STA237 - Activity 5

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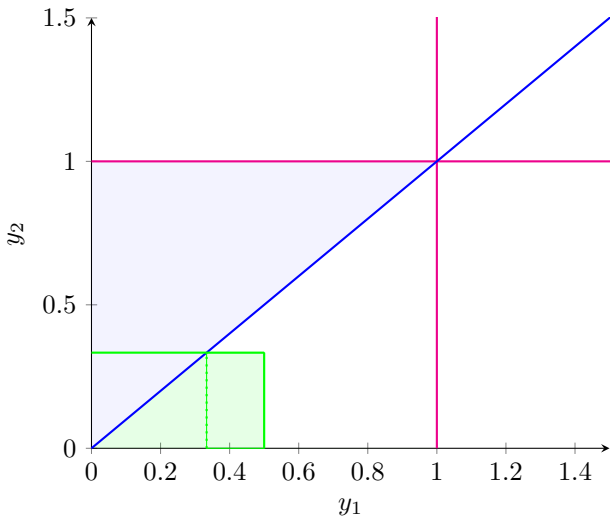
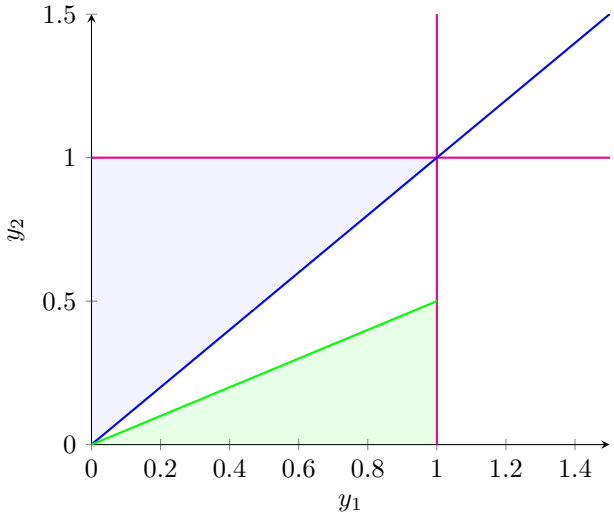
(a) We have the graph:



$$
\begin{aligned}
F(\frac{1}{2},\frac{1}{3}) &= P(Y_1 \le \frac{1}{2}, Y_2 \le \frac{1}{3}) \\
&= \int_0^{\frac{1}{3}} \left[\int_0^{y} {}_1 3y_1 dy_2\right] dy_1 + \int_{\frac{1}{3}}^{\frac{1}{2}} \left[\int_0^{\frac{1}{3}} 3y_1 dy_2\right] dy_1 \\
&= \int_0^{\frac{1}{3}} 3y_1 \left[y_2\right]_0^{y_1} dy_1 + \int_{\frac{1}{3}}^{\frac{1}{2}} 3y_1 [y_2]_0^{\frac{1}{3}} dy_1 \\
&= \int_0^{\frac{1}{3}} 3y_1^2 dy_1 + \int_{\frac{1}{3}}^{\frac{1}{2}} y_1 dy_1 \\
&= [y_1^3]_0^{\frac{1}{3}} + [\frac{y_1^2}{2}]_{\frac{1}{3}}^{\frac{1}{2}} \\
&= (\frac{1}{3})^3 + \frac{1}{2}[\frac{1}{2}^2 - \frac{1}{9}^2] \\
&= \frac{1}{27} + \frac{1}{2} \cdot \frac{9-4}{36} \\
&= \frac{1}{27} + \frac{5}{72} \\
&= \frac{23}{216} \\
&\approx 0.1065
\end{aligned}
$$

(b) We have the graph:



$$
\begin{aligned}
P\left(Y_2 \leq \frac{Y_1}{2}\right) &= \int_0^1 \left[\int_0^{\frac{y_1}{2}} 3y_1 dy_2\right] dy_1 \\
&= \int_0^1 3y_1 \left[y_2\right]_0^{\frac{y_1}{2}} dy_1 \\
&= \int_0^1 \frac{3y_1^2}{2} dy_1 \\
&= \frac{1}{2}\left[y_1^3\right]_0^1 \\
&= \frac{1}{2}
\end{aligned}
$$