PP 5. $\dfrac{1}{\sqrt{x^2 + 2x + 2}}$.

$$x^2 + 2x + 2 = x^2 + 2x + 1 + 2 - 1$$
$$= (x+1)^2 + 1$$

$$G.I = \int \frac{1}{\sqrt{(x+1)^2 + 1}}\, dx$$
$$= \log\left|(x+1) + \sqrt{x^2 + 2x + 2}\right| + C$$

PP 6. $\int \dfrac{dx}{\sqrt{9x - 4x^2}}$.

$$9x - 4x^2 = -(4x^2 - 9x)$$
$$= -4\left(x^2 - \frac{9}{4}x\right)$$
$$= -4\left(x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64}\right)$$
$$= -4\left(\left(x - 9/8\right)^2 - \frac{81}{64}\right)$$
$$= 4\left(\frac{81}{64} - (x - 9/8)^2\right)$$

$$G.I = \int \frac{dx}{\sqrt{4\left(\left(\frac{9}{8}\right)^2 - (x - 9/8)^2\right)}} = \frac{1}{2}\int \frac{dx}{\sqrt{(9/8)^2 - (x - 9/8)^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{x - 9/8}{9/8} + C \qquad \int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\frac{x}{a} + C$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x - 9}{9}\right) + C$$

2. LONG ANSWER & VERY LONG ANSWER TYPE

PP 4. $\dfrac{1}{9x^2 + 6x + 5}$.

$$I = \int \frac{1}{9x^2 + 6x + 5}\, dx = \int \frac{1}{(3x+1)^2 + 2^2}$$

Let $(3x + 1) = t \quad \Rightarrow \quad 3dx = dt$

$$I = \int \frac{1}{(3x+1)^2 + 2^2}\, dx = \frac{1}{3}\int \frac{1}{t^2 + 2^2}\, dt = \frac{1}{3}\left[\frac{1}{2}tan^{-1}\left(\frac{t}{2}\right)\right] + C = \frac{1}{6}tan^{-1}\left(\frac{t}{2}\right) + C$$

PP 5. $\int \frac{dx}{\sqrt{(7-6x-x^2)}}$.

Here, $7-6x-x^2 = 7-(x^2+6x+9-9) = 7-(x^2+6x+9-9)$
$= 16-(x^2+6x+9) = 16-(x+3)^2 = (4)^2-(x+3)^2$

$$\therefore I = \int \frac{1}{\sqrt{7-6x-x^2}}dx = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t \quad \Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx = \int \frac{1}{\sqrt{(4)^2-(t)^2}}dt = sin^{-1}\left(\frac{t}{4}\right) + C = sin^{-1}\left(\frac{x+3}{4}\right) + C$$

PP 8. $\int \frac{dx}{\sqrt{x^2+2x+2}}$.

$$I = \int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$$

$$= \int \frac{dx}{(x+1)^2+1}dx = [tan^{-1}(x+1)] + C$$

PP 9. $\int \frac{1}{\sqrt{(x-1)(x-2)}}$.

$(x-1)(x-2)$ can be written as x^2-3x+2

Therefore $x^2-3x+2 = x^2-3x+\frac{9}{4}-\frac{9}{4}+2 = \left(x-\frac{3}{2}\right)^2-\frac{1}{4} = \left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$$I = \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t \quad \Rightarrow \quad dx = dt$

$$= \int \frac{1}{\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}}dt = \log\left|t+\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}\right| + C = \log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right| + C$$

PP 10. $\frac{4x+1}{\sqrt{2x^2+x-3}}$.

Let $4x+1 = A\frac{d}{dx}(2x^2+x-3)+B \quad \Rightarrow 4x+1 = A(4x+1)+B \quad \Rightarrow 4x+1 = 4Ax+A+B$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \quad \Rightarrow A = 1$ and $A + B = 1 \quad \Rightarrow B = 0$

Let $2x^2+x-3=t \quad \Rightarrow (4x+1)dx = dt$

$$I = \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int \frac{1}{\sqrt{t}}dt = 2\sqrt{t}+C = 2\sqrt{2x^2+x-3}+C$$

PP 11. $\int \frac{dx}{(1-6x-9x^2)}$.

