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2.14 Spring 2018 - Prof. Trumper
5/17/18

2.14 Lab Report - Final Design Project - Part 2

Decoupling

For inputs, given:

$$F_d = \frac{F_1 + F_2}{2}$$

$$\tau_d = (F_2 - F_1)2L$$

Therefore:

$$2F_d = F_1 + F_2 \quad \Rightarrow \quad F_1 = 2F_d - F_2$$

$$\tau_d = 2L(F_2 - F_1) \quad \Rightarrow \quad F_2 = F_d + \frac{1}{4L}\tau_d$$

$$F_1 = 2F_d - \frac{1}{4L}\tau_d - F_d = F_d - \frac{1}{4L}\tau_d$$

$$\boxed{\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-1}{4L} \\ 1 & \frac{1}{4L} \end{bmatrix} * \begin{bmatrix} F_d \\ \tau_d \end{bmatrix}}$$

For outputs, given:

$$\theta = \frac{x_2 - x_1}{2L}$$

$$x_{cg} = \frac{x_1 + x_2}{2}$$

Therefore:

$$\boxed{\begin{bmatrix} x_{cg} \\ \theta \end{bmatrix} = \begin{bmatrix} {}^{1}/_{2} & {}^{1}/_{2} \\ -{}^{1}/_{2L} & {}^{1}/_{2L} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

State Space Model

Decoupled model in MATLAB:

Already have

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + B \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

So let

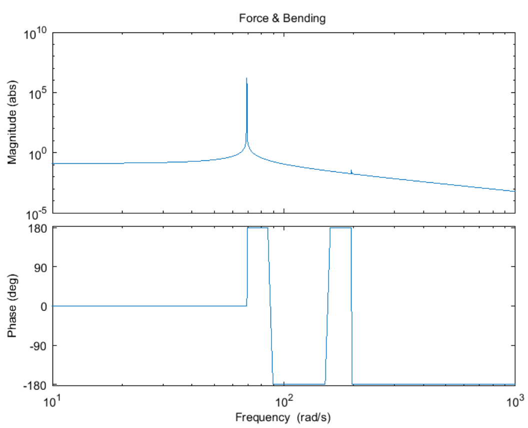
$$E \begin{bmatrix} x_{cg} \\ \dot{x}_{cg} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = F \begin{bmatrix} F_d \\ \tau_d \end{bmatrix} \qquad \Rightarrow$$

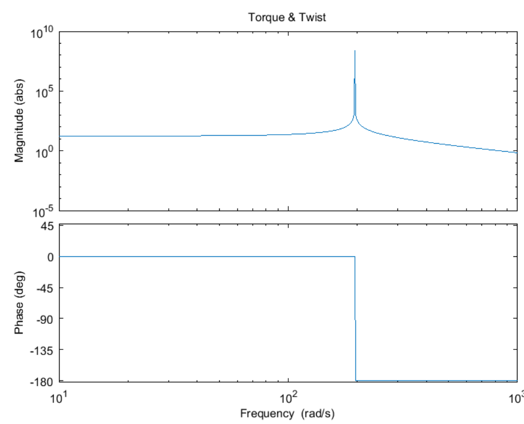
$$\begin{bmatrix} \dot{x}_{cg} \\ \ddot{x}_{cg} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = E^{-1} * A * E \begin{bmatrix} x_{cg} \\ \dot{x}_{cg} \\ \theta \\ \dot{\theta} \end{bmatrix} + E^{-1} * B * F \begin{bmatrix} F_d \\ \tau_d \end{bmatrix}$$

Such that $A_{new} = E^{-1} * A * E$ and $B_{new} = E^{-1} * B * F$

(similar conversions must occur for the C and D matrices)

This state-space model gives the following return ratio bode plots:

