Tutorial 7

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1a)

Let X be the time it takes until the light fails, where $E(X) = 2$. Then, let Y be the time until the computer crashes, with $E(Y) = 3$. These follow an exponential distribution, so we have:

$$f(x) = \frac{1}{2}e^{-x/2}, \quad x \geq 0$$

$$f(y) = \frac{1}{3}e^{-x/3}, \quad x \geq 0$$

Because we are calculating the probability of both X and Y, we can write:

$$\begin{aligned} X \perp\!\!\!\perp Y \implies f(x,y) &= f(x) \cdot f(y) \\ &= \left(\frac{1}{2}e^{-x/2}\right) \cdot \left(\frac{1}{3}e^{-x/3}\right) \\ &= \frac{1}{6}e^{-x/2\ -\ y/3}, \qquad \text{if } x \geq 0, y \geq 0 \end{aligned}$$

Note that:

$$F(X) = \int_0^x \frac{1}{\beta}e^{-t/\beta}dt P(X \leq x) = \left[e^{-t/\beta}\right]_x^0 = 1 - e^{-x/\beta} P(X \geq x) = e^{-x/\beta} \quad \text{if } x \geq 0$$

Thus, we have:

$$\begin{aligned} P(X > 2, Y > 2) &= P(X > 2) \cdot P(Y > 2) \\ &= e^{-2/2} \cdot e^{-2/3} \\ &= e^{-5/3} \\ &\approx 0.1889 \end{aligned}$$

1b)

We define our variables:

```
S = 200000 # Simulation size
beta1 = 2 # Mean for lights failing (X)
beta2 = 3 # Mean for computer crashing (Y)
x = rexp(S, 1/beta1) # Simulated random variable X
y = rexp(S, 1/beta2) # Simulated random variable Y
```

Now, we can calculate the simulated probability of (a):

```r
count_a = rep(0, S)
for (i in 1:S) {
  if ((x[i] > 2) & (y[i] > 2)) { # Checks if it takes more than 2 hours for the failures to happen
    count_a[i] = 1  # Marks it as a success if true
  }
}
sum(count_a)/S  # Simulated probability
```

```
## [1] 0.188145
```

1c)

We want to find $P(Y - X \geq 1)$. We have:

$$
\begin{aligned}
P(Y - X \geq 1) &= \int_1^\infty \left[\int_0^{y-1} \frac{1}{6} e^{-x/2 \; - \; y/3} dx \right] dy \\
&= \int_1^\infty \frac{1}{3} e^{-y/3} \left[e^{-x/2} \right]_{y-1}^{0} dy \\
&= \frac{1}{3} \int_1^\infty e^{-y/3} \left(1 - e^{-(y-1)/2} \right) dy \\
&= \frac{1}{3} \int_1^\infty e^{-y/3} dy - \frac{1}{3} e^{1/2} \int_1^\infty e^{-5y/6} dy \\
&= \left[e^{-y/3} \right]_\infty^1 - \frac{1}{3} e^{1/2} \frac{\left[e^{-5y/6} \right]_\infty^1}{5/6} \\
&= e^{-1/3} - \frac{2}{5} e^{1/2 - 5/6} \\
&= \frac{3}{5} e^{-1/3} \\
&\approx 0.4299
\end{aligned}
$$

1d)

```r
count_b = rep(0, S)
for (i in 1:S) {
  if ((y[i] - x[i]) >= 1) {
    count_b[i] = 1
  }
}

sum(count_b)/S # Simulated Probability
```

```
## [1] 0.43125
```