$$\int \frac{1}{1-6x-9x^2}dx = \frac{1}{9}\int \frac{1}{\frac{1}{9}-\frac{6}{9}x-x^2}dx$$

$$= \frac{1}{9}\int \frac{1}{\frac{1}{9}-\frac{2}{3}x-x^2}dx$$

$$\frac{1}{9}-\frac{2}{3}x-x^2 = -\left(x^2+\frac{2}{3}x-\frac{1}{9}\right)$$

$$= -\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}-\frac{1}{9}\right)$$

$$= -\left((x+1/3)^2-\frac{2}{9}\right) = \frac{2}{9}-(x+1/3)^2$$

$$= \left(\frac{\sqrt{2}}{3}\right)^2-(x+1/3)^2$$

$$\therefore I = \frac{1}{9}\int \frac{1}{(\sqrt{2}/3)^2-(x+1/3)^2}dx$$

$$= \frac{1}{9}\cdot\frac{1}{2\cdot\sqrt{2}/3}\log\left|\frac{\frac{\sqrt{2}}{3}+x+\frac{1}{3}}{\frac{\sqrt{2}}{3}-x-\frac{1}{3}}\right|+c$$

$$= \frac{1}{6\sqrt{2}}\log\left|\frac{\sqrt{2}+3x+1}{\sqrt{2}-3x-1}\right|+c$$

PP 12. $\int \frac{(3x+1)}{(2x^2-2x+3)}dx$ ***-Aw -***

$$\int \frac{3x+1}{2x^2-2x+3}dx = \frac{1}{2}\int \frac{3x+1}{x^2-x+3/2}dx$$

$$3x+1 = A(2x-1)+B$$

$$\Rightarrow 3 = 2A \qquad \boxed{A = 3/2}$$

$$1 = -A+B$$

$$B = 1+A \qquad \boxed{B = 5/2}$$

$$= 1+3/2 = 5/2$$

$$= \frac{3}{4}\log|x^2 - x + \tfrac{3}{2}| + \frac{5}{4} I$$

$$I = \int \frac{1}{x^2 - x + 3/2}\,dx$$

$$x^2 - x + \tfrac{3}{2} = x^2 - x + \tfrac{1}{4} + \tfrac{3}{2} - \tfrac{1}{4}$$
$$= (x - \tfrac{1}{2})^2 + \tfrac{5}{4}$$
$$= (x - 1/2)^2 + \left(\tfrac{\sqrt{5}}{2}\right)^2$$

$$\therefore I = \int \frac{1}{(x-1/2)^2 + (\sqrt{5}/2)^2}\,dx$$

$$= \frac{1}{\sqrt{5}/2} \tan^{-1}\left(\frac{x - 1/2}{\sqrt{5}/2}\right)$$

$$= \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$$

$$\therefore \text{G.I} = \frac{3}{4}\log|x^2 - x + \tfrac{3}{2}| + \frac{5}{4}\cdot\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + C$$

$$= \frac{3}{4}\log|x^2 - x + \tfrac{3}{2}| + \frac{\sqrt{5}}{2}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + C$$

PP 13. $\int \frac{(x^2 + 5x + 3)}{(x^2 + 3x + 2)}\,dx$

$$\int \frac{x^2 + 5x + 3}{x^2 + 3x + 2}\,dx$$

$$\begin{array}{r|l} & 1 \\ x^2 + 3x + 2 & x^2 + 5x + 3 \\ & x^2 + 3x + 2 \\ \hline & 2x + 1 \end{array}$$

$$\text{G.I} = \int 1 + \frac{2x+1}{x^2 + 3x + 2}\,dx$$

$$= \int 1\,dx + \int \frac{2x+1}{x^2 + 3x + 2}\,dx$$

$$= x + I \quad \text{where}$$

$$I = \int \frac{2x+1}{x^2 + 3x + 2}\,dx$$

let $2x + 1 = A \frac{d}{dx}(x^2 + 3x + 2) + B$

$2x + 1 = A(2x + 3) + B$

eq: coeff's of like terms

$2 = 2A$ $\quad \boxed{A = 1}$

$1 = 3A + B$ $\quad \boxed{B = -2}$

$B = 1 - 3A$

$= 1 - 3 = -2$

$$\therefore I = \int \frac{1(2x+3)-2}{x^2+3x+2}\,dx$$

$$= \int \frac{2x+3}{x^2+3x+2}\,dx - \int \frac{2}{x^2+3x+2}\,dx$$

$$x^2+3x+2 = x^2+3x+\frac{9}{4}+2-\frac{9}{4}$$

$$= \left(x+\tfrac{3}{2}\right)^2-\frac{1}{4} = \left(x+\tfrac{3}{2}\right)^2-\left(\tfrac{1}{2}\right)^2$$