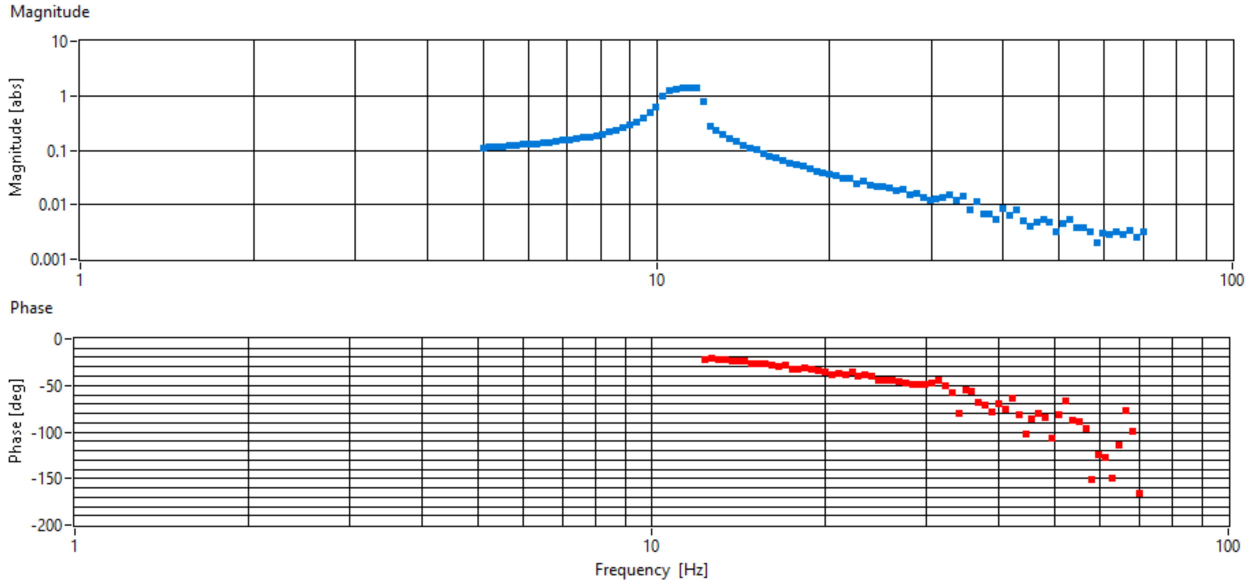




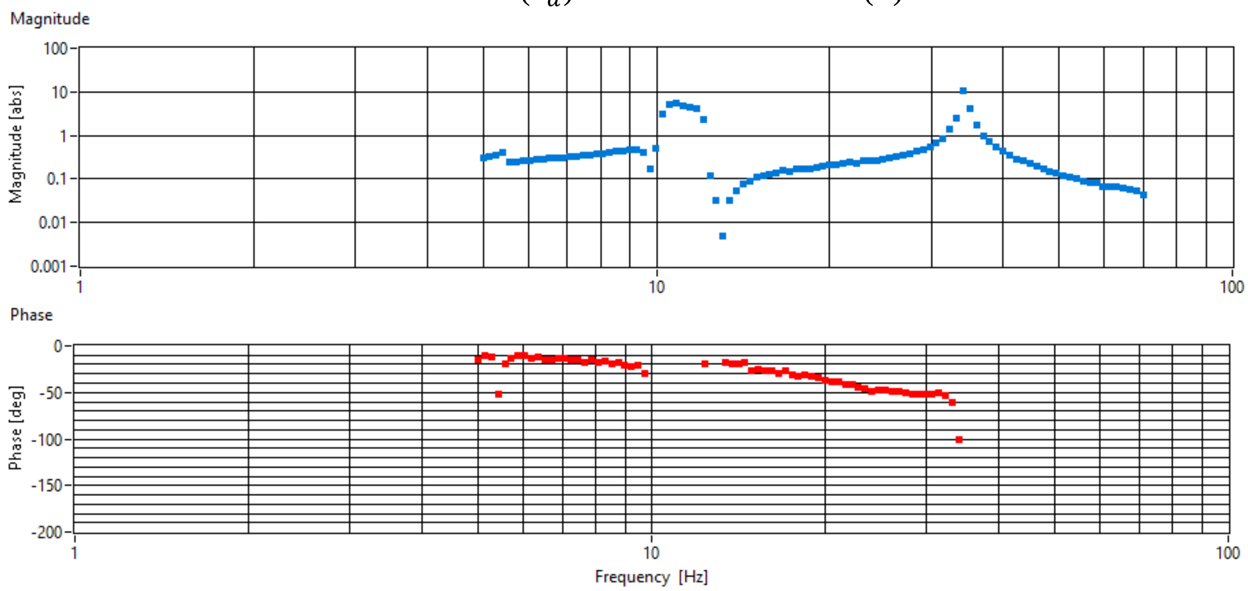
Plant Bode Plots

Using the DSA, these plant bode plots were measured physically:

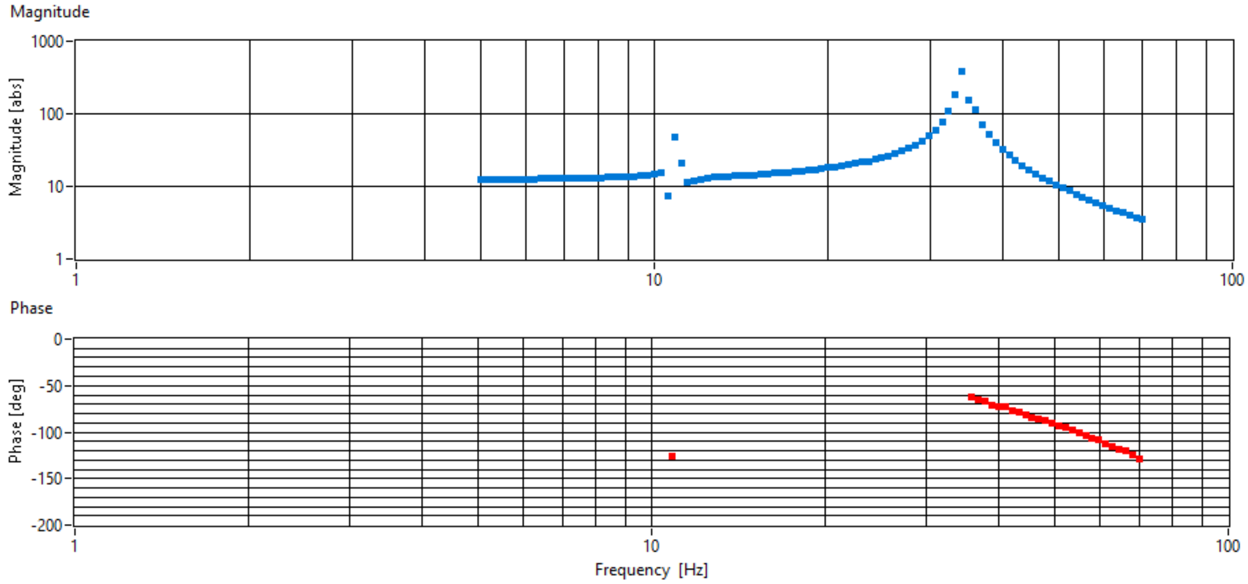
Bend control (F_d) to bend measurement (x_{cg})



