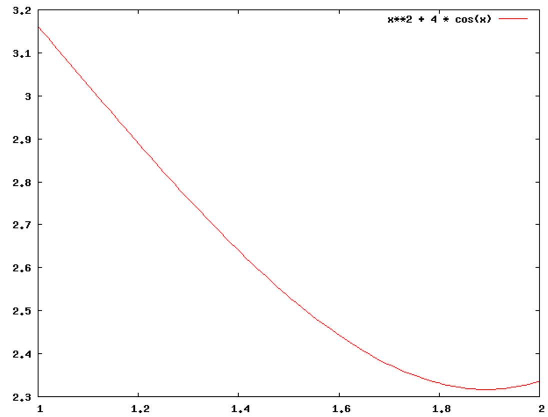
Matt Jibson
EG 520
HW 2

7.2 a.



b. $(0.61803)^N \leq 0.2/1 \Rightarrow N = 4$.

Iteration k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	1.3820	1.6180	2.6607	2.4292	[1.3820, 2.0000]
2	1.6180	1.7639	2.4292	2.3437	[1.6180, 2.0000]
3	1.7639	1.8541	2.3437	2.3196	[1.7639, 2.0000]
4	1.8541	1.9098	2.3196	2.3171	[1.8541, 2.0000]

c. $F_{N+1} \geq \frac{1+2\epsilon}{0.2} \Rightarrow N = 4$.

Iteration k	ρ_k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	0.3750	1.3750	1.6250	2.6688	2.4239	[1.3750, 2.0000]
2	0.4000	1.6250	1.7500	2.4239	2.3495	[1.6250, 2.0000]
3	0.3333	1.7500	1.8750	2.3495	2.3175	[1.7500, 2.0000]
4	0.5000	1.8750	1.8625	2.3175	2.3186	[1.7500, 1.8625]

The numbers found in the above tables were computed using a program authored by me as implementations of the two methods.

8.1

8.3 $u_k = r(1-\rho)^k, r = \text{range}, \rho = \frac{3-\sqrt{5}}{2} \Rightarrow \frac{r(1-\rho)^{(k+1)}}{(r(1-\rho)^{(k)})^p} = \frac{r}{r^p}(1-\rho)^{k+1-kp} = r^{(1-p)}(1-\rho)^{k(1-p)+1}$, hence the order of convergence is 1 since for $p < 1$ the sequence converges to 0 and if $p > 1$ it converges to ∞.

8.13

8.17

9.1 a. $g(x) = \nabla f(x^{(k)}) = 4(x-x_0)^3, F(x) = 12(x-x_0)^2, F(x)^{-1} = \frac{1}{12(x-x_0)^2}, x^{(k+1)} = x^{(k)} - F(x^{(k)})^{-1}g^{(k)} = x^{(k)} - \frac{1}{12(x-x_0)^2}4(x^{(k)}-x_0)^3 = x^{(k)} - \frac{1}{3}(x^{(k)}-x_0)$

b. $y^{(k+1)} = |x^{(k+1)} - x_0| = |x^{(k)} - \frac{1}{3}(x^{(k)} - x_0) - x_0| = |\frac{2}{3}x^{(k)} - \frac{2}{3}x_0| = \frac{2}{3}|x^{(k)} - x_0| = \frac{2}{3}y^{(k)}$

c. For any x_0, the shape of the function is always U-like, with one solution at $f(x) = 0$, and all other values > 0. A change in x_0 only shifts the function left or right. Hence, the next term will always point in the direction of x_0.

d. $\frac{|x^{(k+1)}|}{|x^{(k)}|^p} = \frac{|\frac{2}{3}y^{(k)}|}{|y^{(k)}|^p} = \frac{2}{3}y^{k(1-p)}$. If $p < 1$, the sequence converges to 0, if $p > 1$, the sequence converges to ∞, hence the order of convergence is 1.

e. There is no x^* such that $\nabla f(x^*) = 0$, hence the theorem does not hold.

9.3 a. At $[1, 1]^T, f(x) = 0$. And, since both terms where x_1 and x_2 are used are squared and hence positive, and the only remaining operations are multiplation and addition of positive numbers, there is no way for the function to return a value < 0. Hence, $[1, 1]^T$ is the unique global minimizer.

b. $f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \nabla f(x^k) = [-400(x_2 - x_1^2)x_1 - 2(1 - x_1), 200(x_2 - x_1^2)]^T$,

$$F(x) = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ x_1 - 400 & 200 \end{bmatrix},$$

$$F(x)^{-1} = \frac{1}{(1200x_1^2 - 400x_2 + 2)(200) - (-400x_1)(x_1 - 400)} \begin{bmatrix} 200 & 400x_1 \\ -x_1 + 400 & 1200x_1^2 - 400x_2 + 2 \end{bmatrix}$$

Iter 1: $g^{(0)} = [-2, 0], F(x^{(0)})^{-1} = [[0.5, 0], [1, 0.005]], x^{(1)} = [1, 2]$
Iter 2: $g^{(1)} = [-400, 200], F(x^{(1)})^{-1} = [[-0.0025, -0.00505], [-0.00504, -0.005075]], x^{(2)} = [1, 1]$

c. Iter 1: $g^{(0)} = [-2, 0], x^{(1)} = [0.1, 0]$
Iter 2: $g^{(1)} = [-1.4, -2], x^{(2)} = [0.17, 0.1]$