ECON 5803 — Spring Semester 2018: Homework Assignment 2

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Due Date: 2/12

Problem 1: Solving the Production Model

Suppose the production function at the core of our model is given by $Y = F(K, L) = AK^{\alpha}(L)^{1-\alpha}$. The total amount of labor, $\bar{L}$, as well as capital, $\bar{K}$, is fixed and households supply these resources inelastically (i.e. they don't care what the price for these resources is). Finally, assume that firms operate in a perfectly competitive market, use the production function stated above, and hire capital and labor (at the real rental rate r and the real wage rate w) such that their profits are maximized. **Note:** The steps below are the general cook recipe to solve for a static competitive equilibrium. Remember the order of the steps as we will use them over and over again throughout this class. (1) Find the demand functions for all goods; (2) find the supply functions for all goods; (3) find the market clearing prices; (4) find the equilibrium values for all remaining endogenous variables.

(a) List all endogenous and exogenous variables.

(b) Write down the firms' maximization problem.

(c) Solve for labor and capital demand. Graph the corresponding demand function assuming that the "other input" is fixed.

(d) Add the labor and capital supply curves to the two graphs from (c).

(e) Find the market clearing real wage and real rental rate.

(f) Find the equilibrium quantities of capital, labor, and output.

Problem 2: The Black Death

In the middle of the fourteenth century, an epidemic known as the Black Death killed about a third of Europe's population, about 34 million people. While this was an enormous tragedy, the macroeconomic consequences might surprise you: Over the next century, wages are estimated to have been *higher* than before the Black Death.

(a) Use the production model from class (or problem 1) to explain why wages might have been higher.

(b) Can you attach a number to your explanation? In the model, by how much would wages rise if a third of the population died from disease? Make sure to state and justify any assumptions you made beyond the ones stated in problem 3 to come up with that number.

Problem 3: The Empirical Fit of the Production Model

In class, we investigated the empirical fit of the production model from problem 1 assuming that the production function is given by $F(K, L) = AK^{\alpha}L^{1-\alpha}$. This production function turned out to be convenient because it predicts that the equilibrium labor share is equal to $1 - \alpha$. This allowed us to pick $\alpha = 1/3$, given that historically the labor share was roughly around 2/3 for the vast majority of countries.

However, in a recent paper (http://qje.oxfordjournals.org/content/early/2013/10/24/qje.qjt032) Loukas Karabarbounis and Brent Neiman from the University of Chicago document that the labor share was in fact declining worldwide at least since 1975. Therefore, they analyze a model using a constant elasticity of substitution (CES) production function, which allows for a time varying labor share. They use a production function similar to $Y = F(K, L) = (\gamma(AK)^{\eta} + (1-\gamma)L^{\eta})^{\frac{1}{\eta}}$, with $0 < \gamma < 1$ and $\eta \leq 1$, and their empirical estimates are consistent with the following choice of parameters: $\eta = .21875$ and $\gamma = 0.5$.

In this problem I would like you to compare the empirical fit of the two alternative production structures. To do so, please take the following steps:

(a) Download the Penn World Tables 8.1 (PWT) in `Stata` format from http://www.rug.nl/research/ggdc/data/pwt/pwt-8.1.

(b) Use the following variables for your analysis: $Y =$ `cgdpe`, $L =$ `pop`, $K =$ `ck`. These output and capital variables are measured in current PPPs, which practically means in units of "current

US output". That way these numbers are all expressed in the same units and are comparable across countries. Other than `year`, and maybe `country` and/or `countrycode` you don't really need any other variables. So you can safely drop the rest.

(c) Replicate the tables on slide 33 from my class slides on cross-country income differences with predictions for both production functions. Note that the numbers won't exactly match up because you are using a newer, more up to date version of the PWT.

