CTA200 Assignment #3

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1. Question 1

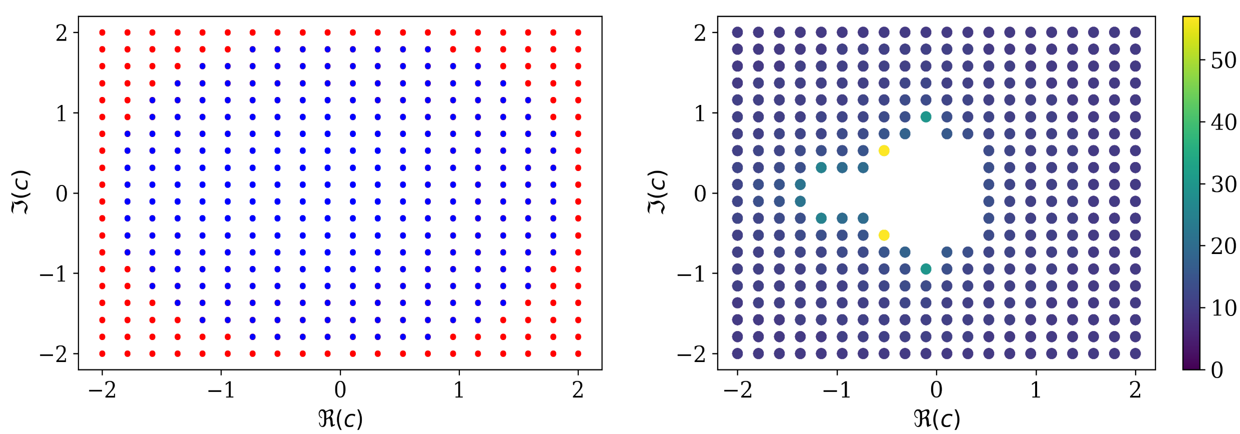
The first part of this assignment involved iterating over a complex grid, and plotting the bounded and divergent solutions in two different manners. A grid of complex points was formed with the constraints $-2 < x < 2$ and $-2 < y < 2$, using the `numpy.outer()` function, to create a 20x20 matrix of real and a second 20x20 matrix of complex points. The sum of these two matrices was then computed to obtain a grid of complex numbers within the desired constraints. Next each point within the grid was inputted into the function `iterate.py`, which iterates the equation shown in equation 1 with a maximum of 10 iterations.

$$z_{i+1} = z_i^2 + c \text{ such that } c = x + iy \tag{1}$$

The function then computes the absolute length given in equation 2, and categorizes whether the point remains bounded under the condition that $|z| <= 2$, otherwise the point diverges. The function then returns the points which remain bounded and the points which diverge and the number of iterations at which the point diverges.

$$|z|^2 = \Re(z)^2 + \Im(z)^2 \tag{2}$$

Figure 1a displays the points that diverge under these conditions in red and the points which remain bounded in blue plotted on the complex plane and Figure 1b displays the points that diverge on the complex plane with a color bar indicating the iteration number at which the given point diverged.



(a) (b)

Fig. 1.—: Bounded points (blue) and divergent points (red) plotted on the complex plane (a) and divergent points with iteration number color bar (b) for all points within a complex grid defined by $-2 < x < 2$ and $-2 < y < 2$.

2. Question 2

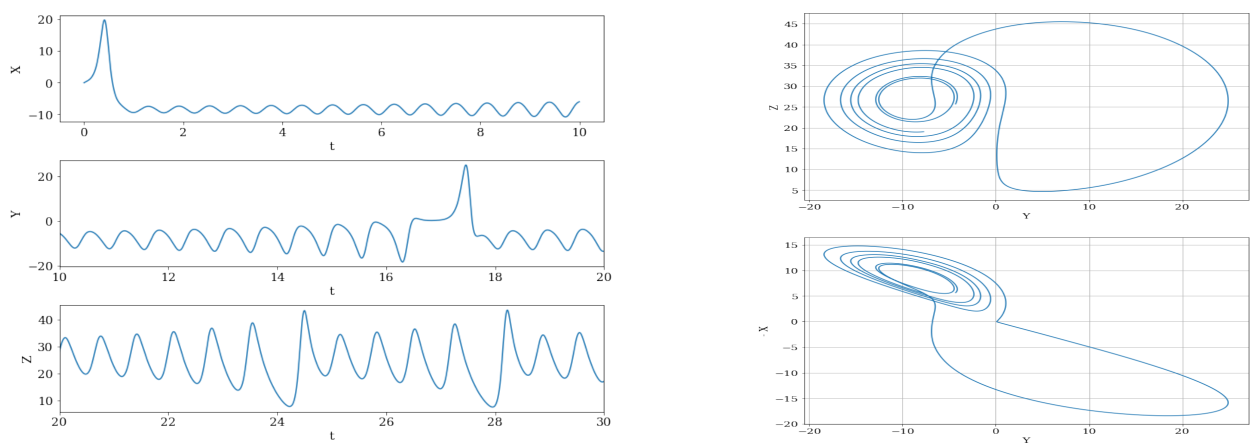
The second part of this assignment involved recreating the results found in Lorenz (1963) by modeling the behavior of the atmosphere. Lorenz (1963) defines the three equations shown below.