$$I = \int \frac{2x+3}{x^2+3x+2}\,dx - 2\int \frac{dx}{\left(x+\tfrac{3}{2}\right)^2-\left(\tfrac{1}{2}\right)^2}$$

$$= \log|x^2+3x+2| - 2\cdot\frac{1}{2\cdot\tfrac{1}{2}}\log\left|\frac{x+\tfrac{3}{2}-\tfrac{1}{2}}{x+\tfrac{3}{2}+\tfrac{1}{2}}\right| + C$$

$$= \log|x^2+3x+2| - 2\log\left|\frac{2x+2}{2x+4}\right| + C$$

$$= \log|x^2+3x+2| - 2\log\left|\frac{x+1}{x+2}\right| + C$$

$$\therefore \text{h.I} = x + \log|x^2+3x+2| - 2\log\left|\frac{x+1}{x+2}\right| + C$$

PP 14. $\int \frac{\sin x}{(1-4\cos^2 x)}\,dx$

Putting $\cos x = t$ and $-\sin x\, dx = dt$, we get

$$\int \frac{\sin x}{(1-4\cos^2 x)}\,dx = -\int \frac{dt}{(1-4t^2)}$$

$$= -\frac{1}{4}\cdot\int \frac{dt}{\left(\frac{1}{4}-t^2\right)} = -\frac{1}{4}\cdot\int \frac{dt}{\left\{\left(\frac{1}{2}\right)^2-t^2\right\}}$$

$$= -\frac{1}{4}\times\frac{1}{\left(2\times\frac{1}{2}\right)}\log\left|\frac{\frac{1}{2}+t}{\frac{1}{2}-t}\right| + C$$

$$= -\frac{1}{4}\log\left|\frac{1+2t}{1-2t}\right| + C = -\frac{1}{4}\log\left|\frac{1+2\cos x}{1-2\cos x}\right| + C.$$

PP 16. $\int \frac{x^2 - 1}{x^2 + 4} dx$

Let $I = \int \frac{x^2 - 1}{x^2 - 4} dx$

Let $I = \int \frac{x^2 - 1}{x^2 + 4} dx$

$= \int \frac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx$

$= \int \frac{x^2 + 4}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx$

$= \int dx - 5\int \frac{1}{x^2 + (2)^2} dx$

$I = x - 5 \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$ $\left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right]$

$I = x - \frac{5}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$

3. HOT QUESTIONS

PP 3. $\frac{5x - 2}{1 + 2x + 3x^2}$.

Let $5x - 2 = A\frac{d}{dx}(1 + 2x + 3x^2) + B \quad \Rightarrow 5x - 2 = A(2 + 6x) + B$

Equating the coefficient of x and constant term on both sides, we have

$5 = 6A \quad \Rightarrow A = \frac{5}{6}$ and $2A + B = -2 \quad \Rightarrow B = -\frac{11}{3}$

Therefore, $5x - 2 = \frac{5}{6}(2 + 6x) + \left(-\frac{11}{3}\right)$

$\therefore \mathrm{I} = \int \frac{5x - 2}{1 + 2x + 3x^2} dx = \int \frac{\frac{5}{6}(2 + 6x) + \left(-\frac{11}{3}\right)}{1 + 2x + 3x^2} dx$

$= \frac{5}{6}\int \frac{(2 + 6x)}{1 + 2x + 3x^2} dx - \frac{11}{3}\int \frac{1}{1 + 2x + 3x^2} dx$

Let $I_1 = \int \frac{(2 + 6x)}{1 + 2x + 3x^2}$ and $I_2 = \int \frac{1}{1 + 2x + 3x^2} dx$

$\therefore \mathrm{I} = \int \frac{5x - 2}{1 + 2x + 3x^2} dx = \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \ldots(1)$

