The Volatility Advantages of Large Labor Markets*

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Abstract

Firms' labor demand is more volatile in larger cities. We propose and test a novel explanation for this finding. Faster hiring conditions attract productive firms with more volatile activity to denser locations where they can swiftly downsize or expand. We estimate a model of firm location choice using French data and show that (i) firm volatility is almost as predictive of location choice as productivity; (ii) both dimensions reinforce each other. This mechanism reduces the productivity–density gradient among volatile firms. Imperfectly correlated firm-level shocks, combined with higher operating costs induced by density, generate matching economies.

JEL-Classification: J23, J63, J64, R12, R23

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When I do nothing
I cost less money
Than when I'm working
Or so they tell me.
Bernard Lavilliers, *Les Mains d'Or.*[1]

1 Introduction

The productivity-density nexus is a central tenet of economic geography and urban economics (Combes et al., 2012, Gaubert, 2018). One of the channels through which agglomeration economies operate is the matching channel, whereby high human densities facilitate the quality and speed of the hiring process (Duranton and Puga, 2004). This channel is particularly beneficial for high-productivity firms because their opportunity costs of operating with limited capacity are higher. Therefore, the complementarity between employer productivity and local hiring conditions gives rise to a labor market pooling externality (Bilal, 2023). While heterogeneity in firm productivity is a key component of this mechanism, the literature has largely neglected another dimension of heterogeneity that is also pervasive in the data, namely the *volatility* of firm activity. However, Krugman (1992) already pointed out the potential benefits of labor market pooling for firms with volatile and imperfectly correlated labor demand. When labor demand fluctuates, it is easier for firms to hire when neighboring firms are downsizing.

In this paper, we study agglomeration patterns when firms are heterogeneous along two dimensions, productivity and volatility. We do so in the context of a stylized model and in the data. In the model, firms are willing to adjust their size positively or negatively in response to idiosyncratic shocks. High job-filling rates reduce the cost of these fluctuations, more so for high-volatility firms. We investigate how this complementarity between volatility and local hiring conditions interacts with sorting patterns along the productivity dimension when firms can either hold labor demand constant or choose to adjust to shocks. In the data, we first provide evidence of a systematic correlation between the density of cities and the average volatility of firms there, conditional on productivity. We then estimate a model of location choices to quantify the relative importance of productivity and volatility in shaping location choice decisions. The results are in line with the model's predictions and confirm the role of volatility in firms' location choices.

[1]Song by a popular French blue-collar singer. The original lyrics are "Quand je fais plus rien, moi / Je coûte moins cher / Que quand j'travaillais, moi / D'après les experts."

We begin by documenting new evidence on firm productivity, firm volatility, and local (working-age) population density. We use French administrative data at the worker and firm level over the period 2009-2019, identifying (large) cities with (dense) commuting zones. Our first key finding is that intra-firm employment volatility is higher in denser cities, even after controlling for various relevant firm characteristics such as sector, size, and age. This correlation is quantitatively significant and is hardly reduced when we also control for firm productivity. The second important empirical result is that we find a flatter productivity density gradient among firms with high employment volatility. The elasticity of firms' average productivity with respect to density is reduced by one-third when moving from the first to the last ventile of the volatility distribution.

We propose a simple search model inspired by Mortensen and Pissarides (1994) to rationalize these facts. Firms in the model differ in their productivity and the volatility of their sales. The economy alternates between good and bad states, and the variance of sales induced by these cycles is heterogeneous across firms. Firms can mitigate the impact of volatility by adopting three different employment strategies. The first one aims at maintaining employment levels even in bad times and is chosen by the most productive firms. In the second, the firm freezes hiring in bad states. Freezing hiring avoids facing operating costs in bad states at the cost of entering good states with vacant positions. In the third one, firms "churn": they adopt a turnover strategy, firing workers when hit by bad shocks and hiring only when their demand is high. The latter strategy is preferred by the most volatile firms (fixing productivity) and by the least productive firms (fixing volatility). Compared to a model that does not incorporate volatility and its impact on firms' employment strategies, our model thus shows a weaker selection on productivity.

The model then allows us to analyze where firms choose to locate. The crucial trade-off for firms is that large cities are expensive to operate in — because of higher labor costs or rents — but allow firms to find workers more quickly when they are needed — when firms experience a positive demand shock. Intuitively, locating in a large city provides "insurance" against volatility because larger cities offer lower adjustment costs for firms. This mechanism is particularly beneficial for high-productivity firms, which have the most to gain from being able to hire more quickly. It is also stronger when a large component of volatility is idiosyncratic, as firms that downsize free up workers that can be hired by expanding firms. Therefore, the model predicts that (i) firms sort positively on productivity and volatility; (ii) these two dimensions reinforce each other; and (iii) the resulting gradient of firm productivity with city density decreases with firm volatility. Since firms are more likely to churn - and thus loosen the market — when they face higher operating costs, this model provides a microfoundation for matching economies based on the existence of urban costs.

Motivated by our theoretical results, we estimate a model of firm location choice and compare the impact of heterogeneity in productivity and volatility on location choice. The model is first estimated on all firms observed in a cross section of the data, using their productivity and volatility, measured after at least 5 years of existence, as inputs into location decisions. However, this specification does not allow us to distinguish between entry decisions and survival probabilities. In addition, employment volatility is endogenous to the firm's location choice: According to our model, higher operating costs induced by density will drive some firms to adopt the churning strategy. Outside the model, we cannot rule out the possibility that employment volatility is also driven by a higher probability of job quits in larger cities, where job-to-job transitions are more frequent.

For those reasons, we also estimate a model restricted to firms that were born during the sample period and we propose a novel strategy to measure the exogenous component of a firm's employment volatility. The measure is similar to a shift-share and combines information on the firm's product portfolio with the time series of international demand at the product level. Intuitively, the exogenous component of volatility is driven by firms specializing in products for which demand is more or less volatile. We measure the expected volatility of demand resulting from a firm's decision to produce a given portfolio of products, which we assume is exogenous to its location choice. Both sets of estimates yield consistent results. We show that more volatile firms are more likely to locate in denser commuting zones. Moreover, firm (demand) volatility is almost as predictive of firm location choice as firm productivity. Finally, consistent with theory, our estimates show that volatility and productivity are complementary in firm location choice.

Relationship to the Literature — Many studies, such as Combes (2000) and Gaubert (2018), provide theory and evidence that more productive firms sort into larger cities. Our work presents a new mechanism for agglomeration economies based on the combination of firm productivity and volatility: matching economies arise endogenously from firms' hiring and firing decisions when they face more expensive operating costs. This mechanism may also help explain why relatively unproductive firms can survive in denser areas (Combes et al., 2012), in addition to the mechanisms already proposed in the literature.[2] A complementary

[2]Another mechanism, also based on firm entry, is that higher entry costs in larger cities shield unproductive firms from competition from other firms if entry is decided before productivity is realized (Melitz,

mechanism that is also consistent with our argument even in the absence of productivity differences is labor market pooling: if demand volatility is uncorrelated across firms, there is a clear advantage for firms to agglomerate because they can hire more workers in good times. This source of agglomeration economies, already recognized by Marshall, was popularized by Krugman (1992). However, this argument has remained largely theoretical.

To the best of our knowledge, the main existing attempt to provide reduced-form evidence for this channel is the study by Overman and Puga (2010). In their static model, firms do not know their productivity before entering a market: productivity is affected by an idiosyncratic shock with known variance. Firms' profits are convex to this shock because firms hire more when the shock is positive, and expected profits thus increase with the variance of the shock. Yet, since wages rise with local demand, firms with higher variance will be all the more profitable when there are many firms, to counteract the effect of individual positive shocks on the local wage level.[3] Therefore, the model predicts that groups of firms with more variability in labor demand will be more agglomerated, a prediction borne on sector-level data. We instead test our model on individual data and we explore the interaction between productivity and volatility on sorting patterns.

By focusing on hiring frictions, this paper also contributes to the literature on the relationship between city size and unemployment. While current leading models of spatial labor markets (Bilal, 2023, Kuhn et al., 2021) posit that more productive firms select into more productive locations, resulting in a negative correlation between average firm productivity and local unemployment rates, they do not directly relate these observations to city size. In the data, large cities are characterized by a higher share of high-productivity firms, but they do not have lower unemployment rates.[4] One reason for this could be that the mobility of unemployed workers acts as a balancing force in the spatial equilibrium (Gaigne and Sanch-Maritan, 2019). However, churning strategies of firms provide an alternative explanation: if firms in large labor markets have higher structural volatility and consequently higher employment volatility, there may be more aggregate labor turnover and also more unemployment. This mechanism would mitigate the effect of the agglomeration of more productive firms in larger cities.

More generally, this paper complements the literature on the spatial dimension of matching

2003, Heise and Porzio, 2023).

[3]Contrary to our setting, firms do not face hiring frictions. In this respect, we are closer in spirit to the seminal model of Helsley and Strange (1990), which derives agglomeration economies from the matching process of workers to firms.

[4]See Appendix Figure B.3 for the case of French commuting zones.

in cities, which has so far largely focused on the worker side (Gan and Zhang, 2006, Bleakley and Lin, 2012, Schmutz and Sidibé, 2019, Dauth et al., 2022, Papageorgiou, 2022) with some incomplete evidence on firms (Glaeser et al., 1992, Henderson et al., 1995, Combes, 2000, Duranton, 2007, Findeisen and Südekum, 2008). In contrast to recent work on the worker side, we abstract from worker heterogeneity. Therefore, we do not address the impact of city size on the level of match assortativeness and we focus on hiring speed as the sole determinant of agglomeration economies.[5] Moreover, in contrast to the existing literature on the firm side, we do not consider structural characteristics of the economy, such as sectoral composition. Instead, we focus on the heterogeneity of firms, conditional on the sector in which the firm operates. We incorporate two dimensions of heterogeneity that affect the first and second moments of firms' labor demand. In doing so, we draw inspiration from the macroeconomic literature, which has long discussed heterogeneity across firms in productivity *and* volatility.[6] We enrich this literature by introducing a novel, firm-specific shifter of employment volatility. Finally, we enrich the literature on labor market churning (Burgess et al., 2000, Nekoei and Weber, 2020, Weingarden, 2020) with a focus on the spatial dimension.[7]

The remainder of the paper is organized as follows: in Section 2, we provide descriptive evidence that employment volatility increases with city size and that the productivity gradient with respect to city size decreases with employment volatility; in Section 3, we present a simple model of firm decisions where employment volatility and location choice are jointly determined. The model predicts that firms sort across space based on the volatility of their activity, and we formally test this prediction in Section 4. Section 5 concludes.

[5] We also abstract from the decision of workers to quit their jobs, which is not observed in our data. Using survey data on U.S. firms, Weingarden (2020) estimates that at least one-third of firm churning is actually initiated by the employer through layoffs. This figure is arguably a lower bound, since employers may have a financial incentive to get workers to quit rather than lay them off. Weingarden (2020) shows that this component of churning is acyclical, unlike worker quits, which fits well with our modeling assumption of firm-specific shocks.

[6] Comin and Philippon (2006) and Comin and Mulani (2006) document the rise in firm-level volatility among publicly traded US firms in the second half of the 20th century. Davis et al. (2007) instead show diverging trends between public and private firms. In this literature, firm volatility is explained by a combination of aggregate shocks and firm idiosyncratic fluctuations. di Giovanni et al. (2014) provide evidence that a large component of individual firm volatility is driven by idiosyncratic shocks that reflect a combination of demand and supply-side factors.

[7] Note that our results also echo some results in the trade literature such as Cuñat and Melitz (2012) showing that countries with more flexible labor markets specialize in sectors with higher volatility.

2 Motivating facts on firms' spatial patterns

2.1 Data

Sample selection — The empirical analysis exploits matched employer-employee data for France over the period from 2009 to 2019 (DADS Postes). This data allows us to characterize the level and volatility of a firm's labor demand, at the monthly level.[8] For each employer-employee relationship, we know the type of contract (permanent or short-term), the number of hours and associated earnings, and the worker's occupation. On the employer's side, we know the location of each establishment, as well as the sector of activity and date of creation. Finally, the data can be matched with two additional yearly firm-level datasets, namely balance-sheet data used to estimate productivity (FARE) and a production survey (EAP) that provides additional information on the firm's portfolio of products.[9]

The analysis focuses on firms in manufacturing, construction, and services (including non-tradable services).[10] We use information on the address of the establishment to assign each plant to a commuting zone. Our sample includes plants in mainland France, which is composed of 280 different commuting zones. Since the focus is on how firms locate across local labor markets, we aggregate plant-level information at the level of a commuting zone, i.e., firms with multiple plants in the same commuting zone are treated as a single plant. In the main analysis, we focus on the January 2015 cross-section of the data, which corresponds to the midpoint of our period, but other reference points yield similar results. We focus on firms located in a single commuting zone because some key variables for our analysis (productivity and demand volatility) can only be calculated at the firm level due to data availability. We also restrict the sample to firms with non-missing data on the key variables of interest (productivity, employment volatility, 2-digit industry, firm age, and firm size). This leaves us with 365,041 firms. Table A.1 in the appendix details the sample selection process.

Each commuting zone is characterized by its population density, which is defined as the size of its working-age population divided by its area (in square kilometers). The working-age

[8] In the rest of the paper, we use a measure of employment equal to the full-time equivalent, based on the number of days worked in each month. We consider a full-time worker as an employee that works 30 days in each month.

[9] The EAP survey is exhaustive for firms in the manufacturing sector above a size threshold of 20 employees. Merging the employer-employee linked data with the this survey introduces severe censoring. The stylized facts discussed in this section exploit the full sample and we restrict the analysis to firms in the EAP survey when we need an exogenous measure of volatility, in section 4.3.

[10] We exclude the public sector, agriculture, forestry, and fishing, finance and insurance, energy and waste production and distribution, artistic activities, overseas activities, and household services.

Table 1: Population density and firms by commuting zone

	Density	Number of firms
Mean	150.75	1,304
Std. Dev.	496.48	3,723
25th percentile	39.74	447
50th percentile	68.69	682
75th percentile	122.32	1,193

Notes: Summary statistics based on the January 2015 cross-section. Density is measured by working age population divided by the commuting zone's area in squared kilometers, for the year 2015.

population is taken from the census at the municipality level, where the breakdown of the population by age and municipality is available at a 5-year frequency. For years in which the population is not available, we use data from the previous non-missing year. The area of commuting zones is based on INSEE 2020 shapefiles (base des zones d'emploi). Table 1 provides statistics on the distribution of commuting zones and the number of firms in each location for the January 2015 cross-section.

Productivity — Firms differ in size, which is typically explained in the literature by some randomness in firm productivity. In the data, we estimate firms' total factor productivity $\phi_{f,t}$ using the Levinshon-Petrin estimation technique with the Ackerberg et al. (2015) correction. Productivity is estimated as the residual of a production function equation including capital and three types of labor distinguished by their skill levels (Combes et al., 2012). Details of the estimation are provided in the Appendix A.2.

Employment volatility — In our model, firms are also heterogeneous in terms of the volatility of their labor demand, due to a combination of structural factors and their endogenous workforce management decisions. We use the panel dimension of the dataset to characterize the volatility of a firm's labor demand. Following Davis et al. (2006), we define a firm's volatility as:

$$\sigma_{f,t} = \sqrt{\frac{1}{2\omega+1}\sum_{\tau=-\omega}^{\omega}(\gamma_{f,t+\tau} - \bar{\gamma}_{f,t})^2} \tag{1}$$

where $\gamma_{f,t}$ is the year-on-year monthly growth rate of labor demand and $\bar{\gamma}_{f,t}$ is the mean growth rate computed over the $(2\omega+1)$-month period centered around date t. Our baseline measure uses a 35-month window, centered around January 2015. The variable is constructed using the total number of employees as our measure of labor demand. This measure of employment volatility captures second moments in the time-series of labor demand at firm-level, thus treating symmetrically upward and downward adjustments. As our focus is on the potential sources of labor market pooling, we focus on the idiosyncratic component of volatility, and thus residualize $\gamma_{f,t}$ in the sector×month×year dimensions.[11] However, our results are robust to using a simpler measure of employment volatility based on actual growth rates. The reason is that the vast majority of firm-level dispersion in volatility is driven by idiosyncratic shocks (di Giovanni et al., 2014).

