

Registration of 3D and 2D images

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Minimally invasive surgical procedures

Traditional orthopedic surgical procedures are usually highly invasive and thus involve an increased risk of patient infections, and long-term and costly patient recovery. Figure 1 *left* shows a traditional (invasive) approach to the treatment of degenerative spine conditions, e.g. scoliosis of Figure 2 *left*. During this procedure the spine is stabilized by the insertion of pedicular screws, which are mutually fastened by connecting rods (Figure 2 *right*). To reduce patient risks, treatment costs and the time required for patient recovery, the orthopedic surgery can be performed by minimally invasive procedures (Figure 1 *right*), which involve the use of modern imaging systems in the phases of planning, navigation and treatment delivery. Before treatment, the planning of the spine stabilization procedure is usually based on the visualization and analysis of the structures of interest in 3D computed tomography (CT) images, and to define the optimal trajectory of pedicular screws (Figure 3). During the procedure the X-ray imaging system (Figure 4) enables a real-time acquisition of 2D intra-operative images with high contrast between the bony structures and surrounding tissue. The insertion of pedicular screws requires a highly accurate determination of the pose of the vertebra of interest so that the surgeon can drill into the vertebra body only through a small incision made in the skin and the underlying muscles (Figure 1 *right*). While performing these tasks the surgeon relies on the 2D X-ray images and a mental 3D reconstruction of the vertebra's shape and pose, all of which requires extensive training and careful preparation before the procedure. The success of the whole procedure critically depends on the physician's mental 3D reconstruction, because even a minor deviation from the planned trajectory of the pedicular screw could cause irreversible damage to the spinal cord and the patient. The key enabling technology to obtain accurate pose of the vertebra in 3D and to transfer the treatment plan from the preoperative 3D image into the intra-operative 2D images is 3D to 2D image registration. In this assignment you will implement and evaluate a method for 3D-2D image registration of 3D CT and 2D X-ray images of the spine vertebrae and transfer the trajectory of the pedicular screw defined in the 3D CT image to the 2D X-ray image.

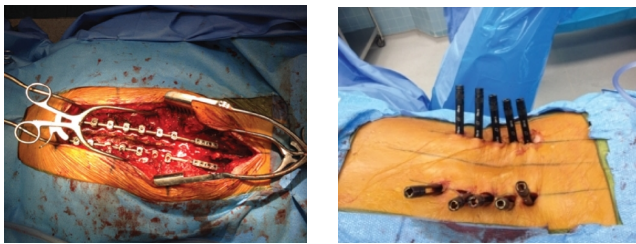


Figure 1: Spine stabilization by a traditional invasive (*left*) and modern minimally invasive (*right*) procedure.

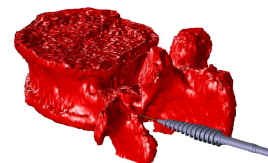


Figure 3: A 3D model of vertebra extracted from preoperative CT image and the planned trajectory for the insertion of pedicular screw.

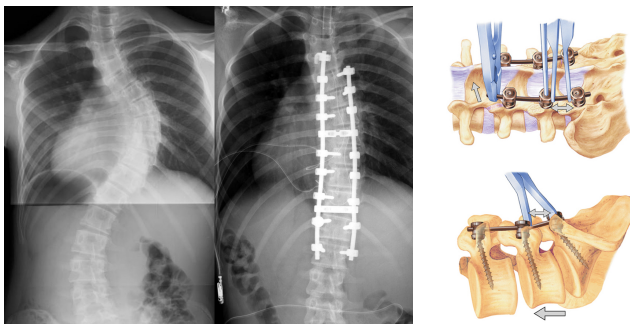


Figure 2: Patient with scoliotic spine (*left*) and the applied treatment by spine stabilization with pedicular screws (*right*).

