Matt Jibson
ST 521
HW 2

1. (a) Markov chain - state space is $\{0, 1, 2, 3, 4, 5, 6\}$.

$$P = \begin{pmatrix} 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 2/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 3/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 4/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

   (b) Markov chain - state space is $\{0, 1, 2, \dots\}$.

$$P = \begin{pmatrix} 5/6 & 1/6 & 0 & \cdots & \\ 0 & 5/6 & 1/6 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

   (c) Not Markov chain since current state depends on its distance from the previous six.

   (d) Markov chain - state space is $\{0, 1, 2, \dots\}$.

$$P = \begin{pmatrix} \frac{1}{6} & \frac{1}{6}(\frac{5}{6})^1 & \frac{1}{6}(\frac{5}{6})^2 & \cdots & \\ 0 & \frac{1}{6} & \frac{1}{6}(\frac{5}{6})^1 & \frac{1}{6}(\frac{5}{6})^2 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

2.

$$P = \begin{pmatrix} .1 & .3 & .2 & .4 \\ 0 & .4 & .2 & .4 \\ 0 & 0 & .6 & .4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.

$$P^3 = \begin{pmatrix} .478 & .264 & .258 \\ .36 & .256 & .384 \\ .57 & .18 & .25 \end{pmatrix}, P^4 = \begin{pmatrix} .4636 & .254 & .2824 \\ .444 & .2256 & .3304 \\ .524 & .222 & .254 \end{pmatrix}$$

   $P(\{X_3 = 1 | X_0 = 0\}) = 0.264$
   $P(\{X_4 = 1 | X_0 = 0\}) = 0.254$

4. $P(Y_n = s | Y_{n-1} = y_{n-1}) = P(h(X_n) = s | h(X_{n-1}) = h(x_{n-1}))$.
   This does not necessarily work if h is not one-to-one.

5. (a) Markov chain.

   (b) Not Markov chain.

6.