STA237 - Activity 9

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1. We have $y_1, y_2, ..., y_n \sim N(\mu, \sigma)$.
   Let $n = 6$, and $df = n - 1 = 5$. Then:

$$T = \frac{\overline{y} - \mu}{s/\sqrt{n}} \sim t_5$$

   The required probability is equal to:

$$P(|\overline{y} - \mu| < \frac{2s}{\sqrt{n}})$$

   Note that $|x| < a$, and $-a < x < a$. Then, if $x > 0$, $|x| = x$. If $x < 0$, then $|x| = -x$.
   So:

$$\begin{aligned} P(|\overline{y} - \mu| < \frac{2s}{\sqrt{n}}) &= P(\frac{-2s}{\sqrt{n}} < \overline{y} - \mu < \frac{2s}{\sqrt{n}}) \\ &= -2 < \frac{y - \mu}{s/\sqrt{n}} < 2 \\ &= P(-2 < T < 2) \end{aligned}$$

   Then, we use the t distribution table.

$$\begin{aligned} P(-2 < T < 2) &= P(-2.015 < T < 2.015) \\ &= (1 - 0.05) - (0.05) \\ &= 0.9 \end{aligned}$$

2. We know that $n = 10$, and $\sigma^2 = 1$. We have $y_1, y_2, ..., y_{10} \sim N(\mu, \sigma)$. Then:

$$\frac{(n-1)s^2}{\sigma^2} \sim^2_{n-1},$$

   where $df = n - 1 = 9$. Thus:

$$\begin{aligned} P(b_1 \leq s^2 \leq b_2) &= 0.9 \\ P(\frac{(n-1)b_1}{\sigma^2} \leq \frac{(n-1)s^2}{\sigma^2} \leq \frac{(n-1)b_2}{\sigma^2} &= 0.9 \\ P(9b_1 \leq^2 \leq 9b_2) &= 0.9 \end{aligned}$$

   Using the Chi-Squared Distribution Table, we have $9b_2 = 16.919$, and $9b_1 = 3.325$. Now, we compute b_1, b_2:

$$\begin{aligned} 9b_1 &= 3.325 \\ b_1 &= 0.369 \\ 9b_2 &= 16.919 \\ b_2 &= 1.88 \end{aligned}$$