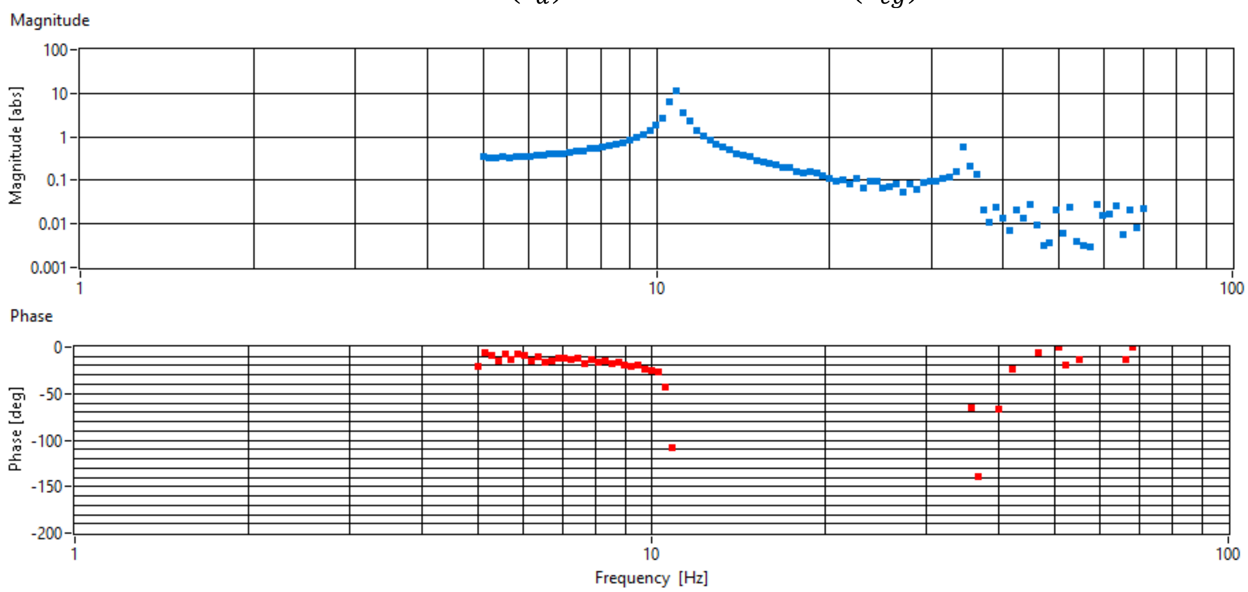
Bend control (F_d) to twist measurement (θ)



Twist control (τ_d) to twist measurement (θ)

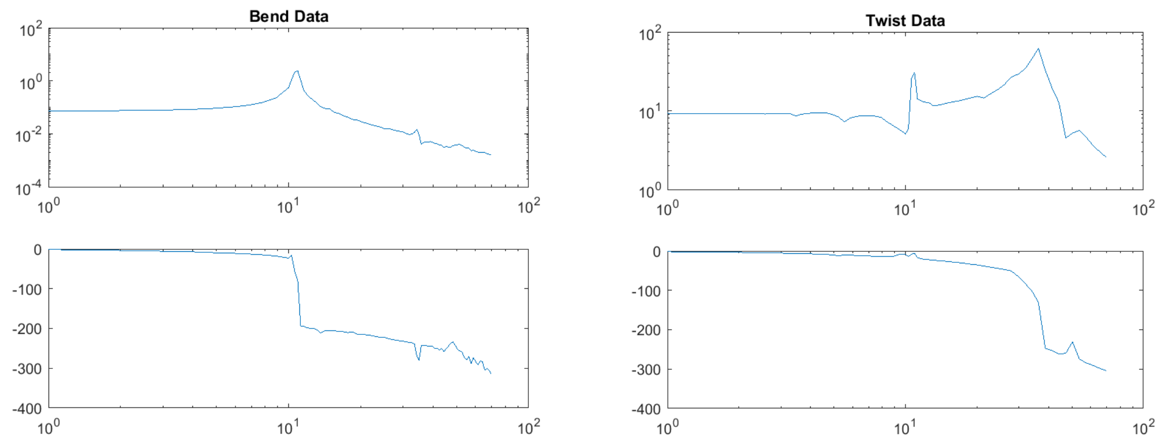


Twist control (τ_d) to bend measurement (x_{cg})

