

Package ‘MsdeParEst’

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Title Parametric estimation in mixed-effects stochastic differential equations

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Description Parametric estimation in stochastic differential equations with random effects in the drift, or in the diffusion or both. Approximate maximum likelihood methods are used.

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Imports MASS, sde, moments, mvtnorm, methods, graphics

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MsdeParEst-package	<i>Parametric estimation in mixed-effects stochastic differential equations</i>
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Description

Parametric estimation in mixed-effects stochastic differential equations

Details

This package is dedicated to parametric estimation in the following mixed-effects stochastic differential equations:

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t),$$

$j = 1, \dots, M$, where the $(W_j(t))$ are independant Wiener processes and the $(X_j(t))$ are observed without noise. The volatility function $a(x)$ is known and can be either $a(x) = 1$ (Ornstein-Uhlenbeck process) or $a(x) = \sqrt{x}$ (Cox-Ingersoll-Ross process).

Different estimation methods are implemented depending on whether there are random effects in the drift and/or in the diffusion coefficient:

1. The diffusion coefficient is fixed $\sigma_j \equiv \sigma$ and the parameters in the drift are Gaussian random variables:
 - (a) either $\alpha_j \equiv \alpha$ and $\beta_j \sim N(\mu, \Omega), j = 1, \dots, M$,
 - (b) or $\beta_j \equiv \beta$ and $\alpha_j \sim N(\mu, \Omega), j = 1, \dots, M$,
 - (c) or $(\alpha_j, \beta_j) \sim N(\mu, \Omega), j = 1, \dots, M$.

μ, Ω and potentially the fixed effects σ, α, β are estimated as proposed in [1] and [4]. The extension to mixtures of Gaussian distributions is also implemented by following [3].

2. The coefficients in the drift are fixed $\alpha_j \equiv \alpha$ and $\beta_j \equiv \beta$ and the diffusion coefficient $1/\sigma_j^2$ follows a Gamma distribution $1/\sigma_j^2 \sim \Gamma(a, \lambda), j = 1, \dots, M$. a, λ , and potentially the fixed effects α and β are estimated by the method published in [2].

3. There are random effects in the drift and in the diffusion, such that $1/\sigma_j^2 \sim \Gamma(a, \lambda)$ and
- (a) either $\alpha_j \equiv \alpha$ and $\beta_j|\sigma_j \sim N(\mu, \sigma_j^2\Omega)$,
 - (b) or $\beta_j \equiv \beta$ and $\alpha_j|\sigma_j \sim N(\mu, \sigma_j^2\Omega)$,
 - (c) or $(\alpha_j, \beta_j)|\sigma_j \sim N(\mu, \sigma_j^2\Omega)$.
- a, λ, μ, Ω and potentially the fixed effects α and β are estimated by following [5].

Some examples and more details on how to use the different functions of the package are available on the package's vignette by typing `vignette("MsdeParEst")` in the R console.

References

- [1] Maximum Likelihood Estimation for Stochastic Differential Equations with Random Effects, Delattre, M., Genon-Catalot, V. and Samson, A. *Scandinavian Journal of Statistics* 40(2) 2012 **322-343**.
- [2] Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, Delattre, M., Genon-Catalot, V. and Samson, A. *ESAIM:PS* 19 2015 **671-688**.
- [3] Mixtures of stochastic differential equations with random effects: application to data clustering, Delattre, M., Genon-Catalot, V. and Samson, A. *Journal of Statistical Planning and Inference* 173 2016 **109-124**.
- [4] Parametric inference for discrete observations of diffusion processes with mixed effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *Stochastic Processes and their Applications*. To appear. (Available pre-publication hal-0133263, 2016).
- [5] Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01446063* 2017.

 bx

Computation Of The Drift Coefficient

Description

Computation of the drift coefficient

Usage

`bx(x)`

Arguments

`x` vector of data

Value

`b` The drift is $b(x, \phi) = \phi_1 b_1(x) + \phi_2 b_2(x)$, the output is the vector $(b_1, b_2)^t$

class.mixture.pred-class

S4 class for the parametric estimation results when the random effects in the drift follow mixture of normal distributions

Description

S4 class for the parametric estimation results when the random effects in the drift follow mixture of normal distributions

Slots

estim object of class Mixture.fit.class containing the results of the model estimation

phipred numeric 1, 2, or c(1,2)

Xpred matrix of predicted trajectories (dimensions)

idexpred matrix of values on which the estimation of the density of the random effects is done

class.pred-class

S4 class for the estimation results in the mixed SDE with random effects in the drift, in the diffusion or both

Description

S4 class for the estimation results in the mixed SDE with random effects in the drift, in the diffusion or both

Slots

estim object of class Fit.class containing the results of the model estimation

phipred matrix of simulated values for the random effects in the drift that are used for prediction (dimensions)

Xpred matrix of simulated trajectories used for prediction (dimensions)

idexpred vector of indexes of the true trajectories that are used for prediction

contrastGamma	<i>Contrast based on the Euler approximation of the likelihood for parameter estimation when there is one random effect in the diffusion coefficient and no random effect in the drift.</i>
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Description

Computation of the contrast based on the Euler approximation of -2 log-likelihood for the estimation of the mixed SDE:

$$dX_j(t) = (\alpha - \beta X_j(t))dt + \sigma_j a(X_j(t))dW_j(t)$$

with Gamma distribution for $1/\sigma_j^2$ and fixed parameters in the drift.

