Homework 9.

Problem #1 (Central Limit Theorem)

In similar fashion to what I did for sample proportion in the lecture code, proceed to use R and generate the following values:

$$X_1, X_2, ..., X_{10,000} \sim Uniform(-2, 2)$$

subsequently demonstrating (once again, using R) the aspects of sampling distribution for $\bar{X}$, such as its

- shape (is it bell-shaped, or not quite?)
- mean (is it unbiased?)
- standard deviation (does it correspond to theoretical st. dev.?)

For more details on Uniform distribution (e.g. its mean, variance, etc) check https://en.wikipedia.org/wiki/Uniform_distribution_(continuous).

Do it for:

a. $n = 2$;

b. $n = 50$.

Problem #2 (Coverage of Confidence Interval)

1. Write your own function that takes as arguments:

- total # of observations,
- number of successes
- confidence level (default at 0.95)

and calculates the confidence interval for population proportion, outputting a 2-element vector (left and right end of the interval, respectively).

2. Proceed to generate $10,000$ values from $Bin(n = 1000, p = 0.6)$ distribution, and use your function from part 1 in order to calculate & record the 95% and 90% intervals for each of those $10,000$ values. Calculate the % of times your confidence intervals actually contain the true population proportion $p = 0.6$. Is it what you expect for 95% and 90% intervals, accordingly?

Some code to start you off:

```r
set.seed(1)

n.sim <- 10000
prob <- 0.6
size <- 100
# Placeholders for left (first column) and right (second column) ends
# of our CIs.
ci.95 <- matrix(0, nrow=n.sim, ncol=2)
ci.90 <- matrix(0, nrow=n.sim, ncol=2)

# Here you will need to
# 1) generate the 10,000 values from Bin(1000,0.6),
# 2) loop through those and feed them as input to your confidence level function from part 1
#    (for cases of 95% and 90%)
#...

# That's an example of how you calculate the % of times your confidence interval
# contains the true parameter
mean(ci.95[,1] < prob & ci.95[,2] > prob)
#...
```

Problem #3 (Make sure to use R when appropriate for carrying out the calculations, showing your work)

7.7

7.14

7.15 (not necessary to provide the graph, but make sure to play with the app at https://istats.shinyapps.io/sampdist_cont/)

7.20

8.6

8.13

8.16

8.29

8.34

8.37