

ST521, Assignment 3
Due Thursday, March 6

1. Consider a Markov chain with state space $\{0, 1, 2\}$ and probability transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 3/4 & 0 & 1/4 \end{pmatrix}.$$

Compute P^n .

2. Consider a Markov chain with state space $\{0, 1, 2, 3\}$ with probability transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ .1 & .6 & .1 & .2 \\ .2 & .3 & .4 & .1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Starting in state 1, determine the probability that the chain ends in state 0.

(b) Determine the mean time to absorption.

3. Consider a Markov chain with state space $\{0, 1, 2, 3, 4\}$ and probability transition matrix

$$P = \begin{pmatrix} q & p & 0 & 0 & 0 \\ q & 0 & p & 0 & 0 \\ q & 0 & 0 & p & 0 \\ q & 0 & 0 & 0 & p \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

where $p + q = 1$, $p, q \geq 0$. Determine the expected time to reach state 4 starting from state 0.

4. One fourth of the married couples in a far-off society have no children at all. The other three-fourths of families have exactly three children, each child equally likely to be a boy or a girl. What is the probability that the male line of descent of a particular husband will eventually die out?
5. Consider a branching process whose offspring follow the geometric distribution $p_k = (1 - c)c^k$, $0 < c < 1$, $k = 0, 1, \dots$. Determine the probability of eventual extinction.
6. A population grows according to a branching process where the offspring follow a distribution with probability generating function

$$p(s) = .15 + .05s + .03s^2 + .07s^3 + .4s^4 + .25s^5 + .05s^6.$$

Will the species survive? With what probability?

7. Consider a branching process associated with probability generating function $\phi(s) = as^2 + bs + c$, where $a, b, c > 0$ and $a + b + c = 1$. Compute the probability of extinction.

8. Consider a branching process X_n with mean number of offspring μ . Let W_n be the total number of individuals up to and including the n^{th} generation and let W be the total number of all individuals.
 - (a) For what values of μ is W finite?
 - (b) Compute $E(W_n)$ and say what happens as $n \rightarrow \infty$.
9. Write a program that computes realizations of a branching process with a given first generation distribution. Repeat the computations presented in Example 2.5.2, page 97 in the notes