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EG 520
HW 3

10.1 To show Q-conjugacy we must show that $\boldsymbol{d}^{(j+1)T}\boldsymbol{Q}\boldsymbol{d}^{(k+1)} = 0$ for any $j \neq k$. Let

$$\boldsymbol{d}^{(j+1)} = \boldsymbol{p}^{(j+1)} - \Sigma_{i=0}^{j} \frac{\boldsymbol{p}^{(j+1)T}\boldsymbol{Q}\boldsymbol{d}^{(i)}}{\boldsymbol{d}^{(i)T}\boldsymbol{Q}\boldsymbol{d}^{(i)}}\boldsymbol{d}^{(i)}, \boldsymbol{d}^{(k+1)} = \boldsymbol{p}^{(k+1)} - \Sigma_{i=0}^{k} \frac{\boldsymbol{p}^{(k+1)T}\boldsymbol{Q}\boldsymbol{d}^{(i)}}{\boldsymbol{d}^{(i)T}\boldsymbol{Q}\boldsymbol{d}^{(i)}}\boldsymbol{d}^{(i)}$$

10.6

10.7 a. $f(x) = \frac{5}{2}x_1^2 + \frac{1}{2}x_2^2 + 2x_1x_2 - 3x_1 - x_2$,

$$\boldsymbol{Q} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

b. $\boldsymbol{x}^{(0)} = [0,0]^T, \boldsymbol{g}(\boldsymbol{x}) = \nabla f(x) = \boldsymbol{Qx - b} = [5x_1 + 2x_2 - 3, 2x_1 + x_2 - 1]^T$
$\boldsymbol{g}^{(0)} = [-3.0, -1.0]^T, \boldsymbol{d}^{(0)} = [3.0, 1.0], \alpha_0 = 0.1724, \beta_0 = 0.0000, \boldsymbol{x}^{(1)} = [0.5172, 0.1724]^T$
$\boldsymbol{g}^{(1)} = [-0.0689, 0.2068]^T, \boldsymbol{d}^{(1)} = [0.0832, -0.2021], \alpha_1 = 5.8000, \beta_1 = 0.0048, \boldsymbol{x}^{(2)} = [1, -1]^T$
$\boldsymbol{g}^{(2)} = [0,0]^T$, as expected. (Solved using my own program.)

c. $5x_1 + 2x_2 - 3 = 0$
$2x_1 + x_2 - 1 = 0$
$x_2 = 1 - 2x_1$
$5x_1 + 2 - 4x_1 - 3 = 0$
$x_1 = 1$
$2 + x_2 - 1 = 0$
$x_2 = -1$
Analytical solution is the same.

11.1

11.6