Tutorial 6

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We have:

$$f(x) = \begin{cases} 2e^{-2x} & x \leq 0; \\ 0 & \text{otherwise} \end{cases}$$

1a)

To get the distribution function of X, we calculate the integral:

$$\begin{aligned} F(x) &= \int_0^x 2e^{-2t} dt \\ &= \left[e^{-2t}\right]_x^0 \\ &= 1 - e^{-2x} \end{aligned}$$

Hence, we have $F(x) = 1 - e^{-2x}$ for $x \geq 0$.

1b)

We want the inverse of $F(x)$, so we have:

$$\begin{aligned} F(x) &= 1 - e^{-2x} \\ y &= 1 - e^{-2x} \\ 1 - y &= e^{-2x} \\ \ln(1 - y) &= -2x \\ x &= -\frac{1}{2} \ln(1 - y) \end{aligned}$$

Take the inverse function and use uniform distribution in order to sample the function. We know $f(x)$ looks similar to the exponential distribution $\lambda e^{-\lambda x}$, so let $\lambda = 2$.

```
lambda = 2
n = 100000 # The number of samples.
y = runif(n, 0, 1) # Random uniform distribution function
x = (-1/lambda) * log(1 - y)
z = seq(0, 6, by=0.01)

# Exact density function
fz = dexp(z, lambda) # Exponential distribution
```

1c)

```
hist(x, prob=TRUE, main="Probability Histogram", xlab="x", ylab="Probability")
lines(z, fz)
```