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EG 510
HW 6

1. Find the dual of:

$$\begin{array}{ll} \text{maximize} & z = 6x_1 - 3x_2 - 2x_3 + 5x_4 \\ \text{subject to} & 4x_1 + 3x_2 - 8x_3 + 7x_4 = 11 \\ & 3x_1 + 2x_2 + 7x_3 + 6x_4 \geq 23 \\ & 7x_1 + 4x_2 + 3x_3 + 2x_4 \leq 22 \\ & x_1, x_2 \geq 0, x_3 \leq 0, x_4 \text{ unrestricted} \end{array}$$

Convert to canonical form:

$$\begin{array}{ll} \text{maximize} & z = 6x_1 - 3x_2 - 2x_3 + 5x_4 \\ \text{subject to} & 4x_1 + 3x_2 - 8x_3 + 7x_4 \geq 11 \\ & -4x_1 - 3x_2 + 8x_3 - 7x_4 \geq -11 \\ & 3x_1 + 2x_2 + 7x_3 + 6x_4 \geq 23 \\ & -7x_1 - 4x_2 - 3x_3 - 2x_4 \geq -22 \\ & x_1, x_2 \geq 0, x_3 \leq 0, x_4 \text{ unrestricted} \end{array}$$

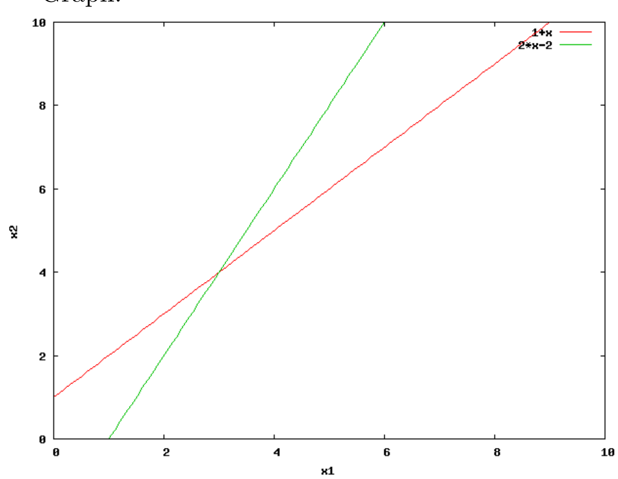
Dual is:

$$\begin{array}{ll} \text{minimize} & w = 11y_1 - 11y_2 + 23y_3 - 22y_4 \\ \text{subject to} & 4y_1 - 4y_2 + 3y_3 - 7y_4 \leq 6 \\ & 3y_1 - 3y_2 + 2y_3 - 4y_3 \leq -3 \\ & -8y_1 + 8y_2 + 7y_3 - 3y_4 \geq -2 \\ & 7y_1 - 7y_2 + 6y_3 - 2y_4 = 5 \\ & y_1, y_2, y_3, y_4 \geq 0 \end{array}$$

2. Find dual of:

$$\begin{array}{ll} \text{maximize} & z = -x_1 - x_2 \\ \text{subject to} & -x_1 + x_2 \geq 1 \\ & 2x_1 - x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{array}$$

Graph:

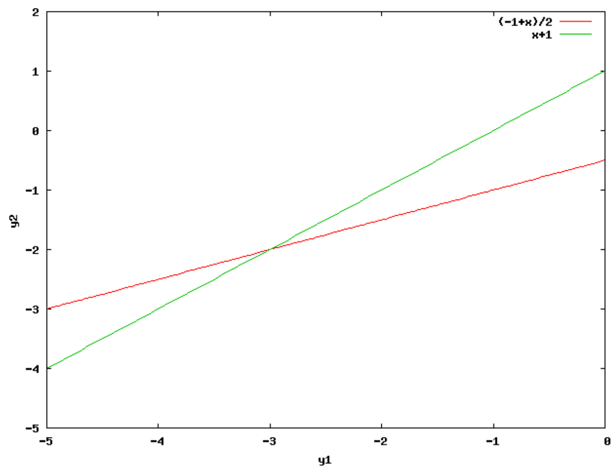


At $\mathbf{x} = (0, 1)^T$, z is maximized at -1.

Dual is:

$$\begin{array}{ll} \text{minimize} & w = y_1 + 2y_2 \\ \text{subject to} & -y_1 + 2y_2 \geq -1 \\ & y_1 - y_2 \geq -1 \\ & y_1 \leq 0, y_2 \geq 0 \end{array}$$

Graph:



At $\mathbf{y} = (-1, 0)^T$, w is minimized at -1. Strong duality is verified: both optima are -1.

