Supplementary Material

1 Problem definition

The steps performed in the theoretical analysis are detailed as follows:

1. Determine the probability density function (pdf) f_Z of $Z = X_1 + (X_2 - X_3)$. First, we proceed by determining the pdf of Y, obtained as the difference of the random variables involved in the mutation, $Y = X_2 - X_3$: .

$$f_Y(y) = \int_{-\infty}^{\infty} f_2(x_2) f_{X_3}(x_2 - y) dx_2 \tag{1}$$

Then the pdf of Z can be computed using the convolution product:

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(y) f_{X_1}(z - y) dy \tag{2}$$

2. Compute the bound violation probability using f_Z, as:

$$P(Z < a) = \int_{-\infty}^{a} f_Z(z) dz \tag{3}$$

$$P(Z > b) = 1 - \int_{-\infty}^{b} f_Z(z) dz \tag{4}$$

2 Assumptions

1. Without loss of generality, we consider the bounds $a = 0$ and $b = 1$.
2. Population corresponding to generation g, $P(g)$ consists of two subpopulations: $P(g) = P_w(g) \cup P_c(g)$, where:
   - $P_w(g)$ is the subpopulation corresponding to feasible mutants that were not the subject of any BCHMs.
   - $P_c(g)$ is the subpopulation corresponding to mutants whose components were corrected using `exp_c` (*Exponentially Confined*).
3. It is assumed that points in $P_w(g)$ are uniformly distributed in [0,1].
4. The probability density of the distribution of points in $P_c(g)$:
   - In case of lower bound violation

$$f(c) = \frac{e^{R-c}}{e^R - 1}, \quad c \in [0, R]; \tag{5}$$

   - In case of upper bound violation

$$f(c) = \frac{e^{c-R}}{e^{(1-R)} - 1}, \quad c \in [R, 1]. \tag{6}$$

3 Computation

3.1 Computing the pdf of the difference

- $Y = X_2 - X_3$;
- Cases analyzed:

Case	X_2	X_3	**Probability**
1.	$\in P_c$	$\in P_w$	$p_v \cdot (1 - p_v)$
2.	$\in P_w$	$\in P_c$	$p_v \cdot (1 - p_v)$
3.	$\in P_c$	$\in P_c$	p_v^2

Table 1. Combinations of elements involved in difference and their corresponding probabilities

- In the paper is presented the case when the lower bound was violated in the previous generation by elements in P_c. For the following computation in this subsections it was assumed that the **upper bound was exceeded**, i.e. points are distributed according to Eq. (6).

Case 1. $X_2 \in P_c$, $X_3 \in P_w$, with the corresponding probability density functions:

$$f_{X_2}(x_2) = \frac{e^{x_2 - R}}{e^{1-R} - 1} \qquad , x_2 \in [R, 1] \tag{7}$$

$$f_{X_3}(x_2 - y) = 1 \qquad , x_2 \in [y, y+1] \tag{8}$$

Proof.

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X_2}(x_2) f_{X_3}(x_2 - y) dx_2 = \begin{cases} k \cdot \int_R^{y+1} e^{x_2} dx_2 & , y \in [R-1, 0] \\ k \cdot \int_R^1 e^{x_2} dx_2 & , y \in [0, R] \\ k \cdot \int_y^1 e^{x_2} dx_2 & , y \in [R, 1] \end{cases}$$

$$= \begin{cases} \dfrac{e^R - e^{1+y}}{e^R - e} & , y \in [R-1, 0] \\ 1 & , y \in [0, R] \\ \dfrac{e^y - e}{e^R - e} & , y \in [R, 1] \end{cases} \tag{9}$$

with $k = \dfrac{e^{-R}}{e^{1-R} - 1}$.

Let us consider the functions that will be used in further computation:

$$g_1(y) = \frac{e^R - e^{1+y}}{e^R - e} \qquad , g_1 : [-1, R-1] \to [0,1] \tag{10}$$

$$h_1(y) = \frac{e^y - e}{e^R - e} \qquad , h_1 : [R, 1] \to [0,1] \tag{11}$$

Case 2. $X_2 \in P_w$, $X_3 \in P_c$, with the corresponding probability density functions:

$$f_{X_2}(x_2) = 1 \qquad , x_2 \in [0,1] \tag{12}$$

$$f_{X_3}(x_2 - y) = \frac{e^{x_2 - y - R}}{e^{1-R} - 1} \qquad , x_2 \in [y + R, y + 1] \tag{13}$$

Proof.