In Appendix tables B.1 and B.2, we compare our baseline measure with alternatives capturing slightly different aspects of the firm's employment volatility. While the baseline measure rests on employment, thus on adjustments at the extensive margin, we show that the correlation with the volatility of hours is high, at .85. Pure intensive margin adjustments, through the number of hours per employee, are not the main factor at the root of a firm's labor demand fluctuations. Likewise, one may be concerned that certain type of contracts, most notably short-term contracts, are particularly well-suited to help the firm smooth out the impact of fluctuations in demand. The correlation of our baseline measure with a measure of volatility recovered solely from the growth of permanent contracts is however high, at .75. The volatility of open-ended contracts is still substantial, only 5% lower than the volatility of overall employment at the sample mean. Finally, our baseline volatility measure correlates highly with alternatives using slightly different strategies for identifying the idiosyncratic component of volatility. The most sensitive robustness check is obtained from statistics computed on month-on-month, instead of year-on-year, growth rates. Mechanically, the average volatility recovered from month-on-month growth rates is an order of magnitude smaller. However, its cross-sectional correlation with the baseline is still high at .70. In our baseline, we neglect month-on-month fluctuations that may to a large extent come from a sector-specific seasonality.

Descriptive statistics — Table 2 contains descriptive statistics on the baseline sample of firms. In January 2015, the sample is composed of 365,041 firms that we observe over at least 35 consecutive months. As expected, firms display significant heterogeneity in size, employment volatility, and productivity. Appendix Figure B.1 shows that, conditional on

[11] Monthly growth rates of labor demand are winsorized at the 1st and 99th percentile within each of 6 firm size classes. These 6 firm size classes identify firms below 2 employees, between 2 and 9 employees, between 10 and 49 employees, between 50 and 249 employees, between 250 and 4,999 employees, and plants of 5,000 and above employees.

its size, the median firm with 2-10 employees adjusts its labor demand (up or down) by approximately 0.8 employees per month, on average.

Table 2: Distribution of employment volatility and productivity

	Employment	$\log \sigma$	$\log \phi$
Mean	10.11	-2.03	3.19
Std. Dev.	20.82	1.15	0.72
25th percentile	2.00	-2.45	2.79
50th percentile	5.00	-1.84	3.20
75th percentile	9.87	-1.31	3.61

Notes: The variables are calculated for the January 2015 cross section of the dataset. Employment is the number of employees. Productivity is based on 2015 balance-sheet data. Volatility is computed using a 35-month window centered around January 2015 and the formula in equation (1). Statistics are calculated on a cross-section of 365,041 firms.

Table 3 shows how our measure of employment volatility correlates with important covariates. First, as seen in column (1), older firms are less volatile, which is a standard pattern in firm-level data (Davis et al., 2007). This relationship can reflect a form of internal diversification of risks when firms age and grow.[12] Appendix Figure B.2 illustrates this pattern in more detail, showing that most of the age variation takes place within the first four years of a firm's life cycle, while volatility stabilizes afterwards. In Section 4, we will use the firm's volatility after at least five years of existence as a proxy for the firm's steady state volatility.

Second, more productive firms are also less volatile (column 2). This empirical correlation is then taken into account and we systematically examine the effect of a firm's volatility, *conditional on its productivity*. Consistent with intuition, firms with a higher share of temporary contracts are more volatile, but controlling for this share does not significantly affect the other estimates (column 3). In addition, controlling for sector fixed effects (column 4) and commuting zone fixed effects (column 5) does not significantly increase the explanatory power of the model once the other controls are included. In column (6), we further control for the average firm growth over the 35 consecutive periods used in the calculation of employment volatility, corresponding to $\bar{\gamma}$ in eq. (1). This increases the explanatory power of the model, but does not affect the direction of the other effects described in the previous columns. In column (7) we report results from the same model as in column (6), but now focusing on a subset of manufacturing firms for which we have information on demand volatility, a measure that we will use in Section 4. Although the cross-section is much

[12] Note that in this table we control for firm size class fixed effects for consistency with the rest of the analysis. Firm size is negatively correlated with employment volatility.

smaller, the qualitative patterns are unchanged.

Table 3: Firm employment volatility: correlates

	Dep. Var: log Employment volatility						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log Age	-0.323	-0.327	-0.311	-0.290	-0.290	-0.171	-0.137
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.009)
log Productivity		-0.061	-0.047	-0.024	-0.035	-0.049	-0.051
		(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.012)
% fixed-term contracts			1.291	1.229	1.221	0.983	1.604
			(0.015)	(0.015)	(0.015)	(0.014)	(0.087)
Size class FE	✓	✓	✓	✓	✓	✓	✓
Sector FE				✓	✓	✓	✓
CZ FE					✓	✓	✓
Average growth						✓	✓
Adjusted R2	0.112	0.114	0.132	0.148	0.151	0.239	0.251
Sample	Full	Full	Full	Full	Full	Full	EAP
N. firms	365,041	365,041	365,041	365,041	365,041	365,041	20,419

Notes: the table shows the conditional correlation between our baseline measure of employment volatility and the firm's age, productivity and dependence on fixed-term contracts. The table contains OLS coefficients and their estimated standard errors in parentheses.

2.2 Motivating stylized facts

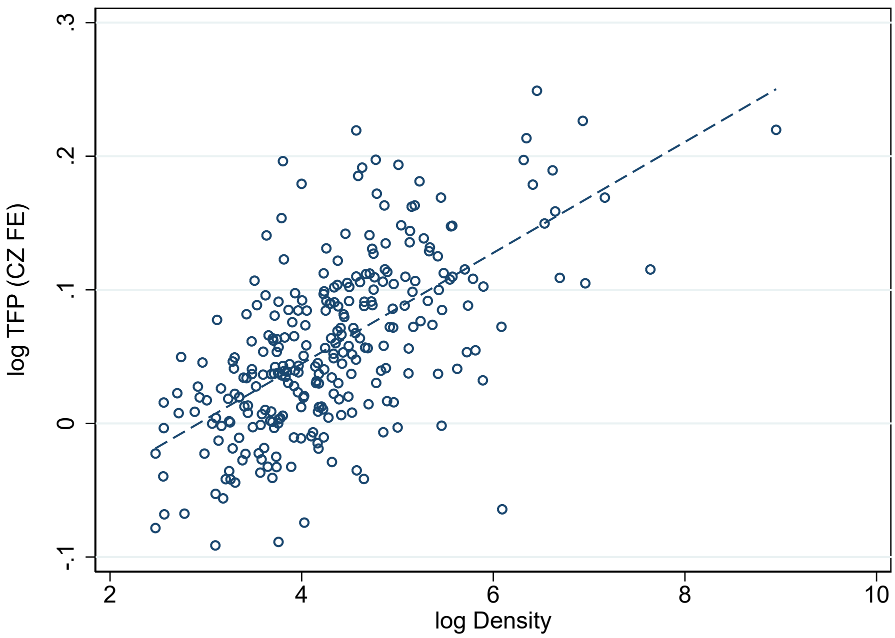
The productivity density gradient — The literature in economic geography has long discussed agglomeration patterns of firms over space. We first reproduce the evidence focusing on the productivity-density correlation (Combes et al., 2012). More precisely, we run the following regression based on the cross-section of firms observed in January 2015:

$$\log \phi_f = X_f \beta + FE_{M(f)} + \varepsilon_f \tag{2}$$

where X_f is a set of controls and $FE_{M(f)}$ denotes a set of fixed effects for each commuting zone. In this equation, the fixed effect captures the average productivity of firms in any commuting zone, once controlling for the heterogeneity that correlates with the control variables, namely the firm's 2-digit sector of activity, its size class and age.

Figure 1 illustrates the correlation between the conditional average productivity of firms and the population density of the commuting zone. As expected, the correlation is positive and significant, consistent with the view that dense commuting zones attract more productive firms, on average. As mentioned in the introduction, there is a vast literature explaining the correlation using various theoretical frameworks. A strand of the literature notably points to

Figure 1: The productivity advantage of large cities

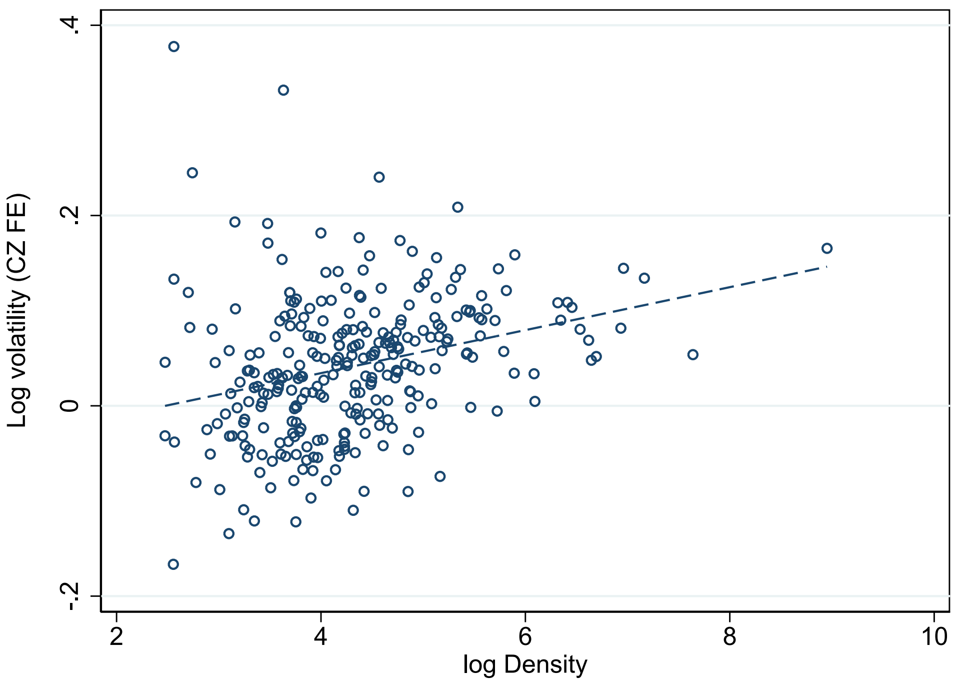


Notes: The figure shows the correlation between the mean productivity of firms and the density of the commuting zone where firms locate. Mean productivity is based on 2015 balance-sheet data. The correlation is conditional on the following firm characteristics: sector, size class, age, average growth during the period over which employment volatility is computed ($\bar{\gamma}$). The slope is 0.042 (the adjusted R^2 is 0.3847) and the slope is significantly different from 0 at 1%.

the role of matching economies through pooling externalities: Locations with higher meeting rates are most beneficial to high-productivity firms that are able to hire more quickly (Bilal, 2023). To the extent that pooling externalities are part of the story, we shall expect that the benefit is also larger for more volatile firms, conditional on productivity. As shown in Section 2.1, firms are indeed strongly heterogeneous in terms of the volatility of their labor demand, which may thus affect spatial location patterns.

The volatility density gradient — We provide preliminary evidence for a role of employment volatility in Figure 2. As in Figure 1, we first recover an estimate of firms' average employment volatility at the commuting zone level. We run a regression similar to eq. (2), using the log of employment volatility as the LHS variable. We then correlate this measure for conditional average employment volatility with the density of the commuting zone. Here as well, the conditional correlation is positive and significant, consistent with the intuition that pooling externalities are particularly valuable for volatile firms, which may then agglomerate in denser commuting zones. Importantly, the set of controls now includes the firm's productivity, which implies that a positive correlation exists beyond and above

Figure 2: The volatility advantage of large cities



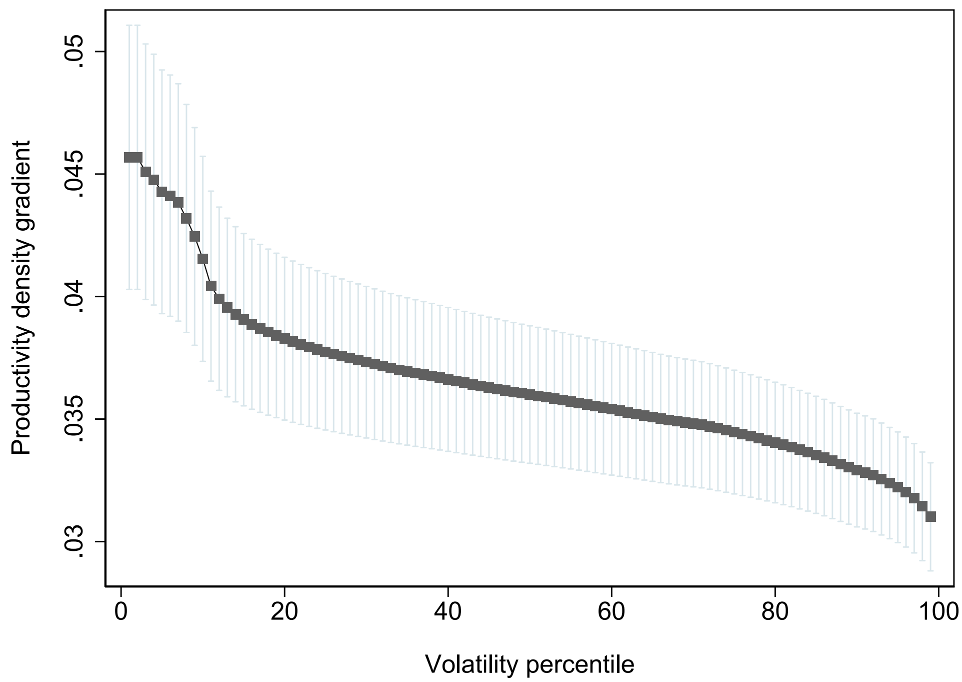
Notes: The figure shows the correlation between the mean volatility of firms and the density of the commuting zone where they locate. Volatility is measured by the standard deviation of the firm's labor demand year-on-year growth. Mean volatility is based on the January 2015 cross section of firms and is conditional on the following firm characteristics: sector, size class, firm age, firm average growth ($\bar{\gamma}$), log productivity. The slope is 0.023 (the adjusted R^2 is 0.0808) and the slope is significantly different from 0 at 1%.

the productivity-density nexus that the literature before us has documented.[13]

The productivity density gradient by volatility — Finally, Figure 3 provides a third motivating stylized fact that directly tackles the joint correlation between density, employment volatility and productivity. Instead of recovering the correlation between firms' attributes and the density of the firm's commuting zone in two stages, we now directly introduce density in eq. (2). The downside is that we can no longer control for unobserved heterogeneity between commuting zones using fixed effects. However, we can now interact density with a measure of the firm's employment volatility to estimate how the productivity-density correlation varies depending on the firm's volatility. The coefficient on the interaction is negative and strongly significant, which implies that the tendency of high-productivity firms to agglomerate in dense cities is less pronounced within the set of more volatile firms. Quantitatively, the cross-correlation is non-negligible: the elasticity of

[13]This correlation survives if we use the volatility of monthly growth rates instead of measuring the variance of year-on-year growth rates as we do in the rest of the paper.

Figure 3: The productivity-density gradient, along the distribution of volatility



Notes: The figure shows the conditional correlation between log productivity of firms and the density of the commuting zone where they locate, along the distribution of firms' log employment volatility. Productivity is conditional on the following firm characteristics: sector, size class, firm age and firm average growth of employment. The estimated equation includes the log density of the commuting zone where the firm is located, log employment volatility, and the interaction of log density and log employment volatility. Volatility is measured by the standard deviation of the firm's residualized labor demand growth. Data is based on the January 2015 cross section of firms. Whiskers indicate 95% confidence intervals.

firms' average productivity to density drops by a third when moving from the first to the last ventile of the volatility distribution.

Overall, the evidence in this section confirms that denser cities attract a pool of firms that are systematically different from the rest of the population in terms of their productivity but also the volatility of their labor demand. In the next section, we build a model that helps understand these agglomeration patterns.

3 Volatility and firm location: theory

We lay out a simple model of the impact of volatility on firms' location decisions. The model provides a micro-foundation of employment volatility based on firms' hiring and firing decisions and helps understand the trade-offs associated with firms' location choice: in particular, it shows why some firms may prefer locating in a denser city, even if that means operating under higher operating costs. The model's main prediction reads as follows: if firms sort across space based on their structural volatility because hiring is faster in denser cities, employment volatility will increase with density and the productivity-density gradient will be lower for firms with higher volatility. Expressions and proofs are provided in Appendix C.

3.1 Framework

We consider a simplified version of the canonical search-and-matching model proposed by Mortensen and Pissarides (1994), where single-job, risk-neutral, profit-maximizing firms face sales shocks and hiring frictions. The economy operates at a steady state and time is continuous. We focus on a partial equilibrium, leaving the worker problem aside. In particular, workers are homogeneous, their location is fixed, they do not search when employed and they do not bargain over wages. We also make the simplifying assumption that firms cannot adjust their labor demand at the intensive margin, by paying overtime or using part-time contracts. As discussed in Section 2.1, the extensive margin is a quantitatively important source of volatility at firm-level.

Set-up — Firms are heterogeneous in terms of their mean productivity $\phi > 0$ and their volatility $\varepsilon \in [0, 1]$, both known ex ante and independent from each other. We assume that firms are price takers and cannot adjust their price to sales shocks.[14] If we normalize price to 1, this means that sales fluctuate in any period between $\phi(1+\varepsilon)$ in the high state $(t = h)$ and $\phi(1 - \varepsilon)$ in the low state $(t = l)$ at an exogenous rate ξ that measures the structural volatility of the economy.