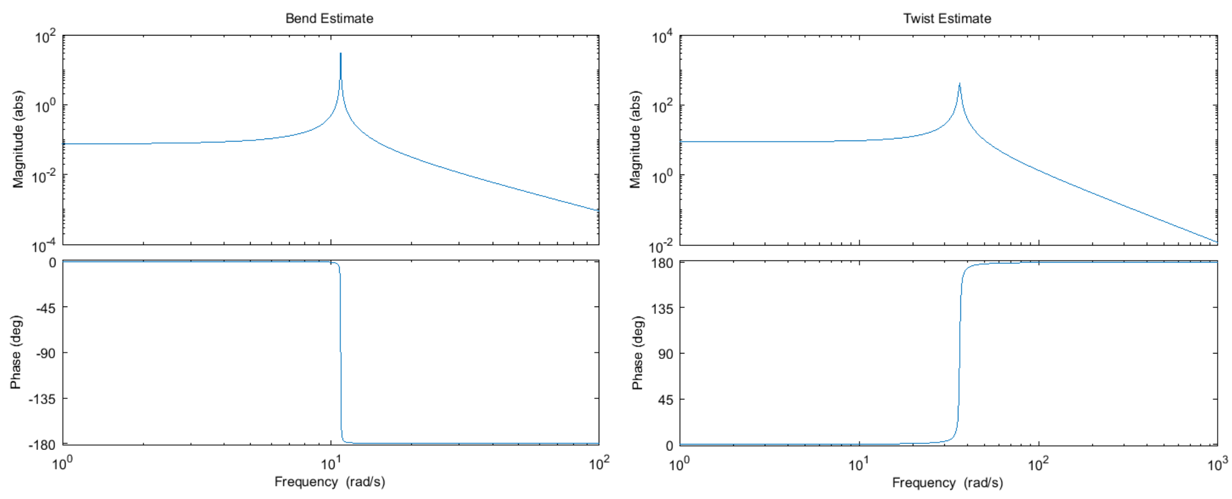


Controller Design

The following decoupled measured bode plots were put into MATLAB to estimate a transfer function for the return ratio.



The estimated return ratio transfer function (using tfest in MATLAB, plus a constant gain to make it match the DSA data) has the following bode plots:



To design the controller, we want to move the crossover frequency far to the right of the resonance peak, and increase the phase margin (for stability). To do this, a lead compensator was designed.

$$G_{lead}(s) = \frac{\alpha\tau s + 1}{\tau s + 1}$$

In addition, we want to include an integrator term to eliminate steady-state error. To this end, a lag compensator was added.

$$G_{lag}(s) = \frac{T_i s + 1}{T_i s}$$

Thus, the controllers are of the form

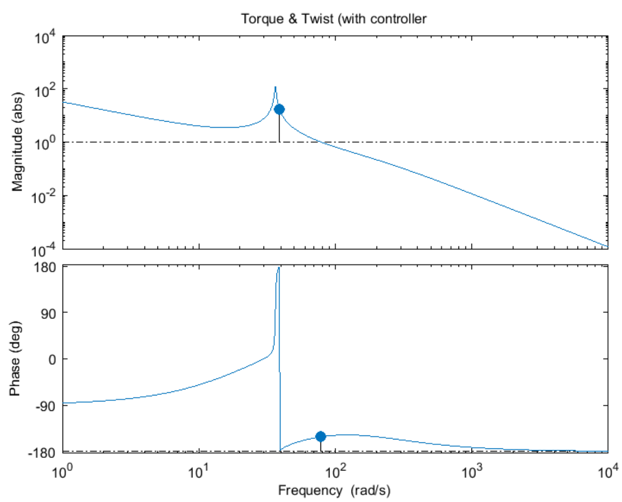
$$G_c(s) = K * G_{lead} * G_{lag}$$

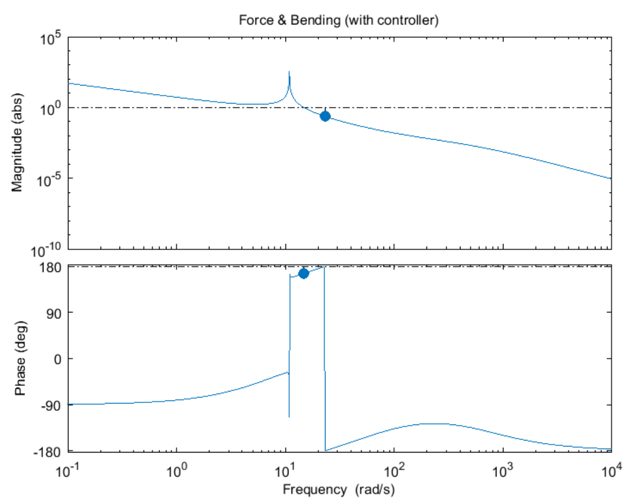
For the lead compensator, α was chosen such that the maximum phase added to the system is large, and τ was chosen such that the maximum phase occurs at the crossover frequency. For the lag compensator, T was chosen such that the change from -1 to 0 magnitude slope occurs at the resonance peak (so as to make the DC gain large). Finally, a gain K was selected such that the crossover frequency occurs after the resonance peak. However, we do not want the gain K to be absurdly large, because the response is limited by physical parameters of the system. To balance these priorities, the following values were chosen:

$$\alpha_1 = 10$$
$$\tau_1 = 0.00145$$
$$T_1 = 0.1445$$
$$K_{p1} = 10$$
$$K_{i1} = 0.006$$

$$\alpha_2 = 10$$
$$\tau_2 = 0.005$$
$$T_2 = 0.0275$$
$$K_{p2} = 0.1$$
$$K_{i2} = 0.006$$

