Matt Jibson
ST 521
HW 4

1. $P_{00}^n > 0$ for all n. State 1 is not periodic as it can only be the current state during the initial state. $P_{22}^n > 0$ for $n \geq 4$. $P_{33}^n > 0$ for $n \geq 3$. $P_{44}^n > 0$ for $n \geq 2$. $P_{55}^n > 0$ for $n \geq 1$. Hence $d(i) = 1$ for all states.

2. States 1, 2, 3, and 4 are recurrent since once we are in states $\{1, 3\}$ or $\{2, 4\}$, we cannot leave the group. States 5 and 6 are transient since there is a > 0 chance that we will enter the other 4 states, which would mean we will never return to state 5 or 6.

3. For $r = 1, P_{ii} = 1$, states ≥ 1 are recurrent, positive, and aperiodic. Since $f_{ii}(0) = 1, f_{ii}(n) = 0$ for $n > 0$, $\mu_i = 1$. State 0 is transient (assuming $a_0 \neq 1$) hence $\mu_0 = \infty$. If $a_0 = 1$, state 0 acts like the other states.

   For $r < 1$ states ≥ 1 are recurrent and aperiodic.

4. At $p = 0$, P is the identity matrix and all states are recurrent, positive (since $f_{ii}(0) = 1, f_{ii}(n) = 0, n > 0$, hence $\mu_i = 1$), and aperiodic (since $d(i) = 1$), so the states are ergodic.

   At $p = 0.25$ (analysis is the same for $0 < p < 0.5$),

$$P = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

   and all states are recurrent. $P_{ii}^n > 0$ for all n, so states are aperiodic.

5. $f_{00}(1) = 1 - a, f_{00}(2) = ab, f_{00}(3) = (1 - a)ab, f_{00}(4) = (1 - a)^2 ab$. Assuming $0 < a, b < 1$, both states are clearly recurrent. Hence for $n \geq 2, f_{00}(n) = (1 - a)^{n-2} ab$. Since $P_{00}^n \not\to 0$ as $n \to \infty$, state 0 is positive and hence $\mu_i < \infty$. $\mu_0 = E(T_0 | X_0 = 0) = \Sigma_{n=1}^{\infty} f_{00}(n) = (1 - a) + \Sigma_{n=2}^{\infty} (1 - a)^{n-2} ab$.

6. If all other states i not s communicate with s, then for some $n \geq 1, p_{is}^n > 0$, hence all states have a > 0 probability of jumping to s, and hence being absorbed. When this happens, $P(X_n = i | X_0 = i) = 0$, hence all states other than s are transient.

7. If and only if a state i is transient then for some $n \geq 1, P(X_n = i | X_0 = i) < 1$, hence there were a finite number of visits to i and so the mean number of visits to i will be finite. Thus state i is recurrent if and only if for some $n \geq 1, P(X_n = i | X_0 = i) = 1$, hence there will always be another visit to i, so the mean number of visits is infinite.