Upon entry, firms choose a location or city defined by a density $M > 0$. City choice determines firms' operating costs $R(M) \geq 0$ and job-filling rate $\mu(M) \geq 0$. $R(M)$ is a local index that combines all costs associated with maintaining an active position.[15] Importantly, firms that are not actively producing do not have to pay these costs. For example, if $R(M)$ represents the price of renting capital or real estate, this assumption means that there are no frictions on the capital market. We further assume that $R(M)$ and $\mu(M)$ are both increasing in M.[16]

[14]See Section 3.4 for the discussion of an extended model with an explicit formulation of entry and demand. While in our base setup presented here demand and productivity shocks may be homeomorphic, the extended model targets directly demand shocks that are the focus of our empirical work and eschews productivity shocks.

[15]It may encompass wages, but those do not depend on firms' individual characteristics (ϕ, ε) in order to keep the focus on hiring decisions.

[16]While $R'(M) > 0$ is easily justified by a congestion argument (commuting costs, inelastic housing

Strategies — Conditional on their location, firms also choose a strategy s, which in this context corresponds to a specific action to take in the low state. Firms can choose between three strategies $s \in \{B, W, C\}$. According to the "Business as usual" strategy (hereafter, denoted by B), if a firm is hit by a bad shock, it will keep paying its workforce or it will keep trying to hire. However, if operating costs are too high, the firm will seek to mitigate them by limiting the amount of time spent active in the low-production state. According to the "Wait-and-see" strategy (hereafter, denoted by W), if an active firm is hit by a bad shock, it will keep paying its workforce and wait for better times; yet, vacant firms, when hit by a bad shock, will postpone hiring until they have reached a high state again. Finally, according to the "Churning" strategy (hereafter, denoted by C), if a firm is hit by a bad shock, it will become idle. This means that it will wait if it is vacant and fire and wait if it is active.

Recursive formulation — Given their choice of city and strategy (M, s), firms alternate between being vacant (V), active (A) or idle (I). They decide whether to operate or hire while in a low state or not and this determines the firm's transition to a low state when posting a vacancy (with value $W_s(\phi, \varepsilon, M)$) or when filled (with value $C_s(\phi, \varepsilon, M)$). For any strategy s, firms' value functions are thus summarized as follows:

$$\begin{aligned}
rV_s^h(\phi,\varepsilon,M) &= -c + \mu(M)[A_s^h(\phi,\varepsilon,M) - V_s^h(\phi,\varepsilon,M)] && (3)\\
&+ \xi[W_s(\phi,\varepsilon,M) - V_s^h(\phi,\varepsilon,M)] \\
rV_s^l(\phi,\varepsilon,M) &= -c + \mu(M)[A_s^l(\phi,\varepsilon,M) - V_s^l(\phi,\varepsilon,M)] && (4)\\
&+ \xi[V_s^h(\phi,\varepsilon,M) - V_s^l(\phi,\varepsilon,M)] \\
rA_s^h(\phi,\varepsilon,M) &= \phi(1+\varepsilon) - R(M) + \delta[V_s^h(\phi,\varepsilon,M) - A_s^h(\phi,\varepsilon,M)] && (5)\\
&+ \xi[C_s(\phi,\varepsilon,M) - A_s^h(\phi,\varepsilon,M)] \\
rA_s^l(\phi,\varepsilon,M) &= \phi(1-\varepsilon) - R(M) + \delta[W_s(\phi,\varepsilon,M) - A_s^l(\phi,\varepsilon,M)] && (6)\\
&+ \xi[A_s^h(\phi,\varepsilon,M) - A_s^l(\phi,\varepsilon,M)] \\
rI_s(\phi,\varepsilon,M) &= \xi[V_s^h(\phi,\varepsilon,M) - I_s(\phi,\varepsilon,M)] && (7)
\end{aligned}$$

where r is the interest rate, c is the vacancy cost and δ is the exogenous component of the match destruction rate. Both c and δ are assumed to be fixed over time and constant across firms. Strategies determine the values of either posting a vacancy in the low state

supply), the sign of $\mu'(M)$ is more contentious because it depends directly on how many firms there are in each location, and what they do. For ease of exposition, we describe the framework in partial equilibrium, whereby those two local factors are not impacted by firms' decisions, and we defer the discussion on the endogenous determination of $\mu(M)$ to Section 3.4.

$(W_s(\phi, \varepsilon, M))$ or being active in the low state $(C_s(\phi, \varepsilon, M))$, as summarized in Table 4.

Table 4: Strategies and values of low state

	$W_s(\phi, \varepsilon, M)$	$C_s(p, \varepsilon, M)$
Business as usual	$V_B^l(\phi, \varepsilon, M)$	$A_B^l(\phi, \varepsilon, M)$
Wait-and-see	$I_W(\phi, \varepsilon, M)$	$A_W^l(\phi, \varepsilon, M)$
Churning	$I_C(\phi, \varepsilon, M)$	$I_C(\phi, \varepsilon, M)$

Entry, location choice and employment volatility — Since firms do not know in which state they will enter nor the state in any other period after entry, their expected profit at entry is given by $\mathbb{E}_s(\phi, \varepsilon, M) = 0.5 \times [V_s^h(\phi, \varepsilon, M) + W_s(\phi, \varepsilon, M)]$. Conditional on location, the *preferred strategy* s^* is thus the one that maximizes expected profit: $s^*(\phi, \varepsilon, M) = \underset{s}{\operatorname{argmax}} [\mathbb{E}_s(\phi, \varepsilon, M)]$.

For ease of exposition, we normalize the outside option to zero. Note that even preferred strategies may not be adopted if they yield a negative expected profit. In that case, the firm does not enter. Finally, under some conditions (detailed below), the model delivers a mapping $M^*(\phi, \varepsilon)$ between firms' characteristics and location:

$$M^*(\phi, \varepsilon) = \begin{cases} \underset{M}{\operatorname{argmax}} \left[\mathbb{E}_{s^*(\phi,\varepsilon,M)}(\phi, \varepsilon, M)\right] \text{ if } \mathbb{E}_{s^*(\phi,\varepsilon,M^*(\phi,\varepsilon))}(\phi, \varepsilon, M^*(\phi, \varepsilon)) \geq 0 \\ \{\emptyset\} \text{ otherwise} \end{cases} \tag{8}$$

Productivity ϕ and volatility ε, together with strategy s and location M determine volatility of employment $\sigma^l(\phi, \varepsilon, M^*(\phi, \varepsilon), s^*(\phi, \varepsilon))$. The model predicts that under reasonable parametric conditions, churning may indeed be associated with higher employment volatility, as summarized in Proposition 1.[17]

PROPOSITION 1. ***Churning and employment volatility*** —Firms that adopt the churning strategy have a higher employment volatility if the structural volatility of the economy is low enough.

3.2 Solution

Firms jointly choose s and M. Yet, for exposition purposes, we solve the model in three steps. First, we detail how firms' characteristics determine their strategy choice, for a given

[17] For high values of ξ, the model features the degenerate prediction that churning firms will mostly oscillate between the idle and the vacant states, with low associated volatility.

location. Then, we compare strategy choices between different cities. Finally, we solve the general model.

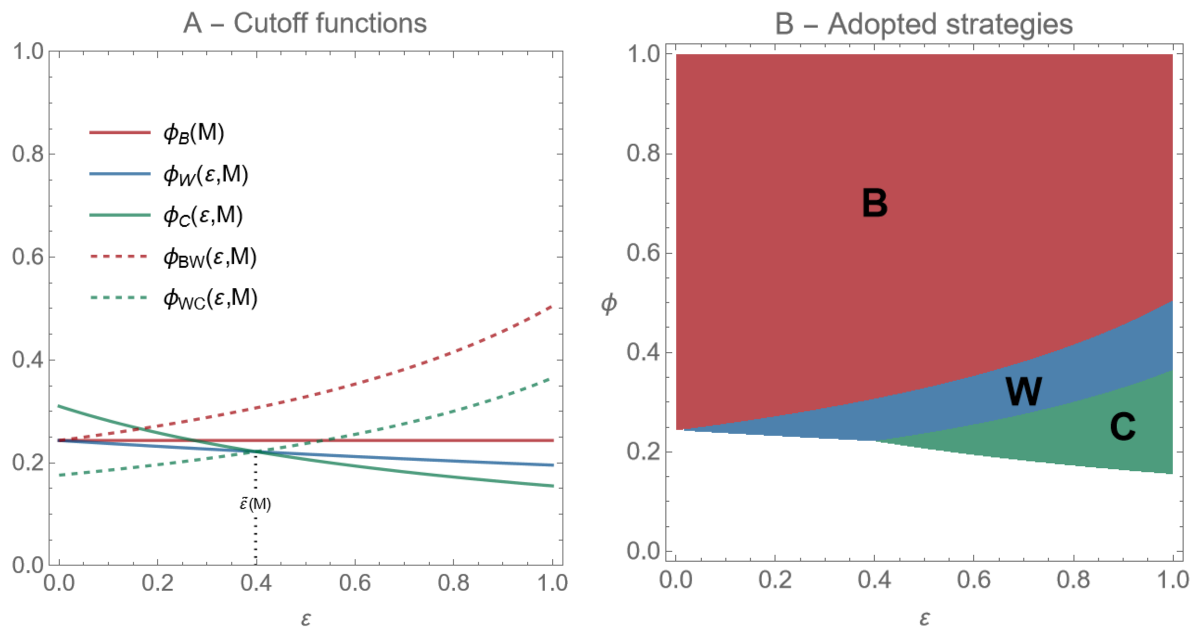
Strategy choice — If we solve the system (3-7), we can make two observations: first, quite naturally, expected profit increases with productivity, regardless of the strategy; second, higher productivity is more profitable under strategy B than under strategy W, and under strategy W than under strategy C. Therefore, strategy choice is determined by five productivity cutoffs: three *selection cutoffs* that determine whether a given strategy is *feasible*, and two *switching cutoffs* that determine which strategy is preferred.

Strategy s is feasible for a type-(ϕ, ε) firm if ϕ is greater than the selection cutoff $\phi_s(\varepsilon, M)$. Under strategy B, the selection cutoff $\phi_B(\varepsilon, M)$ does not depend on ε and may therefore be denoted $\phi_B(M)$. As is usual in this type of models, sales must cover both operating costs and the vacancy cost at entry and following any exogenous separation. Under strategy W, the selection cutoff $\phi_W(\varepsilon, M)$ is lower than under strategy B if $\varepsilon > 0$, and it decreases with ε. This strategy can therefore accommodate more volatile firms that have lower productivity in the low state compared to less volatile firms: by waiting, the firm mitigates the consequences of being in the low state. Finally, under strategy C, the selection cutoff $\phi_C(\varepsilon, M)$ is even more sensitive to ε than under strategy W: $\partial\phi_C(\varepsilon, M)/\partial\varepsilon < \partial\phi_W(\varepsilon, M)/\partial\varepsilon$. However, the selection cutoff also entails a fixed cost $c\xi/\mu(M)$, which corresponds to the additional time spent vacant. Therefore, only highly volatile firms may be able to churn. In particular, churning only allows for the entry of less productive firms if their volatility exceeds a given cutoff $\tilde{\varepsilon}(M)$, which depends on both local and common parameters.

We then turn to the conditions that determine when firms adopt a churning strategy over alternative strategies. In what follows, an *adopted strategy* is both preferred and feasible. We denote by $\phi_{BW}(\varepsilon, M)$ and $\phi_{WC}(\varepsilon, M)$, with $\phi_{BW}(\varepsilon, M) > \phi_{WC}(\varepsilon, M)$, the corresponding cutoffs. Both cutoffs, as well as the difference between the two, are convex increasing functions of ε. Regarding the B strategy, we can note that $\forall\varepsilon, \phi_{BW}(\varepsilon, M) > \phi_B(M)$. Therefore, if strategy B is preferred, it is also feasible, and therefore, adopted. Conversely, strategies W or C may be preferred, yet unfeasible, if $\phi_{WC}(\varepsilon, M) < \phi_W(\varepsilon, M)$ or $\phi_{WC}(\varepsilon, M) < \phi_C(\varepsilon, M)$. We use a calibration to illustrate the working of the model.[18] The productivity cutoffs are

[18] This calibration is somewhat arbitrary, even though we aim for realism for some aspects. The time unit is a year and we set $r = 3\%$. The match destruction rate δ is set to 10%, and the probability of switching between high and low demand states is set to 20%. The vacancy cost is set to 10% of a maximum productivity level $\overline{\phi}$, which is set to 1. We consider a cost function $R(M) = 0.2M^{0.1}$. This 10% elasticity stems from the addition of the 3% of urban costs calibrated by Combes et al. (2019) and 7% elasticity of raw wages (Ahlfeldt and Pietrostefani, 2019). Finally, we consider a worker finding rate given by $\mu(M) = 0.3M^{0.05}$.

Figure 4: Strategy choice for a given city



Calibration: $\xi = 0.2$, $r = 0.03$, $\delta = 0.1$, $\mu(M) = 0.3M^{0.05}$, $R(M) = 0.2M^{0.1}$, $c = 0.1$ and $\overline{\phi} = 1$. We set $M = 1$. **Panel A**: The figure represents the three minimum productivity cutoffs and the two strategy-switching cutoffs as a function of volatility ε. **Panel B**: The figure represents the set of (ε, ϕ) combinations associated with each adopted strategy. The blank section corresponds to combinations that are not feasible, regardless of the strategy.

represented in Panel A in Figure 4.

Equipped with these definitions, we can fully characterize the distribution of adopted strategies as a function of ϕ and ε. They are represented in Panel B in Figure 4 in the form of the three regions labeled B, W, and C.[19] Panel B highlights our first two key results that hold for a fixed city, as summarized in Proposition 2:

PROPOSITION 2. ***Strategy choice*** — In a given city,

2.1 Churning is adopted by more volatile, less productive firms.

2.2 Very volatile firms may churn even if they are quite productive. Conversely, low-productivity firms may be able to operate if they are volatile enough.

The joint strategy/location problem — The next step is understanding how density interacts with firms' productivity, volatility, and strategy choice. To proceed, we make three

[19]Note that one strategy may never be adopted, depending on the parameters. In particular, W disappears when $c \to 0$. Conversely, C disappears for large enough values of c.

further assumptions:

Assumption 1. Churning happens in equilibrium.[20]

Assumption 2. For each strategy, selection on productivity does not decrease with density.[21]

Assumption 3. For each strategy, there exists an optimal level of density.[22]

Those assumptions restrict the analysis to cases where the model is both relevant (Assumption 1), realistic (Assumption 2), and analytically well-defined (Assumption 3). Under those assumptions, we can perform comparative statics of strategy choice under different city sizes, which yields the following results, summarized in Proposition 3:

Proposition 3. ***Comparative statics*** — If cities are heterogeneous in density,

3.1 Denser cities have a higher share of churning firms.

3.2 Low-productivity firms are more volatile in denser cities.

Results 3.1 and 3.2 can also be gauged by comparing adopted strategies in the (ε, ϕ) plane for different levels of density, as we do in Panel A of Appendix Figure C.1. In line with result 2.2, even productive firms may churn in denser cities if they are very volatile. In addition, higher volatility is more conducive to the entry of low-productivity firms, as shown by a steeper lower bound of the colored area.

Finally, we study the firm location choice, and how churning interacts with the spatial sorting of firms based on their productivity. This requires solving a global maximization problem, to identify the density chosen by firms, conditional on their productivity ϕ and volatility ε. While the combinations of (ϕ, ε) associated with strategy choice are only defined implicitly, the envelope theorem ensures that Proposition 2 is robust to firms' location choice. Panel B in Appendix Figure C.1 illustrates this result. In particular, more volatile firms are more

[20] This assumption is verified under the condition $\tilde{\varepsilon}(M) < 1$, which is equivalent to $c/\mu(M) < R(M)/\xi$. In words, this means that the expected vacancy cost is lower than the operating costs paid by the firm when it is operating in the low state. For simplicity, we will even assume a stronger condition, stating that $\forall M \geq 0, R(M) > c$ and $\mu(M) > \xi$. Note that this assumption means that both $R(M)$ and $\mu(M)$ feature a fixed positive component, or that there is a lower bound for density, as we do in our calibration.

[21] The most binding condition is for strategy C, where it is equivalent to: $\forall M \geq 0, R'(M) \geq (r + \delta + \xi)\mu'(M)/\mu(M)^2$. For simplicity, and using Assumption 1, we will even assume a stronger condition on the ratio of the elasticity of each function: $\forall M \geq 0, \epsilon_{R,M}/\epsilon_{\mu,M} > (r + \delta + \xi)/\xi$.

