.17E-11	1.49E-06	9.26E-11	1.55E-06	8.53E-11	4.374	6.376	0.635	6.028
LSHADE_expC_B	4.88E-07	9.58E-11	1.33E-06	9.13E-11	0.0001	9.01E-11	5.737	7.161	0.836	7.760
LSHADE_beta	5.61E-07	9.13E-11	1.24E-06	9.57E-11	0.0004	9.33E-11	4.631	5.546	0.841	5.703
LSHADE_sat	5.99E-07	9.66E-11	1.81E-06	9.11E-11	2.77E-05	7.57E-11	3.834	8.906	1.424	8.438
LSHADE_vectB	0.994	9.46E-11	38.581	8.65E-11	0.019	4.63E-11	3.762	9.41E-11	0.351	9.80E-11
LSHADE_adaptive_linear_0.9	6.172	9.27E-11	4.377	9.18E-11	**0.0**	0.277	8.122	9.51E-11	**0.672**	9.35E-11
LSHADE_adaptive_linear_0.5	8.158	9.08E-11	29.124	9.03E-11	**1.07E-10**	0.155471	7.927	8.06E-11	**0.758**	7.72E-11
LSHADE_adaptive_linear_0.0	8.361	9.12E-11	11.884	9.33E-11	**3.44E-12**	0.111781	6.872	9.04E-11	**0.301**	9.24E-11
LSHADE_adaptive_beta	11.939	9.10E-11	2.387	9.73E-11	**2.53E-10**	0.081	6.538	8.57E-11	**0.742**	1.034

TABLE I
Last Error and varPop, averaged values, for LSHADE coupled with different BCHMs for optimizing functions f_3, f_4, f_5, f_{16}, and f_{23}, instance 1, BBOB

II. Adaptive BCHM with Favouring Saturation

This section presents the results obtained from applying the adaptive BCHM operator with a focus on favouring saturation. One of the main issues encountered by the adaptive variants during the optimization process is the rapid loss of diversity in the population, which led to premature termination of the algorithm due to the stopping condition involving a threshold for the variance.

To mitigate this issue, the relative difference between the variance from the preceding generation and the current generation was computed throughout the generations and an experimental threshold of 0.25 was established.. Subsequently, we leveraged the outcomes depicted in Figure 4, which illustrates that the saturation (sat) method maintains higher variance values in comparison to other corrections.

Overall, this section highlights the improvements achieved through the adaptive strategy with favouring saturation, showcasing the effectiveness of maintaining diversity to enhance optimization performance.

A. Adaptive linear with $\alpha = 0.5$

Run	f3			f4			f5			f16			f23		
	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop
1	576	4.83E-07	9.80E-11	748	9.92E-07	8.93E-11	27	0	0.023	376	3.154	9.98E-11	362	0.129	9.79E-11
2	540	1.989	9.41E-11	623	16.914	9.13E-11	38	3.24E-10	1.52E-05	287	5.190	9.82E-11	445	0.348	9.51E-11
3	618	4.75E-07	9.57E-11	564	24.873	9.25E-11	30	0	0.002	316	3.049	9.75E-11	297	0.137	9.99E-11
4	600	4.17E-07	8.75E-11	760	1.04E-06	9.98E-11	41	0	6.69E-06	269	12.679	2.53E-11	207	0.742	9.81E-11
5	611	3.21E-07	9.32E-11	584	12.934	8.85E-11	50	2.02E-05	5.27E-11	215	7.433	9.93E-11	157	1.900	-5.17E-15

TABLE II
Last iteration, error and varPop values as average on 5 independent runs for L-SHADE coupled with adaptive linear correction mechanism, $\alpha = 0.5$, $k = 0.3$ adjustment for probability of selecting sat, relative diversity threshold $= 0.25$

