Matt Jibson
ST 521
HW 1

1. (a) pmf: $P_X(i) = P(X = i) = \frac{1}{6}(\frac{5}{6})^{i-1}$
   (b) cdf: $F(X) = P(X \leq x) = \sum_{j \leq x} \frac{1}{6}(\frac{5}{6})^{j-1}$
   (c) $E(X) = \sum_{k=0}^{\infty} k\frac{1}{6}(\frac{5}{6})^{k-1} = 6$

2. $P(Y = 0) = 0.22153, P(Y = 1) = 0.41142, P(Y = 2) = 0.2742, P(Y = 3) = 0.0815, P(Y = 4) = 0.0107, P(Y = 5) = 0.000495, P(Y \geq 6) = 0$
   $P(X = 0) = 0.6588, P(Y = 1) = 0.2995, P(Y = 2) = 0.0399, P(Y = 3) = 0.00174, P(X = 4) = 0.0000185, P(Y \geq 5) = 0$

3. P(k Heads | N tosses) = $\binom{N}{k}p^k(1-p)^{N-k}$
   P(j Tails | N tosses) = $1 - \binom{N}{N-j}p^{N-j}(1-p)^j, k = N - j \Rightarrow \binom{N}{k}p^k(1-p)^{N-k}$
   P(j Tails $\cap$ k Heads) =
   P(j Tails) P(k Heads) = $\binom{N}{k}p^k(1-p)^{N-k} * (1 - \binom{N}{k}p^k(1-p)^{N-k}) =$

4. $P(testPos|haveHep) = 0.99 = \frac{P(testPos \cap haveHep)}{P(haveHep)} = \frac{X}{0.0001} \Rightarrow P(testPos \cap haveHep) = 0.000099$
   $P(testPos) = (0.99 * 1 + 0.05 * 9999)/10000 = 0.050094$
   $\boxed{P(haveHep|testPos) = \frac{P(haveHep \cap testPos)}{P(testPos)} = \frac{0.000099}{0.050094} = 0.001976}$

5. (a) $P(allH|firstH) = \frac{P(allH \cap firstH)}{P(firstH)} = \frac{P(\{HH\})}{P(\{HH,HT\})} = \frac{1/4}{1/2} = 0.5$
   (b) $P(allH|oneH) = \frac{P(allH \cap oneH)}{P(oneH)} = \frac{P(\{HH\})}{P(\{HH,HT,TH\})} = \frac{1/4}{3/4} = \frac{1}{3}$

6. $E(T|N) = N\lambda$
   $E(E(T|N)) = E(T) = \frac{M}{2}\lambda$

   Above are my answers. But, during my work I came up with the following equations. Are they right?
   cond pmf: $P_{T|N}(t|n) = \frac{P(T=t,N=n)}{P(N=n)} = \frac{e^{-\lambda}\lambda^t/t!}{1/n}$
   $E(T|N) = \Sigma_{j=0}^{\infty} jP_{T|N}(j|n) = \Sigma_{j=0}^{\infty} j\frac{e^{-\lambda}\lambda^j/j!}{1/n} = N\lambda$