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X_2}(x_2) f_{X_3}(x_2 - y) dx_2 = \begin{cases} k \cdot \int_R^{y+1} e^{x_2 - y} dx_2 & , y \in [-1, -R] \\ k \cdot \int_R^1 e^{x_2 - y} dx_2 & , y \in [-R, 0] \\ k \cdot \int_y^1 e^{x_2 - y} dx_2 & , y \in [0, 1-R] \end{cases}$$

$$= \begin{cases} \dfrac{e^{-y} - e}{e^R - e} & , y \in [R-1, 0] \\ 1 & , y \in [0, R] \\ \dfrac{e^R - e^{1-y}}{e^R - e} & , y \in [R, 1] \end{cases} \tag{14}$$

with $k = \dfrac{e^{-R}}{e^{1-R} - 1}$.

Let us consider the functions that will be used in further computation:

$$g_2(y) = \frac{e^{-y} - e}{e^R - e} \qquad , g_2 : [-1, R-1] \to [0,1] \tag{15}$$

$$h_2(y) = \frac{e^R - e^{1-y}}{e^R - e} \qquad , h_2 : [R, 1] \to [0,1] \tag{16}$$

Case 3. $X_2 \in P_c$, $X_3 \in P_c$, with the corresponding probability density functions:

$$f_{X_2}(x_2) = \frac{e^{x_2 - R}}{e^{1-R} - 1} \qquad , x_2 \in [R, 1] \tag{17}$$

$$f_{X_3}(x_2 - y) = \frac{e^{x_2 - y - R}}{e^{1-R} - 1} \qquad , x_2 \in [y + R, y + 1] \tag{18}$$

Proof.

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X_2}(x_2) f_{X_3}(x_2 - y) dx_2 = \begin{cases} k^2 \cdot \int_R^{y+1} e^{2 \cdot x_2 - y} dx_2 & , y \in [R-1, 0] \\ k^2 \cdot \int_{y+R}^1 e^{2 \cdot x_2 - y} dx_2 & , y \in [0, 1-R] \end{cases}$$

$$= \begin{cases} \dfrac{e^{-y} \cdot \left(e^{-2 \cdot R} + e^{2+2 \cdot y}\right)}{2 \cdot (e - e^R)^2} & , y \in [R-1, 0] \\ \dfrac{e^{-y} \cdot \left(e^{2} - e^{2+2 \cdot (R+y)}\right)}{2 \cdot (e - e^R)^2} & , y \in [0, 1-R] \end{cases} \tag{19}$$

with $k = \dfrac{e^{-R}}{e^{1-R} - 1}$.

Let us consider the functions that will be used in further computation:

$$g_3(y) = \frac{e^{-y} \cdot \left(e^{-2 \cdot R} + e^{2+2 \cdot y}\right)}{2 \cdot (e - e^R)^2} \quad , g_3 : [R-1, 0] \to [0, 1] \tag{20}$$

$$h_3(y) = \frac{e^{-y} \cdot \left(e^{2} - e^{2+2 \cdot (R+y)}\right)}{2 \cdot (e - e^R)^2} \quad , h_3 : [0, 1-R] \to [0, 1] \tag{21}$$

3.2 Computing the pdf of mutant and the bound violation probability

- $Z = X_1 + Y$, with $Y = X_2 - X_3$;
- Cases analyzed:

Case	X_R	X_2	X_3	**Probability**
I*.	$\in P_w$	$\in P_c$	$\in P_w$	$p_v \cdot (1-p_v)^2$
II**.	$\in P_c$	$\in P_c$	$\in P_w$	$p_v^2 \cdot (1-p_v)$
III.	$\in P_w$	$\in P_c$	$\in P_c$	$p_v^2 \cdot (1-p_v)$
IV.	$\in P_c$	$\in P_c$	$\in P_c$	$(1-p_v)^3$
V.	$\in P_c$	$\in P_w$	$\in P_w$	$p_v \cdot (1-p_v)^2$
VI*.	$\in P_w$	$\in P_w$	$\in P_c$	$p_v \cdot (1-p_v)^2$
VII**.	$\in P_c$	$\in P_w$	$\in P_c$	$p_v \cdot (1-p_v)^2$

Table 2. Combinations of elements involved in mutation and their corresponding probabilities

- We continue the computation under the assumption of **upper bound violation** in the previous generation for points in $P_c(g)$, then we complete the cases under the assumption of lower bound exceed that were mentioned in the paper but not addressed.