B. Adaptive linear with $\alpha = 0.0$

Run	f3			f4			f5			f16			f23		
	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop
1	470	39.798	9.88E-11	668	7.959	9.46E-11	33	0	0.001	269	7.211	8.72E-11	401	1.396	9.11E-11
2	643	3.74E-07	9.09E-11	715	3.979	9.88E-11	56	4.11E-07	2.12E-11	285	9.391	9.79E-11	229	2.142	9.76E-11
3	645	5.55E-07	9.07E-11	716	8.49E-07	9.36E-11	31	0	0.0004	298	2.905	8.82E-11	316	0.076	8.92E-11
4	339	64.691	9.57E-11	517	54.722	9.51E-11	68	3.08E-06	8.53E-11	225	6.874	8.77E-11	112	2.955	3.70E-14
5	386	32.833	9.72E-11	767	7.33E-07	9.44E-11	52	6.34E-08	3.05E-11	344	0.704	9.97E-11	406	0.147	9.77E-11

TABLE III

LAST ITERATION, ERROR AND VARPOP VALUES AS AVERAGE ON 5 INDEPENDENT RUNS FOR L-SHADE COUPLED WITH ADAPTIVE LINEAR CORRECTION MECHANISM, $\alpha = 0.0$, $k = 0.3$ ADJUSTMENT FOR PROBABILITY OF SELECTING SAT, RELATIVE DIVERSITY THRESHOLD $= 0.25$

C. Adaptive linear with $\alpha = 0.9$

Run	f3			f4			f5			f16			f23		
	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop
1	224	56.618	9.05E-11	707	9.15E-07	9.03E-11	37	2.07E-10	9.08E-06	330	9.030	8.65E-11	51	2.331	4.56E-15
2	355	36.339	9.10E-11	752	7.60E-07	9.39E-11	61	1.13E-07	4.44E-11	327	1.880	9.24E-11	270	0.973	9.68E-11
3	631	2.84E-07	9.30E-11	645	12.934	9.41E-11	55	0	6.78E-09	296	8.634	9.28E-11	74	2.810	-5.15E-15
4	635	2.40E-07	9.22E-11	471	32.833	9.59E-11	42	0	4.28E-08	366	6.913	9.96E-11	316	0.847	9.96E-11
5	496	13.929	9.82E-11	751	9.08E-07	9.56E-11	24	1.74E-09	0.012	366	7.643	9.57E-11	82	3.072	6.99E-16

TABLE IV

LAST ITERATION, ERROR AND VARPOP VALUES AS AVERAGE ON 5 INDEPENDENT RUNS FOR L-SHADE COUPLED WITH ADAPTIVE LINEAR CORRECTION MECHANISM, $\alpha = 0.9$, $k = 0.3$ ADJUSTMENT FOR PROBABILITY OF SELECTING SAT, RELATIVE DIVERSITY THRESHOLD $= 0.25$

D. Adaptive Beta

Run	f3			f4			f5			f16			f23		
	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop	It	Error	varPop
1	330	51.643	8.81E-11	759	7.90E-07	9.53E-11	31	0	0.02	160	9.087	8.43E-11	390	0.148	9.50E-11
2	531	5.969	8.67E-11	767	7.20E-07	9.34E-11	27	0	0.019	339	1.696	9.58E-11	39	2.760	-7.42E-15
3	301	31.857	9.88E-11	702	9.949	8.88E-11	26	3.55E-15	0.011	339	11.08	9.50E-11	447	0.406	9.95E-11
4	474	10.944	8.90E-11	473	25.868	9.58E-11	27	0	0.004	344	5.656	1.27E-11	80	2.494	1.65E-15
5	289	36.339	9.86E-11	740	7.47E-07	9.51E-11	21	0	0.272	137	13.463	5.99E-11	273	0.396	9.09E-11

TABLE V

LAST ITERATION, ERROR AND VARPOP VALUES AS AVERAGE ON 5 INDEPENDENT RUNS FOR L-SHADE COUPLED WITH ADAPTIVE LINEAR CORRECTION MECHANISM, BETA ADAPTATION STRATEGY, $k = 0.3$ ADJUSTMENT FOR PROBABILITY OF SELECTING SAT, RELATIVE DIVERSITY THRESHOLD $= 0.25$