Usage

```
contrastGamma(a, lambda, U, V, S, K, drift.fixed)
```

Arguments

a	value of the shape of the Gamma distribution.
lambda	value of the scape of the Gamma distribution.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
drift.fixed	values of the fixed effects in the drift.

Value

L	value of the contrast
---	-----------------------

References

Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, M. Delattre, V. Genon-Catalot and A. Samson, *ESAIM: Probability and Statistics 2015*, Vol 19, **671 – 688**

Parametric inference for discrete observations of diffusion processes with mixed effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint 2016*, *hal-01332630*

contrastNormal	<i>Computation of the contrast used for the estimation of the normal conditional distribution of the random effects in the drift in mixed SDE with random effects in the drift and in the diffusion coefficient</i>
----------------	---

Description

Computation of the contrast used for the estimation of the parameters of the Gaussian conditional distribution of the random effects in the drift α_j, β_j when the SDE includes random effects in the drift and in the diffusion coefficient:

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t).$$

Usage

```
contrastNormal(mu, omega, U, V, S, K, estimphi, drift.random)
```

Arguments

mu	value of the mean of the Gaussian distribution.
omega	value of the standard deviation of the Gaussian distribution.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
estimphi	matrix of the M x 2 estimated parameters (α_j, β_j) .
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

L	value of the contrast
---	-----------------------

References

Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint*, [hal-01446063](#).

discr	<i>Simulation Of Random Variables</i>
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Description

Simulation of (discrete) random variables from a vector of probability (the nonparametrically estimated values of the density renormalised to sum at 1) and a vector of real values (the grid of estimation)

Usage

```
discr(x, p)
```

Arguments

x	n real numbers
p	vector of probability, length n

Value

y a simulated value from the discrete distribution

EM	<i>EM algorithm for mixtures of stochastic differential equations with random effects in the drift and a fixed effect in the diffusion coefficient</i>
----	--

Description

EM algorithm for parameter estimation in the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$$

with random effects in the drift α_j, β_j following a mixture of Gaussian distributions.

Usage

```
EM(U, V, S, K, drift.random, start, Niter = 10, drift.fixed = NULL,
   sigma = NULL)
```

Arguments

U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
start	list of starting values: mu, omega, mixt.prop, respectively for the mean and the standard deviation of the Gaussian distributions and the mixing proportions. mixt.prop is a vector of length N, where N stands for the number of mixture components. mu is a N x 2 matrix, first (resp. second) column is the mean of α_j (resp. β_j) if α_j (resp. β_j) is random, the fixed effect value otherwise. omega is a N x 2 matrix, the components corresponding to a fixed effect should be set to 0.
Niter	number of iterations. Defaults to 10.
drift.fixed	NULL if the fixed effects in the drift are estimated, vector of the N values of the fixed effect otherwise. Default to NULL.
sigma	value for the diffusion parameter if known (not estimated), NULL otherwise. Defaults to NULL.

Value

mu	estimated value of the mean at each iteration of the algorithm. Niter x N x 2 array.
omega	estimated value of the standard deviation at each iteration of the algorithm. Niter x N x 2 array.
mixt.prop	estimated value of the mixture proportions at each iteration of the algorithm. Niter x N matrix.
sigma	value of the diffusion parameter.
probindi	posterior component probabilities. M x N matrix.
BIChere	BIC indicator
AIChere	AIC indicator

References

Mixtures of stochastic differential equations with random effects: application to data clustering, M. Delattre, V. Genon-Catalot and A. Samson, *Journal of Statistical Planning and Inference* 2016, Vol 173, **109–124**

EstParamGamma	<i>Estimation In Mixed Stochastic Differential Equations with fixed effects in the drift and one random effect in the diffusion coefficient</i>
---------------	---

Description

Parameter estimation of the mixed SDE with Gamma distribution of the diffusion random effect and fixed effects in the drift:

$$dX_j(t) = (\alpha - \beta X_j(t))dt + \sigma_j a(X_j(t))dW_j(t), 1/\sigma_j^2 \sim \Gamma(a, \text{lambda}),$$

done with [likelihoodGamma](#).

Usage

```
EstParamGamma(U, V, S, SigDelta, K, drift.param = NULL)
```

Arguments

U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS).
K	number of times of observations.
drift.param	values of the fixed effects in the drift if know, NULL if the fixed effects are estimated. Defaults to NULL.