Let $1 + 2x + 3x^2 = t \quad \Rightarrow \quad (2 + 6x)dx = dt$

$I_1 = \int \frac{dt}{t} = \log|t| \quad \Rightarrow I_1 = \log|1 + 2x + 3x^2| \quad \ldots(2)$

$$I_2 = \int \frac{1}{1+2x+3x^2} dx$$

$1+2x+3x^2$ can be written as $1+3\left(x^2+\frac{2}{3}x\right)$

Therefore, $1+3\left(x^2+\frac{2}{3}x\right) = 1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$

$$= 1+3\left(x+\frac{1}{3}\right)^2 - \frac{1}{3} = \frac{2}{3}+3\left(x+\frac{1}{3}\right)^2 = 3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right] = 3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$$

$$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]} = \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}} tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right] = \frac{1}{3}\left[\frac{3}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right] \frac{1}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$$

Substituting the values from (2) and (3) in equation (1), we have

$$\therefore \text{I} = \int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6}[\log|1+2x+3x^2|] - \frac{11}{3}\left[\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right] + C$$

$$= \frac{5}{6}\log|1+2x+3x^2| - \frac{11}{3\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$$

$$I_2 = \int \frac{1}{1+2x+3x^2} dx$$

$1+2x+3x^2$ can be written as $1+3\left(x^2+\frac{2}{3}x\right)$

Therefore, $1+3\left(x^2+\frac{2}{3}x\right) = 1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$

$$= 1+3\left(x+\frac{1}{3}\right)^2 - \frac{1}{3} = \frac{2}{3}+3\left(x+\frac{1}{3}\right)^2 = 3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right] = 3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$$

$$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]} = \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}} tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right] = \frac{1}{3}\left[\frac{3}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right] \frac{1}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$$

Substituting the values from (2) and (3) in equation (1), we have

$$\therefore \text{I} = \int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6}[\log|1+2x+3x^2|] - \frac{11}{3}\left[\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right] + C$$

$$= \frac{5}{6}\log|1+2x+3x^2| - \frac{11}{3\sqrt{2}} tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$$

PP 4. $\dfrac{6x+7}{\sqrt{(x-5)(x-4)}}$.

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}} dx$$

Let $6x+7 = A(2x-9)+B$

$6 = 2A \quad \boxed{A=3}$

$7 = -9A+B$

$B = 7+9A = 7+27 = 34 \quad \boxed{B=34}$

$$\therefore G.I = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\,dx$$

$$= 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}\,dx + 34\int \frac{1}{\sqrt{x^2-9x+20}}\,dx$$

$$= 3I_1 + 34I_2$$

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}\,dx \qquad \text{put } x^2-9x+20 = u$$

$$(2x-9)\,dx = du$$

$$\boxed{\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}+c}$$

$$\therefore I_1 = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} = 2\sqrt{x^2-9x+20}$$

$$I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}\,dx$$

$$x^2-9x+20 = x^2-9x+\frac{81}{4}+20-\frac{81}{4}$$

$$= \left(x-\frac{9}{2}\right)^2-\frac{1}{4} = \left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\,dx$$

$$= \log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right|$$

$$\therefore G.I = 3\times 2\sqrt{x^2-9x+20} + 34\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right|$$

$$= 6\sqrt{x^2-9x+20} + 34\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right| + c$$

PP 6. $\dfrac{x+2}{\sqrt{x^2+2x+3}}$.

$$I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$

$$=\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx+\frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx+\int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$Let\ I_1=\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx\ \ and\ \ I_2=\int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$\Rightarrow I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}I_1+I_2$$

Now, $I_1=\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

$Let\ \ x^2+2x+3=t \quad \Rightarrow (2x+2)dx=dt$

$$I_1=\int \frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{x^2+2x+3} \qquad \dots(2)$$

$$I_2=\int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Here, $x^2+2x+3=x^2+2x+1+2 \quad =(x+1)^2+\left(\sqrt{2}\right)^2$

So, $I_2=\int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}dx \ =log\left|(x+1)+\sqrt{x^2+2x+3}\right| \qquad \dots(3)$

Using equations (2) and (3) in (1), we have

$$I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\left[2\sqrt{x^2+2x+3}\right]+\log\left[(x+1)+\sqrt{x^2+2x+3}\right]+C$$

$$=\sqrt{x^2+2x+3}+\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$$

PP 7. $\frac{x+3}{x^2-2x-5}$

Let $(x+3)=A\frac{d}{dx}(x^2-2x-5)+B \quad \Rightarrow (x+3)=A(2x-2)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A=1 \ \Rightarrow A=\frac{1}{2} \quad \text{and} \quad -2A+B=3 \ \Rightarrow B=4$$

