STA237 Notes

Contents

1 Introduction

1.1 Basic Definitions

1. Scientific Question - A question created by an experimenter.
2. Experiment - A task to collect information in order to answer a scientific question.
3. Sample Space (Ω) - The set of all possible outcomes or results of an experiment.
   For example, $\Omega = \{H, T\}$ is the sample space of tossing a coin.
4. Subsets of the sample space are called events.
   Events all use typical set operations (complements, union, intersection, etc.).

1.2 Properties of Events

1. We call events A, B mutually exclusive if A, B have no outcomes in common. That is, $A \cap B = \emptyset$
2. **Demorgan's Law** - For any two events A, B, we have $(A \cup B)^c = A^c \cap B^c$, and $(A \cap B)^c = A^c \cup B^c$.
3. A **Probability Function** (P) on a finite sample space Ω assigns to each event in A in Ω a number $P(A)$ in $[0, 1]$ such that:
   (a) $P(\Omega) = 1$, and
   (b) $P(A \cup B) = P(A) + P(B)$, if A, B are disjoint.
   The number $P(A)$ is the probability for which A occurs.

   Suppose we had two events A, B, and $P(A) \cap P(B) \neq \emptyset$. We have:

   (a) Elements of ONLY A: $A \cap B^c$
   (b) Elements of A AND B: $A \cap B$
   (c) Elements of ONLY B: $B \cap A^c$

   Then:

   (a) $P(A) = P(A \cap B^c) + P(A \cap B)$
   (b) $P(B) = P(B \cap A^C) + P(A \cap B)$
   (c) $P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(B \cap A^c)$
   Then: $P(A \cup B) = P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(A \cap B)$
   $= P(A) + P(B) - P(A \cap B)$

   Therefore, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

   We know that $P(A) \subseteq P(\Omega)$, and the complement A^c is mutually exclusive. $P(\Omega) = 1$, and thus:

   $$P(\Omega) = 1 = P(A^c) + P(A)$$

   Therefore: $P(A^c) = 1 - P(A)$.
4. A and B are **independent** if $P(A \cap B) = P(A) \cdot P(B)$.

1.2.1 Axioms

Suppose Ω is a sample space associated with an experiment. To every event A in Ω, we assign a number $P(A)$ (called the probability of A), so that the following axioms hold:

1. Axiom 1: $P(A) \geq 0$
2. Axiom 2: $P(S) = 1$
3. Axiom 3: If $A_1, A_2, ..., A_n$ form a sequence of pairwise mutually exclusive events in Ω (that is, $A_i \cap A_j = \emptyset$ if $i \neq j$), then
$$P(A_1 \cup A_2 \cup ... \cup A_n) = \sum_{i=1}^{n} P(A_i)$$

1.3 Tools for Counting Sample Points

With m elements $a_1, a_2, ..., a_m$, and $b_1, b_2, ..., b_n$, it is possible to form $mn = m \times n$ pairs containing one element from each group.

An ordered arrangement of r distinct objects is called a **permutation**. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol P_r^n. That is:

$$P_r^n = n(n-1)(n-2)...(n-(r+1)) = \frac{n!}{(n-r)!}$$

The number of unordered subsets of size r chosen (without replacement from n available objects is:

$$\binom{n}{r} = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

Sometimes it is denoted as C_r^n.

2 Conditional Probability

Conditional probability is the likelihood of an event occurring based on the occurrence of a previous event. That is, for two events R, L, the conditional probability of R given L is $P(R|L)$.
It is denoted by:
$$P(A|C) = \frac{P(A \cap C)}{P(C)},$$

provided $P(C) > 0$.

Note that $P(R|L) + P(R^c|L) = 1$:

$$\begin{aligned} P(R|L) + P(R^c|L) &= \frac{P(A \cap C}{P(C)} + \frac{P(A^c \cap C}{P(C)} \\ &= \frac{P(C)}{P(C)} && \text{Since } P(A), P(A^c) \text{ are mutually exclusive, the union of the intersections is } P(\\ &= 1 \end{aligned}$$

For example, suppose we had the following events:

1. L: Born in a long month (31 days)
$L = \{Jan, Mar, May, Jul, Aug, Oct, Dec\}$;

2. R: Born in a month with letter r
   $R = \{Jan, Feb, Mar, Apr, Sep, Oct, Nov, Dec\}$

This means that the conditional probability of R given L is:

$$\begin{aligned} P(R|L) &= \frac{1/3}{7/12} \\ &= \frac{4}{7} \end{aligned}$$

2.0.1 Multiplication Rule

For any events A, C:

$$\begin{aligned} P(A|C) &= \frac{P(A \cap C)}{P(C)} \\ P(A \cap C) &= P(A|C) \cdot P(C) \end{aligned}$$