[22] This assumption means that $\forall s, \exists M > 0$ s.t. $\partial \mathbb{E}_s(\phi, \varepsilon, M)/\partial M = 0$. As for Assumption 2, this assumption will be met if the ratio of the cost elasticity to the matching elasticity is high enough.

likely to adopt the churning strategy, more productive firms are more likely to adopt the business-as-usual strategy, and low-productivity, high-volatility firms are more likely to be able to operate if they adopt the churning strategy.

3.3 Volatility and the sorting of firms

This framework allows us to study how the joint strategy/location optimization problem at the individual firm level translates into aggregate sorting patterns of firms across space. Conditional on selection and strategy choice, the spatial sorting of firms is implicitly defined by the optimal productivity/volatility-density relationship described by *sorting cutoffs* $\phi_s^*(\varepsilon, M) = \underset{\phi}{\text{argmax}}\,[\mathbb{E}_s(\phi, \varepsilon, M)]$ and $\varepsilon_s^*(\phi, M) = \underset{\varepsilon}{\text{argmax}}\,[\mathbb{E}_s(\phi, \varepsilon, M)]$.

These sorting cutoffs illustrate how matching economies work in this model. Since high-productivity and high-volatility firms have more to gain from being able to hire more quickly, there is positive sorting with respect to productivity and volatility, for a given strategy. In addition, given the multiplicative structure between ϕ and ε, high-productivity (resp., high-volatility) firms have all the more to gain from locating in denser cities if they are more volatile (resp., productive). Formally, we have $\forall \varepsilon \in [0,1], \partial\phi_B^*(\varepsilon, M)/\partial M \geq \partial\phi_W^*(\varepsilon, M)/\partial M \geq \partial\phi_C^*(\varepsilon, M)/\partial M$. Therefore, even if more productive firms sort into denser cities, the share of churning firms also increases with density and the productivity-density gradient decreases with firm volatility.

These patterns are summarized in Proposition 4:

Proposition 4. ***Predictions*** — If firms choose their location in order to maximize their expected profit upon entry,

4.1 More productive and more volatile firms sort into denser cities.

4.2 Productivity and volatility are complementary in city choice.

4.3 The share of churning firms increases with density and the productivity-density gradient is flatter for more volatile firms.

Prediction 4.3 echoes the aggregate sorting patterns described in Section 2. Appendix Figure C.2 illustrates this prediction for the same calibration of the model, for a density spanning between 1 and 10.[23] Panel A, consistent with Figure 2, displays the share of churning firms as

[23] Note that we need to specify the underlying distributions of ϕ and ε. For simplicity, we assume that they are both uniformly distributed over $[0,1]$.

an increasing function of density. In Panel B, we measure the productivity-density gradient along the distribution of firm volatility. Consistent with Figure 3, this gradient decreases with firm volatility. As for predictions 4.1 and 4.2, they will be tested in Section 4.

3.4 Discussions

In order to maintain analytical tractability, the model rests on several simplifying assumptions. We briefly discuss here the robustness of its conclusion to a more general framework.

Endogenous matching rate — In the presentation, it was assumed that the worker meeting rate was not affected by firms' location decisions and strategies. However, such assumption is not internally consistent, because the worker meeting rate depends on the local market tightness, which is, itself, an equilibrium outcome. A priori, the impact of churning on market tightness is ambiguous. On the intensive margin, churning firms lay off workers while remaining idle, which loosens the market; conversely, churning may also allow more firms to enter. To recover market tightness, we need to define a fixed point problem that describes steady-state conditions and to specify a process for firm entry, in order to determine the equilibrium firm-to-worker ratio. In Appendix C.4, we describe a way of tackling this extended model. Using simulations, we show that, under plausible parametric assumptions on the matching technology, the resulting worker meeting rate is an increasing concave function of density, even if the positive effect of churning on firm entry mitigates the magnitude of agglomeration economies.

Firm size heterogeneity and demand — In the presentation, we did not model demand nor allowed for firm size heterogeneity linked with productivity. In Appendix C.5, we embed this model in a framework à-la Melitz (2003), where monopolistically competitive firms face a CES demand system and draw heterogeneous productivity and demand volatility upon entry. This extended model allows us to better understand the underlying differences in the behavior of firms that face demand shocks and either adopt the Business-as-usual or the Churning strategy. Under the former, firms adjust their prices, while under the latter, prices are independent from individual demand shocks, which are then passed on employment. We show that under plausible parametric restrictions, the main predictions of the base model carry through: churning makes it possible for lower productivity firms to enter, the share of volatile firms increases with density, and the productivity-density gradient is flatter for volatile firms.

4 Volatility and firm location: empirical evidence

In this section, we turn back to our data and describe two tests of the main predictions of the model. These tests rely on different assumptions and proxies for firm characteristics introduced in Section 3.

4.1 Empirical strategy

Our empirical framework is based on a location choice model estimated with a conditional logit estimator (CLM). We start with the January 2015 cross-section and select firms that are between 5 and 10 years old in January 2015.[24] This leaves us with a sample of 88,168 firms.

While the theoretical model assumed a continuum of densities, we now consider a discrete set of locations $\mathcal{M} = \{M\}$. Conditional on the firm's decision to enter the French market, we model the choice of a location as a function of the firm's and the location's attributes. We borrow the notations from Section 3 and denote $\mathbb{E}^*(\phi_f, \varepsilon_f, M) = \mathbb{E}_{s^*(\phi_f,\varepsilon_f,M)}(\phi_f, \varepsilon_f, M)$ for brevity. Assuming that the expected inter-temporal profit in each location can be decomposed into a deterministic and a random component e_{fM}, one can write the probability of a firm f choosing a location M as:

$$\begin{aligned} \mathbb{P}\left(f \text{ chooses } M | e_{fM}\right) &= \mathbb{P}\left(\mathbb{E}^*(\phi_f, \varepsilon_f, M) + e_{fM} > \max_{M' \neq M} \left\{\mathbb{E}^*(\phi_f, \varepsilon_f, M') + e_{fM'}\right\}\right) \\ &= \frac{\exp\left[\mathbb{E}^*(\phi_f, \varepsilon_f, M)\right]}{\sum\limits_{M' \in \mathcal{M}} \exp\left[\mathbb{E}^*(\phi_f, \varepsilon_f, M')\right]} \end{aligned}$$

where the second line uses the assumption that e_{fM} are i.i.d. draws from a type-1 extreme value distribution.

Our model predicts the choice between all commuting zones to be a function of the size of operating costs $R(M)$ and the job-filling rate $\mu(M)$ as well as their interaction with firms' productivity ϕ_f and volatility ε_f. Following the theoretical model, the CLM considers the role of commuting zone density, and its interaction with firms' characteristics, TFP and volatility.[25] We obtain productivity and employment volatility from the January 2015 cross

[24] The lower age limit excludes firms that are still in the early stages of their life cycle in 2015, when their employment volatility is a poor measure of their long-term (structural) volatility, as discussed in section 2.1 and illustrated in Appendix Figure B.2. The upper age limit is more arbitrary. It is intended to homogenize the sample with respect to the broader economic context in which the location decisions were made.

[25] In all rigor, employment volatility should not be denoted by ε, but by σ. We introduce a potential

section, focusing on the same variables that we have analysed in Section 2.2. We also control for other commuting zone characteristics that are important for firm location decisions, namely two measures of workforce skill (the share of managers, and the share of college graduates).[26] In order to verify that other local characteristics correlated with density do not drive our results, we test the robustness of our estimates to the inclusion of CZ fixed effects.

Localization economies — The identification assumption behind the CLM is that other firm characteristics that are correlated with volatility and productivity do not interact with density in determining location choice. However, this assumption is unlikely to be true in general. In particular, there is ample evidence that firms benefit from having other firms in the same industry operating in the same area (Combes and Gobillon, 2015). Therefore, we also control for a measure of localization economies measured in 2015. Localization economies measure a firm's sectoral network based on Mayer et al. (2010). This sectoral network is calculated as the total number of firms in the same industry located in each potential commuting zone in the year y corresponding to the chosen cross-section.[27] More precisely, localization economies are defined as:

$$SectoralNetwork^{s}_{i,y} = \sum_{y}\sum_{a} D^{s}_{ai,y} \tag{9}$$

where $D^{s}_{ai,y}$ is a dummy variable equal to one for all firms a of sector s located in commuting zone i and created in year y. The count of firms in each sector and commuting zone only includes firms with positive employment. Sectors are defined at the 2-digit level of the French sector nomenclature (Nomenclature d'activités française – NAF rév. 2).

4.2 Results

Sorting on volatility and productivity — Estimation results are summarized in Table 5. In column (1), we show the coefficient associated with the (log of) density of the commuting zone, and we confirm the tendency of firms to agglomerate in denser commuting zones, even after controlling for other commuting zone characteristics and our measure of localization economies. In columns (2) and (3), we then interact density with the model's

candidate for ε in Section 4.3.

[26]The share of managers is calculated from DADS-INSEE, where managers are defined by 1-digit occupation (CS1) equal to 2 or 3. The share of college graduates is obtained from Census data. Both of these variables are measured in 2015.

[27]In the sample used in section 4.3, we cumulate the sectoral network variables over the years preceding the firm's creation.

Table 5: Results of the location choice model

	Dep. Variable: CZ choice						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
CZ density M	0.464	0.465	0.464	0.464		0.464	
	(0.004)	(0.004)	(0.004)	(0.004)		(0.004)	
- $\times$ Productivity		0.064		0.064	0.061	0.065	0.061
		(0.002)		(0.002)	(0.002)	(0.002)	(0.002)
- $\times$ Volatility			0.041	0.042	0.040	0.042	0.040
			(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
- $\times$ Volatility $\times$ Productivity						0.005	0.004
						(0.002)	(0.002)
CZ characteristics	✓	✓	✓	✓		✓	
Localization economies	✓	✓	✓	✓	✓	✓	✓
CZ Fixed effects					✓		✓
Pseudo R2	0.1353	0.1364	0.1357	0.1369	0.1569	0.1369	0.1569
N. observations	25M	25M	25M	25M	25M	25M	25M

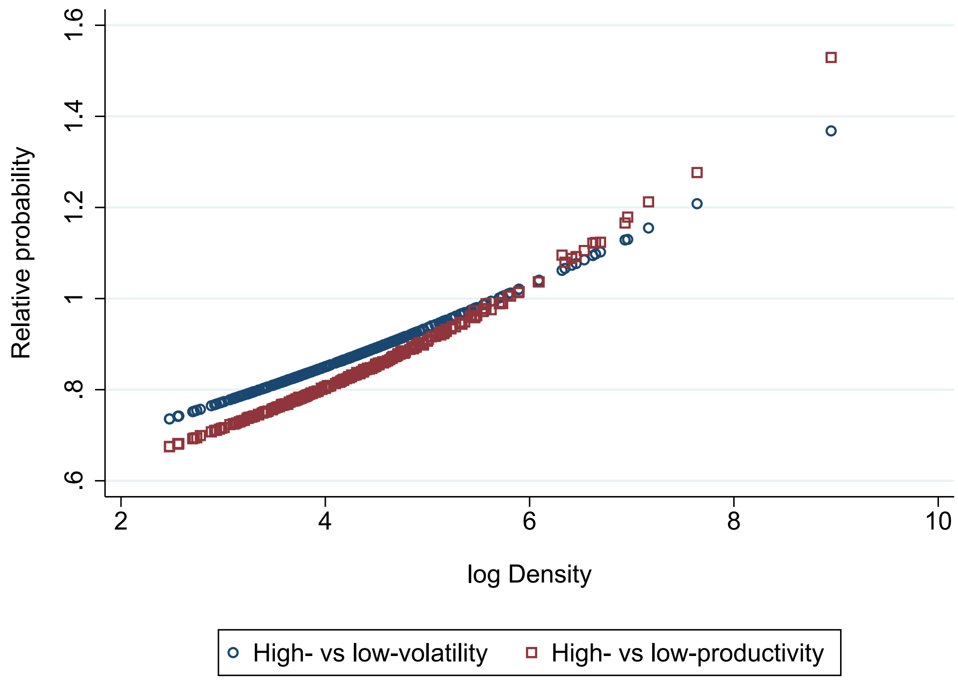
Notes: Coefficient estimates from a conditional logit model with firm fixed effects. The sample is based firms in the January 2015 cross-section with age 5-10 years (88,168 firms, resulting in $N = 24,687,040$ observations). M is the log of CZ density, volatility is the standardized value of employment volatility, and productivity is the standardized value of productivity. Standard errors in parentheses. All estimates are statistically significant at the 5% level.

relevant firms characteristics, namely productivity and volatility. Column (2) confirms previous results in the literature, showing that more productive firms are more likely to locate in denser cities. In column (3), we find that volatile firms are also more likely to locate in dense cities. In columns (4), we simultaneously consider the two interaction terms. Finally column (5) further controls for CZ fixed effects and solely identifies the coefficients on the interaction terms.

Results point to a quantitatively similar impact of productivity and volatility on location patterns, which is stable across specifications. Namely, in our baseline model (column 4) the elasticity of the odds of choosing a specific location to the density of this commuting zone increases from .41 to .53 when moving from the first to the ninth decile of the distribution of productivity. The effect is similar (from .42 to .52), when moving along the distribution of employment volatility. These results are robust to the inclusion of CZ fixed effects (column 5) and suggest that prediction 4.1 is verified in the data.

The heterogeneity in the determinants of location choices along the distribution of firms is further illustrated in Figure 5, which compares the predicted probabilities of locating in a particular commuting zone, for firms at the 75th percentile relative to the 25th percentile of the distribution of firms' productivity and employment volatility. The patterns recovered

Figure 5: Heterogeneity in location choices, along the distributions of productivity and volatility

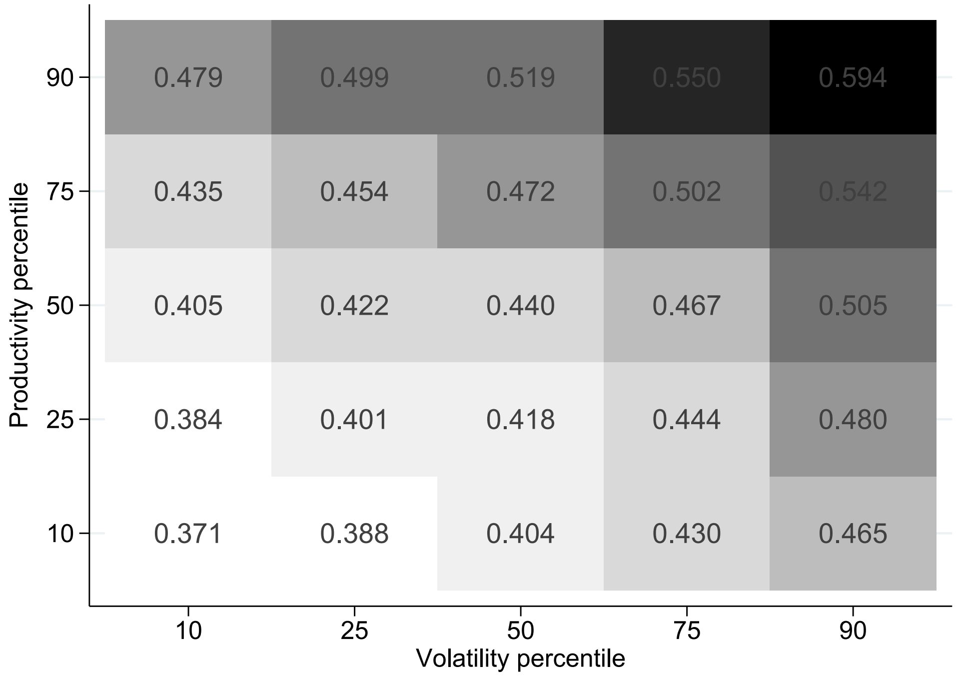


Notes: The figure shows the mean probability of locating in each commuting zone, for high-productivity (respectively, high-volatility) firms in relative terms with low-productivity (respectively low-volatility) firms. The cut-offs are based on firms at the 25th and 75th percentile of each distribution. The probabilities are recovered from the estimation of the model in column (4) of Table 5.

from heterogeneous productivity and volatility are very similar. In both cases, the conditional location probabilities are roughly equal at a density of around 150, which corresponds to the level observed in commuting zones in the top 25th percentile of the population density distribution. Above this level, both high productivity and high volatility firms are more likely to locate in denser cities. However, this figure also suggests that sorting remains higher along the productivity dimension, especially for the highest level of density, which corresponds to the city of Paris.

Combined effects — Finally, we turn to Prediction 4.2, whereby productivity and volatility are complementary in firms' location choices. In other words, more productive (respectively, volatile) firms are all the more likely to sort into denser locations when they are more volatile (respectively, productive). This prediction stems from the multiplicative structure between ϕ and ε in the model. Therefore, by looking at the impact of the interaction between firm productivity and volatility on firms' location choice, we can gauge the importance of complementarity between those two dimensions.

