Scalar Matrix: It is a special kind of diagonal matrix, in which the diagonal contains the same element.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

The identity matrix is also an example of a scalar matrix.

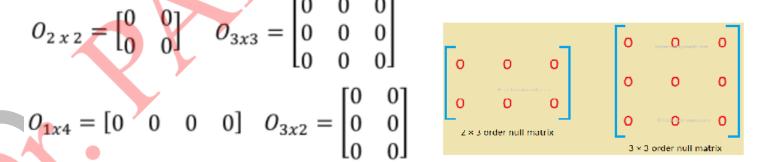
Identity Matrix: Identity Matrix is the matrix which is n × n square matrix where the diagonal consist of 1's and the other elements are all zeros. It is also called as a Unit Matrix. We represent it by I.

$$I_{1} = \begin{bmatrix} 1 \end{bmatrix} \quad I_{2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad I_{n} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Null Matrix: Null Matrix is a matrix having zero as each of its elements.

$$O_{2 \times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad O_{3 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$O_{1x4} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \quad O_{3x2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$



The Inverse of a Matrix

Inverse matrix:

A matrix **A** of dimension $\mathbf{n} \times \mathbf{n}$ (should be square matrix only) is called invertible if and only if there exists another matrix **B** of the same dimension, such that

$$AB = BA = I$$
,

where **I** is the is the identity matrix of the same order.

Matrix **B** is known as the **inverse of matrix A** and symbolically we represents **B** by A^{-1}

$$AA^{-1} = A^{-1}A = I.$$

Definitions:

- Invertible Matrix: An invertible matrix is a square matrix that has an inverse.
- Non-Singular matrix: A matrix is said to be non-singular, if its determinant value is not equal to zero, i.e. $|A| \neq 0$. Hence, inverse of any matrix is possible if and only if its determinant is non-zero. Therefore, Non-Singular matrix also known as Invertible Matrix.
- **Singular matrix:** A matrix is said to be singular, if its determinant is zero, i.e. |A|=0. Hence, inverse of Singular matrices are not possible. Therefore, **Singular matrices** are known as **Non-Invertible**.

Properties of inverse matrices:

$$\frac{1}{2}$$
 $(A^{-1})^{-1} = A$

$$\frac{3}{k}$$
 $(kA)^{-1} = \frac{1}{k}A^{-1}$, for any $k \neq 0$

$$\Psi (AB)^{-1} = B^{-1}A^{-1}$$

$$\frac{\pi}{2}$$
 $(A^n)^{-1} = (A^{-1})^n$, for any $n \in \mathbb{Z}^+$.

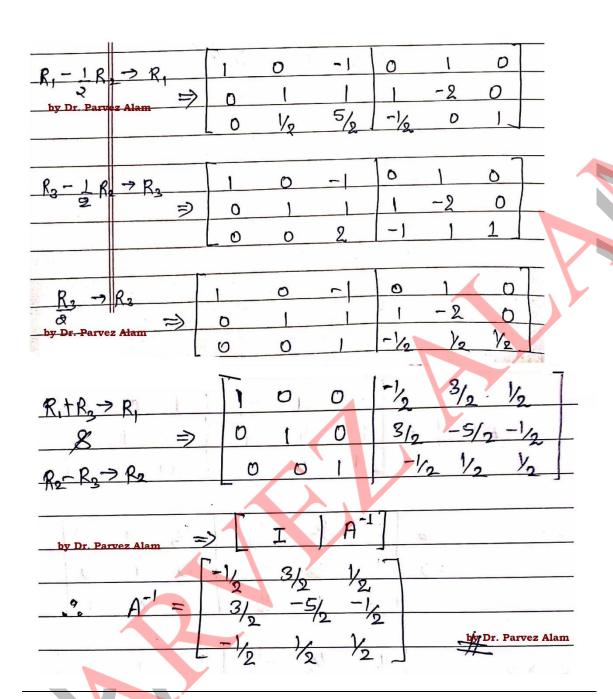
Finding inverse matrices by Gauss-Jordan Elimination method:

1. First we write the matrix A in the following form



2. Then we convert it into the Reduced Row Echelon, hence we get invers of A as

O. Find the inxerse of motivausing Grows-Jordon, method: $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$



Exercise 1: Find the inverse of following matrices;

(1).
$$A = \begin{bmatrix} 1 & 0 & 3 \\ 8 & 1 & -1 & 1 \\ 3 & -1 & 1 & 2 \\ -1 & 8 & 3 & -1 \end{bmatrix}$$
 (2). $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$ (3). $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 0 & 2 \\ 6 & 8 & 14 \end{bmatrix}$

Note: If you can't get $[I/A^{-1}]$ form, then it means the matrix is not invertible (singular). See the Ex. (3), it's not possible to get $[I/A^{-1}]$ form for this.



Dr. PARVEZ ALAM

Finding solution of a system of linear equations after finding inverse of matrix by using Gauss Jordan method

As we know that a system of 'm' linear equations of 'n' variables

$$egin{array}{l} a_{11}x_1+a_{12}x_2+a_{13}x_3+\cdots+a_{1n}x_n=b_1\ a_{21}x_1+a_{22}x_2+a_{23}x_3+\cdots+a_{2n}x_n=b_2\ a_{31}x_1+a_{32}x_2+a_{33}x_3+\cdots+a_{3n}x_n=b_3\ &dots\ a_{m1}x_1+a_{m2}x_2+a_{m3}x_3+\cdots+a_{mn}x_n=b_m \end{array}$$

can be written into the matrix from AX=B as

$$AX = B \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

where,
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Therefore, equation (1) can be written left multiplying of A^{-1} ;

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$
(2)

So, we can find solution of the system by finding A^{-1} and multiplying by B into A^{-1}

Note: If you can't get $[I/A^{-1}]$ form, then it means the coefficient matrix A is not invertible (singular), then the system could be have **no solution** or could be have **infinitely many solutions**.

Find the solution of following system of Line con.

ale com avonite the given system as AX = B

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by finding invoke of coeff mutrix

2x1 + x2-x3 = 5

x1+x2+2x3=3

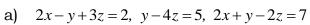
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Gauss - Jordan method.

(3.

Som: .

Exercise: Write the following system of linear equations as AX=B and then solve it by computing A^{-1} using the Gauss Jordan Elimination method.



b) x+2y+2z=10, 2x-2y+3z=1, 4x-3y-+5z=4

