

# Boolean Logic

## Lecture Notes

Boolean algebra is the conversion of logical statements to a mathematical expression, where some of the same laws of mathematics apply. Simply put, it is a way to represent an "if this, then that..." statement (which is conveyed as an English statement) as a formula.

The only values in boolean algebra are *true and false*.

We usually say something like this A AND B. We can also say something like A OR C. In those statements A, B, and C all represent the values true or false.

## AND Operations

**AND** is considered an operation.

If both values on either side of AND are true, then the whole statement is true.

Ex. A AND B. If A is true, and B is true, then the statement A AND B will equate to true. If A is false, however, then the whole statement is considered false. The same would hold true if B were false.

NOTE: The only case where an AND statement is true, is if both operands are true.

Table of AND evaluations:

a	b	a AND b
True	True	True
True	False	False
False	True	False
False	False	False

## OR Operations

**OR** is also considered an operation in boolean algebra.

If any of the values on other side of OR are true, then the whole statement is true.

For example, A OR B. If A is true and B is false, then the statement is true. Conversely, if A is false and B is true, then the statement is still true (because B is true, and only one of the values need to be true).

NOTE: the only case where an OR statement is false is if both operands are false.

Table of OR evaluations:

a	b	a OR b
True	True	True
True	False	True
False	True	True
False	False	False

## XOR Operations

**XOR** is an operation.

An XOR operation is like OR, except if both operands are true, then the whole statement is considered false.

Table of XOR evaluations:

a	b	a XOR b
True	True	False
True	False	True
False	True	True
False	False	False

## XAND / XNOR Operations

XAND (or XNOR) is an operation in boolean algebra.

An XAND operation is like AND, except if both operands are false it will return true. It will act the same when the values are both true or if they are opposites of each other (one true the other false).

Table of XAND evaluations:

a	b	a XAND b
True	True	True
True	False	False
False	True	False
False	False	True

## Law of Associativity

$$X \text{ AND } (Y \text{ AND } Z) = (X \text{ AND } Y) \text{ AND } Z$$

$$X \text{ OR } (Y \text{ OR } Z) = (X \text{ OR } Y) \text{ OR } Z$$

## Law of Distributivity

$$X \text{ AND } (Y \text{ OR } Z) = (X \text{ AND } Y) \text{ OR } (X \text{ AND } Z)$$

$$X \text{ OR } (Y \text{ AND } Z) = (X \text{ OR } Y) \text{ AND } (X \text{ OR } Z)$$

## Law of Absorption

$$X \text{ AND } (X \text{ OR } Y) = X$$

$$X \text{ OR } (X \text{ AND } Y) = X$$

## DeMorgan's Laws

$$\text{NOT } (X \text{ OR } Y) = (\text{NOT } X) \text{ AND } (\text{NOT } Y)$$

$\text{NOT } (X \text{ AND } Y) = (\text{NOT } X) \text{ OR } (\text{NOT } Y)$