Figure 6: Elasticity of the odds of choosing a CZ to CZ density



Notes: Each cell is computed (in %) as $\exp\{0.01[\hat{\beta}_M + \hat{\beta}_{M\varepsilon}P_\varepsilon + \hat{\beta}_{M\phi}P_\phi + \hat{\beta}_{M\varepsilon\phi}P_\varepsilon P_\phi]\} - 1$. Estimates from model (6) of Table 5.

In Column (6) of Table 5, we estimate the impact of the triple interaction between CZ density, firm productivity, and firm demand volatility. The coefficient on the triple interaction is positive and significant and remains so when we further control for CZ fixed effects (column 7). This suggests that Prediction 4.2 is verified in the data. However, the estimated coefficient is small. In order to get a sense of the magnitude of this complementarity, we compute the elasticity of the odds of locating in a given CZ to CZ density, for different quantiles of productivity and volatility. Results are displayed in Figure 6. When going from the first to the ninth decile of productivity (respectively, volatility) the elasticity increases by 29% (respectively, 25%) for firms at the first decile of volatility (respectively, productivity). Conversely, when going simultaneously from the first to the ninth deciles of both productivity and volatility the elasticity increases by 60%, which is slightly higher than the sum of the two marginal effects.

4.3 Robustness: Firm birth and demand volatility

One may be concerned that the previous results are biased for two reasons: First, by selecting firms that are old enough to have reached their long-term employment volatility, the analysis does not distinguish between location decisions upon entry and firm survival, which we do not model. In order to verify whether entry matters, one needs to focus on the event of firm creation. Second, and related, using employment volatility as a factor influencing the location decision is contentious, since it is endogenous to location choices: As argued in Section 3, employment volatility varies with the firm's strategy choice, which depends on the joint impact of the firm's productivity and *structural* volatility, both directly and indirectly through its impact on location decisions. Conditional on these structural characteristics, a dense location may actually cause an increase in employment volatility. Finally, our measure of unemployment volatility does not allow us to distinguish between job quits and layoffs, which might be an issue if the former are correlated with the interaction of firm characteristics with density — for example, if low-productivity firms in larger cities have a harder time retaining some of their workers. We thus test the robustness of our results to using a different measure of volatility.

Our proposed alternative measure of firm volatility uses exogenous variations in export demand for products in the firm's portfolio. Therefore, we focus on the volatility of demand that a firm can expect to face, conditional on the nature of its production. Given the structure of the firm's product portfolio, which we recover from the EAP survey, the volatility of demand in foreign markets can be used as a proxy for the firm's expected demand volatility. To further rule out endogeneity, we use the firm's portfolio of products at entry, i.e. no later than two years after its creation, as the firm's decision to expand its product scope may also be endogenous. For consistency, we also compute productivity over this two-year period. Details are available in Section A.3 in the Appendix. As the EAP survey is not available before 2009, we have to restrict the cross-section to firms that were born posterior to 2009. To increase sample size, we include all firms born until 2018, and end up with a sample of 1,682 firm creations.

Results are displayed in Appendix Table B.3, which follows the same structure as Table 5. All results are confirmed in this smaller subset of location decisions. Consistent with prediction 4.1, both productivity and demand volatility are significant predictors of firms' location choices, although the effect of volatility is less precisely estimated. In addition, their predictive power is still quantitatively equivalent.[28] Appendix Figure B.4 shows that the resulting sorting patterns are also quantitatively similar. Finally, consistent with prediction 4.2, the coefficient on the triple interaction is positive and significant, confirming

[28]The elasticity of the odds of choosing a specific location to the density of this commuting zone increases from .30 to .38 as one moves from the first to the ninth decile of the productivity distribution. The effect is similar (from .31 to .37), when moving along the distribution of demand volatility.

that productivity and volatility are complementary in firms' location choices. Interestingly, however, Appendix Figure B.5 suggests that the effect of volatility and productivity is less symmetric in this selected sample: Low-productivity firms hardly sort by demand volatility, while low-volatility firms still sort by productivity. The structure of our model allows for this possibility: When productivity is close to zero, so is expected profit, regardless of location. Accordingly, whether or not such an asymmetry is observed or not in the data depends on the intensity of firm selection by productivity, which our results suggest is lower in this selected sample.

5 Conclusion

This paper shows that firms with a more volatile activity benefit from locating in denser locations. Higher operating costs associated with density create an incentive for volatile firms to adopt a more flexible workforce management strategy. In turn, by frequently releasing workers, those firms generate a positive externality on other firms, which benefit from easier hiring conditions. This finding opens a fruitful avenue for future research on the determinants of the spatial distribution of economic activity that go beyond static characteristics such as productivity. It provides a novel explanation for the non-negative correlation between city size and unemployment rates, and for the observation that many low-productivity firms are able to operate in large cities. However, our partial-equilibrium analysis leaves many effects aside. For example, workers should be compensated for working in more volatile firms. Conversely, a higher share of volatile firms might mitigate urban congestion costs, if firms are able to adjust their operating expenses. We leave these extensions for further research.

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A Data Appendix

A.1 Sample selection

Table A.1: Sample selection

	Operating in January 2015	Relevant sectors	Single CZ	Non-missing productivity	Non-missing demand
Number of firms	890,088	770,901	609,662	365,041	20,419
		(87%)	(68%)	(41%)	(2%)
Number of plants	1,100,329	934,894	672,556	408,308	22,464
		(85%)	(61%)	(37%)	(2%)
Total employment	11,954,963	10,385,953	5,474,535	3,691,419	483,934
		(87%)	(46%)	(31%)	(4%)

Notes: Number of firms refers to the concept of firm described in the main text, i.e. we aggregate plants of the same firm within each commuting zone. In column (1), we count firms operating in January 2015 with non-missing employment volatility. Column (2) keeps only the relevant sectors for our analysis: manufacturing, construction and service sectors (including non-tradable services). Column (3) drops firms operating in more than one CZ. In column (4) we keep only firms with non-missing productivity and age (i.e. firms for which we do not have information on the date of creation). Column (5) shows the number of firms for which we have information on demand volatility, a variable used in Section 4. Percentages correspond to the share with respect to the numbers in Column (1).

A.2 Total Factor Productivity

In our main results, productivity is calculated using the Levinshon-Petrin estimation technique,[29] with the Ackerberg et al. (2015) correction. We follow Combes et al. (2012) in defining productivity for each firm f and year y as:

$$ln(V_{fy}) = \beta_{0y} + \beta_1 ln(k_{fy}) + \beta_2 ln(l_{fy}) + \sum_{s=1}^{3} \gamma_s l_{sfy} + \phi_{fy} \tag{A.1}$$

where V_{fy} is value added, k_{fy} is capital, l_{fy} is employment.[30] As in Combes et al. (2012), we distinguish between three skill levels: high, intermediate and low, with l_{sfy} the share of skill level s in the firm's overall employment. We estimate the equation separately for each 2-digit sector.[31] To minimize the impact of outliers, we then winsorize productivity at the 1st and 99th percentile within each of the 6 firm size classes described in Section 2.1.[32]

[29]We use the Stata `prodest` command that exploits the control function approach.

[30]Employment data is taken from INSEE-DADS and refers to mean employment, calculated over the months in each year y.

[31]We focus on 12 2-digit sectors, which include manufacturing, construction, and services (including non-tradables).

[32]Firm size is defined based on the mean employment calculated across months of each year. Productivity percentiles by firm size class are calculated over the 2010-2019 sample. After winsorizing, we further clean the data by only keeping productivity if firm revenues are above the 1st percentile and below the 99th percentile, calculated over the 2010-2019 sample.

The skill groups are defined following Combes et al. (2012). The low-skill group includes low-skill blue collars (in craft and manufacturing) and low-skill white collars (sales clerk, employees in personal services). The corresponding occupational codes in the French classification are: 55, "employés de commerce"; 67, "ouvriers non qualifiés de type industriel"; 68, "ouvriers non qualifiés de type artisanal". The intermediate-skill group includes high-skill blue collars (in craft, manufacturing, handling, and transport), and intermediate-skill white collars (administrative employees). In the French standard occupational classification, the following two-digit occupations are included: 52, "employés civils et agents de la fonction publique"; 53, "agents de surveillance"; 54, "employés administratifs d'entreprise"; 62, "ouvriers qualifiés de type industriel"; 63, "ouvriers qualifiés de type artisanal"; 64, "chauffeurs"; and 65, "ouvriers qualifiés de la manutention, du magasinage et du transport." Finally, the high-skill group includes managers (in craft, manufacturing or sales), executive and knowledge workers (executives, scientists, engineers), intermediate professions (intermediate professions in administration and sales firms, technicians, foremen). The group covers the following two-digit occupations are included: 21, "artisans (salariés de leur entreprise)"; 22, "commerçants et assimilés (salariés de leur entreprise)"; 23, "chefs d'entreprise de 10 salariés ou plus (salariés de leur entreprise)"; 31, "professions libérales (exercées sous statut de salarié)"; 34, "professeurs, professions scientifiques"; 35, "professions de l'information, des arts et des spectacles"; 37, "cadres administratifs et commerciaux d'entreprises"; 38, "ingénieurs et cadres techniques d'entreprises"; 46, "professions intermédiaires administratives et commerciales des entreprises"; 47, "techniciens"; and 48, "contremaîtres, agents de maîtrise." Finally, we drop the following non-coded occupations: 99, "non codage"; 00, "allocations assedic."

A.3 Demand volatility

To compute our measure of demand volatility, we use the EAP survey to recover information on the structure of a firm's product portfolio in some base period:[33]

$$w_{fp,0} = \sum_{p} \frac{Sales_{fp,0}}{\sum_{p' \in P_{f,0}} Sales_{fp',0}}$$

where $Sales_{fp,0}$ is the value of product-level sales and $P_{f,0}$ denotes the set of products in the firm's portfolio in base period 0.[34]

We then leverage upon trade data to construct a time series of the synthetic demand growth

[33] The downside is that the use of EAP forces us to focus on a sample of firms in the manufacturing sector, which is not representative of the whole population. See the last Column in Table A.1 for details.

[34] Each product p is measured at the 4-digit level of the CPA 2008 product nomenclature, which can be merged to Eurostat data as seen below. Sales are constructed following EAP documentation as the sum of *Ventes de produits industriels* (VS2 + VF1 + VF2), *Ventes de services industriels* (VF3 or VT1), *Installation et pose de produits industriels* (IR1 + IR2 + IR3 = IT1), *Réparation et maintenance* (RR1 + RR2 + RR3 = RT1).

that a firm can expect to face, given the structure of its product portfolio:

$$\gamma_{f,t}^{D} = \sum_{p' \in P_{f,0}} w_{fp',0} \gamma_{p',t}^{D}$$

where $\gamma_{p',t}^{D}$ is the year-on-year growth of the world demand of product p' recovered from Eurostat trade data at monthly-frequency.[35]

We can finally compute a measure of expected demand volatility:

$$\varepsilon_{f,t} = \sqrt{\frac{1}{2\omega+1} \sum_{\tau=-\omega}^{\omega} (\gamma_{f,t+\tau}^{D} - \bar{\gamma}_{f,t}^{D})^2} \tag{A.2}$$

In comparison with the baseline measure in eq. (1), the advantage of $\varepsilon_{f,t}$ is that it is a measure of volatility that is orthogonal to the firm's hiring strategy, or the structural churning rate in a particular location. From this point of view, it is more exogenous to the firm location choice than the volatility of labor demand. However, strict exogeneity requires that the structure of the firm's portfolio is given, at the time of the location decision. To give credibility to the assumption, we use information on the firm's portfolio of products observed during the first two years of activity. By contrast, world demand used to construct synthetic demand growth is based on the 12 months before and including the firm's creation month.

[35] The monthly import time-series is smoothed using three-month moving averages. World demand refers to imports from all countries in the world excluding France.

B Additional Results

Table B.1: Firm employment volatility: Correlation with alternative measures

	Workers	Hours	Hours per empl.
Baseline	-	0.8492	0.3095
Non-residualized volatility	0.9991	0.8494	0.3111
Residualized by sector×month and CZ×month	0.9999	0.8490	0.3093
Month-on-month growth rates	0.7029	0.6372	0.4392
Growth of permanent contracts	0.7485	0.6513	0.2835

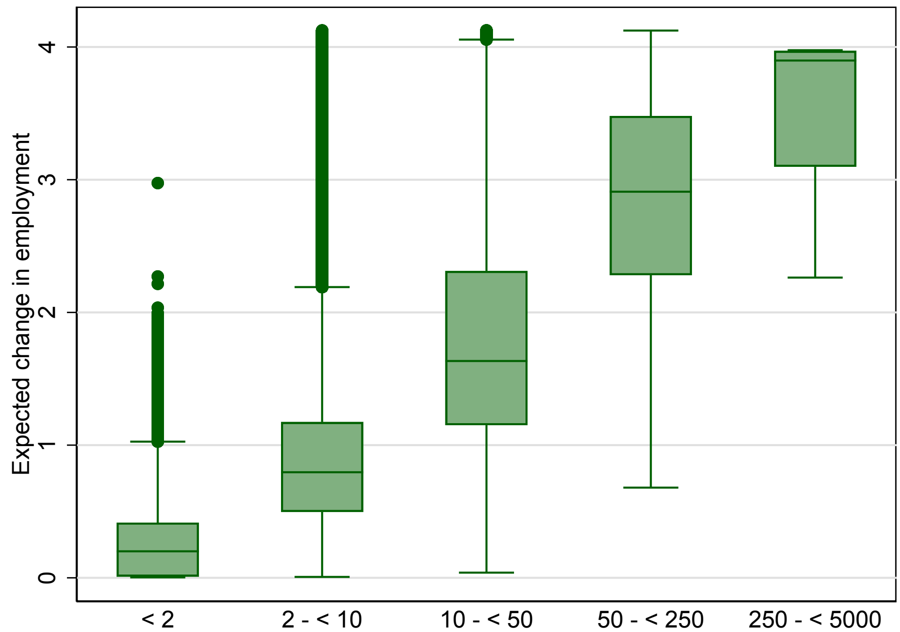
Notes: Correlation coefficients based on various measures of volatility. All measures are computed from the January 2015 cross-section of firms. The baseline measure is described in section 2.1. The "non-residualized volatility" measure is computed as in equation (1) using the growth rate observed in the data instead of focusing on the idiosyncratic component of growth. The measured which is "residualized by sector×month and CZ×month" the residual of an equation that includes CZ fixed effects. The "month-on-month growth rates" measure is computed exactly as the baseline except that the raw data are month-on-month (instead of year-on-year) growth rates. Finally, the "growth of permanent contracts" measure is constructed as the baseline on the restricted set of the firm's permanent contracts. All correlations are statistically significant at the 1% level.

Table B.2: Firm employment volatility: Summary statistics

	Workers		Hours		Hours per empl.	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Baseline	0.210	0.189	0.233	0.219	0.093	0.102
Non-residualized volatility	0.209	0.191	0.231	0.220	0.092	0.103
Residualized by sect×month and CZ×month	0.211	0.189	0.233	0.218	0.093	0.102
Month-on-month growth rates	0.080	0.063	0.085	0.065	0.025	0.019
Growth of permanent contracts	0.199	0.191	0.222	0.215	0.091	0.106

Notes: The statistics are recovered from the cross-section of firms active in January 2015. The baseline measure is described in section 2.1. The "non-residualized volatility" measure is computed as in equation (1) using the growth rate observed in the data instead of focusing on the idiosyncratic component of growth. The measured which is "residualized by sector×month and CZ×month" the residual of an equation that includes CZ fixed effects. The "month-on-month growth rates" measure is computed exactly as the baseline except that the raw data are month-on-month (instead of year-on-year) growth rates. Finally, the "growth of permanent contracts" measure is constructed as the baseline on the restricted set of the firm's permanent contracts.