E. Averaged Adaptive Operator behavior

	$\alpha = 0.5$		$\alpha = 0.0$		$\alpha = 0.9$		Beta	
	Error	varPop	Error	varPop	Error	varPop	Error	varPop
f3	0.397	9.37E-11	27.464	9.46E-11	21.377	9.30E-11	27.351	9.22E-11
f4	10.944	9.23E-11	13.332	9.53E-11	9.153	9.39E-11	7.163	9.37E-11
f5	4.03E-06	0.005	7.12E-07	0.0003	2.30E-08	0.002	7.11E-16	0.065
f16	6.301	8.40E-11	5.417	9.21E-11	6.820	9.34E-11	8.19	6.96E-11
f23	0.651	7.82E-11	1.343	7.51E-11	2.007	3.93E-11	1.241	5.71E-11

TABLE VI

OVERALL BEHAVIOR OF DIFFERENT STRATEGIES AVERAGED OVER 5 RUNS

III. FIGURES

Figures 1 and 2 illustrate the evolution of population variance computed per component, respectively probability of generating infeasible components, during iterations. In figures 3 and 4 are exposed the changes in the distribution probability used in adaptive linear scheme where $\alpha = 0.5$, respectively $\alpha = 0$. Last, figure 5 exposes that maintaining higher variance levels by favoring saturation, leads to more iterations and reduced error compared to the original variant.

