Conic section Assignment

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Problem Statement **- Find the area of the region bounded by the curve $y^2 = x$ and the lines x=1 and x=4 and the axis in the first quadrant**

Solution

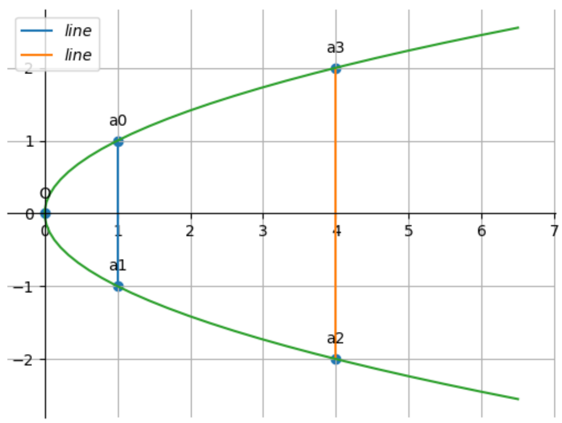

Figure 1: The parabola formed by the curve $y^2 = x$ and the lines x=1 and x=4

The given equation of parabola $y^2 = x$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix}, \tag{3}$$

$$f = 0 \tag{4}$$

The point of intersection of the lines x=1 and x=4 to the parabola is given by

The points of intersection of the line

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbf{R} \tag{5}$$

with the conic section are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{6}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\right) \tag{7}$$

From the line x-1=0 the vectors q,m are taken,

$$\mathbf{q_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{9}$$

by substituting eq(2),(3),(4),(8),(9) in eq(7)

$$\mu_i = 1, -1 \tag{10}$$

substituting eq(8),(9),(10) in eq(6) the intersection points on the parabola are

$$\mathbf{a_0} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

$$\mathbf{a_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{12}$$

From the line x-4=0 the vectors q,m are taken,

$$\mathbf{q_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{13}$$

$$\mathbf{m_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$

by substituting eq(2),(3),(4),(13),(14) in eq(7)

$$\mu_i = 2, -2 \tag{15}$$

substituting eq(13),(14),(15) in eq(6) the intersection points on the parabola are

$$\mathbf{a_3} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{16}$$

$$\mathbf{a_2} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{17}$$

Area of the parabola in between the lines x=1 and x=4 is given by

$$\implies A_1 = \int_0^1 \sqrt{x}\,dx \tag{18}$$

$$\implies A_2 = \int_0^4 \sqrt{x}\,dx \tag{19}$$

$$\implies A_2 - A_1 = \int_0^4 \sqrt{x}\,dx - \int_0^1 \sqrt{x}\,dx \tag{20}$$

$$\implies A_2 - A_1 = 14/3 \tag{21}$$

Construction

Points	**intersection points**
a0	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
a1	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
a3	$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
a2	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$