Therefore, $(x+3)=\frac{1}{2}(2x-2)+4$

$$I=\int \frac{x+3}{x^2-2x-5}dx=\int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx=\frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx+\int \frac{1}{x^2-2x-5}dx$$

Let $I_1=\int \frac{2x-2}{x^2-2x-5}dx$ and $I_2=\int \frac{1}{x^2-2x-5}dx$

$$\Rightarrow I=\int \frac{x+3}{x^2-2x-5}dx=\frac{1}{2}I_1+4\,I_2 \qquad \dots(1)$$

$$I_1=\int \frac{2x-2}{x^2-2x-5}dx$$

Let $x^2 - 2x - 5 = t \quad \Rightarrow (2x-2)dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2 - 2x - 5| \quad \dots(2)$$

$$I_2 = \int \frac{1}{x^2-2x-5}dx = \int \frac{1}{(x^2-2x+1)-6}dx = \int \frac{1}{(x-1)^2+(\sqrt{6})^2}dx$$

$$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \quad \dots(3)$$

Substituting (2) and (3) in (1), we have

$$I = \int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log|x^2-2x-5| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

$$= \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

PP 8. $\frac{5x+3}{\sqrt{x^2+4x+10}}$.

Let $5x+3 = A\frac{d}{dt}(x^2+4x+10) + B \quad \Rightarrow 5x+3 = A(2x+4)+B$

Equating the coefficients of x and constant term, we have

$$2A = 5 \quad \Rightarrow A = \frac{5}{2} \quad \text{and} \quad 4A + B = 3 \quad \Rightarrow B = -7$$

Therefore, $5x+3 = \frac{5}{2}(2x+4) - 7$

$$I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$$

Let $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$

$$\Rightarrow I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \quad \dots(1)$$

Now, $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Let $x^2+4x+10 = t \quad \Rightarrow (2x+4)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx = \int \frac{1}{\sqrt{(x^2+4x+4)+6}}dx = \int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}}dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}}dx$$

$$= \log\left|(x+2)+\sqrt{(x^2+4x+10}\right| \quad \text{... (3)}$$

Using equations (2) and (3) in (1), we have

$$\mathrm{I} = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\left|2\sqrt{x^2+4x+10}\right| - 7\log\left|(x+2)+\sqrt{(x^2+4x+10}\right| + C$$

$$= 5\left|\sqrt{x^2+4x+10}\right| - 7\log\left|(x+2)+\sqrt{(x^2+4x+10}\right| + C$$

PP 10. $\int\sqrt{\frac{a-x}{a+x}}\,dx$

We have,

$$I = \int \sqrt{\frac{a-x}{a+x}}\,dx = \int \sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}\,dx = \int \frac{a-x}{\sqrt{a^2-x^2}}dx$$

$$\Rightarrow \quad I = \int \frac{a}{\sqrt{a^2-x^2}}dx - \int \frac{x}{\sqrt{a^2-x^2}}dx$$

$$\Rightarrow \quad I = a\int \frac{1}{\sqrt{a^2-x^2}}dx + \frac{1}{2}\int \frac{-2x}{\sqrt{a^2-x^2}}dx$$

$$\Rightarrow \quad I = a\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\int \frac{-2x}{\sqrt{a^2-x^2}}dx$$

Putting $a^2 - x^2 = t$, and $-2x\,dx = dt$, we get

$$I = a\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\int \frac{dt}{\sqrt{t}} = a\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right) + C$$

$$\Rightarrow \quad I = a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{t} + C = a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2-x^2} + C$$