Figure B.1: Firm employment volatility: Interpretation



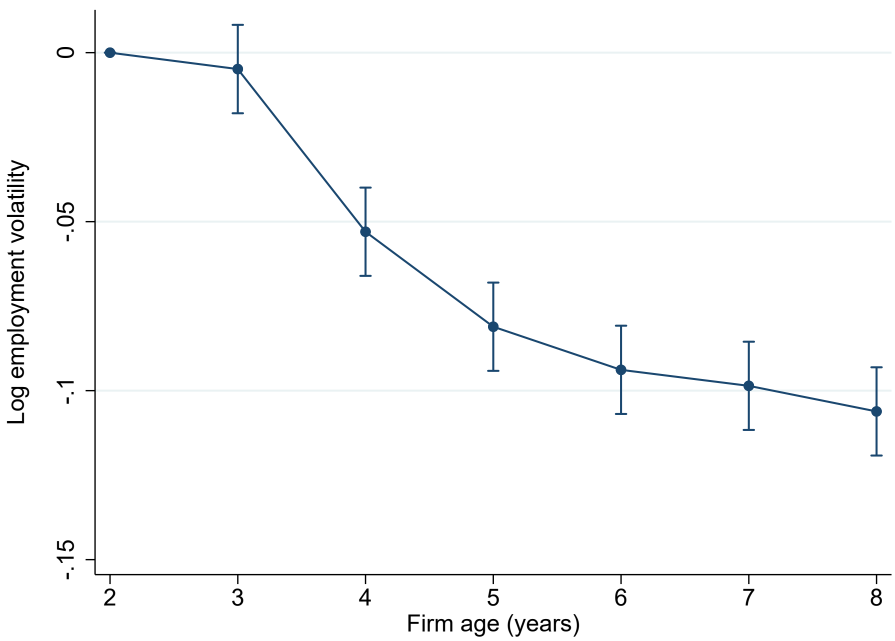
Notes: The figure is based on the January 2015 cross section of single-plant firms. The variable of interest is the expected change in employment computed as the product of the firm's average size and volatility. Results are winsorized at the top 95th percentile.

Table B.3: Results of the location choice model: robustness

	Dep. Variable: CZ choice						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
CZ Density M	0.339	0.339	0.338	0.338		0.337	
	(0.027)	(0.027)	(0.027)	(0.027)		(0.027)	
- × Productivity		0.049		0.048	0.050	0.053	0.055
		(0.019)		(0.018)	(0.018)	(0.015)	(0.015)
- × Volatility			0.034	0.033	0.030	0.027	0.024
			(0.017)	(0.016)	(0.016)	(0.017)	(0.017)
- × Volatility × Productivity						0.030	0.029
						(0.013)	(0.013)
CZ characteristics	✓	✓	✓	✓		✓	
Localization economies	✓	✓	✓	✓	✓	✓	✓
CZ Fixed effects					✓		✓
Pseudo R2	0.0441	0.0446	0.0443	0.0448	0.0451	0.0871	0.0873
N. observations	471K	471K	471K	471K	471K	471K	471K

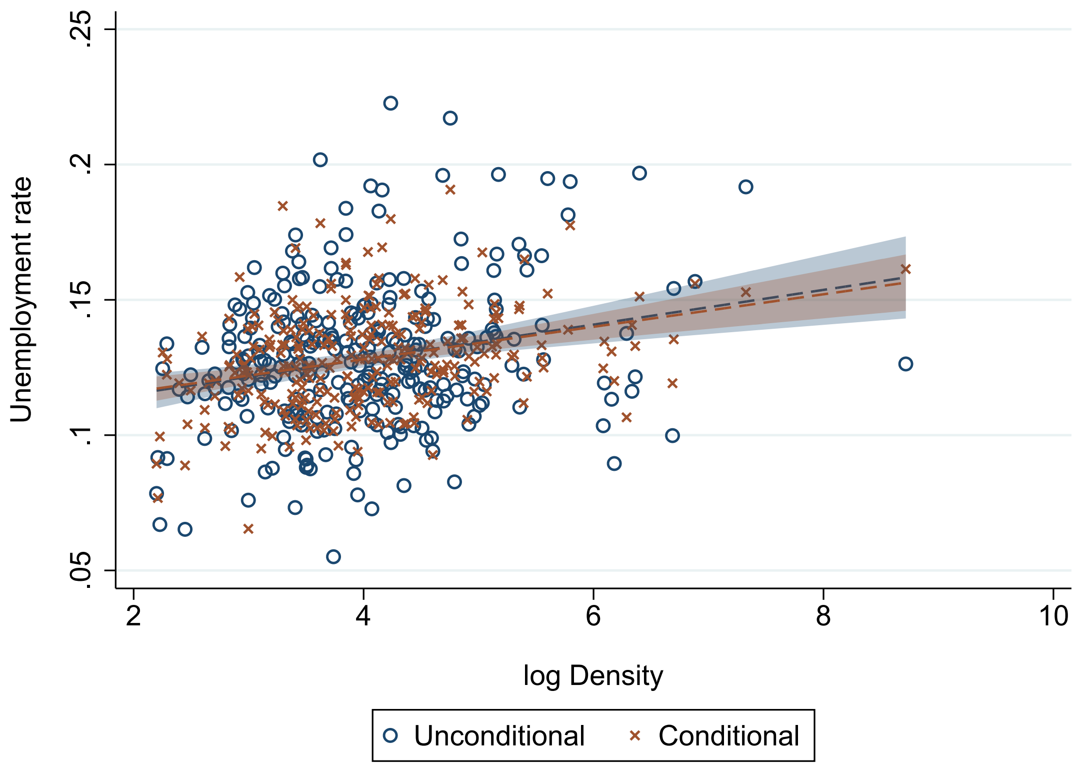
Notes: Coefficient estimates from a conditional logit model with firm fixed effects. The sample is based on all firm entries from January 2010 to December 2019 (1,682 entries, resulting in $N = 470,960$ observations). M is the log of CZ density, volatility is the standardized value of expected demand volatility, and productivity is the standardized value of productivity. Standard errors in round parentheses.

Figure B.2: Firm employment volatility: The effect of age



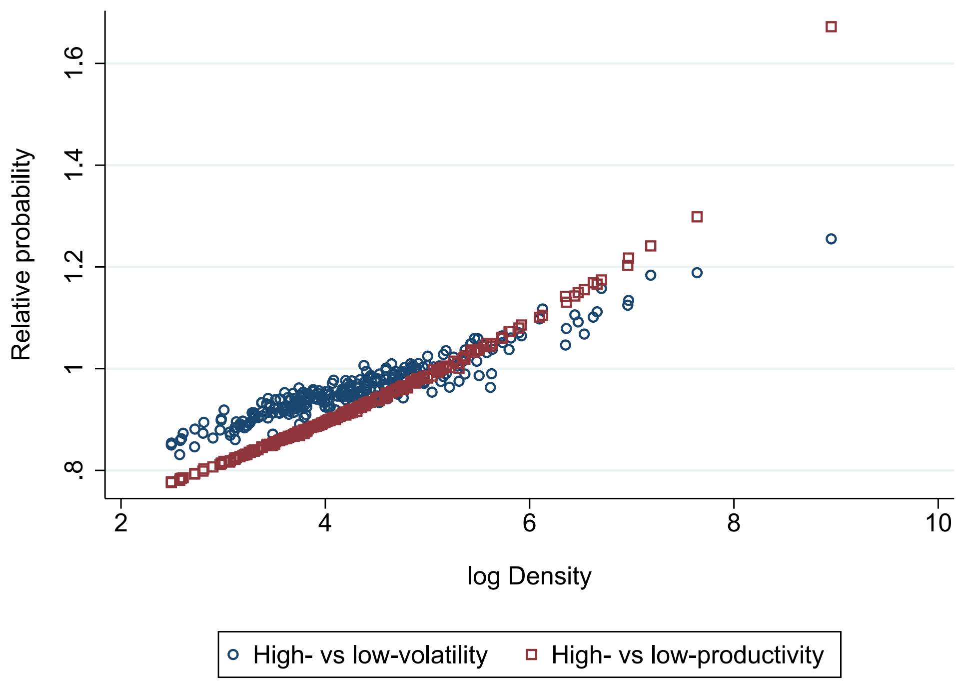
Notes: The figure is based on the panel of firms created in 2010 with non-missing employment volatility (29,168 firms, for a total number of observations of $N = 126,704$). The figure plots age fixed effects from a regression of log employment volatility on age and firm FEs. The reference age category is two years, because volatility is computed over a 35-month window around the date of observation. Whiskers indicate 95% confidence intervals

Figure B.3: Unemployment rate and density



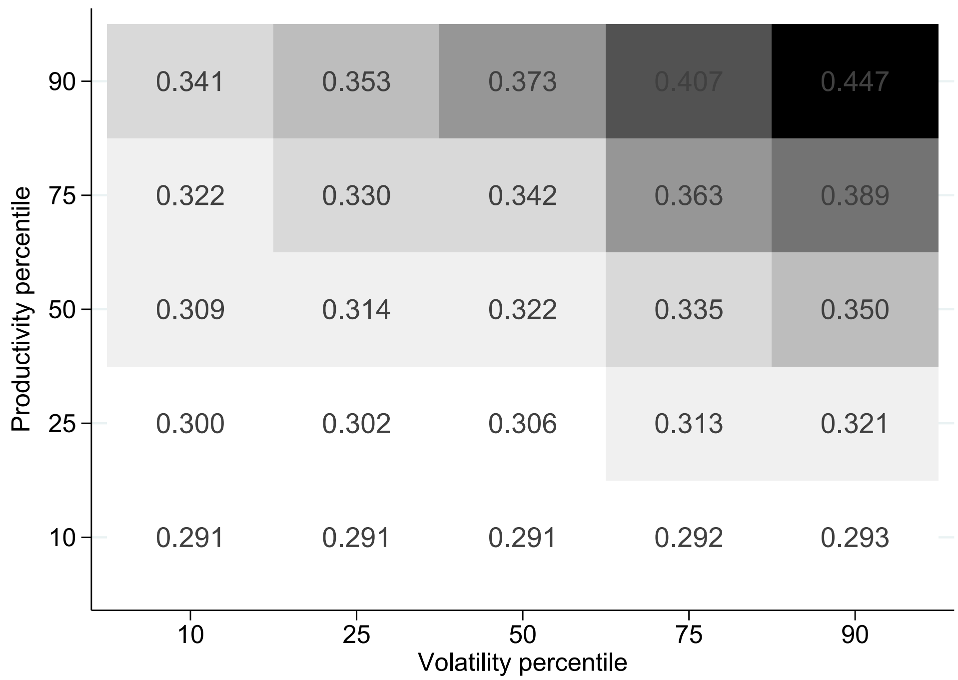
Notes: The figure shows the unemployment rate of the working-age population (aged 15-54) by commuting zone as a function of its (working-age) population density. In red, we provide the correlation after controlling for the share of university graduates in the population above 15, the share of managers among employed workers, the shares of old and young workers in the working-age population, and 22 region fixed effects. In the unconditional case, the slope is equal to 0.0064 and the R-squared is equal to 0.0542. In the conditional case, the slope is equal to 0.0060 and the R-squared is equal to 0.0964. Both slopes are statistically significant at the 1% level. Source: 2018 Census

Figure B.4: Heterogeneity in location choices, along the distributions of productivity and volatility: robustness



Notes: The figure shows the mean probability of locating in each commuting zone, for high-productivity (respectively, high-volatility) firms in relative terms with low-productivity (respectively low-volatility) firms. The cut-offs are based on firms at the 25th and 75th percentile of each distribution. The probabilities are recovered from the estimation of the model in column (4) of Appendix Table B.3.

Figure B.5: Elasticity of the odds of choosing a CZ to CZ density: robustness



Notes: Each cell is computed (in %) as $\exp\{0.01[\hat{\beta}_M + \hat{\beta}_{M\varepsilon}P_\varepsilon + \hat{\beta}_{M\phi}P_\phi + \hat{\beta}_{M\varepsilon\phi}P_\varepsilon P_\phi]\} - 1$. Estimates from model (6) of Appendix Table B.3.

C Theory

C.1 Definitions

Expected profits — Expected profits upon entry are given by:

$$\mathbb{E}_B(\phi,\varepsilon,M) = \frac{\mu(M)[\phi - R(M)] - c(\delta + r)}{r[r+\delta+\mu(M)]} \tag{C.1}$$

$$\mathbb{E}_W(\phi,\varepsilon,M) = \frac{1}{2} \times \frac{(r+\delta+2\xi)\{\mu(M)[\phi - R(M)] - c(r+\delta)\} + \mu(M)(r+\delta)\varepsilon\phi}{r[(r+\delta)(r+\delta+2\xi) + \mu(M)(r+\delta+\xi)]} \tag{C.2}$$

$$\mathbb{E}_C(\phi,\varepsilon,M) = \frac{1}{2} \times \frac{\mu(M)[\phi(1+\varepsilon) - R(M)] - c(r+\delta+\xi)}{r[r+\delta+\xi+\mu(M)]} \tag{C.3}$$

These three expressions are increasing in ϕ. In addition, we can show that:

$$\frac{\partial \mathbb{E}_B(\phi,\varepsilon,M)}{\partial \phi} - \frac{\partial \mathbb{E}_W(\phi,\varepsilon,M)}{\partial \phi} = \frac{(r+\delta)\mu(M)[(1-\varepsilon)(r+\delta+\mu(M)) + 2\xi]}{2r(r+\delta+\mu(M))[(r+\delta)(r+\delta+2\xi) + (r+\delta+\xi)\mu(M)]} > 0$$

$$\frac{\partial \mathbb{E}_W(\phi,\varepsilon,M)}{\partial \phi} - \frac{\partial \mathbb{E}_C(\phi,\varepsilon,M)}{\partial \phi} = \frac{\xi\mu(M)[(1-\varepsilon)(r+\delta+\mu(M) + 2\xi]}{2r(r+\delta+\xi+\mu(M))[(r+\delta)(r+\delta+2\xi) + (r+\delta+\xi)\mu(M)]} > 0$$

Selection cutoffs — Solving for $\mathbb{E}_s(\phi,\varepsilon,M) = 0$ we find:

$$\phi_B(\varepsilon,M) = \phi_B(M) = R(M) + \frac{c(r+\delta)}{\mu(M)} \tag{C.4}$$

$$\phi_W(\varepsilon,M) = \left(\frac{r+\delta+2\xi}{(1+\varepsilon)(r+\delta)+2\xi}\right)\phi_B(M) \tag{C.5}$$

$$\phi_C(\varepsilon,M) = \frac{1}{1+\varepsilon}\left(\phi_B(M) + \frac{c\xi}{\mu(M)}\right) \tag{C.6}$$

Note that $\phi_B(M) \geq \phi_W(M)$ and $\phi_C(\varepsilon,M) \leq \phi_W(M) \iff \varepsilon \geq \tilde{\varepsilon}(M) = \frac{c(r+\delta+2\xi)}{c(r+\delta)+2\mu(M)R(M)}$.

Switching cutoffs — Solving for $\mathbb{E}_B(\phi,\varepsilon,M) = \mathbb{E}_W(\phi,\varepsilon,M)$ and $\mathbb{E}_W(\phi,\varepsilon,M) = \mathbb{E}_C(\phi,\varepsilon,M)$, we find:

$$\phi_{BW}(\varepsilon,M) = \left(\frac{r+\delta+\mu(M)+2\xi}{(1-\varepsilon)(r+\delta+\mu(M))+2\xi}\right)\phi_B(M) \tag{C.7}$$

$$\phi_{WC}(\varepsilon,M) = \frac{R(M)(r+\delta+2\xi+\mu(M))}{(1-\varepsilon)(r+\delta+\mu(M))+2\xi} \tag{C.8}$$

Note that $\phi_{BW}(M) \geq \phi_B(M)$. In addition, we can show that:

$$\phi_{BW}(\varepsilon,M) - \phi_{WC}(\varepsilon,M) = c \times \frac{(r+\delta)(r+\delta+2\xi) + (r+\delta+\xi)\mu(M)}{\mu(M)[(1-\varepsilon)(r+\delta+\mu(M))+2\xi]} > 0$$

so that strategy W is never adopted when $c = 0$. Conversely, since $\partial\phi_C(\varepsilon,M)/\partial c > 0$ and $\partial\phi_{WC}(\varepsilon,M)/\partial c = 0$, there is maximum value of c above which strategy C is never adopted.