2.1 Independent Events

Events A, C are **independent** if and only if the probability of A is the same when we know that C has occurred. That is:

$$P(A|C) = P(A)$$

Then:

$$\begin{aligned} \frac{P(A \cap C)}{P(C)} &= P(A) \\ P(A \cap C) &= P(A) \cdot P(C) \end{aligned}$$

2.2 Partitions

For some positive integer k, let the sets $B_1, B_2, ..., B_k$ be such that:

1. $\Omega = B_1 \cup B_2 \cup ... \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $\imath \neq j$.

Then, the collection of sets $\{B_1, B_2, ..., B_k\}$ is said to be a partition of Ω.

2.2.1 The Law of Total Probability

Suppose that $\{B_1, B_2, ..., B_k\}$ is a partitions of Ω such that $P(B_i) > 0$ for $i = 1, 2, ..., k$. Then, for any event A:

$$\begin{aligned} P(A) &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + ... + P(A|B_k)P(B_k) \\ &= \sum_{i=1}^{k} P(A|B_i)P(B_i) \end{aligned}$$

2.3 Bayes' Theorem

Suppose that $\{B_1, B_2, ..., B_k\}$ is a partition of Ω such that $P(B_i) > 0$, for $i = 1, 2, ..., k$. Then, for any event A:

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

3 Random Variables

Discrete Variables are variables whose values can be measured by counting.
For example, a course mark: 0, 1, 2, ..., 100

Continuous Variables are impossible to count and can never properly be counted.
For example, time or weighs: 25 years, 10, months, ...

Categorical Variables take on a finite number of possible values, assigning units of observation to particular groups on the basis of qualitative properties.

For some event with sample space Ω taking multiple parameters ($e.g., \Omega = \{\sigma_1, \sigma_2\} : \sigma \in \{1, 2\}$), we can calculate the total outcome, i.e., the value of the function $X : \Omega \to \mathbb{R}$, given by:

$$X(\sigma_1, \sigma_2) = \sigma_1 + \sigma_2 \text{ for } (\sigma_1, \sigma_2) \in \Omega$$

We denote the event that the function S attains the value k by:

$$\{X = k\} = \{(\sigma_1, \sigma_2) \in \Omega : X(\sigma_1, \sigma_2) = k\}$$

We call X the **random variable**.
$X : \Omega \to \mathbb{R}$ is a **discrete random variable** if it takes on a finite number of values $a_1, a_2, ..., a_n$, **or** an infinite number of values $a_1, a_2, ...$
The probability that X takes on the value x, $P(X = x)$ is the sum of probabilities of all sample points in Ω that are assigned to the value x (i.e., $P(x) = P(X = x)$). We sometimes denote this as $p(x)$.
Then, the probability distribution of a discrete variable X can be represented by a formula, a table, or a graph that provides $P(X = x)$ for all x.

3.0.1 Result

For any discrete probability distribution, the following must be true:

1. $0 \leq p(x) \leq 1$ for all x
2. $\sum_x p(x) = 1$, where the summation is over all values of x with non-zero probability.

3.1 Expected Values of Random Variables

Let X be a discrete random variable with the probability function $p(x)$. Then, the expected value of X, $E(X)$, is defined as:

$$E(X) = \sum_x xP(x),$$

where $P(x) = P(X - x)$. Note that $E(x) = \mu = \sum_x xP(x)$.

3.1.1 Variance of Random Variables

If X is a random variable with mean $E(X) = \mu$, then the variance of a random variable X is the expected value of $(X - \mu)^2$. That is:

$$\sigma^2 = V(X) = E[(-\mu)^2]$$

The standard deviation of X is the positive square root of $V(X)$, or σ.

3.1.2 Results

1. Let X be a discrete random variable with probability function $p(x)$, and let c be a constant. Then,

$$\begin{aligned} E(c) &= \sum_x c \sum P(x) \\ &= c \cdot 1 \\ &= c \end{aligned}$$

   Therefore, $E(c) = c$.