Case I. $X_1 \in P_w$, with pdf described by (12), $X_2 \in P_c, X_3 \in P_w$, with pdf of $X_2 - X_3$ described by (9):

Proof.

$$f_Z(z) = \begin{cases} \int_{R-1}^{z} g_1(y)dy & , z \in [R-1, 0) \\ \int_{R-1}^{0} g_1(y)dy + \int_{0}^{z} dy & , z \in [0, R] \\ \int_{z-1}^{0} g_1(y)dy + \int_{0}^{R} dy + \int_{R}^{z} h_1(y)dy & , z \in [0, R] \\ \int_{z-1}^{R} dy + \int_{R}^{1} h_1(y)dy & , z \in [0, R] \\ \int_{z-1}^{1} h_1(y)dy & , z \in [0, R] \end{cases}$$

$$= \begin{cases} \frac{-e^{1+z} + e^R \cdot (2 - R + z)}{e^R - e} & , z \in [R-1, 0] \\ 1 - \frac{e^R \cdot (1-R)}{e^R - e} & , z \in [0, R] \\ \frac{-2 \cdot e^z + e \cdot (1+z) + e^R \cdot (z - R)}{e - e^R} & , z \in [R, 1] \\ 1 + R + \frac{e^r + e \cdot R}{e - e^R} - z & , z \in [1, 1+R] \\ 1 + R + \frac{e^r + e \cdot R}{e - e^R} - z & , z \in [1+R, 2] \end{cases} \tag{22}$$

Case II. $X_1 \in P_c$, with pdf described by (7), $X_2 \in P_c, X_3 \in P_w$, with pdf of $X_2 - X_3$ described by (9).
Here we distinguish 2 sub-cases:

- $R \in [0, 0.5]$

  Proof.

$$f_Z(z) = \begin{cases} k \cdot \int_{R-1}^{z-R} g_1(y) \cdot e^{z-y}dy & , z \in [2 \cdot R - 1, R) \\ k \cdot (\int_{z-1}^{0} g_1(y) \cdot e^{z-y}dy + \int_{0}^{z-R} dy) & , z \in [R, 2 \cdot R) \\ k \cdot (\int_{z-1}^{0} g_1(y) \cdot e^{z-y}dy + \int_{0}^{R} e^{z-y}dy + \int_{R}^{z-R} h_1(y) \cdot e^{z-y}dy) & , z \in [2 \cdot R, 1) \\ k \cdot (\int_{z-1}^{R} e^{z-y}dy + \int_{R}^{z-R} h_1(y) \cdot e^{z-y}dy) & , z \in [1, R+1) \\ k \cdot \int_{z-1}^{1} h_1(y) \cdot e^{z-y}dy & , z \in [R+1, 2] \end{cases}$$

$$= \begin{cases} \frac{e^{2 \cdot R} + e^{1+z} \cdot (-2 \cdot R + z)}{(e - e^R)^2} & , z \in [2 \cdot R - 1, R) \\ \frac{e^{2 \cdot R} - 2 \cdot e^{1+R} - e^{1+z} \cdot (z-2)}{(e - e^R)^2} & , z \in [R, 2 \cdot R) \\ \frac{-2 \cdot e^{1+R} + e^z \cdot (1 + 2 \cdot R - z) - e^{1+z} \cdot (z - R)}{e - e^R} & , z \in [2 \cdot R, 1) \\ \frac{e^2 - 2 \cdot e^{1+R} + e^z \cdot (1 + 2 \cdot R - z}{e - e^R} & , z \in [1, R+1) \\ \frac{e^2 + e^z \cdot (z-3)}{e - e^R} & , z \in [R+1, 2] \end{cases} \tag{23}$$

- $R \in [0.5, 1]$

Proof.

