

Problems solved using LINDO.

Production Planning (tomatoes):

```

MAX = 0.082 * AW + 0.082 * BW + 0.066 * AJ + 0.066 * BJ + 0.074 * AP + 0.074 * BP;
AW - 3 * BW > 0;
3 * AJ - BJ > 0;
AW + BW < 14400000;
AJ + BJ < 1000000;
AP + BP < 2000000;
AW + AJ + AP < 600000;
BW + BJ + BP < 2400000;

```

Results:

Global optimal solution found.

Objective value: 225200.0

Total solver iterations: 5

Variable	Value	Reduced Cost
AW	525000.0	0.000000
BW	175000.0	0.000000
AJ	75000.00	0.000000
BJ	225000.0	0.000000
AP	0.000000	0.3200000E-01
BP	2000000.	0.000000

Row	Slack or Surplus	Dual Price
1	225200.0	1.000000
2	0.000000	-0.8000000E-02
3	0.000000	-0.8000000E-02
4	0.1370000E+08	0.000000
5	700000.0	0.000000
6	0.000000	0.1600000E-01
7	0.000000	0.9000000E-01
8	0.000000	0.5800000E-01

1. Red Brand Canners should produce $525,000 + 175,000 = 700,000/18 = 38,887$ cases of canned whole, $75,000 + 225,000 = 300,000/20 = 15,000$ cases of tomato juice, and $2,000,000$ cases of tomato paste. Net profit is $1.48 * 38887 + 1.32 * 15000 + 1.85 * 2,000,000 = \$3,777,352.76$.

2. Since dual price for row 7 is \$0.09, we determine that it is profitable for them to buy at least some of the additional Grade A lot. The sensitivity output:

Ranges in which the basis is unchanged:

Objective Coefficient Ranges			
Variable	Current Coefficient	Allowable Increase	Allowable Decrease
AW	0.8200000E-01	0.1546667	0.2133333E-01
BW	0.8200000E-01	0.4640000	0.2133333E-01

AJ	0.6600000E-01	0.2133333E-01	0.1546667
BJ	0.6600000E-01	0.1422222E-01	0.5155556E-01
AP	0.7400000E-01	0.3200000E-01	INFINITY
BP	0.7400000E-01	INFINITY	0.1600000E-01

Righthand Side Ranges			
Row	Current RHS	Allowable Increase	Allowable Decrease
2	0.0	466666.7	600000.0
3	0.0	1400000.	200000.0
4	0.1440000E+08	INFINITY	0.1370000E+08
5	1000000.	INFINITY	700000.0
6	2000000.	200000.0	466666.7
7	600000.0	600000.0	466666.7
8	2400000.	466666.7	200000.0

From this we see that it is profitable for up to 600,000 lbs of Grade A tomatoes, so we recommend that they buy the entire lot of 80,000.

Dynamics of Toy Train:

MIN = $y + 0.01 * z$;

$x1_1 - 1 > -y$;
 $x1_1 - 1 < y$;
 $u_2 - u_1 > -z$;
 $u_2 - u_1 < z$;

$x1_2 - 2 > -y$;
 $x1_2 - 2 < y$;
 $u_3 - u_2 > -z$;
 $u_3 - u_2 < z$;

$x1_3 - 3 > -y$;
 $x1_3 - 3 < y$;
 $u_4 - u_3 > -z$;
 $u_4 - u_3 < z$;

$x1_4 - 4 > -y$;
 $x1_4 - 4 < y$;
 $u_5 - u_4 > -z$;
 $u_5 - u_4 < z$;

$x1_1 = 0.048374 * u_1$;
 $x1_2 = x1_1 + 0.95163 * x2_1 + 0.048374 * u_2$;
 $x1_3 = x1_2 + 0.95163 * x2_2 + 0.048374 * u_3$;
 $x1_4 = x1_3 + 0.95163 * x2_3 + 0.048374 * u_4$;
 $x1_5 = 5$;

