ST521, Assignment 7
Due Friday, May 16 - my mailbox

1. We model the spread of a disease in a population consists of a fixed number N of individuals. At time $t = 0$ there is one infected individual and $N - 1$ susceptible individuals. Once infected, an individual remains infected forever. In any short time interval of length h, any given infected person will transmit the disease to any given susceptible person with probability $\alpha h + \mathbf{o}(h)$. Let $X(t)$ denote the number of infected individuals in the population at time $t \geq 0$. Describe $X(t)$ as a pure birth process and identify the parameters.

2. A chemical solution contains N A molecules and M B molecules. An irreversible reaction occurs between type A and B molecules, which results in a new compound AB. Suppose that in any small time interval of length h, any particular unbonded A molecule will react with any particular unbonded B molecule with probability $\theta h + \mathbf{o}(h)$. Let $X(t)$ denote the number of unbonded A molecules at time t.

   (a) Model $X(t)$ as a pure death process and give the parameters.

   (b) Assume that $N < M$, so that eventually all the A molecules become bonded. Determine the mean time until this happens.

3. Collards are planted equally spaced in a single row in order to provide an experimental setup for observing the chaotic movements of the flea beetle. A beetle at position k in the row remains on that plant for a random length of time having mean m_k, and then is equally likely to move right to position $k + 1$ or left to position $k - 1$. Model the position of the beetle at time t as a birth-death process having parameters $\lambda_k = \mu_k = 1/(2m_k)$ for $k = 1, 2, \cdots, N-1$. Give plausible assumptions at the ends 0 and N.

4. Determine the stationary distribution for a birth-death process having parameters $\lambda_n = \alpha(n+1)$ and $\mu_n = \beta n^2$ for $n = 0, 1, 2, \cdots$ where $0 < \alpha < \beta$.