$$f_Z(z) = \begin{cases} k \cdot \int_{R-1}^{z-R} g_1(y) \cdot e^{z-y} dy & , z \in [2 \cdot R - 1, R) \\ k \cdot (\int_{z-1}^{0} g_1(y) \cdot e^{z-y} dy + \int_{0}^{z-R} dy) & , z \in [R, 1) \\ k \cdot (\int_{z-1}^{0} g_1(y) \cdot e^{z-y} dy + \int_{0}^{R} e^{z-y} dy + \int_{R}^{z-R} h_1(y) \cdot e^{z-y} dy) & , z \in [1, 2 \cdot R) \\ k \cdot (\int_{z-1}^{R} e^{z-y} dy + \int_{R}^{z-R} h_1(y) \cdot e^{z-y} dy) & , z \in [2 \cdot R, R+1) \\ k \cdot \int_{z-1}^{1} h_1(y) \cdot e^{z-y} dy & , z \in [R+1, 2] \end{cases}$$

$$= \begin{cases} \frac{e^{2 \cdot R} + e^{1+z} \cdot (-2 \cdot R + z)}{(e - e^R)^2} & , z \in [2 \cdot R - 1, R) \\ \frac{e^{2 \cdot R} - 2 \cdot e^{1+R} - e^{1+z} \cdot (z-2)}{(e - e^R)^2} & , z \in [R, 1) \\ 1 & , z \in [1, 2 \cdot R) \\ \frac{e^2 - 2 \cdot e^{1+R} + e^z \cdot (1 + 2 \cdot R - z}{e - e^R} & , z \in [2 \cdot R, R+1) \\ \frac{e^2 + e^z \cdot (z-3)}{e - e^R} & , z \in [R+1, 2] \end{cases} \tag{24}$$

Case III. $X_1 \in P_w$, with pdf described by (12), $X_2 \in P_c, X_3 \in P_c$, with pdf of $X_2 - X_3$ described by (19).
Here we distinguish 2 sub-cases:

- $R \in [0, 0.5]$

  Proof.

$$f_Z(z) = \begin{cases} \int_{R-1}^{z} g_3(y) dy & , z \in [R-1, 0) \\ \int_{r-1}^{0} g_3(y) dy + \int_{0}^{z} h_3(y) dy & , z \in [0, R) \\ \int_{z-1}^{0} g_3(y) dy + \int_{0}^{z} h_3(y) dy & , z \in [R, 1-R) \\ \int_{z-1}^{0} g_3(y) dy + \int_{0}^{1-R} h_3(y) dy & , z \in [1-R, 1 \\ \int_{z-1}^{R} e^{z-y} dy & , z \in [1, 2-R) \end{cases}$$

$$= \begin{cases} \frac{1}{2} \cdot (e - e^R)^2 \cdot [-e^{1+R} + e^{2+z} + e^{-3 \cdot R} \cdot (e - e^{r-z})] & , z \in [R-1, 0) \\ -\frac{1}{2} \cdot e^{-3 \cdot R} \cdot (e^R - e)^3 \cdot (1 + e^{1+3 \cdot R}) - \frac{1}{2} \cdot e^{-z} \cdot (e - e^R)^2 \cdot (e^z - 1) \cdot (-e^2 + e^{2 \cdot R + z}) & , z \in [0, R) \\ -\frac{1}{2} \cdot e^{-2 \cdot R - z} \cdot (e - e^R)^2 \cdot (-e + e^{2+2 \cdot R} + e^z - 2 \cdot e^{2+2 \cdot R + z} - e^{4 \cdot R + z} + e^{1+2 \cdot R + 2 \cdot z} + e^{4 \cdot R + 2 \cdot z}) & , z \in [R, 1-R) \\ \frac{e^2 - 2 \cdot e^{1+R} + e^z \cdot (1 + 2 \cdot R - z}{e - e^R} & , z \in [1-R, 1) \\ \frac{e^2 + e^z \cdot (z-3)}{e - e^R} & , z \in [1, 2-R] \end{cases} \tag{25}$$

- $R \in [0.5, 1]$

  Proof.

$$f_Z(z) = \begin{cases} \int_{R-1}^{z} g_3(y)dy & , z \in [R-1, 0) \\ \int_{r-1}^{0} g_3(y)dy + \int_{0}^{z} h_3(y)dy & , z \in [0, 1-R) \\ \int_{z-1}^{0} g_3(y)dy + \int_{0}^{z} h_3(y)dy & , z \in [1-R, R) \\ \int_{z-1}^{0} g_3(y)dy + \int_{0}^{1-R} h_3(y)dy & , z \in [R, 1) \\ \int_{z-1}^{R} e^{z-y}dy & , z \in [1, 2-R) \end{cases}$$