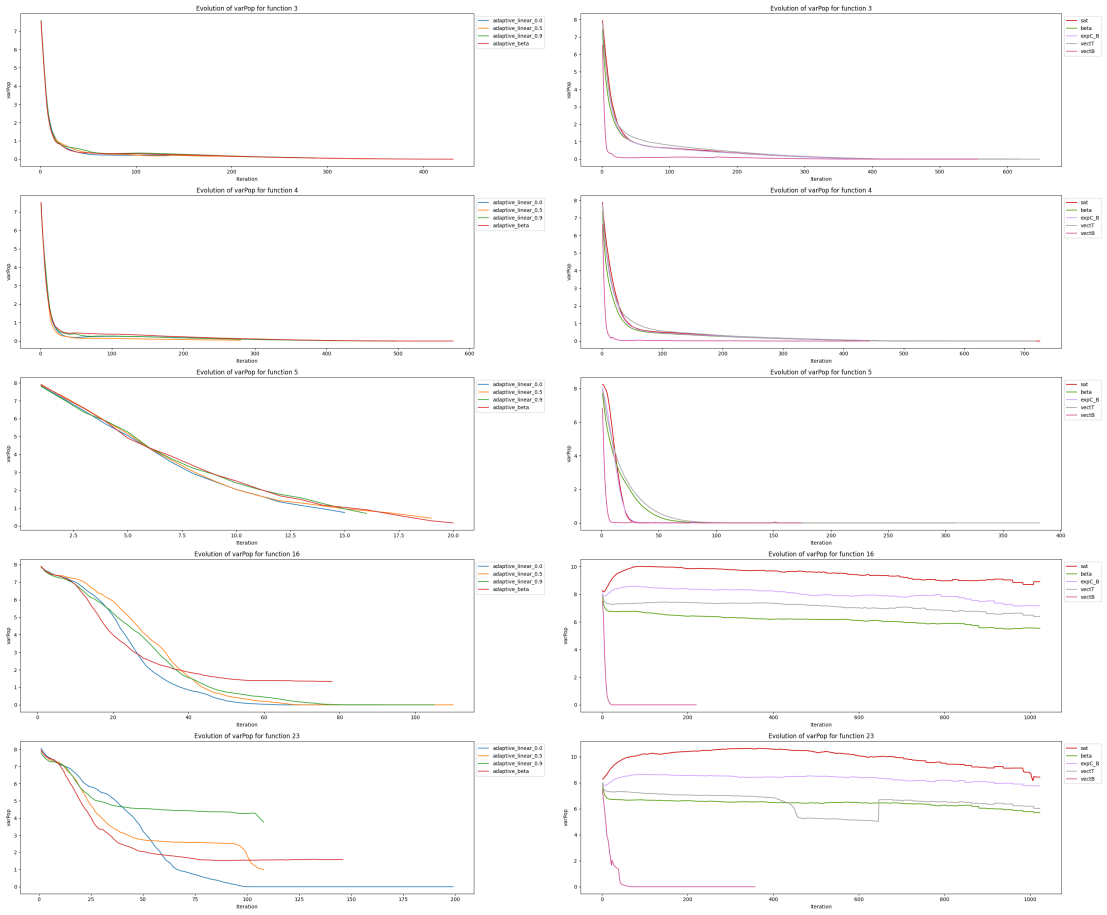

Fig. 1. Comparison of diversity evolution during iterations for LSHADE coupled with adaptive BCHM operator (left) and LSHADE coupled with individual BCHMs (right)

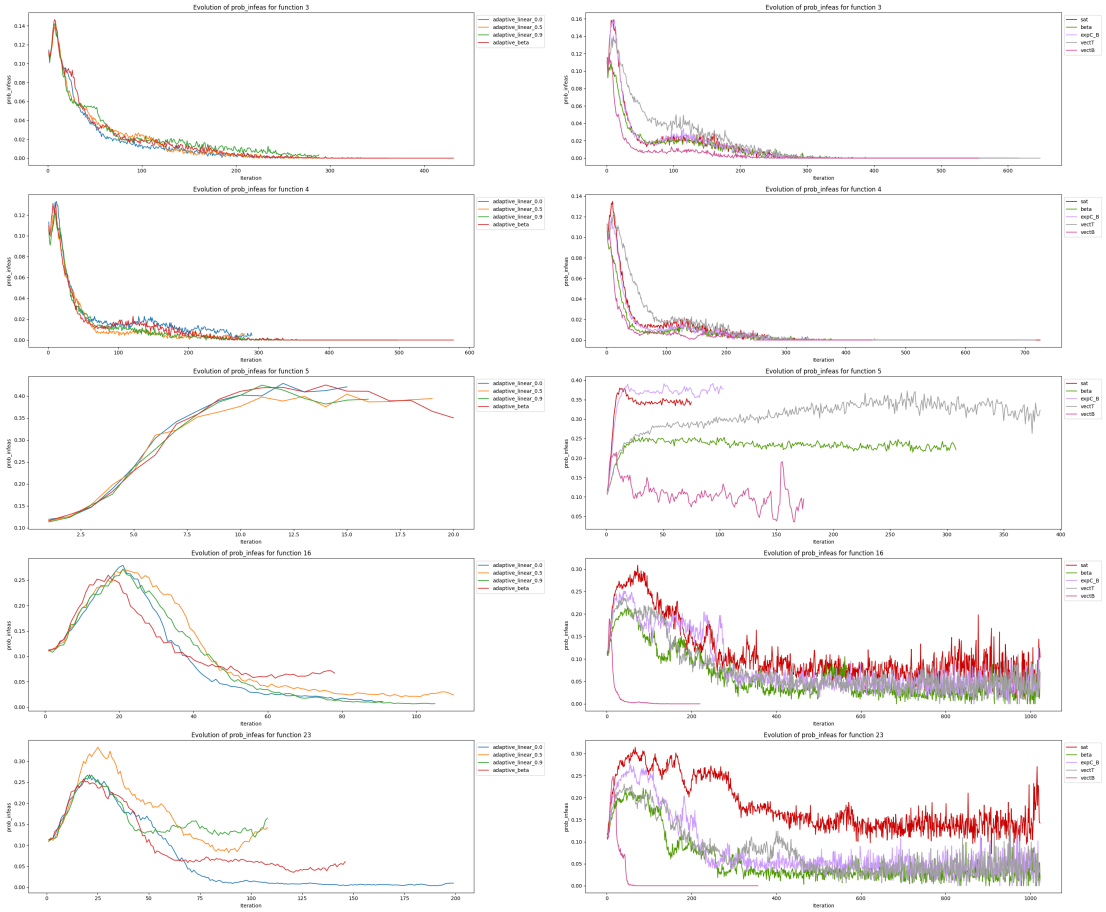

Fig. 2. Comparison of probability of generating infeasible components evolution during iterations for LSHADE coupled with adaptive BCHM operator (left) and LSHADE coupled with individual BCHMs (right)