(d) Use the command `twoway` (http://www.stata.com/manuals13/g-2graphtwoway.pdf) to replicate the figures on slides 31 and 34. How do the predictions of the two production functions differ? Does the CES model fit substantially better or is it pretty much the same? (Hint: To illustrate the differences it is best to draw the predictions of both models in the same graph. Just make sure that the dots you plot have a different color and/or size so you can tell the difference. This link has some examples of how to combine multiple plots in one graph: http://www.ats.ucla.edu/stat/stata/modules/graph8/twoway_scatter_combine/)

(e) Write up your results as if this was a mini paper. Type it in a professional text processing software (e.g. MS Word) and create a PDF or print out your results, exactly how you want them to look like. Below are a few practical tips:

1. Use my sample code from class to get started with the analysis.
2. This page has a number useful graphing examples that may may be helpful: http://www.ats.ucla.edu/Stat/stata/library/GraphExamples/default.htm
3. To save your graphs use the command `graph export` http://www.stata.com/manuals13/g-2graphexport.pdf. I usually save graphs either as PDF or EPS files, because these files can be scaled without quality loss. You can then import the saved files into your document (e.g. in MS Word).
4. To export the tables, it might be most convenient to `browse` exactly the data you want to see. You can pick certain variables, sort the data, etc. Then you can copy/paste the cells from the `Stata` browser window (just mark everything and hit copy) directly into Excel (or Word) and make a table.

5. When producing documents with a "final layout", never send an MS Word file (or any other editable text processor file) electronically (unless you explicitly want the recipient to edit your file). If you are not delivering a printed copy, always make a PDF. The reason is that MS Word files will look completely differently on different computers. So if you care about your layout (which you should!), there is no way of predicting what it will look like on somebody else's computer, especially if they don't have the exact same version of your text processing software.

Problem 4: Physical vs. Human Capital

In 1992, Mankiw, Romer, and Weil estimated that the log of output per worker on average (across 98 countries) relates to the logs of the savings rate, s_i, and population growth, n_i. They run two different specifications and their results for a sample of 75 countries are shown below (standard errors are in parentheses underneath the point estimates):

$$\ln\left(\frac{Y_i}{L_i}\right) = \underset{(1.55)}{5.36} + \underset{(0.17)}{1.31}\ln(s_i) - \underset{(0.53)}{2.01}\ln(n_i + 0.05) \tag{1}$$

$$\ln\left(\frac{Y_i}{L_i}\right) = \underset{(0.15)}{7.10} + \underset{(0.14)}{1.43}\left[\ln(s_i) - \ln(n_i + 0.05)\right] \tag{2}$$

(a) Mankiw, Romer, and Weil (1992) interpret this as an empirical test of the Solow model. Derive two equations of this form directly from the Solow model and state what assumptions need to be made to arrive at these equations. (Hint: Start directly from the model's central equation that characterizes the balanced growth path.)

(b) Assuming Mankiw, Romer, and Weil's point estimates are correct, what range of values for α in the production function do they imply? Show how you infer this range of values from equations (5) and (6) and their theoretical counterparts that you derived in (a). Discuss.

(c) Suppose you estimate the labor share around the world at approximately 2/3. In that case, are your predictions from (c) consistent with Mankiw, Romer, and Weil's (1992) estimates? Discuss.

Problem 5: Consumption in the Solow model

Consider a Solow Model where the production function takes the particular form $F(K, AL) = K^{\psi}(EL)^{1-\psi}$.

(a) Compute the following steady state values: y^*, c^*.

(b) Since there is no explicit notion of "welfare" in the Solow model, it is somewhat hard to judge whether any particular steady state is a "good" or a "bad" equilibrium (from the viewpoint of the people living in the country under consideration). The best way to measure peoples' "happiness" within the Solow model is to look at consumption (assuming that more consumption is preferred to less).

Remember that, depending on the parameter values $(\alpha, \delta, n, g, s)$, the economy will end up in different steady states. What I'd like you to find in this problem is the steady state level of capital per effective worker, that maximizes steady state consumption. To do so, first show that steady state consumption per effective worker can be written as

$$c(k^*) = f(k^*) - sf(k^*)$$

and then find the value of k^* that maximizes consumption. This level of capital per effective worker is often called the "Golden Rule" steady state. Thus we'll call this value k^{GR}. Then compute the golden rule level of consumption per effective worker, c^{GR}.

(c) Suppose you would like to maximize peoples' long run consumption and you had the political power to dictate the aggregate savings rate. Based on the model, what savings rate would you choose? Please compute an exact expression for this savings rate.

Problem 6: Steady State Consumption with the Golden Rule

Consider the model from problem 5, with $\delta = 0.1$, $\psi = 1/3$, and $g = 0.02$.

(a) Write `Matlab` code to simulate the model.

(b) Using your simulation code, verify that your model converges to the highest level of consumption for the golden rule savings rate you found in problem 5. Specifically, illustrate your results in a graph (or a series of graphs). Report your results as professionally as possible and explain/discuss.