STA237 - Tutorial 9

1. (a) We have:

$$f(y) = \begin{cases} \frac{1}{100}e^{-y/100}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Let Y_1, Y_2 be two independent random variables representing the two components. Then, $n = 2$.
We also define the new random variable $X = \min(Y_1, Y_2)$.
We want to find the density function, so first we calculate $F(y)$:

$$\begin{aligned} F(y) &= \int_0^y \frac{1}{100}e^{-t/100}dt \\ &= \left[e^{-t/100}\right]_y^0 \\ &= 1 - e^{-y/100}, \text{ if } y > 0 \end{aligned}$$

Now, we can calculate $f_X(y)$:

$$f_X(y) = n \cdot (e^{-y/100}) \frac{1}{100 \cdot e}^{-y/100}$$

Hence:

$$f_X(y) = \begin{cases} \frac{1}{50}e^{-y/50}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(b) Here, $X = \max(Y_1, Y_2)$. Hence:

$$\begin{aligned} f_X(y) &= n(F(y))^{n-1}f(y) \\ &= 2(1 - e^{-y/100}) \cdot \frac{1}{100}e^{-y/100} \\ &= \frac{1}{50}(e^{-y/100} - e^{-y/50}) \end{aligned}$$

Therefore:

$$f_X(y) = \begin{cases} \frac{1}{50}(e^{-y/100} - e^{-y/50}), & y > 0 \\ 0, & \text{otherwise} \end{cases}$$