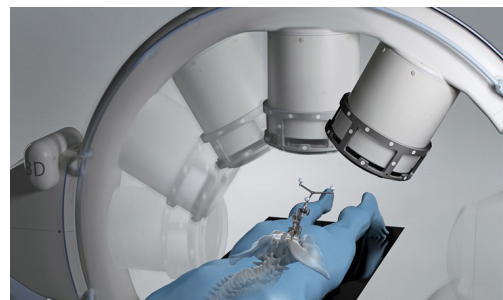


Figure 4: X-ray imaging with a C-arm during surgical procedure.

Registration of 3D and 2D images

Registration of a 3D to a 2D image is concerned with finding an optimal geometric transformation $\mathcal{T} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, which transforms the 3D image V into a pose such that its projection is *consistent* with the projection of the same object on the 2D image P . The main challenge of 3D to 2D image registration is to overcome the dimensional correspondence (3D vs. 2D), which can be addressed in two ways: 1) by projection of 3D information to 2D image space or 2) by reconstruction of 3D image from multiple 2D projections or 2D images. In this assignment you will implement a 3D-2D image registration by projecting the 3D image into 2D ($\mathcal{P} : \mathbb{R}^3 \rightarrow \mathbb{R}^2$) and maximizing the similarity measure SM between the projection $\mathcal{P}(V)$ and the 2D image P .

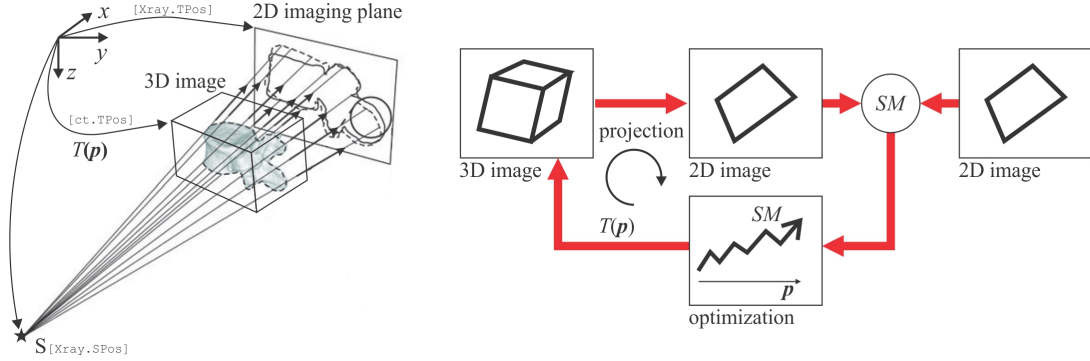


Figure 5: Projection of 3D image into 2D imaging plane (*left*) and image registration by maximization of image similarity measure SM (*right*).

The implementation of projection operator \mathcal{P} depends on the form of the 3D information. In the assignment you will work with the 3D CT image, in which the intensity of each voxel represents the attenuation coefficient $\mu(\mathbf{x})$ of the corresponding structure. For the purpose of 3D to 2D image registration you will use the 3D CT image to simulate the 2D X-ray images, or the so-called digitally reconstructed radiographs (DRRs). The DRRs are obtained by computing the integral of the attenuation coefficient along the line connecting the X-ray source S and the 2D imaging plane, i.e. $\mathcal{P}(V) = \int_l \mu(l) dl$. The projection and symbols are shown in Figure 5 *left*.

The spatial alignment of structures between the projection and 2D X-ray images $\mathcal{P}(V)$ and P , respectively, can be evaluated by a similarity measure SM ; a scalar function whose value is optimal (minimal or maximal) when the positions of the corresponding structures in $\mathcal{P}(V)$ and P are mutually aligned. The similarity measure has to be carefully chosen for a particular application such that it is insensitive to image noise and other image artifacts, but at the same time, highly sensitive to the actual geometric inconsistencies between the two images undergoing registration. In image registration, the following two similarity measures are often used:

- **Correlation coefficient CC :**

$$CC(I, J) = \frac{\sum_x \sum_y (I(x, y) - \bar{I})(J(x, y) - \bar{J})}{\left[\sum_x \sum_y (I(x, y) - \bar{I})^2 \cdot \sum_x \sum_y (J(x, y) - \bar{J})^2 \right]^{1/2}}.$$