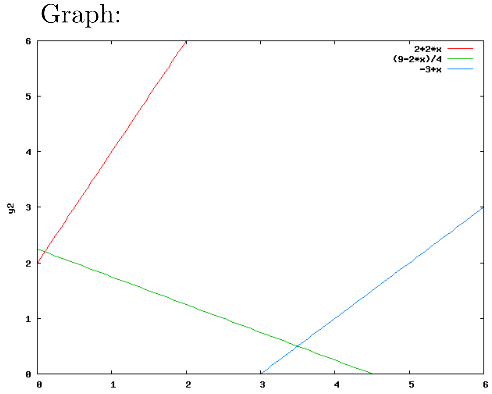
3. Find dual of:

$$\begin{array}{ll} \text{minimize} & z = 2x_1 + 9x_2 + 3x_3 \\ \text{subject to} & -2x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 + 4x_2 - x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Dual is:

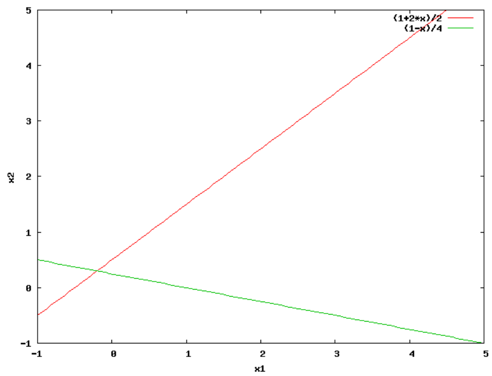
$$\begin{array}{ll} \text{maximize} & w = y_1 + y_2 \\ \text{subject to} & -2y_1 + y_2 \leq 2 \\ & 2y_1 + 4y_2 \leq 9 \\ & y_2 - y_2 \leq 3 \\ & y_1, y_2 \geq 0 \end{array}$$

Graph:



At $\mathbf{y} = (4.5, 0)^T$, w is maximized at 4.5.

In the dual the first two constraints are binding. Using complementary slackness, the last one constraint in the primal must be binding. Graphically solving:



At $\mathbf{x} = (0, 0.5, 0)^T$, z is minimized at 4.5.

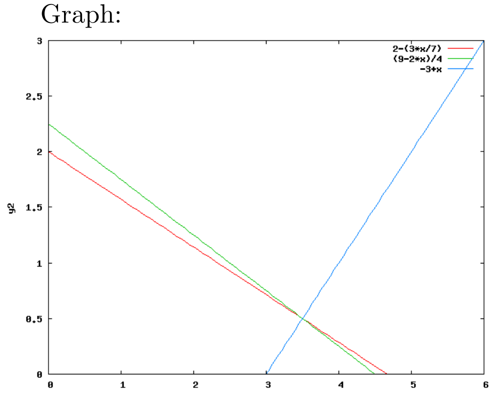
4. Find dual of:

$$\begin{array}{ll} \text{minimize} & z = 2x_1 + 9x_2 + 3x_3 \\ \text{subject to} & \frac{3}{7}x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 + 4x_2 - x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Dual is:

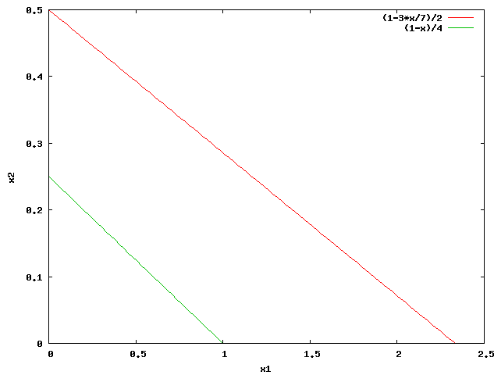
$$\begin{array}{ll} \text{maximize} & w = y_1 + y_2 \\ \text{subject to} & \frac{3}{7}y_1 + y_2 \leq 2 \\ & 2y_1 + 4y_2 \leq 9 \\ & y_2 - y_2 \leq 3 \\ & y_1, y_2 \geq 0 \end{array}$$

Graph:



At $\mathbf{y} = (4.5, 0)^T$, w is maximized at 4.5.

In the dual the first two constraints are binding. Using complementary slackness, the last one constraint in the primal must be binding. Graphically solving:



At $\mathbf{x} = (0, 0.5, 0)^T$, z is minimized at 4.5. (Apparently I did something wrong as I don't think this is the right answer.)