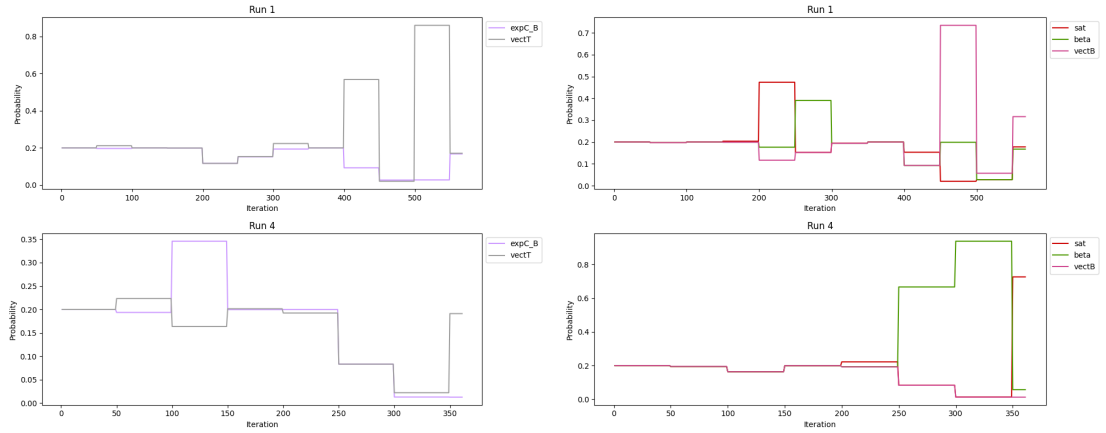

Fig. 3. Evolution of distribution probability for BCHM selection for LSHADE coupled with adaptive linear ($\alpha = 0.5$) operator on f_3

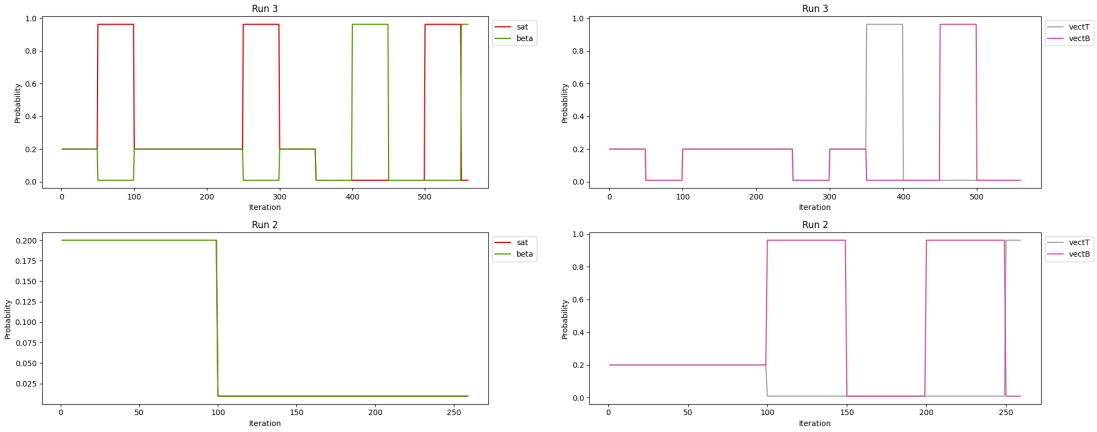

Fig. 4. Evolution of distribution probability for BCHM selection for LSHADE coupled with adaptive linear ($\alpha = 0$) operator on f_3