$x2_1 = 0.95163 * u_1$;
 $x2_2 = 0.90484 * x2_1 + 0.95163 * u_2$;
 $x2_3 = 0.90484 * x2_2 + 0.95163 * u_3$;
 $x2_4 = 0.90484 * x2_3 + 0.95163 * u_4$;
 $x2_5 = 0$;

```
@FREE(u_1);
@FREE(u_2);
@FREE(u_3);
@FREE(u_4);
@FREE(u_5);
```

Results for $c = 0.01$:

Objective value: 0.8983354

Variable	Value	Reduced Cost
Y	0.8514394	0.000000
Z	4.689601	0.000000
X1_1	0.1485606	0.000000
U_2	-1.618517	0.000000
U_1	3.071084	0.000000
X1_2	2.851439	0.000000
U_3	-1.049907	0.000000
X1_3	3.851439	0.000000
U_4	0.000000	0.000000
X1_4	3.851439	0.000000
U_5	4.689601	0.000000
X2_1	2.922536	0.000000
X2_2	1.104198	0.000000
X2_3	0.000000	0.5025365E-03
X1_5	5.000000	0.000000
X2_4	0.000000	0.000000
X2_5	0.000000	0.000000

Results for $c = 0.1$:

Objective value: 0.9656577

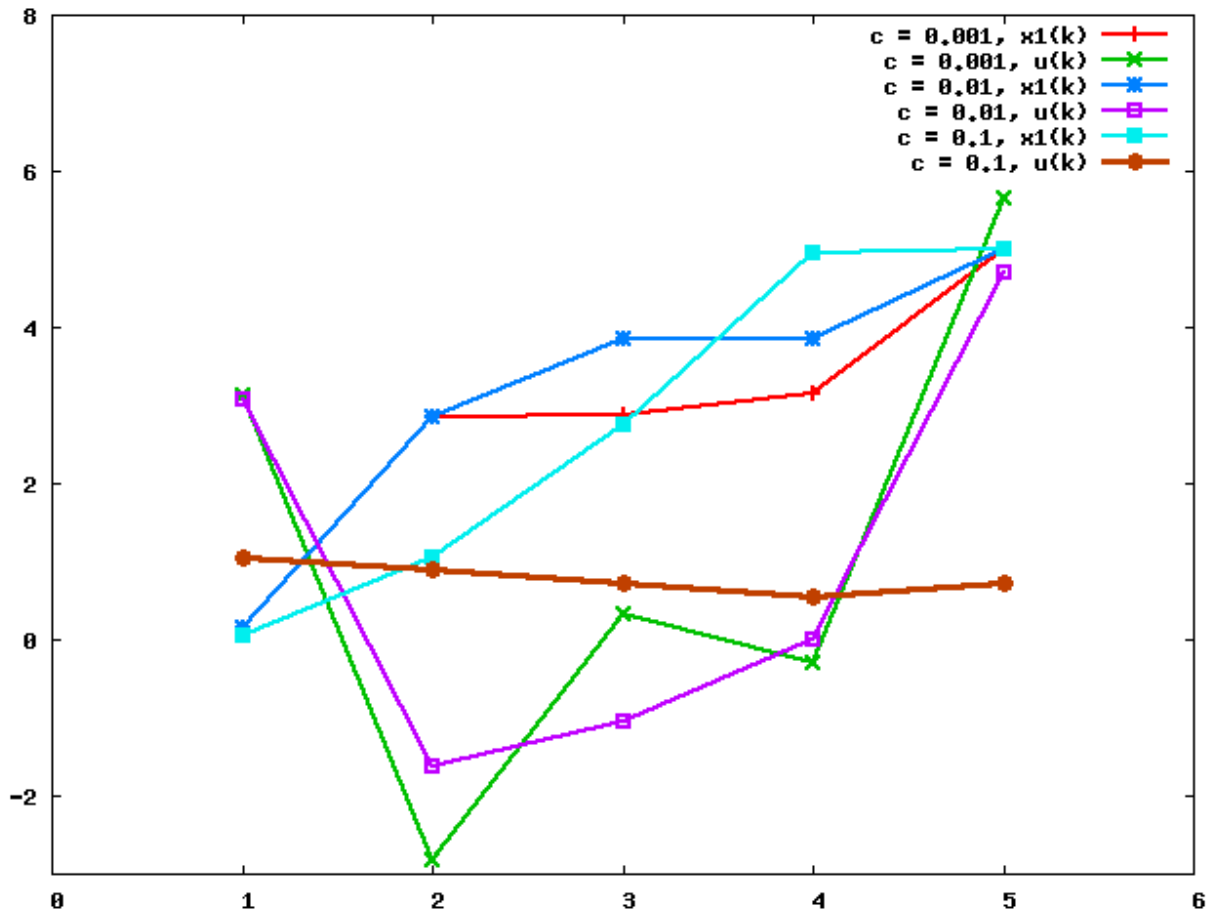
Variable	Value	Reduced Cost
Y	0.9488806	0.000000
Z	0.1677710	0.000000
X1_1	0.5111943E-01	0.000000
U_2	0.8889833	0.000000
U_1	1.056754	0.000000
X1_2	1.051119	0.000000
U_3	0.7212123	0.000000
X1_3	2.756999	0.000000
U_4	0.5534413	0.000000
X1_4	4.948881	0.000000
U_5	0.7212123	0.000000
X2_1	1.005639	0.000000
X2_2	1.755926	0.000000
X2_3	2.275159	0.000000
X1_5	5.000000	0.000000
X2_4	2.585326	0.000000
X2_5	0.000000	0.000000

