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ST 521
HW 4

1. $P_{00}^n > 0$ for all n. State 1 is not periodic as it can only be the current state during the initial state. $P_{22}^n > 0$ for $n \geq 4$. $P_{33}^n > 0$ for $n \geq 3$. $P_{44}^n > 0$ for $n \geq 2$. $P_{66}^n > 0$ for $n \geq 1$. Hence $d(i) = 1$ for all states.

2. States 1, 2, 3, and 4 are recurrent since once we are in states $\{1, 3\}$ or $\{2, 4\}$, we cannot leave the group. States 5 and 6 are transient since there is a > 0 chance that we will enter the other 4 states, which would mean we will never return to state 5 or 6.

3.

4. At $p = 0$, all states are recurrent since P is the identity matrix. At $p = 0.5$,

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix}$$

so all states are recurrent since states 1 and 3 always jumps to state 2 which will jump to states 1 or 3. At $p = 0.25$,

$$P = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

all states are recurrent for the same reason.

5.

6. If all other states i not s communicate with s, then for some $n \geq 1, p_{is}^n > 0$, hence all states have a > 0 probability of jumping to s, and hence being absorbed. When this happens, $P(X_n = i | X_0 = i) = 0$, hence all states other than s are transient.

7. If and only if a state i is transient then for some $n \geq 1, P(X_n = i | X_0 = i) < 1$, hence there were a finite number of visits to i and so the mean number of visits to i will be finite. Thus state i is recurrent if and only if for some $n \geq 1, P(X_n = i | X_0 = i) = 1$, hence there will always be another visit to i, so the mean number of visits is infinite.