ASSIGNMENT #12
EG/M 510
[due 12/10/07]

1. Consider the problem:

minimize $\alpha x_1 - x_2$

subject to:

$$x_1^2 + x_2^2 \le 25$$
$$x_1 - x_2 \le 1$$
$$x_1, x_2 \ge 0$$

(a) Write down the Karush-Kuhn-Tucker conditions for this problem [*hint:* be sure to include **all** constraints].

(b) Show that the KKT conditions are both necessary and sufficient for this problem [*hint:* invoke Theorem 4 to show this]

(c) Using (b), determine the range of coefficient α that maintains the optimality of the solution $\mathbf{x}^T = (4,3)$. [*hint:* use the stationarity conditions to determine what range of α guarantees that $\lambda_1, \lambda_2 \ge 0$] .

2. Show that *constraint qualification* is **not** satisfied **at the optimum** for the following problem [*hint:* you must first find the optimum $\mathbf{x}^*$ to this problem by inspection—**do not** try to use the KKT conditions—the optimum solution should be obvious.]

maximize x_1

subject to:

$$(x_1 - 1)^3 + (x_2 - 2) \le 0$$
$$(x_1 - 1)^3 - (x_2 - 2) \le 0$$
$$x_1, x_2 \ge 0$$

3. Use the KKT conditions to determine whether **or not** the solution $\mathbf{x}^T = (1,1,1)$ is optimal for the following problem:

minimize $2x_1 + x_2^3 + x_3^2$

subject to:

$$2x_1^2 + 2x_2^2 + x_3^2 \ge 4$$
$$x_1, x_2, x_3 \ge 0$$

4. Consider the following problem:

$$\text{maximize } 20x_1 - 20x_1^2 + 50x_2 - 5x_2^2 + 18x_1x_2$$

subject to:

$$x_1 + x_2 \le 6$$
$$x_1 + 4x_2 \le 18$$
$$x_1, x_2 \ge 0$$

(a) Place this problem in the form:

$$\text{minimize } \mathbf{c}^T\mathbf{x} + \tfrac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}$$
$$\text{subject to: } \mathbf{A}\mathbf{x} \ge \mathbf{b}$$
$$\mathbf{x} \ge \mathbf{0}$$

Define: $\mathbf{c}$, $\mathbf{Q}$, $\mathbf{A}$, and $\mathbf{b}$ for this problem:

(b) Solve this problem by hand using Lemke's algorithm.

(c) Check your answer in Part (b) by solving this problem with Solver in EXCEL.