STA237 - Activity 2

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1. (a) Using the probability density function property $\int_{-\infty}^{\infty} f(y)dy = 1$, we calculate for $0 \leq y \leq 2$:

$$\begin{aligned}
\int_0^2 c(2-y)dy &= 1 \\
c\int_0^2 2-y\ dy &= 1 \\
c\left[2y - \frac{y^2}{2}\right]_0^2 &= 1 \\
c[4-2] &= 1 \\
2c &= 1 \\
c &= \frac{1}{2}
\end{aligned}$$

Therefore, $f(y) = \frac{1}{2}(2-y)$ for $0 \leq y \leq 2$, and 0 otherwise.

(b) We want to find the distribution function $F(y)$. From (a), we have, for $0 \leq y \leq 2$:

$$\begin{aligned}
F(y) &= P(Y \leq y) \\
&= \int_0^y \frac{1}{2}(2-t)dt \\
&= \frac{1}{2}\left[2t - \frac{t^2}{2}\right]_0^y \\
&= \frac{1}{2}\left(2y - \frac{y^2}{2}\right) \\
&= y - \frac{y^2}{4}
\end{aligned}$$

Hence, our distribution function is:

$$F(y) = \begin{cases} 0 & y < 0 \\ y - \frac{y^2}{4} & 0 \leq y \leq 2 \\ 1 & y > 2 \end{cases}$$

(c) We want to find $P(1 \leq Y \leq 2)$, which is equivalent to $P(Y \leq 2) - P(Y < 1)$. We have:

$$\begin{aligned}
P(Y \leq 2) - P(Y < 1) &= F(2) - F(1) \\
&= (2 - \frac{2^2}{4}) - (1 - \frac{1}{4}) \\
&= 1 - \frac{3}{4} \\
&= \frac{1}{4}
\end{aligned}$$

Therefore, $P(1 \leq Y \leq 2) = \frac{1}{4}$.

(d) We want to find $\mu = E(Y)$.

$$
\begin{aligned}
E(Y) &= \int_0^2 y \cdot \frac{1}{2}(2-y)dy \\
&= \frac{1}{2}\int_0^2 2y - y^2 \ dy \\
&= \frac{1}{2}\left[y^2 - \frac{y^3}{3}\right]_0^2 \\
&= \frac{1}{2}\left(4 - \frac{8}{3}\right) \\
&= 2 - \frac{2}{3}
\end{aligned}
$$

Therefore, $E(Y) = \frac{2}{3}$.

(e) We want to find $V(Y)$. Calculate $E(Y^2)$:

$$
\begin{aligned}
E(Y^2) &= \int_0^2 y^2 \frac{1}{2}(2-y)dy \\
&= \frac{1}{2}\int_0^2 2y^2 - y^3 dy \\
&= \frac{1}{2}\left[\frac{2y^3}{3} - \frac{y^4}{4}\right]_0^2 \\
&= \frac{1}{2}\left(\frac{16}{3} - \frac{16}{4}\right) \\
&= \frac{2}{3}
\end{aligned}
$$

Now, we can calculate the variance:

$$
\begin{aligned}
\sigma^2 = V(Y) &= E(Y^2) - \mu^2 \\
&= \frac{2}{3} - (\frac{2}{3})^2 \\
&= \frac{2}{9}
\end{aligned}
$$