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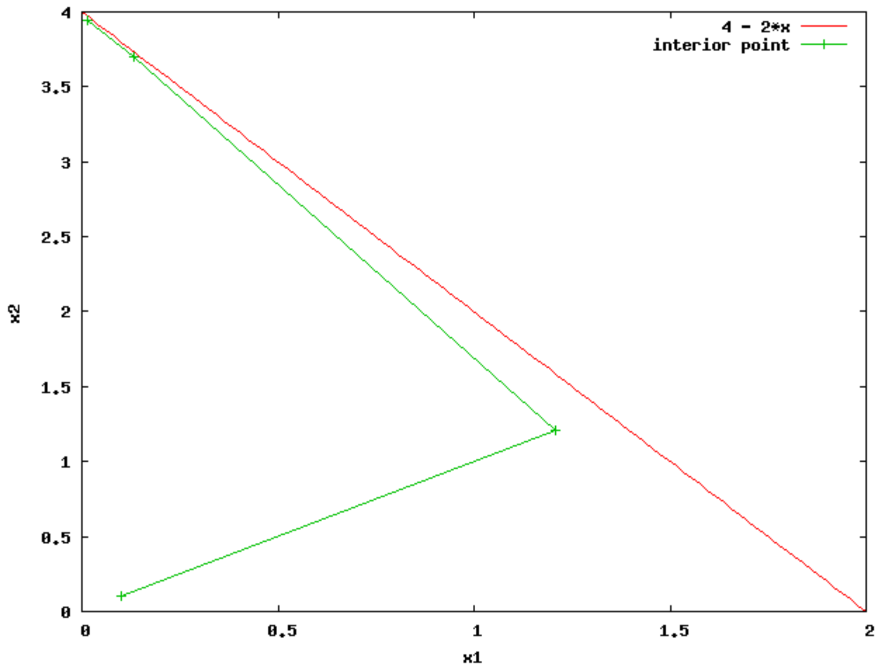
EG 510
HW 11

1. (a) Graphical solution: $x_1 = 0, x_2 = 4, -x_1 - x_2 = -4$:



(b)

$$\mathbf{x}_0 = \begin{bmatrix} 0.1 \\ 0.1 \\ 3.7 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 3.7 \end{bmatrix}, \mathbf{B} = \mathbf{AD} = \begin{bmatrix} 0.2 & 0.1 & 3.7 \end{bmatrix},$$

$$\mathbf{BB}^T = \begin{bmatrix} 13.74 \end{bmatrix}, \mathbf{Dc} = \begin{bmatrix} -0.1 \\ -0.1 \\ 0 \end{bmatrix}, \mathbf{BDc} = \begin{bmatrix} -0.03 \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} -0.02183 \end{bmatrix}, \mathbf{c}_p = \begin{bmatrix} 0.0996 \\ 0.0998 \\ -0.008079 \end{bmatrix}, \theta = 1/0.008079 = 123.778, \alpha = 0.9 \Rightarrow$$

$$\mathbf{x}_1 = \begin{bmatrix} 1.209 \\ 1.212 \\ 0.37 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 1.209 & 0 & 0 \\ 0 & 1.212 & 0 \\ 0 & 0 & 0.37 \end{bmatrix}, \mathbf{B} = \mathbf{AD} = \begin{bmatrix} 2.418 & 1.212 & 0.37 \end{bmatrix},$$

$$\mathbf{BB}^T = \begin{bmatrix} 7.453 \end{bmatrix}, \mathbf{Dc} = \begin{bmatrix} -1.209 \\ -1.212 \\ 0 \end{bmatrix}, \mathbf{BDc} = \begin{bmatrix} -4.392 \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} -0.5894 \end{bmatrix}, \mathbf{c}_p = \begin{bmatrix} -0.2161 \\ 0.4977 \\ -0.2181 \end{bmatrix}, \theta = 1/0.2181 = 4.585, \alpha = 0.9 \Rightarrow$$

$$\mathbf{x}_2 = \begin{bmatrix} 0.1311 \\ 3.7 \\ 0.037 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.1311 & 0 & 0 \\ 0 & 3.7 & 0 \\ 0 & 0 & 0.037 \end{bmatrix}, \mathbf{B} = \mathbf{AD} = \begin{bmatrix} 0.2622 & 3.7 & 0.037 \end{bmatrix},$$