$$= \begin{cases} \frac{1}{2} \cdot (e - e^R)^2 \cdot [-e^{1+R} + e^{2+z} + e^{-3 \cdot R} \cdot (e - e^{r-z})] & , z \in [R-1, 0) \\ -\frac{1}{2} \cdot e^{-3 \cdot R} \cdot (e^R - e)^3 \cdot (1 + e^{1+3 \cdot R}) - \frac{1}{2} \cdot e^{-z} \cdot (e - e^R)^2 \cdot (e^z - 1) \cdot (-e^2 + e^{2 \cdot R + z}) & , z \in [0, 1-R) \\ \frac{1}{2} e^2 \cdot (e - e^R)^2 \cdot [(e - e^R)^2 + e^{-2 \cdot R - z} \cdot (e - e^z) \cdot (1 + e^{1+2 \cdot R + z})] & , z \in [1-R, R) \\ \frac{e^2 - 2 \cdot e^{1+R} + e^z \cdot (1 + 2 \cdot R - z)}{e - e^R} & , z \in [R, 1) \\ \frac{e^2 + e^z \cdot (z-3)}{e - e^R} & , z \in [1, 2-R] \end{cases} \tag{26}$$

Case IV. $X_1 \in P_c$, with pdf described by (7), $X_2 \in P_c, X_3 \in P_c$, with pdf of $X_2 - X_3$ described by (19).

Proof.

$$f_Z(z) = \begin{cases} k \cdot \int_{R-1}^{z-R} g_3(y) \cdot e^{z-y}dy & , z \in [2 \cdot R - 1, R) \\ k \cdot \int_{z-1}^{0} g_3(y) \cdot e^{z-y}dy + \int_{0}^{z-R} h_3(y) \cdot e^{z-y}dy & , z \in [R, 1) \\ k \cdot \int_{0}^{z} h_3(y) \cdot e^{z-y}dy & , z \in [1, 2-R) \end{cases}$$

$$= \begin{cases} \frac{e^{-4 \cdot R - z} \cdot [e^{4 \cdot R} - e^{2+2 \cdot z} + e^{2+4 \cdot R + 2 \cdot z} \cdot (4 \cdot R - 2 \cdot (1+z))]}{4 \cdot (e^R - e)^3} & , z \in [2 \cdot R - 1, R) \\ \frac{e^{-2 \cdot R - z} \cdot [-e^2 + e^{2+4 \cdot R} + e^{2 \cdot z} + 2 \cdot e^{4 \cdot R + 2 \cdot z} \cdot (r - z) + e^{2 \cdot (1+r+z)} \cdot (2 \cdot z - 3)]}{4 \cdot (e^R - e)^3} & , z \in [R, 1) \\ \frac{e^{-z} \cdot [-e^4 + e^{2 \cdot (r+z)} \cdot (-3 + 2 \cdot R = 2 \cdot z)]}{4 \cdot (e^R - e)^3} & , z \in [1, 2-R) \end{cases} \tag{27}$$

Case V. $X_1 \in P_c$, with pdf described by (12), $X_2 \in P_w, X_3 \in P_w$, with pdf of $X_2 - X_3$ described by:

$$f_Y(y) = \begin{cases} 1 + y & , y \in [-1, 0] \\ 1 - y & , y \in [0, 1] \end{cases} \tag{28}$$

Let us consider:

$$m(y) = 1 + y \ , \ m : [-1, 0] \to [0, 1]$$

$$n(y) = 1 - y \ , \ m : [0, 1] \to [0, 1]$$

Proof.

$$f_Z(z) = \begin{cases} k \cdot \int_{-1}^{z-R} m(y) \cdot e^{z-y} dy & , z \in [R-1, 0) \\ k \cdot \int_{z-1}^{z-R} m(y) \cdot e^{z-y} dy & , z \in [0, R) \\ k \cdot [\int_{z-1}^{0} m(y) \cdot e^{z-y} dy + \int_{0}^{z-r} n(y) \cdot e^{z-y} dy] & , z \in [R, 1) \\ k \cdot \int_{z-1}^{z-r} n(y) \cdot e^{z-y} dy & , z \in [1, 1+R) \\ k \cdot \int_{z-1}^{1} n(y) \cdot e^{z-y} dy & , z \in [1+R, 2) \end{cases}$$

$$= \begin{cases} -\dfrac{e^{1+z} + e^R \cdot (-2 + R - z)}{} & , z \in [R-1, 0) \\ 2 + \dfrac{e \cdot (-1 + R)}{e - e^R} & , z \in [0, R) \\ \dfrac{-2 \cdot e^z + e \cdot (1 + z) + e^R \cdot (z - R)}{e - e^R} & , z \in [R, 1) \\ \dfrac{e \cdot (R - 1)}{e - e^R} + R - z & , z \in [1, 1+R) \\ \dfrac{-e^{z-1} + e \cdot (z - 1)}{e - e^R} & , z \in [1+R, 2) \end{cases} \tag{29}$$