- **Mutual information MI :**

$$MI(I, J) = H(I) + H(J) - H(I, J),$$

where $H(I)$ and $H(J)$ are the marginal entropies of the reference $I(x, y)$ and moving $J(x, y)$ image, respectively, and $H(I, J)$ is their joint entropy:

$$\begin{aligned} H(I) &= - \sum_{s_I=0}^{L-1} p_I(s_I) \log p_I(s_I), \\ H(J) &= - \sum_{s_J=0}^{L-1} p_J(s_J) \log p_J(s_J), \\ H(I, J) &= - \sum_{s_I=0}^{L-1} \sum_{s_J=0}^{L-1} p_{IJ}(s_I, s_J) \log p_{IJ}(s_I, s_J), \end{aligned}$$

where s_I in s_J denote the co-occurring discrete intensity values of the reference and moving images $I(x, y)$ and $J(x, y)$, respectively, and L denotes the number of bins.

The marginal probability density functions (PDFs) $p_I(s_I)$ in $p_J(s_J)$ and the joint PDF $p_{IJ}(s_I, s_J)$ is obtained from the associated normalized intensity histograms $h_I(s_I)$, $h_J(s_J)$ and $h_{IJ}(s_I, s_J)$ as:

$$p_I(s_I) = \frac{h_I(s_I)}{N \cdot M}, \quad p_J(s_J) = \frac{h_J(s_J)}{N \cdot M}, \quad p_{IJ}(s_I, s_J) = \frac{h_{IJ}(s_I, s_J)}{N \cdot M}.$$

During the registration of 3D and 2D images the optimization method is used to iteratively update the parameters \mathbf{p} of the geometric transformation $\mathcal{T} = \mathcal{T}(\mathbf{p})$ such that the similarity measure is maximized:

$$\mathbf{p}^* = \operatorname{argmax}_{\mathbf{p}} SM\left(\mathcal{P}(V[\mathcal{T}(\mathbf{p})]), P\right),$$

where \mathbf{p}^* represent the optimal parameters of the geometric transformation $\mathcal{T}(\mathbf{p})$. As the vertebrae are rigid structures you will use the 3D rigid-body transformation, which is defined by six parameters $\mathbf{p} = [t_x, t_y, t_z, \alpha, \beta, \gamma]^T$. The flowchart of 3D to 2D image registration is shown in Figure 5 *right*.

Materials required for the assignments are given in Matlab file `materials.mat` that contains two structures named `ct` in `Xray` representing a 3D CT image of lumbar vertebra L3 and a 2D X-ray image of the lumbar vertebrae L1–L5. The structures contain several fields, i.e. `ct.volume` and `Xray.image` are the respective 3D and 2D image matrices of intensity values, `ct.TPos` and `Xray.TPos` are the geometric transformations of the coordinate systems of 3D and 2D images from the reference coordinate system to the first image element indexed by (1,1), `Xray.SPos` is the position of the X-ray source in the reference coordinate system. The spatial sampling of the 3D and 2D images is 1 millimeter isotropic. The meaning of the geometric quantities is illustrated in Figure 5 *left*. The planned trajectory of the pedicular screw is given by variables `Te` and `Ve`, which represent the entry point and the direction of the screw and the coordinates of which are represented in the coordinate system of the preoperative 3D CT image.

Assignments and questions

Report should include the results, associated figures and graphs and answers to questions. The report should be written in English using the [IEEE template](#) and up to 4 pages. Besides the report, please submit the source codes such that the results in the report can be easily reproduced. The materials and the assignment instructions are prepared for coding in Matlab, however, you can use any other programming language and the materials will be submitted to you in the raw format.