These values result in the following theoretical return ratio bode plots, which are stable:



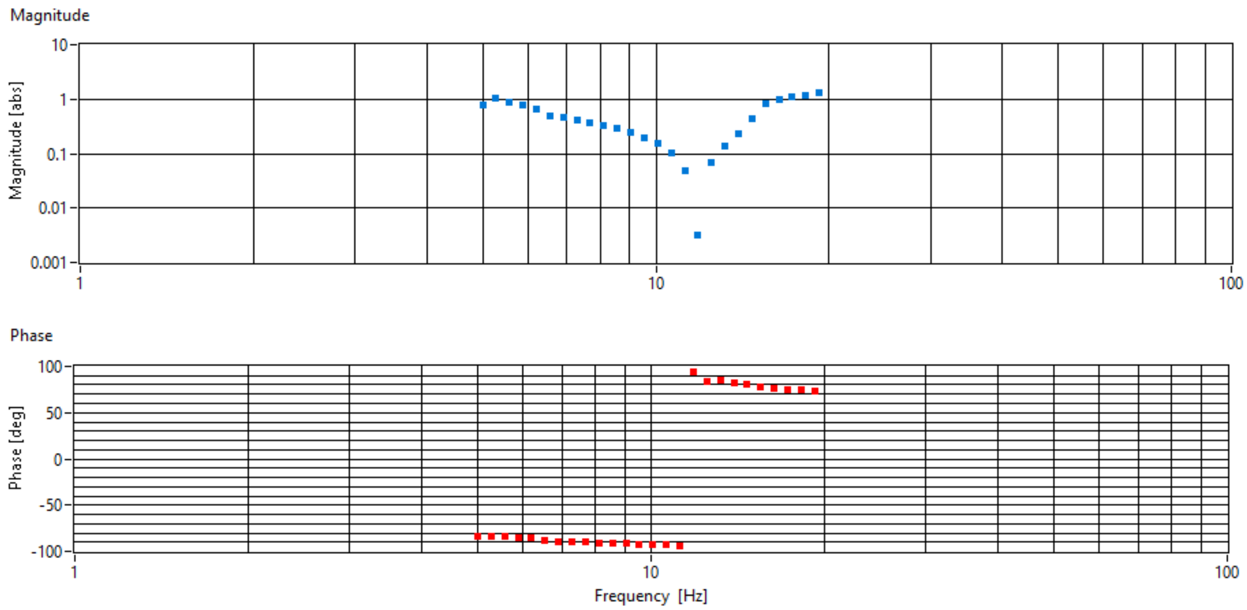


When I input this into LabVIEW, the lag compensator caused the system to vibrate and behave strangely, so in the interests of time I used **only the lead compensators** for the physical system.

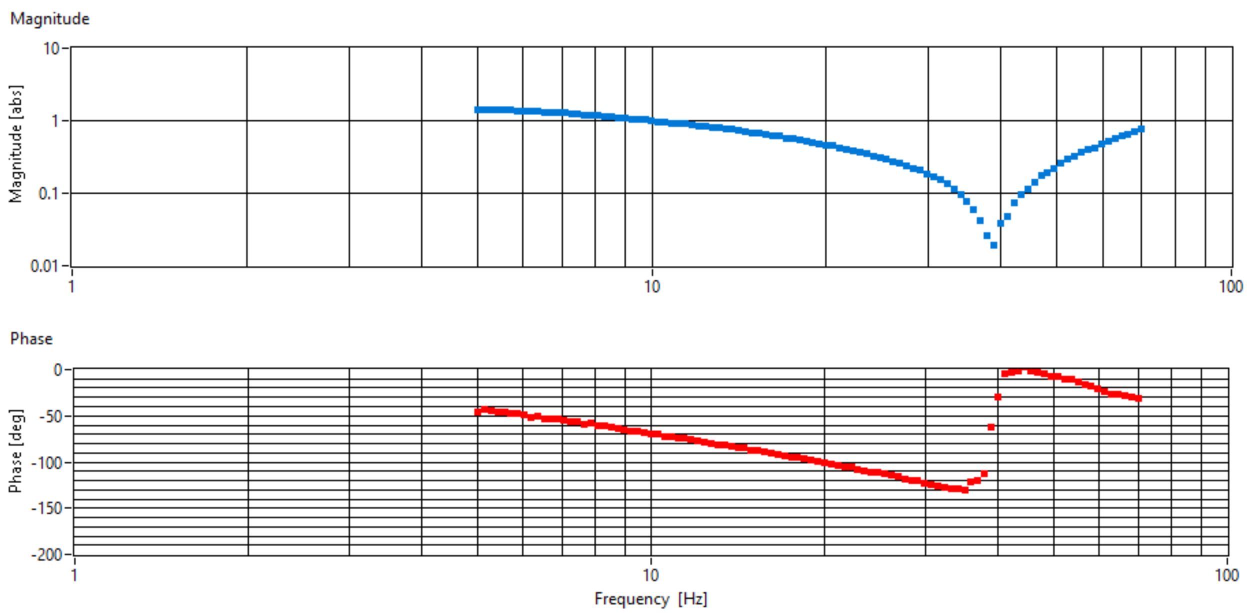
Closed-Loop Return Ratio Bode Plots

Physical return ratio bode plots are the following:

