

(12) Decomponeti 14107 în factori primi

$$\begin{array}{r|l} \sqrt{14107} & 118 \\ \hline 1 & 21 \cdot 1 = 21 \\ 141 & 228 \cdot 8 = 1824 \\ \hline 2007 & \\ 1824 & \\ \hline 183 & \end{array}$$

$$[\sqrt{14107}] = 118 \quad (\text{Fermat})$$

$$t = [\sqrt{14107}] + 1 = 119$$

$$\begin{aligned} t^2 - n &= 119^2 - 14107 = 14161 - 14107 = 54 = \\ &= 6 \cdot 3^2 = (\sqrt{6} \cdot 3)^2 \Rightarrow \end{aligned}$$

$$\begin{array}{r|l} 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$\Rightarrow S = 3\sqrt{6} \Rightarrow \begin{cases} t = 119 \\ S = 3\sqrt{6} \end{cases} \Rightarrow \begin{cases} b = 119 + 3\sqrt{6} \\ a = 119 - 3\sqrt{6} \end{cases}$$

(QS) $F(1) = 119^2 - 14107 = 54 = 6 \cdot 3^2 = 2 \cdot 3^3$

$$F(2) = 120^2 - 14107 = 293$$

$$F(3) = 121^2 - 14107 = 534 = 2 \cdot 3 \cdot 89$$

$$\begin{aligned} F(1) \cdot F(3) &= 2^2 \cdot 3^4 \cdot 89 = 89(2 \cdot 3)^2 = 89 \cdot 18^2 \\ &\equiv 119^2 \cdot 121^2 \equiv 292^2 \pmod{14107} \end{aligned} \quad \Rightarrow$$

$$\Rightarrow 292^2 - 89 \cdot 18^2 \equiv 0 \pmod{14107}$$

$$(292 - \sqrt{89} \cdot 18)(292 + \sqrt{89} \cdot 18) \equiv 0 \pmod{14107}$$

$$(292 - \approx 170)(292 + \approx 170) \equiv 0$$

$$122 \cdot 462 = 14107$$

$$\begin{array}{r|l} 534 & 2 \\ 267 & 3 \\ 89 & \end{array}$$