2. Note that for the variance:

   (a)

$$\begin{aligned} V(c) &= E((c-\mu)^2) \\ &= E((c-c)^2) \\ &= 0 \end{aligned}$$

   (b)

$$\begin{aligned} V(cX) &= c^2 V(X) \\ V(aX+b) &= a^2 V(X) \end{aligned}$$

3. Let X be a discrete random variable with probability function $p(x)$, $g(x)$ be a function of X, and let c be a constant. Then:

$$\begin{aligned} E(cx) &= cE(x) \\ &= E[ax+b] \\ &= aE(x)+b \end{aligned}$$

   Therefore, $E[cg(X)] = cE(g(X))$.

4. Let X be a discrete random variable with probability function $p(x)$, and $g_1(X), g_2(X), ..., g_k(X)$ be k functions of X. Then:

$$E[g_1(X) + g_2(X) + ...g_k(X)] = E[g_1(X)] + E[g_2(X)] + ... + E[g_k(X)]$$

3.2 Distribution Function

The distribution function F of a random variable X is the function $F : \mathbb{R} \to [0,1]$, defined by:

$$F(a) = P(X \leq a) \text{ for } -\infty < a < \infty$$

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STA237 Exercises

Contents

4 Exercises

4.1 Probability

1. Suppose $P(A) = 0.5, P(A \cap B) = 0.2$, and $P(A \cup B) = 0.7$. Find:

   (a) $P(B)$

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
0.7 &= 0.5 + P(B) - 0.2 \\
0.7 &= 0.3 + P(B) \\
P(B) &= 0.4
\end{aligned}
$$

   (b) $P(\text{exactly one of two events occurs})$
   This means the probability of no elements from the intersection. Let this area be $P(O)$.

$$
\begin{aligned}
P(A \cup B) &= P(O) + P(A \cap B) \\
0.7 &= P(O) + 0.2 \\
P(O) &= 0.5
\end{aligned}
$$

   (c) $P(\text{neither event occurs})$
   This means the probability of no elements in A, B. Call this probability $P(X)$.

$$
\begin{aligned}
P(X) &= P(\Omega) - P(A \cup B) \\
&= 1 - 0.7 \\
&= 0.3
\end{aligned}
$$

2. Suppose we throw a coin two times. The sample space for this experiment is:

$$\Omega = \{H, T\} \times \{H, T\} = \{(H, H), (H, T), (T, H), (T, T)\}$$

   If it is a fair coin, then all four outcomes have equal possibilities:

$$P((H, H)) = P((H, T)) = P((T, H)) = P((T, T)) = \frac{1}{4}$$

   What is the probability of getting at least one tail?
   Looking at the sample space, the probability is $\frac{3}{4}$.
   Alternatively, we can solve as follows, where RP represents the required probability:

$$
\begin{aligned}
P(\Omega) &= RP + P(HH) \\
1 &= RP + P(HH) \\
&= RP + P(H)P(H) && \text{Since it is independent.} \\
&= RP + \frac{1}{2} \cdot \frac{1}{2} \\
1 &= RP + \frac{1}{4} \\
RP &= \frac{3}{4}
\end{aligned}
$$

   Suppose we wanted to throw three coins, what would happen to the sample space?

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Then, there are $2 \cdot 2 \cdot 2 = 2^3$ possibilities (to keep on with this pattern, flipping a coin 6 times would have 2^6 possibilities, and so on).
What is the probability of at least one head appearing in a 3-coin toss?

$$\begin{aligned}
P(X) &= 1 - P(TTT) \\
&= 1 - P(T)P(T)P(T) \\
&= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
&= 1 - \frac{1}{8} \\
&= \frac{7}{8}
\end{aligned}$$

3. A survey of students in a class was asked about their birth order (1 = oldest/only child), and which college of the university they were enrolled in.

Birth Order	1 or Only	2 or More	Total
Arts	Science	34	23
57			
Agriculture	52	41	93
Human Ecology	15	28	43
Other	12	17	30
Total	113	110	223

Suppose we selected a random student from this class.