Case VI. $X_1 \in P_w$, with pdf described by (12), $X_2 \in P_w, X_3 \in P_c$, with pdf of $X_2 - X_3$ described by (14):

Proof.

$$f_Z(z) = \begin{cases} \int_{-1}^{z} g_2(y) dy & , z \in [-1, -R) \\ \int_{-1}^{-R} g_2(y) dy + \int_{-R}^{z} dy & , z \in [-R, 0) \\ \int_{z-1}^{-R} g_2(y) dy + \int_{-R}^{0} dy + \int_{0}^{z} h_3(y) dy & , z \in [0, 1-R) \\ \int_{z-1}^{0} dy + \int_{0}^{1-R} h_3(y) dy & , z \in [1-R, 1) \\ \int_{z-1}^{1-R} h_3(y) dy & , z \in [1, 2-R) \end{cases}$$

$$
= \begin{cases} \dfrac{e^{-z}+e\cdot z}{e-e^R} & , z\in[-1,-R) \\ R+z+\dfrac{e^R-e\cdot R}{e-e^R} & , z\in[-R,0) \\ R+z+\dfrac{e^{-z}\cdot[2\cdot e+e^{1+z}\cdot(z-2)+e^{r+z}\cdot(R+z-1)]}{e^R-e} & , z\in[0,1-R) \\ 1-\dfrac{e+e^R\cdot(R-2)}{e^R-e}-z & , z\in[1-R,1) \\ \dfrac{e^{-z}\cdot[-e^2-e^{R+z}\cdot(-3+R+z)]}{e^R-e} & , z\in[1,2-R) \end{cases} \tag{30}
$$

Case VII. $X_1 \in P_c$, with pdf described by (7), $X_2 \in P_w, X_3 \in P_c$, with pdf of $X_2 - X_3$ described by (14).
Here we distinguish 2 sub-cases:

- $R \in [0, 0.5]$

  Proof.

$$
f_Z(z) = \begin{cases} k\cdot\displaystyle\int_{-1}^{z-R} g_2(y)\cdot e^{z-y}dy & , z\in[R-1,0) \\ k\cdot\displaystyle\int_{z-1}^{-R} g_2(y)dy+\int_{-R}^{z-R}\cdot e^{z-y}dy & , z\in[0,R) \\ k\cdot\displaystyle\int_{z-1}^{-R} g_2(y)\cdot e^{z-y}dy+\int_{-R}^{0}\cdot e^{z-y}dy+\int_{0}^{z-R} h_3(y)\cdot e^{z-y}dy & , z\in[R,1-R) \\ k\cdot\displaystyle\int_{z-1}^{0}\cdot e^{z-y}dy+\int_{0}^{z-R} h_3(y)\cdot e^{z-y}dy & , z\in[1-R,1) \\ k\cdot\displaystyle\int_{z-1}^{1-R} h_3(y)\cdot e^{z-y}dy & , z\in[1,2-R) \end{cases}
$$

$$
= \begin{cases} \dfrac{e^{-z}\cdot(e^R-e^{1+z})^2}{2\cdot(e-e^R)^2} & , z\in[R-1,0) \\ \dfrac{e^z}{2}+\dfrac{e^R\cdot(e^z-1)}{e-e^R}-\dfrac{e^2\cdot(\cosh z-1)}{(e-e^R)^2} & , z\in[0,R) \\ -\dfrac{2\cdot e-2\cdot e^2+e^{2-z}+2\cdot e^{1+z}-2\cdot e^{1-R+z}-2\cdot e^{R+z}+e^{2\cdot R+z}+2\cdot e^z\cdot(z-R)}{2\cdot(e-e^R)^2} & , z\in[R,1-R) \\ \dfrac{-e+e^2-e^{1+R}-e^{1+z}+e^{1-R+z}+e^{R+z}+e^z\cdot(r-z)\cdot(R-2)}{(e-e^R)^2} & , z\in[1-R,1) \\ \dfrac{e^{-R}\cdot(e^2+e^{R+z}\cdot(z+R-3)}{(e-e^R)^2} & , z\in[1,2-R) \end{cases} \tag{31}
$$