1. The contrast of the vertebrae in the 2D X-ray image `Xray.image` can be improved by intensity windowing.¹ Transform the intensity values of the 2D X-ray image by linear windowing so as to obtain optimal contrast of the vertebrae. *The windowed 2D X-ray image should be used in all subsequent assignments!*
 - Report the optimal parameters, i.e. the center c and width w , of the linear windowing.
2. Create sampling grids of the 2D X-ray and 3D CT images by using the Matlab function `ndgrid()` and transform the sampling grids into the reference coordinate system by using the respective transformations `Xray.TPos` and `ct.TPos`. Show the transformed 3D and 2D sampling grids and the position of the X-ray source `Xray.SPos` by using the function `plot3()`.
 - Verify the obtained geometric setup of the X-ray source, 3D and 2D images mapped into the reference coordinate system with the help of Figure 6. Please enclose the obtained Figure in the report.
3. Write a Matlab function for 3D rigid-body transformation of the sampling grid. Ensure that the 3D rotations are performed with respect to the center of the 3D CT image. The declaration of the function should be:

```
function [oX, oY, oZ] = rigidTrans( ct, iPar ) ,
```

¹The intensity windowing is performed by $I_{out}(x, y) = \frac{1}{w} [I_{in}(x, y) - (c - \frac{w}{2})]$, where c is the center and w the width of the intensity window. The intensities of I_{in} within the window are rescaled to the full dynamic range of the output image I_{out} , while the intensities I_{in} lower and higher of the intensity window edges are set to minimal and maximal intensity values of the output image I_{out} , respectively.

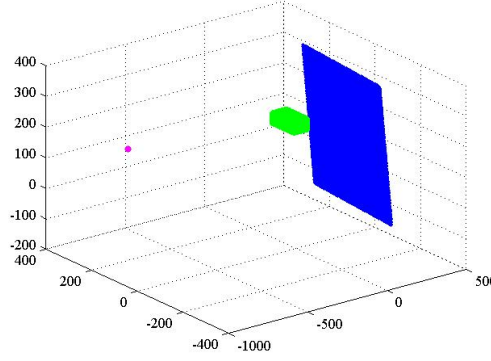


Figure 6: Geometric setup of the X-ray source and the sampling grids of 3D and 2D images in the reference coordinate system.

where the input parameter `ct` represents the structure of the 3D CT image and `iPar` the 6×1 vector of 3D rigid-body parameters. The function outputs `oX`, `oY` and `oZ` that represent the respective transformed x , y and z coordinates of the sampling grid of 3D CT image.

- Similar to the previous assignment show the geometric setup of the X-ray source and the sampling grids of 3D and 2D images in the reference coordinate system, however, the sampling grid of the 3D image should be transformed using `iPar = [0 0 0 90 0 0]`². Please enclose the obtained Figure in the report.
4. Write a function for cone beam projection of 3D CT image into the 2D imaging plane that represents the digitally reconstructed X-ray projection (DRR):

```
function [oImage, oMask] = drr( ct, Xray, iStep ) ,
```

where `ct` and `Xray` are the structures of the 3D CT and 2D X-ray images and `iStep` is the sampling step in *millimeters* along a line from the X-ray source to the intersection the line with the 2D imaging plane. To sample the intensities of the 3D CT image along these lines you can use the Matlab function `interp()`. The `drr()` function should return a 2D DRR image as `oImage` and the corresponding 2D DRR mask image as `oMask`, both of which have dimensions 446×446 (equal to the dimensions of `Xray.image`). The 2D DRR mask is 1 in those points, for which the line from the X-ray source to the imaging plane intersects the 3D image, or 0 otherwise.

- Generate the DRRs of the 3D CT image for different values of the sampling step `iStep`. Report the value of sampling step that you consider optimal and enclose the Figures of the corresponding DRR image and mask. How does the value of the sampling step effect the quality of the DRR image?
5. Extend the `drr()` function by adding a new input parameter `iPar`, i.e. `drr(..., iPar)`. Let `iPar` be a 6×1 vector of the 3D rigid-body parameters $\mathbf{p} = [t_x, t_y, t_z, \alpha, \beta, \gamma]^T$, where the rotations are defined with respect to the center of the 3D image.

