STA237 Notes

1. For A, B being mutually exclusive events, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

2. If A, B are independent, then: $A \cap B) = P(A) \cdot P(B)$.

3. For conditionally dependent events, we have $P(A|C) = \frac{P(A \cap C)}{P(C)}$, if $P(C) > 0$.

4. Conditionally dependent random variables: Suppose we had $P(Y \leq 5 | Y > 2)$. We use the definition of conditional probability to get $\frac{P(Y \leq 5, Y > 2)}{P(Y > 2)}$. Recall that the comma implies 'and'.
   We find the common area, and get $\frac{P(2 < Y \leq 5)}{P(Y > 2)}$. We then need to write this as a cumulative set-up, so we have:
   $$\frac{P(Y \leq 5) - P(Y \leq 2)}{1 - P(Y \leq 2)}$$
   where we can then use the cumulative distribution table.

5. Exponential distribution: We have $Y \sim \exp(\lambda)$ is equal to $f(y) = \lambda e^{-\lambda y}$, for $y \geq 0$.
   If $y \geq 0$,
   $$\begin{aligned} F(y) = P(Y \leq y) &= \int_0^y \lambda e^{-\lambda t} dt \\ &= [e^{-\lambda t}]_y^0 \\ &= 1 - e^{-\lambda y} \\ P(Y \leq y) &= 1 - e^{-\lambda y} \\ P(Y > y) &= e^{-\lambda y} \end{aligned}$$

6. We can partition Ω into a set of partitions $P(B_i) > 0$. Then:
   $$\begin{aligned} P(A) &= \sum_{i=k}^{k} P(A|B_i)P(B_i) \\ P(B_i|A) &= \frac{P(A \cap B_i)}{P(A)} \\ &= \frac{P(A|B_i)P(B_i)}{P(A)} && \text{Baye's Theorem} \end{aligned}$$

7. The probability mass function (pmf) is a function over the sample space giving a certain value (i.e., $f(x) = P(X = x)$.

8. Expected value: $\mu = E(X) = \sum_x xP(x)$.
   We can change the summation to an integral:
   $$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

(a) For exponents, we have:

$$\begin{aligned}E(X) &= \int_0^\infty x\lambda e^{-\lambda x}dx \\ &= [xe^{-\lambda x}]_\infty^0 + \int_0^\infty e^{-\lambda x}dx \\ &= (0-0) + [\frac{e^{-\lambda x}}{\lambda}]_\infty^0 \\ &= \frac{1}{\lambda}\end{aligned}$$

So, $E(X) = \frac{1}{\lambda}$, and $V(X) = \frac{1}{\lambda^2}$.

We can do this for everything except for normal distributions.

9. Variance: $\sigma^2 = E(x-\mu)^2 = E(x^2) - \mu^2$.
The standard deviation of X is $\sqrt{V(X)}$.

10. Binomial Distribution: When we have an experiment where we take items without replacement.
We have N as the number of times the times the experiment is conducted, and n as the sample size.
As an example:

$$P(X=x) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

11. Hypergeometric Distribution: Suppose we had $X \to 2$ blue balls in a draw of 3 balls. This can only be denoted as a success or failure.