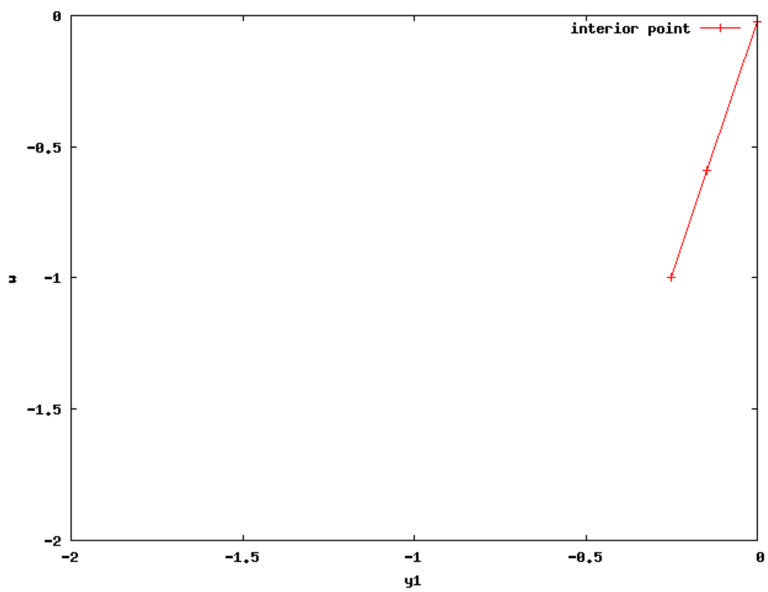
$$\mathbf{B}\mathbf{B}^T = \begin{bmatrix} 13.76 \end{bmatrix}, \mathbf{D}\mathbf{c} = \begin{bmatrix} -0.1311 \\ -3.7 \\ 0 \end{bmatrix}, \mathbf{B}\mathbf{D}\mathbf{c} = \begin{bmatrix} -13.724 \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} -0.9974 \end{bmatrix}, \mathbf{c}_p = \begin{bmatrix} -0.1304 \\ 0.0096 \\ -0.0369 \end{bmatrix}, \theta = 1/0.1304 = 7.6687, \alpha = 0.9 \Rightarrow$$

$$\mathbf{x}_3 = \begin{bmatrix} 0.0131 \\ 3.946 \\ 0.0278 \end{bmatrix}$$

(d) Dual is:

$$\begin{array}{ll} \text{maximize} & w = 4y_1 \\ \text{subject to} & 2y_1 \leq -1 \\ & y_1 \leq -1 \\ & y_1 \leq 0 \\ & y_1 \text{ free} \end{array}$$



The feasible range is $y_1 \leq -1$, and the $\mathbf{w}$'s generated in the affine scaling alrorithm are converging on the dual solution of $y_1 = -1$.

2. Original problem:

$$\begin{array}{ll} \text{minimize} & -x_1 - x_2 \\ \text{subject to} & 2x_1 + x_2 + x_3 = 1 \\ & x_i \geq 0 \end{array}$$

Phase I problem:

$$\begin{array}{ll} \text{minimize} & \lambda \\ \text{subject to} & 2x_1 + x_2 + x_3 + 3\lambda = 1 \\ & \lambda \geq 0, x_i \geq 0 \end{array}$$

Set $x_i, \lambda = 1$:

$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \mathbf{A}\mathbf{D} = \begin{bmatrix} 2 & 1 & 1 & 3 \end{bmatrix},$$

$$\mathbf{B}\mathbf{B}^T = \begin{bmatrix} 15 \end{bmatrix}, \mathbf{D}\mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{B}\mathbf{D}\mathbf{c} = \begin{bmatrix} 3 \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} 0.2 \end{bmatrix}, \mathbf{c}_p = \begin{bmatrix} 0.4 \\ 0.2 \\ 0.2 \\ -0.4 \end{bmatrix}, \theta = 1/0.4 = 2.5, \alpha = 1 \Rightarrow$$

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1.5 \\ 1.5 \\ 0 \end{bmatrix}$$