- Generate the DRR image for the following parameter values:

$$\mathbf{iPar} = [0 \ 20 \ 0 \ 0 \ 0 \ 0], \mathbf{iPar} = [0 \ 0 \ 0 \ 0 \ 45 \ 0], \mathbf{iPar} = [0 \ 0 \ 0 \ 0 \ 0 \ 90].$$

Please enclose the corresponding Figures in the report.

6. In theory, the X-ray intensity represents the integral of attenuation coefficients along the entire line from the X-ray source to the 2D imaging plane. Here, the attenuation coefficients are known only in the coordinates of the sampling grid of 3D CT image, therefore, the obtained DRR image has distorted intensity values. Here we will address this problem i) by assuming that attenuation coefficients of the background are zero and ii) by focusing only on the integration of the attenuation coefficients belonging to the structure of interest, i.e. the vertebra.

²Corresponds to $\mathbf{p} = [t_x, t_y, t_z, \alpha, \beta, \gamma]^T$, where $\alpha = 90^\circ$ and the other parameters are zero. Note that here α, β and γ are given in degrees.

- Consider methods for the correction of the intensity values of 3D CT image that address this problem. Explain your solution in words.
 - Implement the corrections in the `drd()` function and generate the improved DRR image. Please enclose the corresponding Figure in the report.
7. Write a function that computes the correlation coefficient CC between two grayscale input images `iImageI` in `iImageJ`:

```
function oCC = correlationCoefficient( iImageI, iImageJ ) ,
```

The function should return a scalar value `oCC`.

- Compute the values `oCC` between the 2D X-ray image and several DRR images, which are obtained by varying one parameter of the 3D rigid-body transformation at a time. Let translations t_x , t_y and t_z vary in the range from -20 to 20 millimeters with a step of 2 millimeters, and the rotations α , β and γ in the range from -10 to 10° with a step of 1°. Plot the graphs and enclose them in the report.
 - Elaborate on the connection between the value of the correlation coefficient `oCC` and the actual similarity of the 2D X-ray and DRR images. Which values, higher or lower, represent higher similarity between the images? Which are the theoretically minimal and maximal values of correlation coefficient?
8. Write a function that computes the mutual information MI between two grayscale input images `iImageI` in `iImageJ`:

```
function oMI = mutualInformation( iImageI, iImageJ ) ,
```

The function should return a scalar value `oMI`.

- Compute the values `oMI` between the 2D X-ray image and several DRR images, which are obtained by varying one parameter of the 3D rigid-body transformation at a time. Let translations t_x , t_y and t_z vary in the range from -20 to 20 millimeters with a step of 2 millimeters, and the rotations α , β and γ in the range from -10 to 10° with a step of 1°. Plot the graphs and enclose them in the report.
 - Elaborate on the connection between the value of the mutual information `oMI` and the actual similarity of the 2D X-ray and DRR images. Which values, higher or lower, represent higher similarity between the images? Which are the theoretically minimal and maximal values of mutual information?
9. Devise an automated method for the rigid-body registration of 3D to 2D image based on iterative optimization of either the correlation coefficient CC or mutual information MI . For the purpose of iterative optimization you should create an *criterion function*, e.g.:

```
function oSM = criterionFcn( iPar, iType, ct, Xray ) ,
```

which takes as input some arbitrary 3D rigid-body parameters `iPar` and computes the corresponding value of the similarity measure `oSM` between the DRR and 2D X-ray images. Depending on the value of input parameter `iType`={ 'cc', 'mi' } the similarity function can be either CC or MI . Matlab toolboxes implement several optimization routines for finding a minimum of a function such as `fminsearch()` and `fminunc()`, which can be used to optimize your criterion function `criterionFcn()`. However, consider the modifications required so that these functions will correctly optimize, i.e. minimize or maximize, the similarity measure. To obtain the optimal values of the 3D rigid-body parameters \mathbf{p}^* the use of `fminsearch()` function could look something like this:

```
% definition of similarity measure SM(p)
oSM = @(iPar) criterionFcn( iPar, 'cc', ct, Xray );
% parameters of the simplex optimization
opts = optimset('Display','iter',...
               'NumIter',50,...
               'TolX',1e-3,...
               'TolFcn',1e-3 );
% start simplex optimization
iPar_opt = fminsearch( oSM, 1e-2*ones(6,1), opts )
```