(a) What is the probability we select a Human Ecology student?
P(Human Ecology) = $\frac{43}{223}$

(b) What is the probability we select a first-born student?
P(First Born) = $\frac{113}{223}$

(c) What is the probability that the student is a first-born and a Human Ecology student?
P(C) = $P(A \cap B) = \frac{15}{223}$

(d) What is the probability that the student is a first born or a Human Ecology student?

$$\begin{aligned}
P(D) &= P(A \cup B) \\
&= P(A) + P(B) - P(\cap B) \\
&= \frac{43}{223} + \frac{113}{223} - \frac{15}{223} \\
&= \frac{141}{223}
\end{aligned}$$

4. A sample space consists of 5 simple events, E_1, E_2, E_3, E_4, E_5.

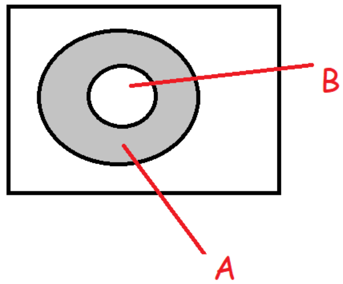
(a) If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$, and $P(E_4) = 2P(E_5)$, find the probabilities of E_4 and E_5.

$$\begin{aligned}
1 &= 0.15 + 0.15 + 0.4 + 2P(E_5) + 2P(E_5) \\
0.3 &= 3P(E_5) \\
P(E_5) &= 0.1
\end{aligned}$$

Then:

$$\begin{aligned}
P(E_4) &= 2(0.1) \\
&= 0.2
\end{aligned}$$

5. If A, B are events, $B \subset A$, $P(A) = 0.6$ and $P(B) = 0.2$, then find $P(A \cap B^c)$.
   To visualize:



$$P(A) = P(A \cap B^c) + P(B)0.6 \qquad = P(A \cap B^c) + 0.2$$
$$P(AcapB^c) = 0.4$$

6. An experiment involves tossing a pair of dice and observing the numbers on the upper faces. Find the number of sample points in Ω and the sample space for the experiment The sample points are $\Omega_{\{}m, n\} = \{1, 2, 3, 4, 5, 6\}$.
   The sample space is taking the cross product of the sample points (i.e., $\Omega_1 \times \Omega_2$). Then:
$$\Omega\{(1, 1), (1, 2), ..., (6, 6)\}$$
   There is a $m \cdot n = 6 \cdot 6 = 36$ chance for a single probability.

7. The names of three employees are randomly drawn without replacement from a bowl containing the names of 30 employees of a small company. The person whose names is drawn first receives \$100, and the individuals whose names are drawn second and third receive \$50 and \$25, respectively. How many samples are associated with this experiment? There are 30 employees and 3 people are picked. So, $r = 3, n = 30$. Hence:
$$\begin{aligned}\Omega &= P_3^{30} \\ &= \frac{30!}{(30-3)!} \\ &= \frac{30!}{27!}\end{aligned}$$

8. Find the number of ways of selecting two applicants out of five.
   Here, $n = 5, r = 2$.
$$\begin{aligned}\binom{5}{2} &= \frac{5!}{2!(5-2)!} \\ &= \frac{5!}{2!3!} \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} \\ &= 10\end{aligned}$$

4.2 Conditional Probability

1. Let $N = R^c$ be the event 'born in a month without r', and L is the event 'born in a long month'. What is the conditional probability $P(N|L)$?
   $P(R^c|L) = \frac{3}{7}$
   Note that $P(R|L) + P(R^c|L) = \frac{4}{7} + \frac{3}{7} = 1$.

2. A survey asked: 'Are you currently in a relationship?', and 'Are you involved in club sports?'. The survey found that 33% were in a relationship, and 25% were involved in sport. 11% said yes to both. Suppose you meet a student who is on a sports team. What is the probability they are also in a relationship?
   Let A be students in a relationship, and B be students in a club sport.
   $P(A) = 0.33$, $P(B) = 0.25$, $P(A \cap B) = 0.11$. We have:

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{0.11}{0.25} \\ &= 0.44 \end{aligned}$$

   We know that $P(A|B) \neq P(A)$, A and B must be dependent events.

3. A bowl contains 6 red and 4 blue balls, A child selects two balls at random. What is the probability at least one of them is red?
   We have:
   (a) $\{R, R\} = \frac{6}{10}, \frac{5}{9}$
   (b) $\{R, B\} = \frac{6}{10}, \frac{4}{9}$
   (c) $\{B, R\} = \frac{4}{10}, \frac{6}{9}$
   (d) $\{B, B\} = \frac{4}{10}, \frac{3}{9}$

   Note that $P(\text{at least one red}) + P(\text{no red}) = 1$, so:

$$\begin{aligned} P(one\_R) &= 1 - P(no\_R) \\ &= 1 - P(BB) \\ &= 1 - P(B_1 \cap B_2) \\ &= 1 - P(B_2|B_1)P(B_1) \\ &= 1 - \frac{3}{9} \cdot \frac{4}{10} \\ &= 1 - \frac{2}{15} \\ &= \frac{13}{15} \end{aligned}$$

4. A recent highway safety study found that in 77% of all accidents, the driver was wearing a seatbelt. Accident reports indicated that 92% of those drivers did not experience serious injury (defined as hospitalization or death), but only 63% of the non-belted were okay.
   (a) What is the probability that a driver was seriously injured?
   (b) What is the probability that a driver who was seriously injured was not wearing a seatbelt?