Bend



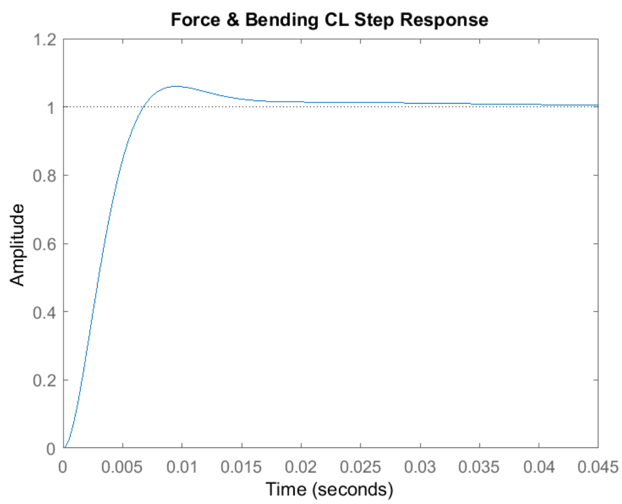
Twist

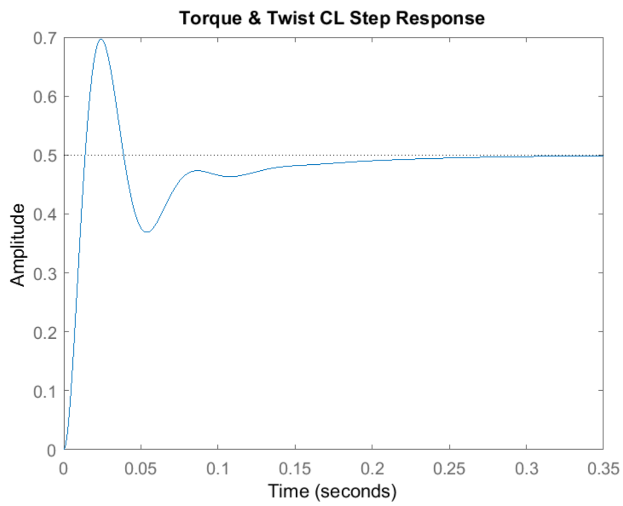


Note that the two bode plots are flipped because I multiplied the definitions of x_{cg} and θ by -1 in LabVIEW.

Step Responses

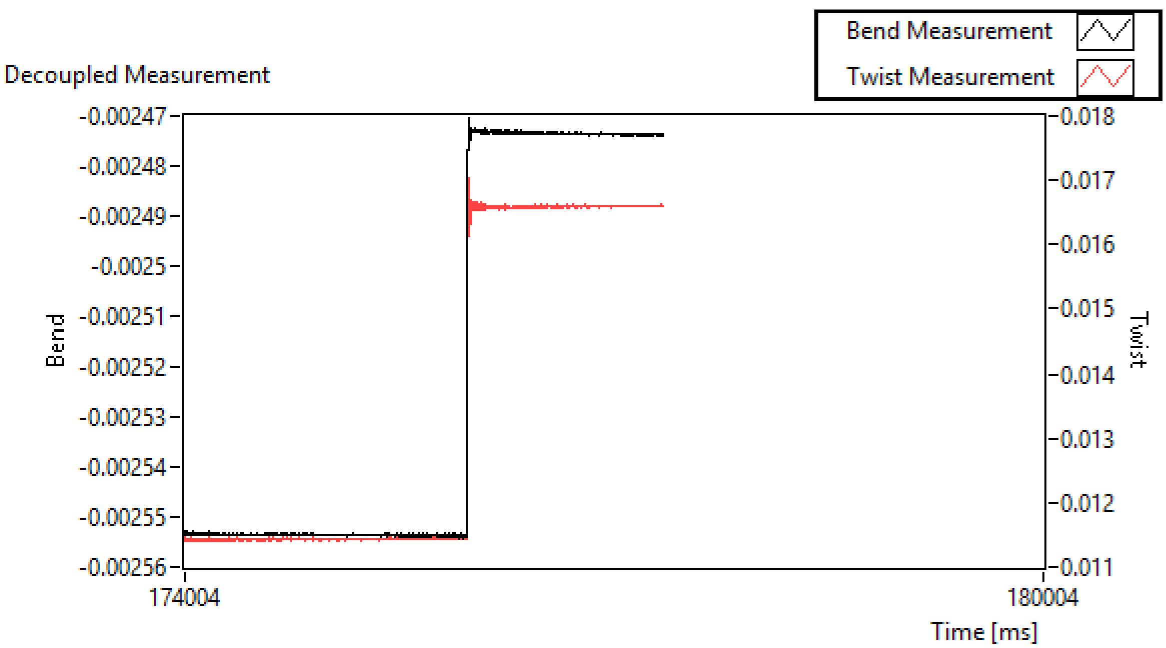
Theoretical closed-loop step responses are the following:

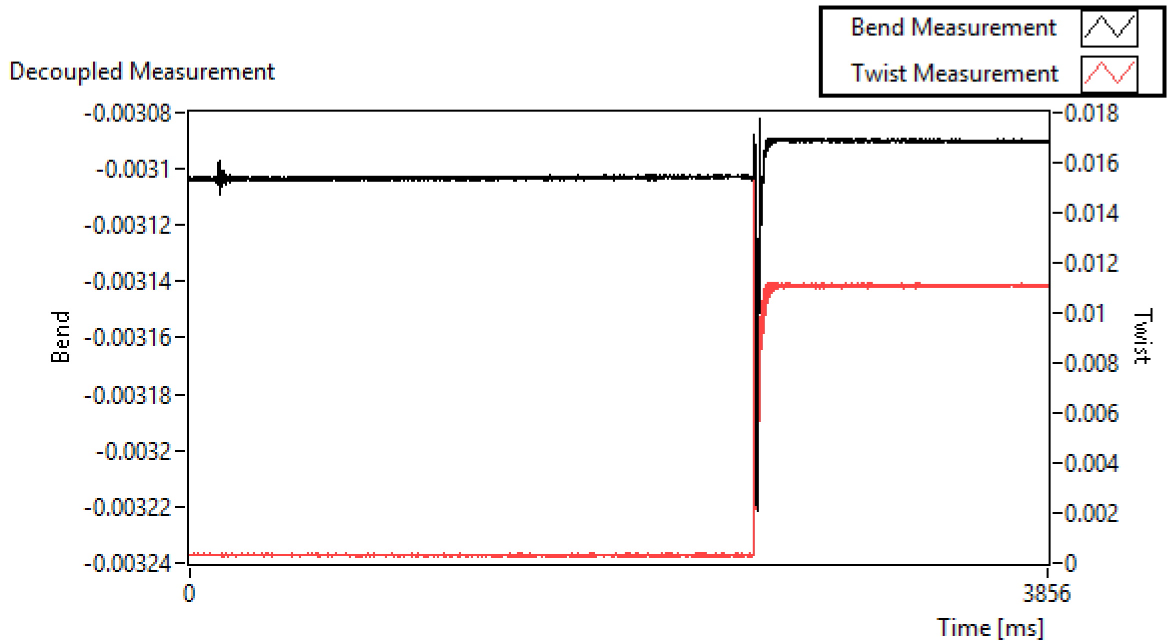




The response is acceptably fast here, as the settling time for bending is close to 0.04 seconds and the settling time for twisting is close to 0.2 seconds.

Physical decoupled step response plots are the following:

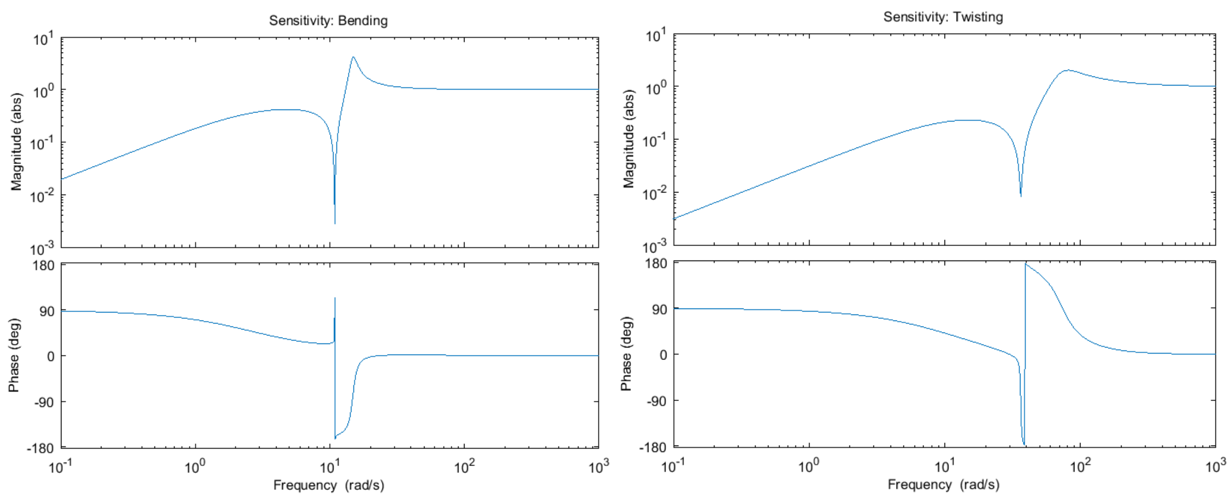




The responses look slower here than in the theoretical model, but they are quite fast from the perspective of the human eye.