- Verify if the 3D-2D registration method was successful by showing the 2D X-ray image with the DRR image obtained with the optimal 3D rigid-body parameters. Is the DRR projection of the 3D CT image in optimal alignment with the vertebra on the 2D X-ray image?
 - Test different values of the parameters of the optimization method, e.g. '`NumIter`', '`TolX`' in '`TolFcn`', and explain their influence on the result of 3D-2D image registration. Based on the obtained experience, select and report the optimal values of these parameters.
 - Report the obtained optimal values of 3D rigid-body parameters \mathbf{p}^* , which result in an optimal registration of 3D CT to 2D X-ray image, and show the corresponding 2D DRR and X-ray images. Please enclose in the report the corresponding Figures for registration with both *CC* and *MI* similarity measures.
 - Which of the two similarity measures results in a more accurate registration? How would you measure the registration accuracy based on the analysis of registered images? Propose a method based for measuring registration accuracy and then evaluate the registration accuracy for *CC* and *MI* similarity measures.
10. The planned trajectory of the pedicular screw is given by variables $\mathbf{T_e}$ and $\mathbf{V_e}$ (see the materials), which represent the entry point and the direction of the screw. The coordinates of $\mathbf{T_e}$ and $\mathbf{V_e}$ are represented in the coordinate system of the preoperative 3D CT image (Figure 3). Use the obtained optimal values of 3D rigid-body parameters \mathbf{p}^* to map the planned trajectory of the pedicular screw into the 2D X-ray image.
- Draw the mapped trajectory of the pedicular screw into the 2D X-ray image using Matlab commands `hold on` and `line()`.

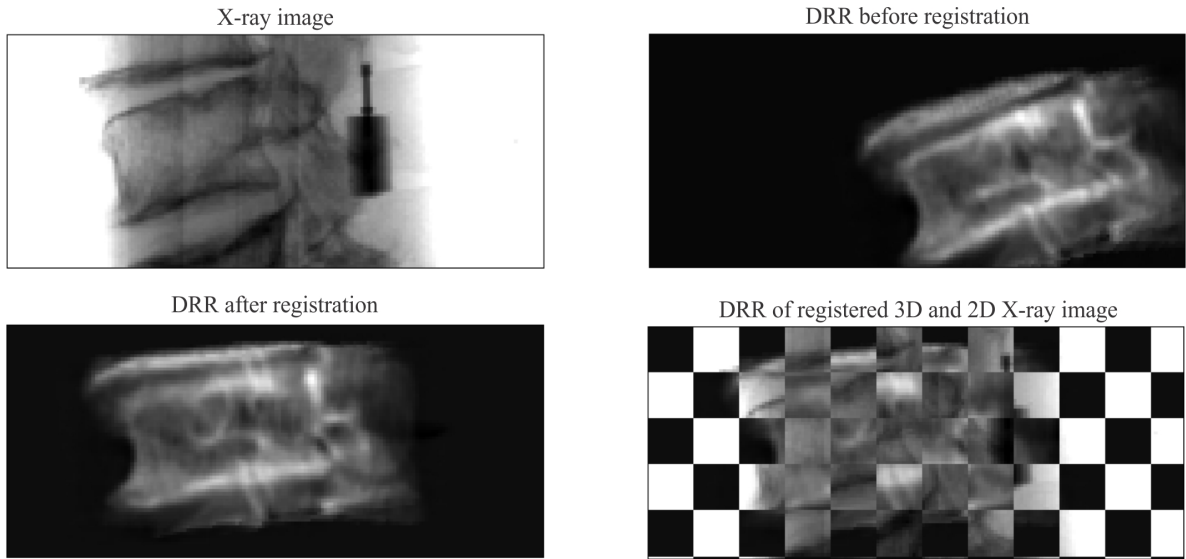


Figure 7: An example of 3D CT to 2D X-ray image registration and a visual verification of registration quality.