# Introduction

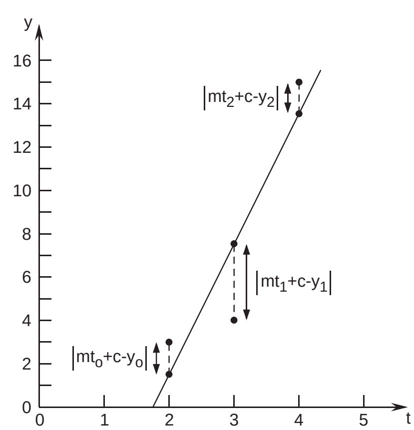
- Optimization $\equiv$ making the best decision.
- Engineering design, management, etc.
- We focus on choices that involve real numbers (examples later).
- What does "best" mean?
- Measure "goodness" by a function $f$.
  (*Cost function* or *objective function*)
- Want to minimize $f$. [Smaller = better]
- $\Omega =$ set of all possible choices.
  (*Feasible set*)
- Optimization problem:

$$\begin{array}{rl} \text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in \Omega \end{array}$$

- What about maximization? [Bigger = better]
- How to solve optimization problem?
  - Analytically
  - Numerically

**Example: Linear Regression**

- Given: points on the plane $(t_0, y_0), \ldots, (t_n, y_n)$.
- Want to find the "line of best fit" through these points.
- Best = minimize the average squared error.

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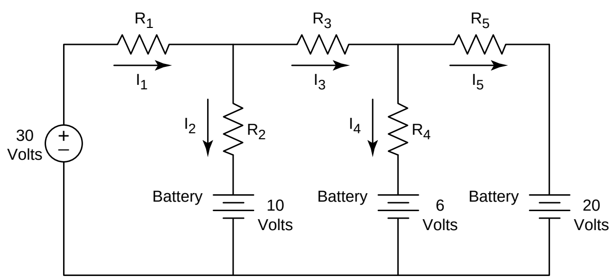
- Equation of line: $y = mt + c$;
- Optimization problem: Find $m$ and $c$ to

$$\text{minimize } \frac{1}{n}\sum_{i=0}^{n-1}(mt_i + c - y_i)^2$$

- Solution: In this case we can find the solution analytically (using least-squares theory).
- Related application: system identification.

**Example: Battery Charger**

- Charger circuit:




- Specifications:

| Current | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ |
|---|---|---|---|---|---|
| Upper Limit (Amps) | 4 | 3 | 3 | 2 | 2 |
| Lower Limit (Amps) | 0 | 0 | 0 | 0 | 0 |

- Design objective: Find $I_1, \ldots, I_5$ to maximize power transferred to batteries;
- Optimization problem:

$$\begin{aligned} \text{maximize} \quad & 10I_2 + 6I_4 + 20I_5 \\ \text{subject to} \quad & I_1 = I_2 + I_3 \\ & I_3 = I_4 + I_5 \\ & I_1 \leq 4 \\ & I_2 \leq 3 \\ & I_3 \leq 3 \\ & I_4 \leq 2 \\ & I_5 \leq 2, \\ & I_1, I_2, I_3, I_4, I_5 \geq 0. \end{aligned}$$

- This is a *linear programming problem.*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- Solution: Use Simplex algorithm, or interior point algorithm.

**Example: Savings in bank**

- Bank interest paid monthly at rate $r$ (compound).
- We wish to deposit some money into the bank every month for $n$ months, such that the total does not exceed $D$ dollars.
- Goal: maximize the total amount of money accumulated at the end of $n$ months.

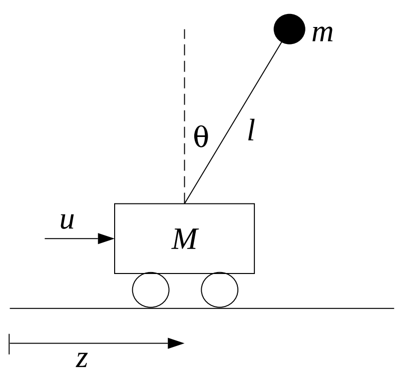
- Let $x_i$ be amount deposited in beginning of $i$th month;
- Optimization problem:

$$\begin{array}{ll} \text{maximize} & (1+r)^n x_1 + (1+r)^{n-1} x_2 + \cdots + (1+r) x_n \\ \text{subject to} & x_1 + \ldots + x_n \leq D \\ & x_1, \ldots, x_n \geq 0 \end{array}$$

- Solution: optimal strategy is to deposit $D$ dollars in the first month (use Karush-Kuhn-Tucker conditions).

**Example: Optimal Control**

Inverted pendulum system:




- Wish to move the cart to a given position in 1 sec., and balance the pendulum.
- Many possible control actions.
- Choose one that minimizes total energy.
- What if control action cannot exceed some fixed limit?

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