ST521, Assignment 5
Due Tuesday, April 15

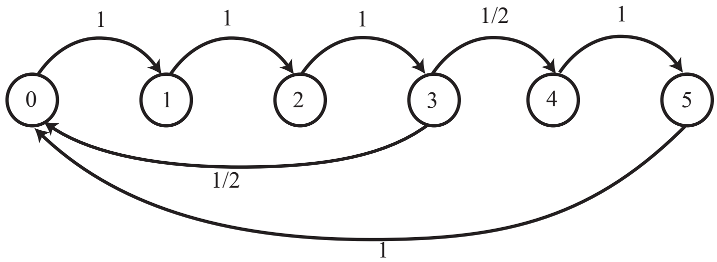
1. Customers arrive at an ATM where there is room for three customers to wait in line. Customers arrive alone with probability 2/3 and in pairs with probability 1/3, but only one may be served at a time. When a pair arrives, if both cannot join the line, then they both leave. Call a completed service or an arrival an "event". Let the state be the number of customers in the system, i.e. serviced and waiting, immediately after an event. Suppose that an event is equally likely to be an arrival or a completed service.

   (a) Compute the transition probability matrix.

   (b) Compute the stationary distribution, if it exists.

   (c) If a customer arrives, what is the probability that she finds the system either empty or full?

2. Consider the Markov chain corresponding to the following graph connecting the states (probabilities shown above arrows connecting states)



   (a) What is the smallest number of steps excluding 0 in which a state can reach itself?

   (b) What is the period of the chain?

   (c) Find the stationary distribution, if it exists.

3. Consider the success run chain in Example 3.2.9 with $q_i = p_i = 1/2$ for all i. Suppose the chain has been running for awhile and and is currently in state 10.

   (a) What is the expected number of steps until the chain is back in state 10?

   (b) What is the expected number of times the chain visits state 9 before it is back to state 10?

4. Let X_n be a Markov chain with probability transition matrix P and state space S, and let A be a subset of S. Let $T_A = \min_{n\geq 0} X_n \in A$ by the first passage time to enter A. Let $\eta_j = \mathbf{P}(T_A < \infty | X_0 = j)$. Show that

$$\eta_j = \begin{cases} 1, & j \in A, \\ \sum_{k\in S} p_{jk}\eta_k, & j \notin A. \end{cases}$$