$$\dot{X} = -\sigma(X - Y) \tag{3}$$

$$\dot{Y} = rX - Y - XZ \tag{4}$$

$$\dot{Z} = -bZ + XY \tag{5}$$

Equations 3, 4 & 5 involve three dimensionless parameters: σ denotes the Prandtl number, r denotes the Rayleigh number and b is a length scale. Equations 3, 4 & 5 were solved with the initial condition $W_0 = [0.0, 1.0, 1.0]$ and parameters $[\sigma, r, b] = [10.0, 28.0, 8.0/3.0]$ with `scipy.integrate.solve_ivp` over $0 \le t \le 60$. Figures 1 & 2 from Lorenz (1963), were then recreated and are displayed in Figure 2.



(a) (b)

Fig. 2.—: Recreated figures from Lorenz (1963).

Figure 2a displays the solution to equation 3 over $0 \le t \le 10$ (top), the solution to equation 4 over $10 \le t \le 20$ (middle) and the solution to equation 5 over $20 \le t \le 30$ (bottom). The solution to equations 3, 4 & 5 were then computed over the time interval $14 \le t \le 19$, these solutions on the Y-Z plane are shown in Figure 2b (top) and solutions on the Y-X plane are shown in Figure 2b (bottom).

Next, the solution of equations 3, 4 & 5, were computed with the initial condition $W_0' = W_0 + [0.0, 1.0\text{e}{-8}, 0.0] = [0.0, 1.00000001, 0.0]$. The difference is defined by $W' - W$ such that W' is the set of solutions with initial condition W_0' and W is the set of solutions with initial condition W_0. Figure 3 displays the distance as a function of time on a semilog plot. The error is represented by an approximately linear line, this result agrees with the Lorenz's findings which is stated to indicate exponential growth.

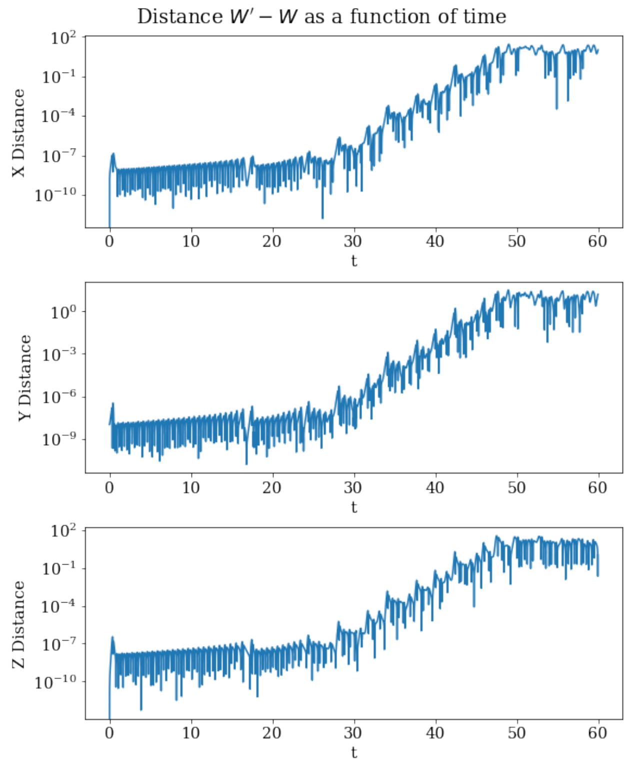


Fig. 3.—: Distance $W' - W$ as a function of time.

REFERENCES

Edward N. Lorenz. Deterministic nonperiodic flow. *Journal of Atmospheric Sciences*, 20(2):130 – 141, 1963. doi: 10.1175/1520-0469(1963)020⟨0130:DNF⟩2.0.CO;2. URL https://journals.ametsoc.org/view/journals/atsc/20/2/1520-0469_1963_020_0130_dnf_2_0_co_2.xml.

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