   Let B is those that are wearing a seat bet, and NB is for those who did not wear a seat belt. Then, let I represent those who experienced a serious injury.
   We have $P(B) = 0.77$, $P(NB) = 1 - 0.77 = 0.23$, and $P(I^c|B) = 0.92$, $P(I^c|NB) = 0.63$.
   Then, $P(I|B) = 0.08$, $P(I|NB) = 0.37$.
   [a)]Want to find $P(I)$:

$$\begin{aligned} P(I) &= P(B \cap I) + P(NB \cap I) \\ &= P(I|B)P(B) + P(I|NB)P(NB) \\ &= 0.08 \cdot 0.77 + 0.37 \cdot 0.23 \\ &= 0.0616 + 0.0851 \\ &= 0.1467 \end{aligned}$$

Want to find $P(NB|I)$.

$$\begin{aligned} P(I|NB) &= \frac{P(NB \cap I}{P(I)} \\ &= \frac{P(I|NB)P(I)}{0.1467} \\ &= \frac{0.23 \cdot 0.37}{0.1467} \\ &= \frac{0.0851}{0.1467} \\ &= 0.58 \end{aligned}$$

5. A student figures he has a 30% chance of being let out of class late. If he leaves class late, there is a 45% chance he will miss the bus. What is the probability that he is let out of class late and misses the bus?
   Let L be the chance he leaves class late. Let M be the chance he misses the bus.
   $P(L) = 0.3$, $P(M|L) = 0.45$. We want to find $P(L \cap M)$:

$$\begin{aligned} P(L \cap M) &= P(M|L)P(L) \\ &= 0.45 \cdot 0.3 \\ &= 0.135 \end{aligned}$$

6. Leah flies from Moncton to Vancouver with a connection in Montreal. The probability her flight leaves on time is 15%. If the flight is on time, the chance her luggage makes the connecting flight in Montreal is 95%. But, if the flight is delayed, the probability the luggage will make it is only 65%.

   (a) Are the first flight leaving on time and the luggage making the connection independent events? Explain.

   (b) What is the probability that Leah's luggage arrives in Vancouver with her?

   Let F be the chance her flight leaves on time. Let L be the luggage being in the connecting flight.
   $P(F) = 0.15$, $P(L|F) = 0.95$, $P(L|F^C) = 0.65$.

   [a)]We have $P(L|F) = 0.95$ and $P(L) = 0.695$, so they are not equal. Therefore, they are dependent events.

$$\begin{aligned} P(L) &= P(L|F)P(F) + P(L|F^c)P(F^c) \\ &= 0.95 \cdot 0.15 + 0.65 \cdot 0.85 \\ &= 0.1425 + 0.5525 \\ &= 0.695 \end{aligned}$$

7. The Ontario Lottery association claims your odds of winning a price on an Instant Win Crossword game are 1 in 3.09. This means that any ticket has 32.4% chance of winning a prize. Every Friday, you buy one Crossword game.

   (a) What is the probability you don't win a prize next Friday?

   (b) What is the probability you don't win a prize 6 Fridays in a row?

   (c) If you haven't won a prize for the past 6 Fridays, what is the probability you win a prize on your next game?

   (d) What is the probability you win on two of your next three games?

   Let W be the chance you win a prize. $P(W) = 0.324$.

   (a) Want to find $P(W^c)$. $P(W^c) = 1 - 0.324 = 0.676$

(b) Want to find $P(W_1^c ... W_6^c)$:

$$\begin{aligned} P(W_1^c ... W_6^c) &= P(W_1^c) \cdot ... \cdot P(W_2^c) \\ &= P(W^c)^6 \\ &= 0.676^6 \\ &= 0.0954 \end{aligned}$$

(c) Want to find $P((W_1^c ... W_6^c)^c)$. So: $P((W_1^c ... W_6^c)^c) = 1 - 0.0954 = 0.9046$.

