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Faculté des Sciences Exactes et l'Informatique
Département de Mathématiques et d'Informatique

UNIVERSITE
Abdelhamid Ibn Badis
MOSTAGANEM

Master 1 ISI
Module : Techniques de programmation avancées

Subject : ***Inverting Matrices***

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Work plans

1. ***HOW TO DESIGN ALGORITHMS***
2. ***DYNAMIC PROGRAMMING***
3. ***MULTITHREADED ALGORITHMS***
4. ***INVERTING MATRICES***
5. ***HOW IT WORKS***
6. ***EXECUTION***

1 HOW TO DESIGN ALGORITHMS

Planning the correct algorithm for a given application is a major innovation.
The design of the algorithm is incremental by asking yourself questions to guide your staring process.

- *Do I really understand the problem?*
- *Can I find a simple algorithm for my problem?*

2

DYNAMIC PROGRAMMING

Is to solve a problem by splitting it into sub-problems, then solving sub-problems, From smallest to large by storing intermediate results.

3 MULTITHREADED ALGORITHMS

Multithread Algorithms execute multiple operations at the same time in order to minimize the execution time.

4 INVERTING MATRICES

Inverse of Matrix for a matrix A is A-1. The inverse of a 2 × 2 matrix can be calculated using a simple formula. Further, to find the inverse of a 3 × 3 matrix, we need to know about the determinant and adjoint of the matrix. The inverse of matrix is another matrix, which on multiplying with the given matrix gives the multiplicative identity.

The inverse of matrix is used of find the solution of linear equations through the matrix inversion method. Here, let us learn about the formula, methods, and terms related to the inverse of matrix.

HOW IT WORKS

Inverse of Matrix Formula

The inverse of matrix can be computed using the inverse of matrix formula, by dividing the adjoint of a matrix by the determinant of the matrix. The inverse of a matrix can be calculated by following the given steps:

- *Step 1: Calculate the minor for the given matrix.*
- *Step 2: Turn the obtained matrix into the matrix of cofactors.*
- *Step 3: Then, the adjugate, and*
- *Step 4: Multiply that by reciprocal of determinant.*

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HOW IT WORKS

With The Law

For a matrix A 3x3 , its inverse $\boldsymbol{A^{-1} = \frac{1}{|A|} \times Adj(A)}$.

$|A|$: is the determinant

$\boldsymbol{Adj(A)}$: is the adjoint of the matrix A.

$$\boldsymbol{A = \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}}$$

$$\boldsymbol{|A| = a11\ (-1)^{1+1}\begin{vmatrix} a22 & a23 \\ a32 & a33 \end{vmatrix} + a12(-1)^{1+2}\begin{vmatrix} a21 & a23 \\ a31 & a33 \end{vmatrix} + a13(-1)^{1+2}\begin{vmatrix} a21 & a22 \\ a31 & a32 \end{vmatrix}}$$

5

HOW IT WORKS

With The Law

Adj A = Transpose of Cofactor Matrix = Transpose of

$$\begin{pmatrix} \mathbf{a11} & \mathbf{a12} & \mathbf{a13} \\ \mathbf{a21} & \mathbf{a22} & \mathbf{a23} \\ \mathbf{a31} & \mathbf{a32} & \mathbf{a33} \end{pmatrix} = \begin{pmatrix} \mathbf{a11} & \mathbf{a21} & \mathbf{a31} \\ \mathbf{a12} & \mathbf{a22} & \mathbf{a32} \\ \mathbf{a31} & \mathbf{a23} & \mathbf{a33} \end{pmatrix}$$

$$\mathbf{A}^{-1} = \frac{\mathbf{1}}{|\mathbf{A}|} \times \begin{pmatrix} \mathbf{a11} & \mathbf{a21} & \mathbf{a31} \\ \mathbf{a12} & \mathbf{a22} & \mathbf{a32} \\ \mathbf{a31} & \mathbf{a23} & \mathbf{a33} \end{pmatrix}$$

5 HOW IT WORKS

With The Example

Find the inverse of the matrix $A = \begin{pmatrix} \mathbf{4} & \mathbf{-2} & \mathbf{1} \\ \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{-1} & \mathbf{2} & \mathbf{6} \end{pmatrix}$

The given matrix is $A = \begin{pmatrix} \mathbf{4} & \mathbf{-2} & \mathbf{1} \\ \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{-1} & \mathbf{2} & \mathbf{6} \end{pmatrix}$

Step - 1: Let us find the determinant of the given matrix using Row - 1 of the above matrix.

$$|\boldsymbol{A}| = \mathbf{4}\begin{vmatrix} \mathbf{0} & \mathbf{3} \\ \mathbf{2} & \mathbf{6} \end{vmatrix} - (\mathbf{-2})\begin{vmatrix} \mathbf{5} & \mathbf{3} \\ \mathbf{-1} & \mathbf{6} \end{vmatrix} + \mathbf{1}\begin{vmatrix} \mathbf{5} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} \end{vmatrix}$$

HOW IT WORKS

With The Example

$$\begin{aligned}|\mathbf{A}| &= 4(0 \times 6 - 3 \times 2) + 2(5 \times 6 - (-1) \times 3) + 1(5 \times 2 - 0 \times (-1)) \\ &= 4(0-6) + 2(30+3) + 1(10-0) \\ &= -24 + 66 + 10\end{aligned}$$

$$|\mathbf{A}| = 52$$

Now, we will determine the adjoint of the matrix A by calculating the cofactors of each element and then taking the transpose of the cofactor matrix.

$$\boldsymbol{Adj(A)} = \begin{pmatrix} \begin{vmatrix} 0 & 3 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 6 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & 5 \\ 6 & -1 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -1 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 5 & 0 \end{vmatrix} & \begin{vmatrix} -2 & 4 \end{vmatrix} & \begin{vmatrix} 4 & -2 \end{vmatrix} \end{pmatrix}$$

