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ST 521
HW 5

1. (a) $S = \{0, 1, 2, 3, 4\}$

$$P = \begin{pmatrix} 0 & 1/3 & 2/3 & 0 & 0 \\ 1/2 & 0 & 1/6 & 1/3 & 0 \\ 0 & 1/2 & 0 & 1/6 & 1/3 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(b) Write down our initial equations:

$$\begin{aligned} \Pi_0 &= \tfrac{1}{2}\Pi_1 \\ \Pi_1 &= \tfrac{1}{3}\Pi_0 + \tfrac{1}{2}\Pi_2 \\ \Pi_2 &= \tfrac{2}{3}\Pi_0 + \tfrac{1}{6}\Pi_1 + \tfrac{3}{4}\Pi_3 \\ \Pi_3 &= \tfrac{1}{3}\Pi_1 + \tfrac{1}{6}\Pi_2 + \Pi_4 \\ \Pi_4 &= \tfrac{1}{3}\Pi_2 + \tfrac{1}{4}\Pi_3 \end{aligned}$$

Write everything in terms of Π_0:

$$\begin{aligned} \Pi_0 &= \Pi_0 \\ \Pi_1 &= 2\Pi_0 \\ \Pi_2 &= \tfrac{10}{3}\Pi_0 \\ \Pi_3 &= \tfrac{28}{9}\Pi_0 \\ \Pi_4 &= \tfrac{17}{9}\Pi_0 \end{aligned}$$

Double checking with $\Pi_4 = \frac{1}{3}\Pi_2 + \frac{1}{4}\Pi_3$ yields $\frac{17}{9}$ on both sides, so things look correct so far. Now sum everything and solve: $\Pi_0 + 2\Pi_0 + \frac{10}{3}\Pi_0 + \frac{28}{9}\Pi_0 + \frac{17}{9}\Pi_0 = 1 \Rightarrow \frac{34}{3}\Pi_0 = 1 \Rightarrow \Pi_0 = \frac{3}{34}$. So, $\Pi = (3/34, 3/17, 5/17, 14/51, 1/6)$.

(c) There is a 3/34 chance she will find it empty and a 1/6 chance she will find it full.

2. (a) For states 0, 1, 2, 3 the smallest number of steps is 4. For states 4, 5 the smallest number of steps is 6.

(b)

(c)