Sorting cutoffs — Solving for $\frac{\partial \mathbb{E}_s(\phi,\varepsilon,M)}{\partial \phi} = 0$ and $\frac{\partial \mathbb{E}_s(\phi,\varepsilon,M)}{\partial \varepsilon} = 0$, we find:

$$\phi_B^*(\varepsilon, M) = \phi_B^*(M) = R(M) - c + \tfrac{R'(M)}{\mu'(M)} \left(\tfrac{\mu(M)(r+\delta+\mu(M))}{(r+\delta)} \right) \tag{C.9}$$

$$\phi_W^*(\varepsilon, M) = \tfrac{(r+\delta+2\xi)R(M)-c(r+\delta+\xi)}{(1+\varepsilon)(r+\delta)+2\xi} + \tfrac{R'(M)}{\mu'(M)} \left(\tfrac{\mu(M)[(r+\delta)(r+\delta+2\xi)+(r+\delta+\xi)\mu(M)]}{(r+\delta)[(1+\varepsilon)(r+\delta)+2\xi]} \right) \tag{C.10}$$

$$\phi_C^*(\varepsilon, M) = \frac{1}{1+\varepsilon} \left[R(M) - c + \tfrac{R'(M)}{\mu'(M)} \left(\tfrac{\mu(M)(r+\delta+\xi+\mu(M))}{(r+\delta)} \right) \right] \tag{C.11}$$

$$\varepsilon_W^*(\phi, M) = \frac{1}{\phi} \left[-(c + \phi - R(M)) - \xi \left(\tfrac{c+2(\phi-R(M))}{r+\delta} \right) + \tfrac{R'(M)}{\mu'(M)} \left(\tfrac{\mu(M)[(r+\delta)(r+\delta+2\xi)+(r+\delta+\xi)\mu(M)]}{(r+\delta)^2} \right) \right] \tag{C.12}$$

$$\varepsilon_C^*(\phi, M) = \frac{1}{\phi} \left[-(c + \phi - R(M)) + \tfrac{R'(M)}{\mu'(M)} \left(\tfrac{(r+\delta+\xi+\mu(M))\mu(M)}{r+\delta+\xi} \right) \right] \tag{C.13}$$

Motion laws — At steady state, the measures of firms in each state are constant, for each strategy. We use bold symbols to distinguish these measures from their corresponding values:

$$\text{Strategy B:} \quad \begin{cases} \mu(M)\boldsymbol{V_B}(M) = \delta \boldsymbol{A_B}(M) \\ \boldsymbol{V_B}(M) + \boldsymbol{A_B}(M) = \boldsymbol{B}(M) \end{cases} \tag{C.14}$$

$$\text{Strategy W:} \quad \begin{cases} [\xi + \mu(M)]\boldsymbol{V_W}(M) = \delta \boldsymbol{A_W^h}(M) + \xi \boldsymbol{I_W}(M) \\ \xi \boldsymbol{I_W}(M) = \xi \boldsymbol{V_W}(M) + \delta \boldsymbol{A_W^l}(M) \\ (\delta + \xi)\boldsymbol{A_W^h}(M) = \mu(M)\boldsymbol{V_W}(M) + \xi \boldsymbol{A_W^l}(M) \\ (\delta + \xi)\boldsymbol{A_W^l}(M) = \xi \boldsymbol{A_W^h}(M) \\ \boldsymbol{V_W}(M) + \boldsymbol{I_W}(M) + \boldsymbol{A_W^h}(M) + \boldsymbol{A_W^l}(M) = \boldsymbol{W}(M) \end{cases} \tag{C.15}$$

$$\text{Strategy C:} \quad \begin{cases} [\mu(M) + \xi]\boldsymbol{V_C}(M) = \delta \boldsymbol{A_C}(M) + \xi \boldsymbol{I_C}(M) \\ \xi \boldsymbol{I_C}(M) = \xi[\boldsymbol{A_C}(M) + \boldsymbol{V_C}(M)] \\ (\delta + \xi)\boldsymbol{A_C}(M) = \mu(M)\boldsymbol{V_C}(M) \\ \boldsymbol{V_C}(M) + \boldsymbol{I_C}(M) + \boldsymbol{A_C}(M) = \boldsymbol{C}(M) \end{cases} \tag{C.16}$$

where $\boldsymbol{B}(M)$, $\boldsymbol{W}(M)$ and $\boldsymbol{C}(M)$ are the respective measures of firms that follow strategies B, W and C.

C.2 Proofs

Proof of Proposition 1 — Employment is a random variable that follows a Bernoulli distribution. The variance of employment is thus

$$\sigma^l(\phi, \varepsilon, M^*(\phi, \varepsilon), s^*(\phi, \varepsilon)) = \mathbb{P}(\phi, \varepsilon, M^*, s^*)\left[1 - \mathbb{P}(\phi, \varepsilon, M^*, s^*)\right]$$

where $\mathbb{P}(\phi, \varepsilon, M^*, s^*)$ is the probability of a position being filled, which follows the steady-state constraints given by Equations C.14–C.16 with $\boldsymbol{B}(M) = \boldsymbol{W}(M) = \boldsymbol{C}(M) = 1$. Employment volatility is higher under strategy C than under strategy B if $\xi < (\mu(M) - 2\delta)(\delta + \mu(M))/(2\delta)$, which is only possible if $\mu(M) > 2\delta$, and is more likely to be satisfied if M is large, given $\mu'(M) > 0$.

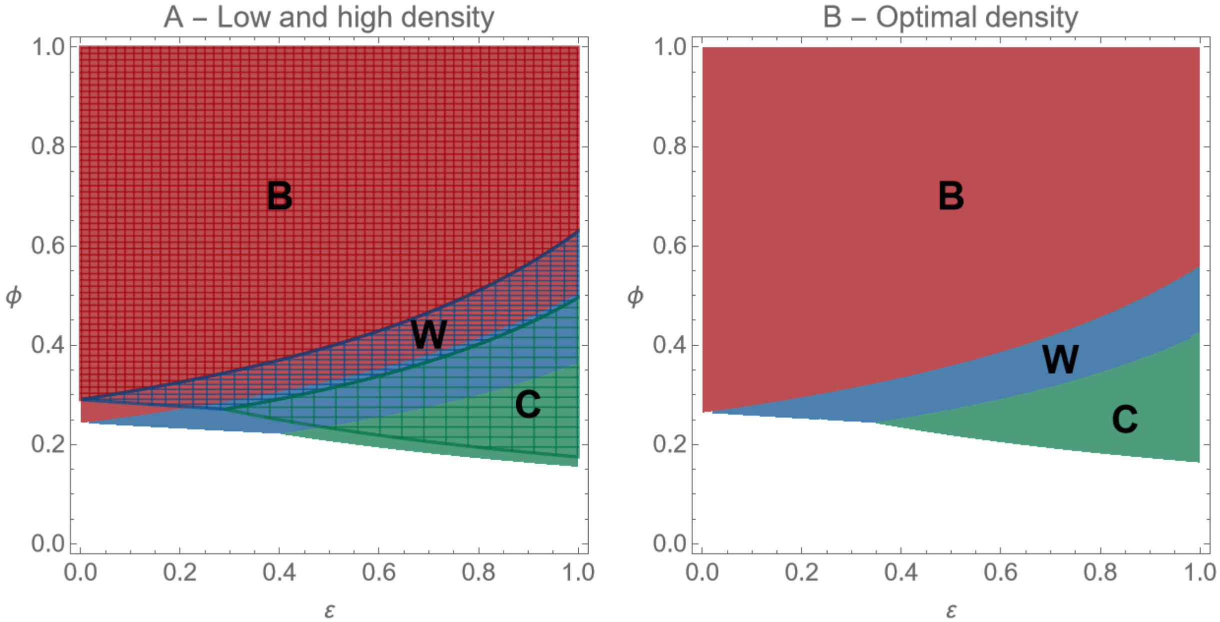
Proof of Proposition 2 — Result 2.1 stems from the facts that $\partial\phi_{WC}(\varepsilon, M)/\partial\varepsilon > 0$ and $\partial\phi_C(\varepsilon, M)/\partial\varepsilon < 0$ and that the selection cutoff for strategy C is the lowest, as long as $\varepsilon \geq \tilde{\varepsilon}(M)$. Result 2.2 stems from the fact that $\partial^2\phi_{WC}(\varepsilon, M)/\partial\varepsilon\partial\varepsilon > 0$.

Proof of Proposition 3 — Result 3.1 is obtained by noticing that, under Assumptions 1 and 2, $\partial\phi_{BW}(\varepsilon, M)/\partial M > 0$. Thus, the area of region B decreases with density. Moreover, under Assumption 2, $\partial\phi_W(\varepsilon, M)/\partial M > 0$. In addition, under no assumption, $\partial[\phi_{BW}(\varepsilon, M) - \phi_{WC}(\varepsilon, M)]/\partial M < 0$. Thus, the area of region W decreases with density. Finally, under no assumption, $\partial\tilde{\varepsilon}(M)/\partial M < 0$. In addition, under Assumption 2, $\partial[\phi_{WC}(\varepsilon, M) - \phi_C(\varepsilon, M)]/\partial M > 0$. Thus, the area of region C increases with density. Result 3.2 can be derived by observing that the productivity-volatility substitution for selection is represented by $\phi_W(\varepsilon, M)$ for $\varepsilon \in [0, \tilde{\varepsilon}(M)]$ and $\phi_C(\varepsilon, M)$ for $\varepsilon \in [\tilde{\varepsilon}(M), 1]$. Then, under Assumption 2, $\partial^2\phi_W(\varepsilon, M)/\partial\varepsilon\partial M < 0$ and $\partial^2\phi_C(\varepsilon, M)/\partial\varepsilon\partial M < 0$. Thus, in denser cities, volatility and productivity are better substitutes for lowering the selection of firms.

Proof of Proposition 4 — Result 4.1 stems from the fact that under Assumption 3, we can show that $\forall s \in \{B, W, C\}, \partial\phi_s^*(\varepsilon, M)/\partial M > 0$ and $\forall s \in \{W, C\}, \partial\varepsilon_s^*(\phi, M)/\partial M > 0$. Therefore, more productive and more volatile firms sort into denser cities. Results 4.2 stems from the fact that $\forall s \in \{W, C\}, \partial^2\phi_s^*(\varepsilon, M)/\partial M\partial\varepsilon < 0$ and $\partial^2\varepsilon_s^*(\phi, M)/\partial M\partial p < 0$. Therefore, more productive (resp., volatile) firms sort into denser cities if they are more volatile (resp., productive). This second result ensures that the share of churning firms increases with density, even though average productivity also increases with density. Then, again under Assumption 3, we can also show that $\forall\varepsilon \in [0, 1], \frac{\partial\phi_B^*(M)}{\partial M} > \frac{\partial\phi_W^*(\varepsilon,M)}{\partial M} > \frac{\phi_C^*(\varepsilon,M)}{\partial M}$. The relationship between productivity and density is stronger when firms choose strategy B, followed by strategy W and then C. Therefore, the productivity-density gradient decreases with volatility, which proves result 4.3.

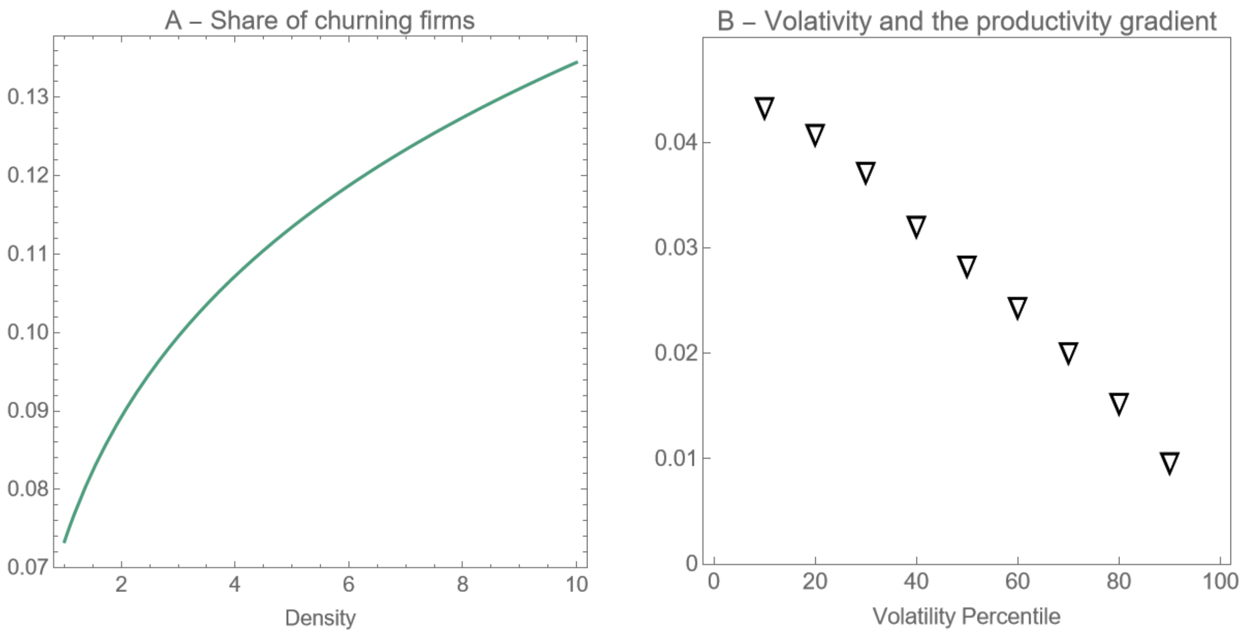
C.3 Additional figures

Figure C.1: Strategy choice and city choice



Calibration: $\xi = 0.2$, $r = 0.03$, $\delta = 0.1$, $c = 0.1$, $\mu(M) = 0.3M^{0.05}$, $R(M) = 0.2M^{0.1}$, $\bar{p} = 1$. **Panel A**: The figure represents the set of (ε, ϕ) combinations associated with each adopted strategy, for a low density ($M = 1$, plain colors), and for a high density ($M = 10$, mesh lines). The letters locate adopted strategies when $M = 10$. **Panel B**: The figure represents the set of (ε, ϕ) combinations associated with each adopted strategy, for the optimal level of density, found by numerical search.

Figure C.2: Volatility and the sorting of firms



Calibration: $c = 0.1$, $\xi = 0.2$, $r = 0.03$, $\delta = 0.1$, $\mu(M) = 0.3M^{0.05}$, $R(M) = 0.2M^{0.1}$, $\overline{p} = 1$. We assume that ϕ and ε are independent and uniformly distributed over $[0, 1]$. The optimum is found by a numerical search. **Panel A:** The share of churning firms is given by $\mathcal{C}(M) = \frac{\int\int \mathbf{1}_{C=s^*(\phi,\varepsilon,M)} h(\phi,\varepsilon) d\phi d\varepsilon}{\sum_s \int\int \mathbf{1}_{s=s^*(\phi,\varepsilon,M)} h(\phi,\varepsilon) d\phi d\varepsilon}$. **Panel B:** For each decile d of the distribution of ε, let $d_\varepsilon = [d - 5\%, d + 5\%]$. The average productivity of firms in this decile of volatility is given by $\phi^*(M, d) = \frac{\sum_s \int\int_{d_\varepsilon} \mathbf{1}_{s=s^*(\phi,\varepsilon,M)} \phi^*_s(\varepsilon, M) h(\phi,\varepsilon) d\phi d\varepsilon}{\sum_s \int\int_{d_\varepsilon} \mathbf{1}_{s=s^*(\phi,\varepsilon,M)} h(\phi,\varepsilon) d\phi d\varepsilon}$.To approximate the productivity-density gradient, we compute the slope of this function at the median density, such that $\nabla(d) = \phi^*(6, d) - \phi^*(5, d)$.

C.4 Endogenous matching rate

In order to recover $\boldsymbol{B}(M)$, $\boldsymbol{W}(M)$ and $\boldsymbol{C}(M)$ in eq. (C.14)-(C.16), we introduce an entry process: In each city, we assume that there is a continuum of latent firms with known distribution $h(\phi, \varepsilon)$. Those firms pay a cost f_E to draw (ϕ, ε). Free entry means that:

$$\forall M, f_E = \int\int \max\left\{0, \max_s\{\mathbb{E}_s(\phi, \varepsilon, M)\}\right\} h(\phi, \varepsilon) d\phi d\varepsilon \tag{C.17}$$

Market tightness is given by the ratio of the number of vacancies to that of unemployed workers:

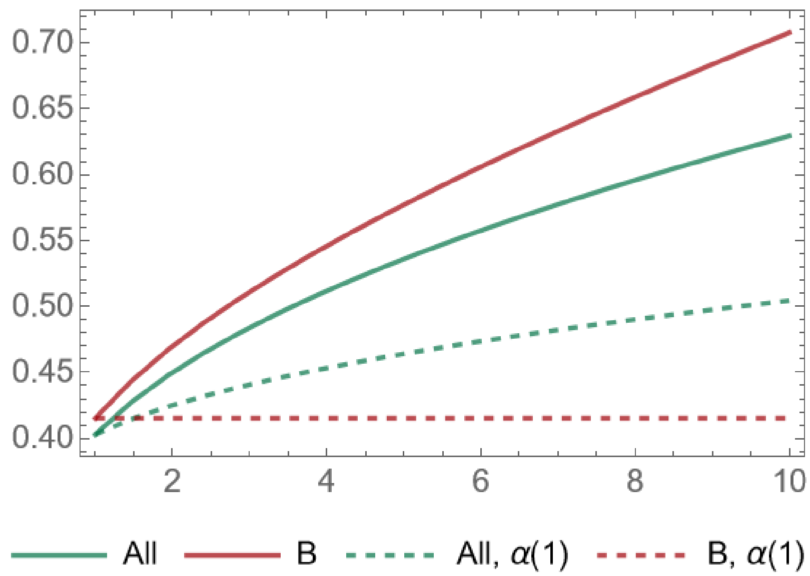
$$\theta(M) = \frac{\sum_s \boldsymbol{V_s(M)}}{M - \sum_s \boldsymbol{A_s}(M)} \tag{C.18}$$

If we denote $\alpha(M) = M/F(M)$ with $F(M) = \boldsymbol{B}(M) + \boldsymbol{W}(M) + \boldsymbol{C}(M)$ the density of firms and use a parametric assumption $\mathcal{M}(\cdot)$ on the matching technology, we can solve numerically for the fixed point given by eq. (C.17)-(C.18) and recover the values of $\alpha(M)$ and $\mu(M)$.