PP 12. $\int \sqrt{\cot x} + \sqrt{\tan x}\;dx$

$$\int\left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$$

$$= \int \sqrt{\tan x} + \frac{1}{\sqrt{\tan x}}\,dx$$

$$= \int \frac{\tan x + 1}{\sqrt{\tan x}}\,dx$$

put $\tan x = t^2$ $\qquad x = \tan^{-1} t^2$

$$dx = \frac{1}{1+t^4}.2t\,dt$$

$$dx = \frac{2t}{1+t^4}dt$$

$$\text{a} \cdot I = \int \frac{t^2+1}{t} \cdot \frac{2t}{1+t^4}\, dt$$

$$= 2\int \frac{t^2+1}{t^4+1}\, dt \qquad \div \text{by } t^2$$

$$= 2\int \frac{1+1/t^2}{(t^2+1/t^2)}\, dt$$

put $t - \frac{1}{t} = u$ then $\quad t^2 + \frac{1}{t^2} = \left(t-\frac{1}{t}\right)^2 + 2$

$$\left(1+\frac{1}{t^2}\right) dt = du \qquad\qquad = u^2 + 2$$

$$\therefore \text{a} \cdot I = 2\int \frac{du}{u^2+2} = 2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\frac{u}{\sqrt{2}}$$

$$= \sqrt{2}\tan^{-1}\frac{u}{\sqrt{2}} + c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) + c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + c$$

Evaluate :

PP 13. $\int \frac{x}{(x^4 + 2x^2 + 3)} dx$

Let $I = \int \frac{x}{x^4 + 2x^2 + 3} dx$

Let $x^2 = t$

$\Rightarrow \qquad 2x\, dx = dt$

$\Rightarrow \qquad x\, dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{t^2 + 2t + 3}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$= \frac{1}{2}\int \frac{dt}{(t+1)^2 + 2}$$

put $t + 1 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2}\int \frac{du}{u^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$$

PP 15. $\int \sqrt{\dfrac{1 - x}{1 + x}}\, dx$

Let $I = \int \sqrt{\dfrac{1-x}{1+x}}dx$

$$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$$

$$= \int \frac{1-x}{\sqrt{1-x^2}}dx$$

Let $1 - x = \lambda \dfrac{d}{dx}\left(1 - x^2\right) + \mu$

$$= \lambda(-2x) + \mu$$

$$1 - x = (-2\lambda)x + \mu$$

Comparing the coefficients of like powers of x,

$$-2\lambda = -1 \qquad \Rightarrow \qquad \lambda = \frac{1}{2}$$

$$\Rightarrow \qquad \mu = 1$$

so, $$I = \int \frac{\frac{1}{2}(-2x) + 1}{\sqrt{1-x^2}}dx$$

$$= \frac{1}{2}\int \frac{(-2x)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$$

$$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}(x) + c\right]$$

PP 18. $\int \frac{2\sin 2\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi}\, d\phi$

C

$$\int \frac{2\sin 2\varphi - \cos\varphi}{6 - \cos^2\varphi - 4\sin\varphi}\, d\varphi$$

$$= \int \frac{2 \cdot 2\sin\varphi\cos\varphi - \cos\varphi}{6 - (1 - \sin^2\varphi) - 4\sin\varphi}\, d\varphi$$

$$= \int \frac{(4\sin\varphi - 1)\cos\varphi}{\sin^2\varphi - 4\sin\varphi + 5}\, d\varphi$$

put $\sin\varphi = u$ $\quad \cos\varphi\, d\varphi = du$

$$G.I = \int \frac{4u - 1}{u^2 - 4u + 5}\, du$$

$$4u - 1 = A\frac{d}{du}(u^2 - 4u + 5) + B$$

$$4u - 1 = A(2u - 4) + B$$

$$4 = 2A \qquad \boxed{A = 2}$$

$$-1 = -4A + B$$

$$B = -1 + 4A = -1 + 8 \qquad \boxed{B = 7}$$

$$G.I = \int \frac{2(2u - 4) + 7}{u^2 - 4u + 5}\, du$$

$$= 2\int \frac{2u - 4}{u^2 - 4u + 5}\, du + 7\int \frac{1}{u^2 - 4u + 5}\, du$$

$$= 2\log|u^2-4u+5| + 7\int \frac{1}{u^2-4u+5}\,du$$

$$u^2-4u+5 = u^2-4u+4+5-4 = (u-2)^2+1$$

$$= (u-2)^2+1$$

$$\therefore G \cdot I = 2\log|u^2-4u+5| + 7\int \frac{1}{(u-2)^2+1^2}\,du$$

$$= 2\log|u^2-4u+5| + 7\tan^{-1}(u-2)$$

$$= 2\log|\sin^2\varphi - 4\sin\varphi + 5| + 7\tan^{-1}(\sin\varphi - 2) + c$$