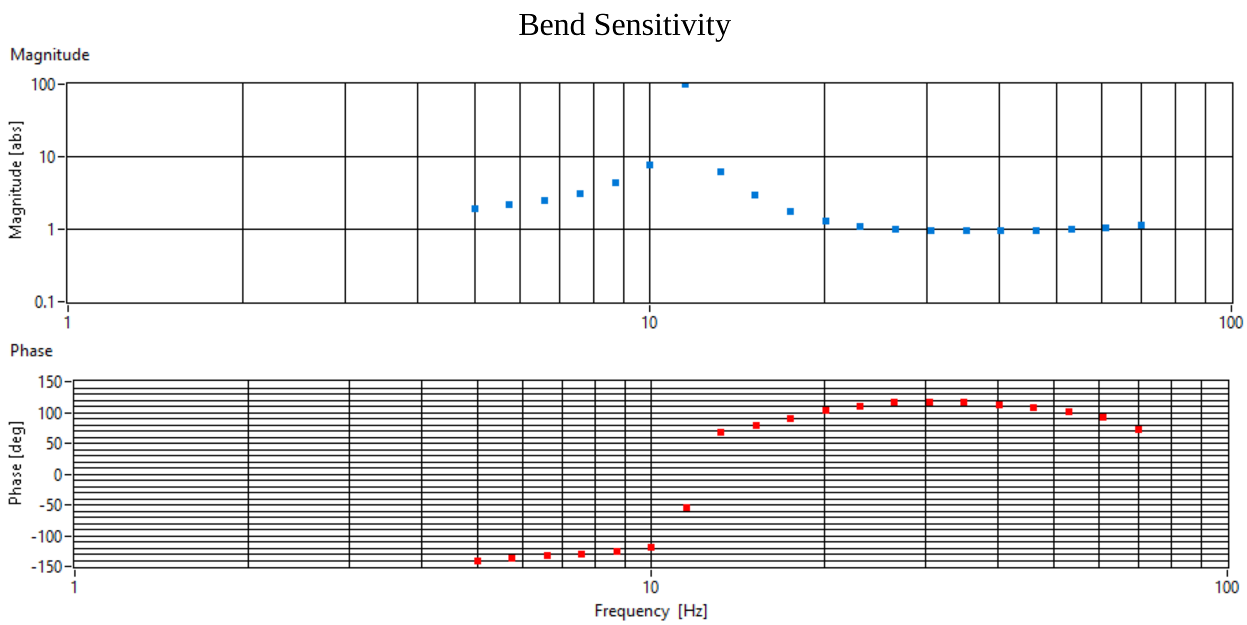
Sensitivity

Theoretical sensitivity bode plots for the decoupled system are the following:

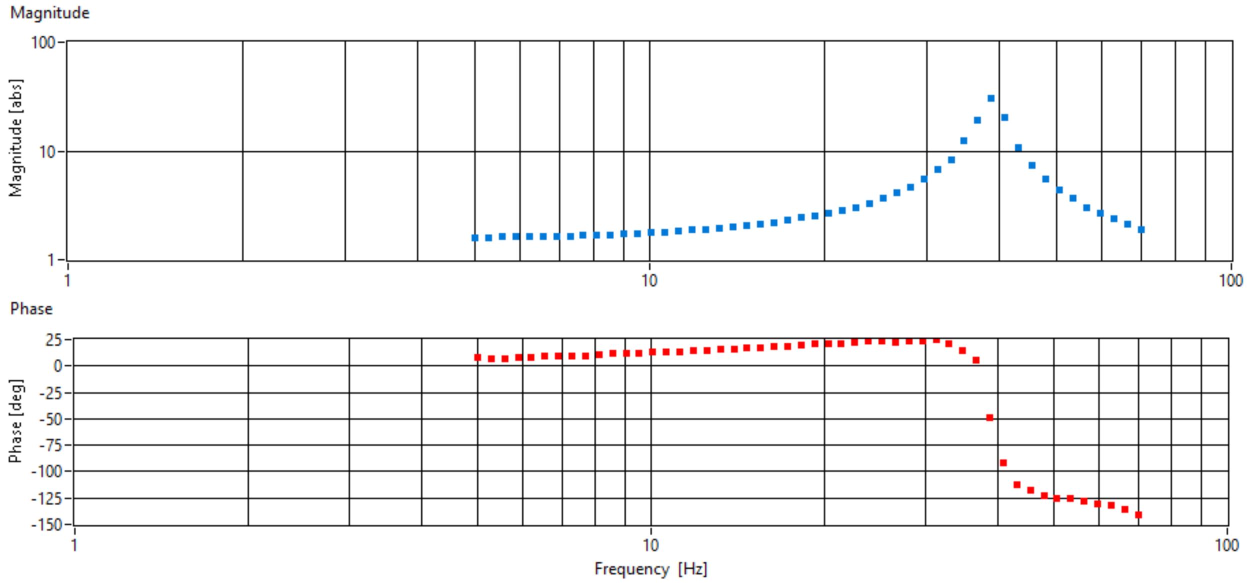


The maximum sensitivity for twisting is around 1.25, which means that the system should have good disturbance rejection. For bending, it is close to 4, which is not as good, but still acceptable.

As for experimental sensitivity bode plots, the following were obtained:



Twist Sensitivity



Here, the two bode plots are again flipped because I multiplied the definitions of x_{cg} and θ by -1 in LabVIEW. If you take this into account, the magnitude plots more or less match those of the theoretical model, and the system has acceptable disturbance rejection.