We illustrate this method with the same calibration as in the main text.[36] The results are shown in Figure C.3, for two different scenarios: either the full model, where firms may follow any of the three strategies, or a restricted model where firms may only follow the B strategy. Under this parametrization, the impact of density on the worker finding rate is higher under the restricted model: the indirect negative effect of churning through firm entry trumps the direct positive effect. Conversely, even if the number of firms does not adjust, firms that are allowed to churn still benefit from better matching conditions in larger cities. To substantiate this point, we simulate two counterfactual situations where α is set to its value for the lowest density ($M = 1$): matching economies, as measured by $d\mu(M)/dM$, drop by half when firms are allowed to churn, and drop to zero when they are not.

[36]We use a Cobb-Douglas matching function $\mathcal{M}(V, U) = \sqrt{VU}$ and we set $f_E = 7.5$ so that the resulting unemployment rate lies between 5% and 10%.

Figure C.3: Endogenous worker finding rate



Calibration: $c = 0.1$, $\xi = 0.2$, $r = 0.03$, $\delta = 0.1$, $R(M) = 0.2M^{0.1}$, $\overline{p} = 1$. We assume that ϕ and ε are independent and uniformly distributed over $[0, 1]$. The value of $\mu(M)$ is obtained as the numerical solution to the fixed point problem described in the text, with a Cobb-Douglas matching technology $\mathcal{M}(V, U) = \sqrt{VU}$ and $f_E = 7.5$. All: $\mu(M)$ is obtained by solving the full model; B: $\mu(M)$ is obtained by solving the restricted model. Continuous lines mean that firm density adjusts to meet the free entry condition. Dashed lines mean that firm density is set to its equilibrium value when $M = 1$.

C.5 Model extension

In this section, we provide an extension to our base model in the spirit of Melitz (2003), introducing monopolistically competitive firms facing a CES demand system that draw heterogeneous productivity and demand volatility upon entry. This connection allows us to (i) explicitly model demand shocks, (ii) introduce firm size, (iii) clarify the link between productivity thresholds and the share of volatile firms active in a given market and (iv) exhibit that the same forces as used in our parsimonious model are needed to generate the main results of the paper. We use the model to perform comparative statics. All proofs and derivations are available upon request.

Assumptions — We analyze a differentiated goods sector in one city of size M. There are M workers-consumers inelastically providing one unit of labor that spend a fraction γ of their income I on the sector's differentiated goods over which they have CES preferences.

There is a unit mass of potential entrants in each city. Firms pay a fixed entry cost $f_E > 0$ after which they learn their productivity ϕ and demand volatility. Productivity is drawn from a Pareto distribution of shape ν and scale $\phi_{min} > 0$. A proportion χ of firms face constant demand, while $(1-\chi)$ are exposed to volatile demand flows. More specifically, the firm observes the demand for its variety at each period

$$q\left(p,\varepsilon\right) = \varepsilon^{\eta}\bar{Q}p^{-\eta}$$

where $\bar{Q}$ is aggregate real consumption and ε is the firm's idiosyncratic demand.[37] The corresponding indirect demand is $p\left(q,\varepsilon\right) = \varepsilon\left(\bar{Q}\right)^{\frac{1}{\eta}}\left(q\right)^{-\frac{1}{\eta}}$ which allows us to link this extension with the base model in the main paper. For the firms with no demand volatility, we normalize $\varepsilon = 1$. Volatile firms alternate between ε_l and ε_h (respectively low and high demand) at a rate ξ. We consider the case where $\frac{\varepsilon_l+\varepsilon_h}{2} = 1$, i.e. volatile and non-volatile firms only differ by the second-moment of their indirect demand process, conditional on their productivity. In our exposition, we further assume that $\varepsilon_l = 0$ and $\varepsilon_h = 2$. These are extreme values that, however, permit analytical characterization of the problem without resorting to simulations.

In the rest of the model, we assume monopolistic competition, i.e. firms are input price takers and view any aggregate parameters (e.g. labor market tightness, or real consumption) as being exogenous.

Once productivity and volatility are revealed, the firm decides whether to pay a set-up cost $f_p > 0$ to produce, then chooses a hiring strategy. In comparison with the stylized model in Section 3, we concentrate on the two extreme strategies, namely Business-as-usual and Churning, which corresponds to the case where c is small. While the baseline model considers one-job firms, we now consider the firm's decision on a measure of job openings, given an exogenous filling rate $\mu(M)$ and separation rate δ for each of these positions. If a worker fills a position, she produces ϕ units of the differentiated good. The firm assesses the present discounted value (PDV) of each created position given operational and hiring costs, as well

[37] We assume that $\nu > \eta - 1$ to ensure finite aggregate productivity levels.

as the job filling and separation rates. When the firm enters, it doesn't know the state of its demand ε, but it immediately chooses the type of each position it creates. A position is permanent if the firm will not fire the employed worker under any circumstances and only an exogenous separation can destroy it. A position will be a churning one if the firm may also fire workers when demand is low.

Business as usual strategy — Firms using this strategy keep employment constant and hence also production $\bar{q}$ and the associated cost. It is straightforward to show that non-volatile firms always choose this strategy. Volatile firms instead choose between Business-as-usual and Churning. Under Business-as-usual, the firm produces at full capacity and adjusts its price to demand shocks.

The employment level $\bar{L}_B$ chosen by a Business-as-usual firm upon entry, assuming it does not know a priori the state of the demand, maximizes expected profits

$$\Pi_B(\phi, \varepsilon, M) = \left[\frac{\mu(M)\left\{[\frac{p_l+p_h}{2}]\phi - R(M)\right\} - c(\delta + r)}{r[r + \delta + \mu(M)]}\right]\bar{L}_B \tag{C.19}$$

subject to $p_t = \varepsilon_t \left(\bar{Q}\right)^{\frac{1}{\eta}} (\bar{q})^{-\frac{1}{\eta}}$, where t corresponds to the low or high state. $\bar{q} = \phi\kappa_B\bar{L}_B$ is the expected (constant) output given the adopted strategy where $\kappa_B = \frac{\mu(M)}{\delta+\mu(M)}$ is the fraction of time a position is filled. The term under brackets is simply the present discounted value of a permanent position and is as in eq. (C.1) in the baseline model (using the fact that the demand shocks average to one across states), except that now we explicitly allow the firm to optimize on its price p.

Letting $\Gamma_1 = \frac{\mu(M)}{r[r+\delta+\mu(M)]}$ and $\Gamma_2 = \frac{\mu(M)R(M)+c(\delta+r)}{r[r+\delta+\mu(M)]}$, we can rewrite the problem of the firm as

$$\max_{\bar{L}_B}\left[\Gamma_1\left(\frac{\phi\kappa_B\bar{L}_B}{\bar{Q}}\right)^{-1/\eta}\phi - \Gamma_2\right]\bar{L}_B$$

The optimal solution gives

$$\begin{aligned}\bar{L}_B &= \left(\frac{\eta}{\eta-1}\frac{\Gamma_2}{\Gamma_1}\right)^{-\eta}\left(\frac{\bar{Q}}{\kappa_B}\right)\phi^{\eta-1}\\ p_t &= \varepsilon_t\frac{\eta}{\eta-1}\frac{\Gamma_2}{\phi\Gamma_1}\end{aligned}$$

Plugging the equilibrium strategies into eq. (C.21) implies:

$$\Pi_B(\phi, \varepsilon, M) = \frac{\Gamma_2}{\eta-1}\left(\frac{\eta}{\eta-1}\frac{\Gamma_2}{\Gamma_1}\right)^{-\eta}\frac{\bar{Q}}{\kappa_B}\phi^{\eta-1}$$

which does not depend on the firm's volatility.

Churning strategy — Consider now a firm that lays off some workers in the low state of demand. Given the firm faces zero demand in the low state ($\varepsilon_l = 0$), it can be shown that if firing some workers is preferred in the low demand state, the firm would want to fire its entire workforce.[38]

Following the same steps as before, the problem of the firm consists in choosing $\bar{L}_C$ to maximize expected discounted profits:

$$\Pi_C(\phi, \varepsilon, M) = \frac{1}{2}\left[\frac{\mu(M)\{p_h\phi - R(M)\} - c(r+\delta+\xi)}{r[r+\delta+\xi+\mu(M)]}\right]\bar{L}_C \tag{C.20}$$

$$= \frac{1}{2}\left[\Phi_1\varepsilon_h\left(\frac{\phi\kappa_C\bar{L}_C}{\bar{Q}}\right)^{-\frac{1}{\eta}}\phi - \Phi_2\right]\bar{L}_C \tag{C.21}$$

where $\Phi_1 \equiv \frac{\mu(M)}{r[r+\delta+\xi+\mu(M)]}$, $\Phi_2 \equiv \frac{\mu(M)R(M)+c(r+\delta+\xi)}{r[r+\delta+\xi+\mu(M)]}$ and $\kappa_C = \frac{\mu(M)}{(\delta+\xi+\mu(M))}$ is the fraction of time a position is filled when the firm is in the high state. In the low state, the firm is idle and receives zero profits. Again, the term under brackets corresponds to the present discounted value of a position, as in eq. (C.3).

We obtain that

$$p_h = \frac{\eta}{\eta-1}\frac{\Phi_2}{\phi\Phi_1}$$

$$\bar{L}_C = \left(\frac{1}{\varepsilon_h}\frac{\eta}{\eta-1}\frac{\Phi_2}{\Phi_1}\right)^{-\eta}\frac{\bar{Q}}{\kappa_C}\phi^{\eta-1}$$

and the expected profit

$$\Pi_C(\phi, \varepsilon, M) = \frac{1}{2}\varepsilon_h^{\eta}\frac{\Phi_2}{\eta-1}\left(\frac{\eta}{\eta-1}\frac{\Phi_2}{\Phi_1}\right)^{-\eta}\frac{\bar{Q}}{\kappa_C}\phi^{\eta-1}. \tag{C.22}$$

In this model, the impact of demand shocks on firms' outcomes depends on their strategy. Under Business-as-usual, the firm's employment is constant and prices are adjusted to shocks. Instead, the churning strategy allows firms to adjust to demand shocks through quantities, and thus prices are independent of demand shocks.[39]

[38] It should be noted here that handling non-zero demand in the low state is straightforward conceptually. In such a scenario, the firm churns over $\bar{L}_C$ and hires $\bar{L}_C^P$ permanent workers to ensure sufficient production to serve demand in the low state. However, given the maximization problem of the firm is then not separable in $\bar{L}_C$ and $\bar{L}_C^P$, it is impossible to get a full characterization for the optimal measures of positions, prices nor the value function. This is why we set $\varepsilon_l = 0$ to obtain an explicit solution to the problem.

[39] Note that this is true in the extreme case in which production is zero in the low state. But if churning firms had to combine permanent and churning positions, their price in the high state would be more complicated to determine as the high state would combine permanent and adjustable positions.

The firm compares the expected gains from pursuing the Business-as-usual strategy and Churning. The latter is preferred if the following holds:

$$\Gamma_2 \left(\frac{\Gamma_1}{\Gamma_2}\right)^{\eta} \kappa_C < \frac{1}{2}\varepsilon_h^{\eta}\Phi_2 \left(\frac{\Phi_1}{\Phi_2}\right)^{\eta} \kappa_B \tag{C.23}$$

The condition depends on structural parameters and location through $R(M)$ and $\mu(M)$ but not on firm productivity ϕ. A sufficient condition is that $R(M) > c$ and $\mu(M) > \xi$, exactly the conditions needed for churning in the base model (see Assumption 1). Moreover, as $\xi \to 0$ firms always prefer to churn. It is only if demand levels change frequently, that firms may prefer to keep workers instead of constantly readjusting the labor force and saving on operating and hiring costs in low states of demand at the expense of waiting to hire when its demand turns high. In the calibration used in Section 3, condition (C.23) is met.

In what follows, we will thus discern between two types of firms: non-volatile ones (that follow the Business-as-usual strategy) and volatile that churn.

Equilibrium distribution of volatile and non-volatile firms — After firms learn their productivity and demand volatility, they decide whether to pay a fixed cost f_p to produce. This defines the cutoff productivity values for firms that break even, conditional on their volatility denoted $\phi^*(0)$ and $\phi^*(\varepsilon)$ respectively:

$$\Pi_B\left(\phi^*(0), 0, M\right) = \Pi_C\left(\phi^*(\varepsilon), \varepsilon, M\right) = f_p. \tag{C.24}$$

This implies a linear relationship between the productivity cutoffs for volatile and non-volatile firms $\phi^*(\varepsilon) = \Omega\phi^*(0)$ where $\Omega^{\eta-1} \equiv 2\varepsilon_h^{-\eta}\frac{\Gamma_2}{\Phi_2}\left(\frac{\Gamma_2}{\Phi_2}\frac{\Phi_1}{\Gamma_1}\right)^{-\eta}\frac{\kappa_C}{\kappa_B}$. If eq. (C.23) holds, $\Omega < 1$ and $\phi^*(\varepsilon) < \phi^*(0)$ i.e. the productivity cut-off is lower for volatile than non-volatile firms, as in the base model of Section 3.

For a firm considering paying entry cost f_E, the expected profit depends on

$$\left[\begin{array}{c} \chi\left[\left(1 - G\left(\phi^*(0)\right)\right)\int_{\phi^*(0)}^{\infty}\left(\Pi_B(\phi, 0, M) - f_p\right)\frac{g(\phi)}{(1-G(\phi^*(0)))}d\phi\right] \\ +(1-\chi)\left[\left(1 - G\left(\phi^*(\varepsilon)\right)\right)\int_{\phi^*(\varepsilon)}^{\infty}\left(\Pi_C(\phi, \varepsilon, M) - f_p\right)\frac{g(\phi)}{(1-G(\phi^*(\varepsilon)))}d\phi\right] \end{array}\right] = f_E$$

Letting $\tilde{\phi}(\cdot) := \left(\frac{1}{1-G(\phi^*(\cdot))}\int\limits_{\phi^*(\cdot)}^{\infty}\phi^{\eta-1}g(\phi)\,d\phi\right)^{\frac{1}{\eta-1}}$, this simplifies to:

$$\left[\chi\left(1 - G\left(\phi^*(0)\right)\right)\left(\left(\frac{\tilde{\phi}(0)}{\phi^*(0)}\right)^{\eta-1} - 1\right) + (1-\chi)\left(1 - G\left(\phi^*(\varepsilon)\right)\right)\left(\left(\frac{\tilde{\phi}(\varepsilon)}{\phi^*(\varepsilon)}\right)^{\eta-1} - 1\right)\right] = \frac{f_E}{f_p} \tag{C.25}$$

Since $\phi^*(\varepsilon) = \Omega\phi^*(0)$ we can obtain the solution for $\phi^*(0)$ from eq. (C.25).

With the Pareto productivity distribution, condition (C.25) becomes

$$\chi\left(\phi^*(0)\right)^{-\nu} + (1-\chi)\left(\phi^*(\varepsilon)\right)^{-\nu} = \frac{f_E}{f_p}\frac{(\nu-\eta+1)}{(\eta-1)}\left(\varphi_{\min}\right)^{-\nu}.$$

The measures of non-volatile firms N_B and volatile firms N_C that respectively use Business-as-usual and Churning strategies are given by $N_B = \chi[1-G(\phi^*(0))]N$ and $N_C = (1-\chi)[1-G(\phi^*(\varepsilon))]N$ where N is the measure of entering firms. The share of non-volatile to volatile firms is determined by the productivity cutoffs $\phi^*(0)$ and $\phi^*(\varepsilon)$. With the Pareto productivity distribution, this is given by

$$\frac{N_B}{N_C} = \frac{\chi}{(1-\chi)}\left(\frac{\phi^*(\varepsilon)}{\phi^*(0)}\right)^{\nu} = \frac{\chi}{(1-\chi)}(\Omega)^{\nu}$$

The share of firms of each type comoves with the relative productivity thresholds of non-volatile and volatile firms. We can show that for η high enough, a faster filling rate $\mu(M)$ and higher operating costs $R(M)$ ceteris paribus (holding the labor market tightness constant) increase the share of volatile churning firms as those benefit more from faster hiring and savings on operating in low demand periods. If we further assume that f_p is increasing with $R(M)$, we can show that for ν high enough, $\frac{\partial(\phi^*(\varepsilon))}{\partial R(M)} > 0$ or selection on productivity increases with city size. Given that $\frac{\partial\Omega}{\partial R(M)} < 0$, the productivity-density gradient is thus flatter for volatile firms.