ST521, Assignment 6
Due Thursday, April 24

1. Determine the limiting distribution for the Markov chain whose transition probability matrix is

$$P = \begin{pmatrix} q & p & 0 & 0 & 0 \\ q & 0 & p & 0 & 0 \\ q & 0 & 0 & p & 0 \\ q & 0 & 0 & 0 & p \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where $p > 0$, $q > 0$, and $p + q = 1$.

2. Consider a random walk Markov chain on state $0, 1, 2, \cdots, N$ with transition probability matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ q_1 & 0 & p_1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2 & 0 & p_2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & q_{N-1} & 0 & p_{N-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

where $p_i + q_i = 1$, $p_i > 0$, $q_i > 0$ for all i. The process is "reflected" back into $\{1, 2, \cdots, N-1\}$ from 0 and N. Determine the limiting distribution.

3. A Markov chain on states $\{0, 1, 2, \cdots\}$ has transition probabilities

$$p_{ij} = \frac{1}{i+2} \text{ for } j = 0, 1, 2, \cdots, i, i+1.$$

Find the stationary distribution.

4. Suppose that minor defects are distributed over the length of a cable as a Poisson process with rate α and that, independently, major defects are distributed over the cable according to a Poisson process of rate β. Let $N(t)$ be the number of defects, either major or minor, in the cable up to length t from 0. Argue that $N(t)$ is a Poisson process of rate $\alpha + \beta$.

5. For a Poisson process $N(t)$, compute the probability that $P(N(t) = 1, 3, 5, \cdots)$, i.e. that a process having rate λ takes an odd value.