- $R \in [0.5, 1]$

Proof.

$$f_Z(z) = \begin{cases} k \cdot \int_{-1}^{z-R} g_2(y) \cdot e^{z-y} dy & , z \in [R-1, 0) \\ k \cdot \int_{z-1}^{-R} g_2(y) dy + \int_{-R}^{z-R} \cdot e^{z-y} dy & , z \in [0, 1-R) \\ k \cdot \int_{z-1}^{z-R} g_2(y) \cdot e^{z-y} dy & , z \in [1-R, R) \\ k \cdot \int_{z-1}^{0} \cdot e^{z-y} dy + \int_{0}^{z-R} h_3(y) \cdot e^{z-y} dy & , z \in [R, 1) \\ k \cdot \int_{z-1}^{1-R} h_3(y) \cdot e^{z-y} dy & , z \in [1, 2-R) \end{cases}$$

$$= \begin{cases} \frac{e^{-z} \cdot (e^R - e^{1+z})^2}{2 \cdot (e - e^R)^2} & , z \in [R-1, 0) \\ \frac{e^z}{2} + \frac{e^R \cdot (e^z - 1)}{e - e^R} - \frac{e^2 \cdot (\cosh z - 1)}{(e - e^R)^2} & , z \in [0, 1-R) \\ -\frac{2 \cdot e - 2 \cdot e^2 + e^{2-z} + 2 \cdot e^{1+z} - 2 \cdot e^{1-R+z} - 2 \cdot e^{R+z} + e^{2 \cdot R+z} + 2 \cdot e^z \cdot (z - R)}{2 \cdot (e - e^R)^2} & , z \in [1-R, R) \\ \frac{-e + e^2 - e^{1+R} - e^{1+z} + e^{1-R+z} + e^{R+z} + e^z \cdot (r - z) \cdot (R - 2)}{(e - e^R)^2} & , z \in [1-R, 1) \\ \frac{e^{-R} \cdot (e^2 + e^{R+z} \cdot (z + R - 3)}{(e - e^R)^2} & , z \in [1, 2-R) \end{cases} \tag{32}$$

- In the followings, we assume **the lower bound was violated** in the previous generation for points in P_c.
- Cases analyzed:

Case	X_R	X_2	X_3	**Probability**
I*.	$\in P_w$	$\in P_c$	$\in P_w$	$p_v \cdot (1-p_v)^2$
II**.	$\in P_c$	$\in P_c$	$\in P_w$	$p_v^2 \cdot (1-p_v)$
III.	$\in P_w$	$\in P_c$	$\in P_c$	$p_v^2 \cdot (1-p_v)$
IV.	$\in P_c$	$\in P_c$	$\in P_c$	$(1-p_v)^3$
V.	$\in P_c$	$\in P_w$	$\in P_w$	$p_v \cdot (1-p_v)^2$
VI*.	$\in P_w$	$\in P_w$	$\in P_c$	$p_v \cdot (1-p_v)^2$
VII**.	$\in P_c$	$\in P_w$	$\in P_c$	$p_v \cdot (1-p_v)^2$

Table 3. Combinations of elements involved in mutation and their corresponding probabilities

First, let us consider:

$$u_1(y) = k \cdot (\sinh(y+1) - \cosh(y+1) + 1) \qquad , u_1 : [-1, R-1] \to [0,1] \tag{33}$$

$$u_2(y) = k \cdot (\sinh(R) - \cosh(R) + 1) \qquad , u_1 : [R-1, 0] \to [0,1] \tag{34}$$

$$u_3(y) = k \cdot (\sinh(R) - \cosh(R) - \sinh(y) + \cosh(y)) \qquad , u_1 : [0, R] \to [0,1] \tag{35}$$

with $k_1 = \dfrac{e^R}{e^R - 1)}$.

As previously, we use the functions in order to simplify the expressions involved in computation of pdf of Z. The functions above correspond to situation when $X_2 \in P_c, X_3 \in P_w$ presented in paper, while bellow we simplify notation in case when $X_2, X_3 \in P_c$:

$$v_1(y) = k_1 \cdot \sinh(R + y) \qquad , v_1 : [-R, 0) \to [0,1] \tag{36}$$

$$v_2(y) = k_1 \cdot \sinh(R - y) \qquad , v_1 : [0, R) \to [0,1] \tag{37}$$

with $k_1 = \dfrac{e^R}{(e^R - 1)^2}$.

