Matt Jibson
ST 521
HW 3

1.

$$P^n = \begin{pmatrix} 1 & 0 & 0 \\ (1/4)^n & (1/2)^n & (1/4)^n \\ (3/4)^n & 0 & (1/4)^n \end{pmatrix}$$

2. (a) $U_{10} = P_{10} + P_{11}U_{10} + P_{12}U_{20}$
$U_{20} = P_{20} + P_{21}U_{10} + P_{22}U_{20}$
or
$U_{10} = 0.1 + 0.6U_{10} + 0.1U_{20}$
$U_{20} = 0.2 + 0.3U_{10} + 0.4U_{20}$
so
$U_{10} = 8/21, U_{20} = 11/21$

(b) $\nu_1 = 1 + .6\nu_1 + .1\nu_2$
$\nu_2 = 1 + .3\nu_1 + .4\nu_2$

$$\begin{pmatrix} .4 & -.1 \\ -.3 & .6 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \nu_1 = \nu_2 = 10/3$$

3. $\nu_i = 1 + p\nu_{i+1} + q\nu_0, i = 0, 1, 2, 3$
$\nu_4 = 0$
$\nu_0 = 1 + q\nu_0 + p(1 + q\nu_0 + p(1 + q\nu_0 + p(1 + q\nu_0 + p(\nu_4)))) =$
$1 + q\nu_0 + p + pq\nu_0 + p^2 + p^2q\nu_0 + p^3 + p^3q\nu_0 = 1 + q\nu_0 + p(1 + q\nu_0) + p^2(1 + q\nu_0) + p^3(1 + q\nu_0) =$
$(1 + q\nu_0)(1 + p + p^2 + p^3) = \nu_0$

4. Probability P_n of having n boys:
$P_0 = 1/4 + (3/4 * 1/8) = 11/32, P_1 = (3/4 * 3/8) = 9/32, P_2 = 9/32, P_3 = (3/4 * 1/8) = 3/32$
$\mu = 9/32 + 2 * 9/32 + 3 * 3/32 = 36/32 > 1.$
$\phi(s) = 11/32 + 9/32s + 9/32s^2 + 3/32s^3, \phi'(1) = \mu$, so the probability of ultimate extinction is $\frac{11}{32} + \frac{9t}{32} + \frac{9t^2}{32} + \frac{3t^3}{32} = t \Rightarrow 32(11 + 9t + 9t^2 + 3t^3) = 0$, which has solutions at $t = 2.38672, 0.306639 \pm 1.20094i$. But $\phi(s)$ is not convex, so does this analysis hold?

5. $p_k = (1 - c)c^k, 0 < c < 1, k = 0, 1, \ldots.$
$\phi(t) = (1 - c)\Sigma_{k=1}^{\infty} c^{k-1}t^k = (1 - c)\frac{t}{1-ct}, \mu = |\phi'(1)| = \frac{1-c}{(1-c)^2} = \frac{1}{1-c} > 1.$
$(1 - c)\frac{t}{1-ct} = t \Rightarrow (1 - c)t = t - ct^2 \Rightarrow -ct + ct^2 = 0 = c(t^2 - t) = ct(t - 1) \Rightarrow \eta = 0$

6. $p'(1) = .05 + 2 * .03 + 3 * .07 + 4 * .4 + 5 * .25 + 6 * .05 = 3.47 > 1.$
$.15 - .95t + .03t^2 + .07t^3 + .4t^4 + .25t^5 + .05t^6 = 0 \Rightarrow \eta = 0.1592928$

7. If $|\phi'(1)| = 2a + b < 1, \eta = 1$. If $|\phi'(1)| > 1, \eta$ is the smallest nonnegative value of $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that is < 1.

8. (a)

(b)

9.