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HOW IT WORKS

With The Example

$$\boldsymbol{Adj(A)} = \begin{pmatrix} \big((0\times 6)-(3\times 2)\big) & \big((1\times 2)-(-2\times 6)\big) & \big((-2\times 3)-(1\times 0)\big) \\ \big((3\times -1)-(5\times 6)\big) & \big((4\times 6)-(1\times -1)\big) & \big((1\times 5)-(4\times 3)\big) \\ \big((5\times 2)-(0\times -1)\big) & \big((-2\times -1)-(4\times 2)\big) & \big((4\times 0)-(-2\times 5)\big) \end{pmatrix}$$

$$\boldsymbol{Adj(A)} = \begin{pmatrix} -6 & 14 & -6 \\ -33 & 25 & -7 \\ 10 & -6 & 10 \end{pmatrix}$$

5

HOW IT WORKS

With The Example

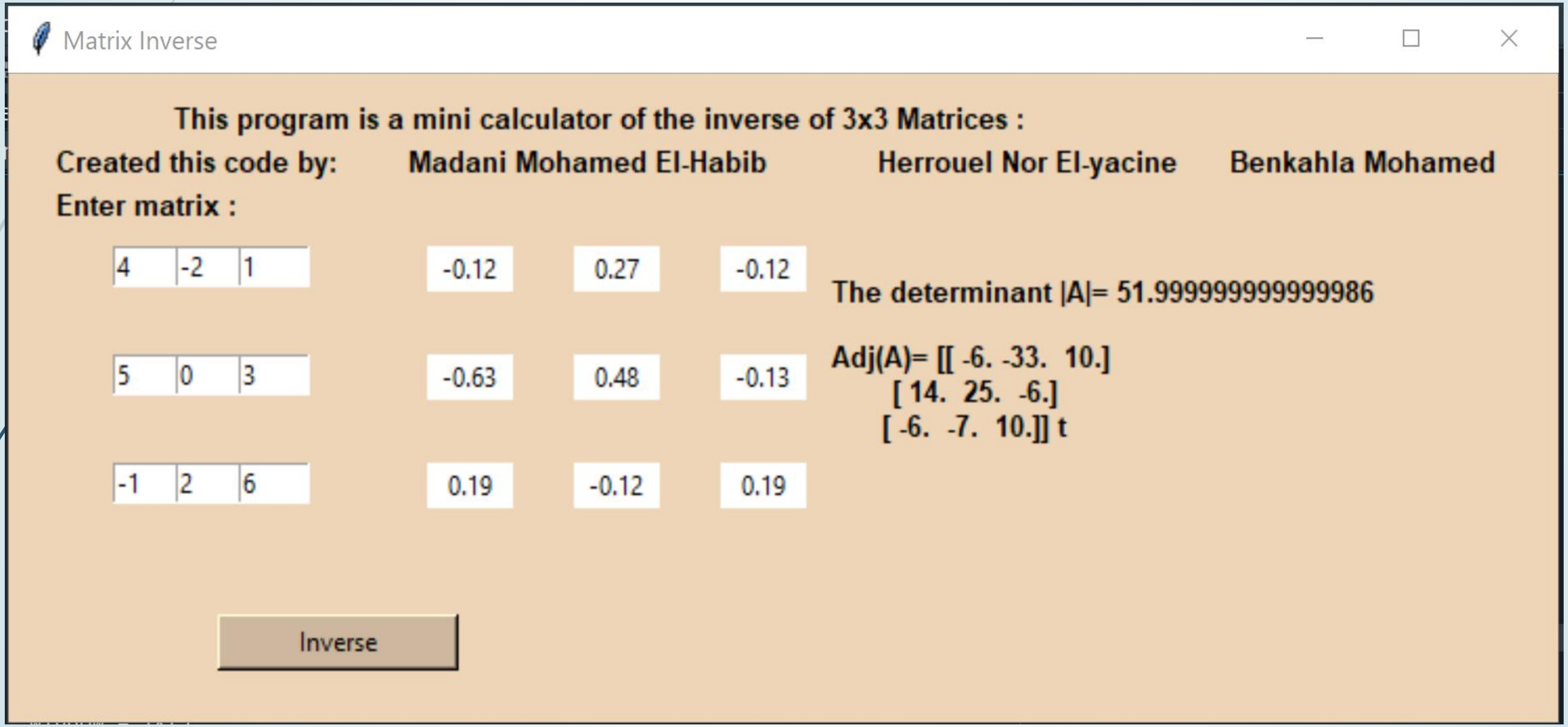
The inverse of matrix A is given by the formula $\boldsymbol{A^{-1}} = \frac{\mathbf{1}}{|\boldsymbol{A}|} \times \boldsymbol{Adj}(\boldsymbol{A})$

$$\mathbf{A^{-1}} = \frac{\mathbf{1}}{\mathbf{52}} \times \begin{pmatrix} \mathbf{-6} & \mathbf{14} & \mathbf{-6} \\ \mathbf{-33} & \mathbf{25} & \mathbf{-7} \\ \mathbf{10} & \mathbf{-6} & \mathbf{10} \end{pmatrix}$$

$$\mathbf{A^{-1}} = \begin{pmatrix} \mathbf{-3/26} & \mathbf{7\,/26} & \mathbf{-3/26} \\ \mathbf{-33/52} & \mathbf{25/52} & \mathbf{-7/52} \\ \mathbf{5/26} & \mathbf{-3/26} & \mathbf{5/26} \end{pmatrix}$$

6 EXECUTION

With The Interface



$Adj\ (A)$ write in transposed matrix form

THE END

THANKS