Since **case I** was presented in the paper, we proceed with **case II**, where $X_1 \in P_c$,$X_2 \in P_c, X_3 \in P_w$. Two cases need to be distinguished:

- $R \in [0, 0.5]$

  Proof.

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(y) \cdot f_{X_1}(z-y)dy$$

$$f_Z(z) = \begin{cases} k^2 \cdot \displaystyle\int_{-1}^{z} u_1(y) \cdot e^{y-z} dy & , z \in [-1, R-1] \\ k^2 \cdot \displaystyle\int_{z-R}^{R-1} u_1(y) \cdot e^{y-z} dy + k \cdot \int_{R-1}^{z} e^{y-z} dy & , z \in [R-1, 2R-1] \\ k^2 \cdot \displaystyle\int z - R^y u_1(y) \cdot e^{y-z} dy & , z \in [2R-1, 0] \\ k \cdot \displaystyle\int_{z-R}^{0} e^{y-z} dy + k^2 \cdot \int_0^z u_2(y) \cdot e^{y-z} dy & , z \in [0, R] \\ k^2 \cdot \displaystyle\int_{z-R}^{R} u_2(y) \cdot e^{y-z} dy & , z \in [R, 2R] \end{cases}$$

$$= \begin{cases} k^2 \cdot (1 - e^{-z-1} \cdot (z+2)) & , z \in [-1, R-1] \\ k \cdot (1 - e^{R-z-1}) + k^2 \cdot [e^{-z-1} \cdot (-2R + z + 1) + e^{R-z-1} - e^{-R}] & , z \in [R-1, 2R-1] \\ k \cdot (1 - e^{-R}) & , z \in [2R-1, 0] \\ k \cdot (e^{-z} - e^{-R}) + k^2 \cdot e^{R-z} \cdot (e^R \cdot z - e^z + 1) & , z \in [0, R] \\ k^2 \cdot [e^{-2R} - e^{-z} \cdot (-2R + z + 1)] & , z \in [R, 2R] \end{cases} \tag{38}$$

- In the second case, when $R \in (0.5, 1]$, we have as consequence that $2 \cdot R - 1 > 0$ and we are lead to the following computation:

Proof.

$$f_Z(z) = \begin{cases} k^2 \cdot \int_{-1}^{z} u_1(y) \cdot e^{y-z} dy & , z \in [-1, R-1] \\ k^2 \cdot \int_{z-R}^{R-1} u_1(y) \cdot e^{y-z} dy + k \cdot \int_{R-1}^{z} e^{y-z} dy & , z \in [R-1, 0] \\ k^2 \cdot \int_{z-R}^{R-1} u_1(y) \cdot e^{y-z} dy + k \cdot \int_{R-1}^{0} e^{y-z} dm + k^2 \cdot \int_{0}^{y} u_3(y) \cdot e^{y-z} & , z \in [0, 2R-1] \\ k \cdot \int_{z-R}^{0} e^{y-z} dy + k^2 \cdot \int_{0}^{z} u_3(y) \cdot e^{y-z} dy & , z \in [2R-1, R] \\ k^2 \cdot \int_{z-R}^{R} u_3(y) \cdot e^{y-z} dy & , z \in [R, 2R] \end{cases}$$

$$= \begin{cases} k^2 \cdot (1 - e^{-z-1} \cdot (z+2)) & , z \in [-1, R-1] \\ k \cdot (1 - e^{R-z-1}) + k^2 \cdot [e^{-z-1} \cdot (-2R + z + 1) + e^{R-z-1} - e^{-R}] & , z \in [R-1, 0] \\ k^2 \cdot [e^{-z-1} \cdot (-2R + z + 1) + e^{R-z-1} - e^{R}] + k \cdot (e - e^{R}) \cdot e^{-z-1} + & \\ \quad + k^2 \cdot [e^{-z} \cdot (R - z) + e^{-R-1}] & , z \in [0, 2R-1] \\ k \cdot (e^{-z} - e^{-R}) + k^2 \cdot e^{R-z} \cdot (e^{R} \cdot z - e^{z} + 1) & , z \in [2R-1, R] \\ k^2 \cdot [e^{-2R} - e^{-z} \cdot (-2R + z + 1)] & , z \in [R, 2R] \end{cases} \tag{39}$$