Ejercicio 3

Usando $f(x)=x^3$ determinar el polinomio interpolador de Lagrange $L(x)$

• Pol. lineal $P_1(x)$ en los nodos $x_0=-1$, $x_1=0$

$$P_1(x) = y_0 \frac{(x-x_1)}{(x_0-x_1)} + y_1 \frac{(x-x_0)}{(x_1-x_0)}$$

$$= -1 \frac{(x)}{-1} + \left[0 \frac{(x)}{1}\right]$$

$$= x$$

• Pol. cuadrático $P_2(x)$ en $x_0=-1$, $x_1=0$, $x_2=1$

$$P_2(x) = y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$+ y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = -1 \frac{(x)(x-1)}{(-1)(-2)} + 0 + 1 \frac{(x+1)(x)}{(2)(1)}$$

$$= -1 \frac{(x)(x-1)}{2} + 1 \frac{x(x+1)}{2}$$

$$= \frac{-x^2+x+x^2+x}{2}$$

$$= \frac{2x}{2} = x$$

• Pol. cúbico $P_3(x)$ en $x_0=-1$, $x_1=0$, $x_2=1$, $x_3=2$

$$P_3(x) = y_0 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ y_2 \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= -1 \frac{(x)(x-1)(x-2)}{(-1)(-2)(-3)} + 0 + 1 \frac{(x+1)(x)(x-2)}{(2)(1)(-1)} + 8 \frac{(x+1)(x)(x-1)}{(3)(2)(1)}$$

$$= \frac{x(x-1)(x-2)}{6} - \frac{x(x+1)(x-2)}{2} + 8 \frac{x(x+1)(x-1)}{6}$$

$$= \frac{x(x-1)(x-2) - 3[x(x+1)(x-2)] + 8[x(x+1)(x-1)]}{6}$$

• $x^2 - x(x-2) = x^3 - 2x^2 - x^2 + 2x = x^3 - 3x^2 + 2x$

• $x^2 + x(x-2) = x^3 - 2x^2 + x^2 - 2x = x^3 - x^2 - 2x$

• $x^2 + x(x-1) = x^3 - x^2 + x^2 - x = x^3 - x$

$$= \frac{x^3-3x^2+2x-3x^3+3x^2+6x+8x^3-8x}{6}$$

$$= \frac{\cancel{6}x^3}{\cancel{6}} = x^3$$

- Pol. cuadrático $Q_2(x)$ en $x_0=0$, $x_1=1$, $x_2=2$

$Q_2 = (x)(x-1)(x-2)$

$= x^2-x(x-2)$

$= x^3-2x^2-x^2+2x$

$= x^3-3x^2+2x$

Escribir para los siguientes funciones el término del error E_3 del polinomio interpolador cúbico $P_3(x)$ en los nodos $x_0=-1$, $x_1=0$, $x_2=3$, $x_3=4$

- $f(x) = 4x^3-3x+2$

$E_3 = Q_3 \frac{f^{(4)}(c)}{4!} = (x+1)(x)(x-3)(x-4)\frac{f^{(4)}(c)}{24} = ⊛$

$(4x^3-3x+2)' = 12x^2-3$

$(4x^3-3x+2)'' = 24x$

$(4x^3-3x+2)''' = 24$

$(4x^3-3x+2)'''' = 0$

$⊛ (x+1)x(x-3)(x-4)\frac{0}{24} = 0$

- $f(x) = x^4-2x^3$

$(x^4-2x^3)' = 4x^3-6x^2$

$(x^4-2x^3)'' = 12x^2-12x$

$(x^4-2x^3)''' = 24x-12$

$(x^4-2x^3)'''' = 24$

$E_3 = (x+1)x(x-3)(x-4)\frac{f^{(4)}(c)}{24}$

$= (x+1)x(x-3)(x-4)\frac{24}{24}$

$= x^2+x(x-3)(x-4)$

$= x^3 \overbrace{-3x^2+x^2}^{-2x^2} -3x(x-4)$

$= x^4-4x^3-2x^3+8x^2-3x^2+12x$

$= x^4-6x^3+5x^2+12x$

$f(x) = x^5-5x^4$

$(x^5-5x^4)' = 5x^4-20x^3$

$(x^5-5x^4)'' = 20x^3-60x^2$

$(x^5-5x^4)''' = 60x^2-120x$

$(x^5-5x^4)'''' = 120x-120$

$E3 = (x+1)x(x-3)(x-4)\frac{120(c)-120}{24}$

$= (x+1)x(x-3)(x-4)\,5(c-1)$