Results for $c = 0.001$:

Objective value: 0.8545683

Variable	Value	Reduced Cost
Y	0.8486068	0.000000
Z	5.961462	0.000000
X1_1	0.1513932	0.000000
U_2	-2.831823	0.000000
U_1	3.129639	0.000000
X1_2	2.848607	0.000000
U_3	0.3326581	0.000000
X1_3	2.864699	0.000000
U_4	-0.3010023	0.000000
X1_4	3.151393	0.000000
U_5	5.660460	0.000000
X2_1	2.978259	0.000000
X2_2	0.000000	0.1413404E-02
X2_3	0.3165674	0.000000
X1_5	5.000000	0.000000
X2_4	0.000000	0.000000
X2_5	0.000000	0.000000

Graph:



From the graph, it appears that the $c = 0.01$ solution is the best compromise between accuracy and smoothness.

Regression Analysis:

1. Chebychev Norm ($p = \infty$):

```
MIN = e;
B * 0 + a + e > 1;
B * 0 + a - e < 1;
B * 1.5 + a + e > 1.8;
B * 1.5 + a - e < 1.8;
B * 2.5 + a + e > 1.3;
B * 2.5 + a - e < 1.3;
B * 4.3 + a + e > 2.2;
B * 4.3 + a - e < 2.2;
B * 5.9 + a + e > 3.1;
B * 5.9 + a - e < 3.1;
B * 7.5 + a + e > 3.3;
B * 7.5 + a - e < 3.3;
B * 9.4 + a + e > 2.8;
B * 9.4 + a - e < 2.8;
```

Results:

Variable	Value	Reduced Cost
E	0.5304348	0.000000
B	0.2173913	0.000000
A	1.286957	0.000000