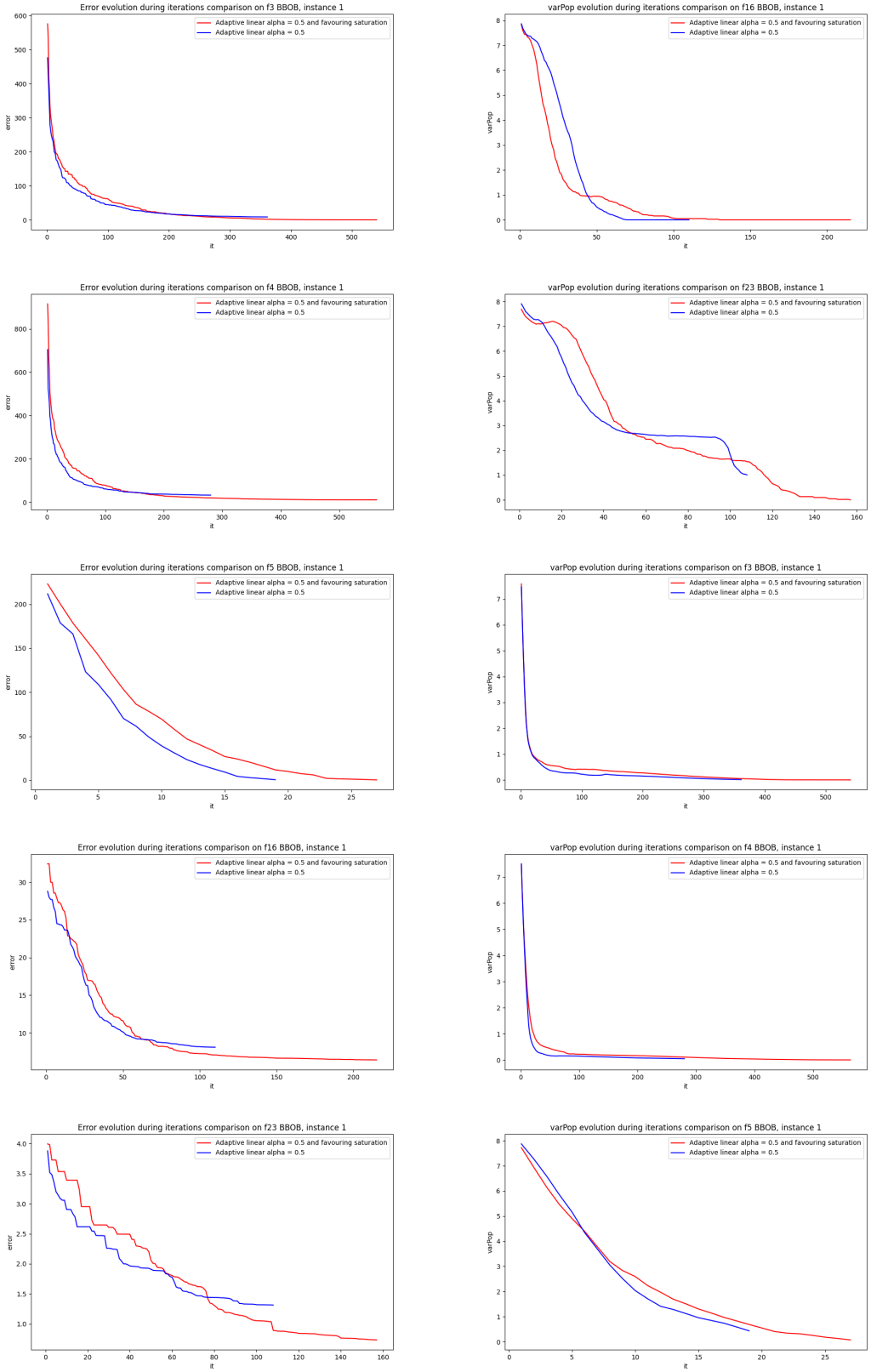

Fig. 5. Comparison of error and varPop evolution during iterations for L-SHADE coupled with adaptive linear correction mechanism with $alpha = 0.5$ and $k = 0.3$ adjustment for probability of selecting `sat` method when relative diversity falls below the threshold of 0.25