ST521, Assignment 4
Due Thursday, March 27

1. Determine the period of each state of the Markov chain with state space $S = \{0, 1, 2, 3, 4, 5\}$ and transition probability matrix

$$P = \begin{pmatrix} 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1/3 & 1/3 & 0 & 1/3 \end{pmatrix}$$

2. Determine the transient and recurrent states for the Markov chain with state space $S = \{0, 1, 2, 3, 4, 5\}$ and

$$P = \begin{pmatrix} 1/3 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 3/4 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 4/5 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{pmatrix}$$

3. Let X be a Markov chain on $\{0, 1, 2, \cdots\}$ with probability transition matrix having entries $P_{0j} = a_j$ for $j \geq 0$, $P_{ii} = r$ and $P_{i,i-1} = 1 - r$ for $i \geq 1$, and all other entries zero, where $0 \leq a_i \leq 1$, $\sum_i a_i = 1$, and $0 \leq r \leq 1$. Classify the states of the chain, and find their mean recurrence times.

4. Classify the states of the Markov chain with state space $\{0, 1, 2\}$ and probability transition matrix

$$P = \begin{pmatrix} 1 - 2p & 2p & 0 \\ p & 1 - 2p & p \\ 0 & 2p & 1 - 2p \end{pmatrix}$$

where $0 \leq p \leq 1/2$. *Note there are different cases!*

5. A two state Markov chain with state space $S = \{0, 1\}$ has the transition probability matrix

$$P = \begin{pmatrix} 1 - a & a \\ b & 1 - b \end{pmatrix}.$$

Determine the first return distributions $f_{00}(n)$ and the mean time to return μ_0.

6. Let X be a Markov chain containing an absorbing state s with which all other states i communicate, i.e., $p_{is}^n > 0$ for some $n = n(i)$. Show that all states other than s are transient.

7. Show that a state i is recurrent if and only if the mean number of visits of the chain to i, having started at i, is infinite.