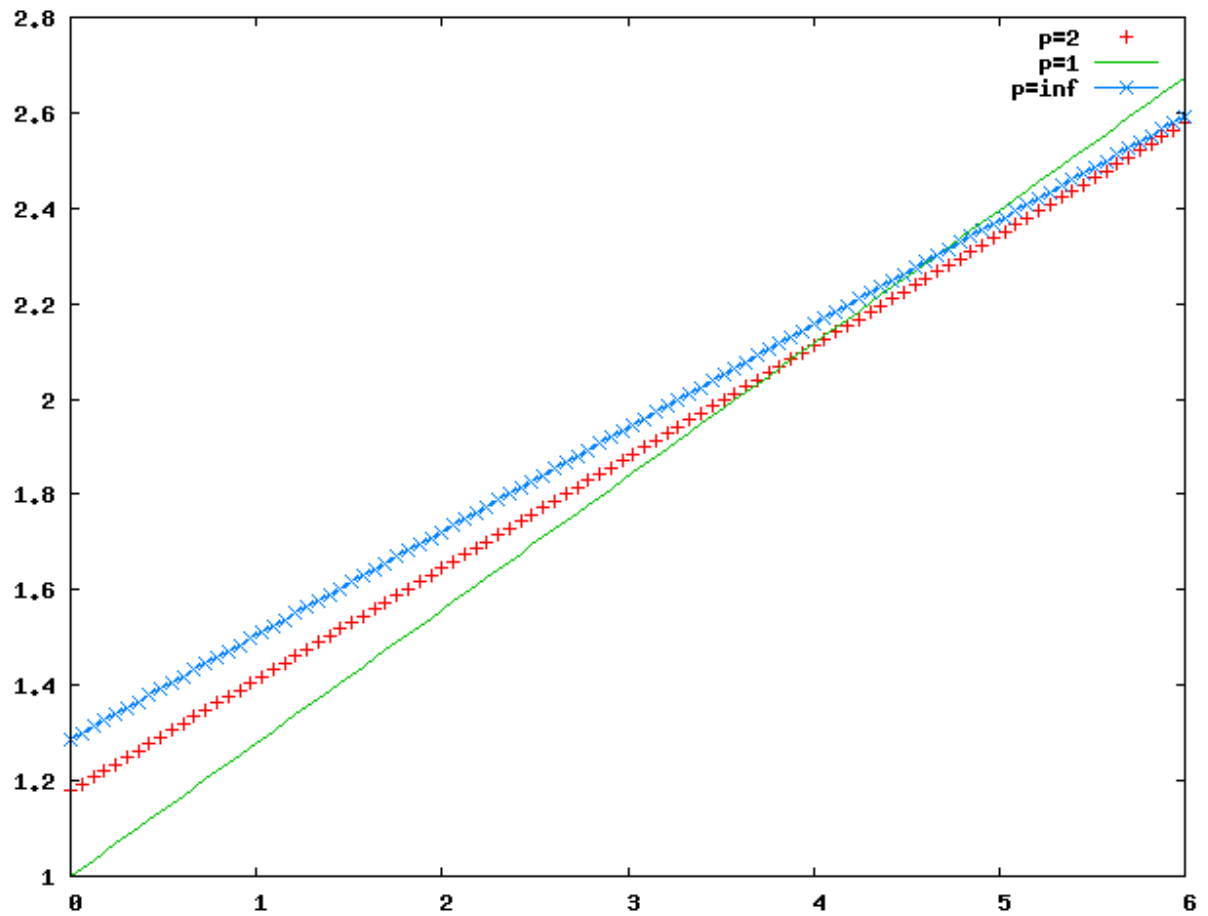
Minimum Absolute Value ($p = 1$):

```
MIN = e1p + e1m + e2p + e2m + e3p + e3m + e4p + e4m + e5p + e5m + e6p + e6m + e7p + e7m;
B * 0 + a + e1p - e1m = 1;
B * 1.5 + a + e2p - e2m = 1.8;
B * 2.5 + a + e3p - e3m = 1.3;
B * 4.3 + a + e4p - e4m = 2.2;
B * 5.9 + a + e5p - e5m = 3.1;
B * 7.5 + a + e6p - e6m = 3.3;
B * 9.4 + a + e7p - e7m = 2.8;
```

Results:

Variable	Value	Reduced Cost
E1P	0.000000	1.302326
E1M	0.000000	0.6976744
E2P	0.3813953	0.000000
E2M	0.000000	2.000000
E3P	0.000000	2.000000
E3M	0.3976744	0.000000
E4P	0.000000	1.697674
E4M	0.000000	0.3023256
E5P	0.4534884	0.000000
E5M	0.000000	2.000000
E6P	0.2069767	0.000000
E6M	0.000000	2.000000
E7P	0.000000	2.000000
E7M	0.8232558	0.000000
B	0.2790698	0.000000
A	1.000000	0.000000

For Least-Squares Analysis, after setting up the problem using the LINEST function, I get results:
 $y = 0.2331x + 1.1787$. Data graph:



2. All reduced costs are > 0 , so solutions are unique.