(d) Want to find $P(W_1 W_2 W_3^c)$:

$$\begin{aligned} P(W_1 W_2 W_3^c) &= 0.324 \cdot 0.3424 \cdot 0.676 \\ &= 0.071 \end{aligned}$$

4.3 Random Variables

1. Suppose we roll two dice. List the outcomes:

   (a) $\{X = 8\} = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$.
   $P(X = 8) = \frac{5}{36}$

   (b) $\{X = 3\} = \{(1,2), (2,1)\}$.
   $P(X = 3) = \frac{2}{36} = \frac{1}{18}$

   (c) $\{X = 13\} = \{\}$
   $P(X = 13) = 0$

2. Suppose the death rate in a year is 1 out of 1000 people, and that 2 out of 1000 people suffer some kind of disability. Then, we display the probability model for X = company payout in a table:

Policyholder Outcome	Payout x	P(X = x)
Death	10000	1/1000
Disability	5000	2/1000
Neither	0	

   What is $P(X = 0)$?
   We have $X = 0, 5000, 10000$, and $P(X = 10000) = \frac{1}{1000}$, $P(X = 5000) = \frac{2}{1000}$. We calculate:

$$\begin{aligned} P(X = 0) &= 1 - \frac{1}{1000} - \frac{2}{1000} \\ &= \frac{997}{1000} \end{aligned}$$

3. Take the previous example, and determine $E(X)$, where X is the company payout.
   Recall: $X = 0, 5000, 10000$, and $P(X = 10000) = \frac{1}{1000}$, $P(X = 5000) = \frac{2}{1000}$, and $P(X = 0) = \frac{997}{1000}$.
   To calculate $E(X)$:

$$\begin{aligned} E(X) &= (10000 \cdot \frac{1}{1000}) + (5000 \cdot \frac{2}{1000}) + (0 \cdot \frac{997}{1000}) \\ &= 10 + 10 \\ &= 20 \end{aligned}$$

   Next, suppose we wanted to calculate the variance of X.
   We have:

   (a) $10000 - 20 = 9980$

(b) $5000 - 20 = 4980$

(c) $0 - 20 = -20$

So, we have:

$$\begin{aligned}\sigma^2 &= \sum_x (x-\mu)^2 P(x) \\ &= (9980^2 \cdot \frac{1}{1000}) + (4980^2 \cdot \frac{2}{1000}) + ((-20)^2 \cdot \frac{997}{1000}) \\ &= 149600\end{aligned}$$

Then:

$$\begin{aligned}\sigma = SD(X) &= \sqrt{V(X)} \\ &= \sqrt{149600} \\ &= \$386.78\end{aligned}$$

4. If $E(X) = 10$ and $V(X) = 3$, then:

   (a) Calculate $E(9x + 11)$.

$$\begin{aligned}E[9x+11] &= 9E(x) + 11 \\ &= (9 \cdot 10) + 11 \\ &= 90 + 11 = 101\end{aligned}$$

   (b) Calculate $V(9x + 11)$.

$$\begin{aligned}V(9x+11) &= 9^2 V(x) \\ &= 9^2 \cdot 3 \\ &= 243\end{aligned}$$

   (c)

$$\begin{aligned}\sigma^2 &= E(X-\mu)^2 \\ &= E(x^2 - 2\mu x + \mu^2) \\ &= E(x^2) - 2\mu E(X) + \mu^2 \\ &= E(x^2) - 2\mu^2 + \mu^2 \\ &= E(x^2) - \mu^2\end{aligned}$$

   (d) If $E(X) = 10$ and $E(X^2) = 160$:

$$\sigma^2 = 160 - 10^2 = 60$$

5. The probability distribution for a random variable Y is given by the following table. Find the mean, variance, and standard deviation of Y.

y	p(y)
0	1/8
1	1/4
2	3/8
3	1/4

Then:

$$\begin{aligned}
F(0) &= P(Y \leq 0) \\
&= P(Y = 0) = \frac{1}{8}
\end{aligned}$$

$$\begin{aligned}
F(1) &= P(Y \leq 1) \\
&= P(Y = 0) + P(Y = 1) \\
&= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}
\end{aligned}$$

$$\begin{aligned}
F(2) &= P(Y \leq 2) \\
&= \frac{3}{8} + \frac{1}{4} + \frac{1}{8} = \frac{6}{8}
\end{aligned}$$

$$\begin{aligned}
F(3) &= P(Y \leq 3) \\
&= \frac{6}{8} + \frac{1}{4} = 1
\end{aligned}$$