ST521, Assignment 2
Due Thursday, February 22

1. A die is rolled repeatedly. Which of the following are Markov chains? Give reasons for your answer if it is not and give the probability transition matrix if it is.

   (a) The largest number X_n appearing up to the n^{th} role.

   (b) The number X_n of sixes in n rolls.

   (c) At time n, the time X_n since the most recent six.

   (d) At time n, the time X_n until the next six.

2. The random variables $Y_1, Y_2, \cdots$ are independent and have the common probability mass function

k	0	1	2	3
P({Y=k})	.1	.3	.2	.4

   Set $X_0 = 0$ and $X_n = \max\{Y_1, \cdots, Y_n\}$ be the largest Y observed up to time n. Determine the probability transition matrix for the Markov chain X_n

3. A Markov chain X_n with states $\{0, 1, 2\}$ has transition probability matrix

$$P = \begin{pmatrix} .7 & .2 & .1 \\ 0 & .6 & .4 \\ .5 & 0 & .5 \end{pmatrix}.$$

   Determine the conditional probabilities $P(\{X_3 = 1 | X_0 = 0\})$ and $P(\{X_4 = 1 | X_0 = 0\})$.

4. Let X_n be a Markov chain with state space S and suppose that $h : S \to T$ is a one-to-one function. Show that $Y_n = h(X_n)$ defines a Markov chain with state space T. Does this necessarily work if h is not one-to-one?

5. Let X be a Markov chain. Which of the following are Markov chains?

   (a) X_{m+j} for some $j \geq 0$.

   (b) X_{2m}.

6. Let X_n be a Markov chain. Show that for $1 < j < n$,

$$P(\{X_j = k | X_i = x_i,\ i = 1, 2, \cdots, j-1, j+1, \cdots, n\}) \\ = P(\{X_j = k | X_{j-1} = x_{j-1}, X_{j+1} = x_{j+1}\})$$