Value

mu	values of the fixed effects in the drift.
a	estimated value of the shape of the Gamma distribution.
lambda	estimated value of the scale of the Gamma distribution.
BIChere	BIC indicator.
AIChere	AIC indicator.

References

Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, M. Delattre, V. Genon-Catalot and A. Samson, *ESAIM: Probability and Statistics 2015*, Vol 19, **671 – 688**

EstParamNormal	<i>Estimation In Mixed Stochastic Differential Equations with random effects in the drift and fixed effect in the diffusion coefficient</i>
----------------	---

Description

Estimation of the parameters of the mixed SDE:

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t),$$

with Normal distribution of the random effects in the drift α_j, β_j and fixed parameter σ in the diffusion. Done with [likelihoodNormal](#).

Usage

```
EstParamNormal(U, V, S, SigDelta = 0, K, drift.fixed = NULL, sigma = NULL,
  drift.random, discrete = 1)
```

Arguments

U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS). Required only if discrete = 1. Defaults to 0.
K	number of times of observations.
drift.fixed	NULL if the fixed effect in the drift is estimated, value of the fixed effect otherwise. Default to NULL.
sigma	value of the fixed effect in the diffusion if known (not estimated), NULL otherwise. Defaults to NULL.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
discrete	1 for discrete observations, 0 otherwise. If discrete = 0, the exact likelihood associated with continuous observations is discretized. If discrete = 1, the likelihood of the Euler scheme of the mixed SDE is computed. Defaults to 1.

Value

mu	estimated value of the mean of the Normal distribution
omega	estimated value of the standard deviation of the Normal distribution
sigma	value of the diffusion coefficient
BIChere	BIC indicator
AIChere	AIC indicator

References

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

EstParamNormalGamma	<i>Estimation In Mixed Stochastic Differential Equations with random effects in the drift and in the diffusion coefficient</i>
---------------------	--

Description

Estimation of the parameters of the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t)$$

with Normal conditional distribution of the random effects in the drift $\alpha_j, \beta_j | \sigma_j \sim N(\mu, \sigma_j^2 \Omega)$ and the square root of an inverse Gamma distributed random effect in the diffusion $\sigma_j^2 \sim \text{Gamma}(a, \lambda)$.

Usage

```
EstParamNormalGamma(U, V, S, SigDelta, K, drift.random, drift.fixed = NULL)
```

Arguments

U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS).
K	number of times of observations.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
drift.fixed	NULL if the fixed effect(s) in the drift is (are) estimated, value of the fixed effect(s) otherwise. Default to NULL.

Value

mu	estimated value of the mean of the Normal distribution
omega	estimated value of the standard deviation of the Normal distribution
a	estimated value of the shape of the Gamma distribution.
lambda	estimated value of the scale of the Gamma distribution.
BIChere	BIC indicator
AIChere	AIC indicator

References

Estimaton of the joint distribution of random effects for a discretely observed diffusion with random effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint*, hal-01446063.

Fit.class-class	<i>S4 class for the estimation results in the mixed SDE with random effects in the drift, in the diffusion or both</i>
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Description

S4 class for the estimation results in the mixed SDE with random effects in the drift, in the diffusion or both

Slots

model character 'OU' or 'CIR'
 drift.random numeric 0, 1, 2, or c(1,2)
 diffusion.random numeric 0 or 1
 gridf matrix of values on which the estimation of the density of the random effects in the drift is done
 gridg matrix of values on which the estimation of the density of the random effects in the diffusion is done
 mu numeric estimator of the mean μ of the drift random effects
 omega numeric estimator of the variance of the drift random effects
 a numeric estimator of the shape of the Gamma distribution for the diffusion random effect
 lambda numeric estimator of the scale of the Gamma distribution for the diffusion random effect
 sigma2 numeric estimated value of σ^2 if the diffusion coefficient is not random
 index index of the used trajectories
 estimphi matrix of the estimator of the drift random effects
 estimps2 vector of the estimator of the diffusion random effects σ_j^2
 estimf estimator of the (conditional) density of ϕ , matrix form
 estimg estimator of the density of ϕ , matrix form
 estim.drift.fix 1 if the user asked for the estimation of fixed parameter in the drift
 estim.diffusion.fix 1 if the user asked for the estimation of fixed diffusion coefficient
 discrete 1 if the estimation is based on the likelihood of discrete observations, 0 otherwise
 bic numeric bic
 aic numeric aic
 times vector of observation times, storage of input variable
 X matrix of observations, storage of input variable

likelihoodGamma	<i>Computation of the Euler approximation of -2 log-likelihood when there is one random effect in the diffusion coefficient and fixed effects in the drift.</i>
-----------------	---

Description

Computation of the Euler approximation of -2 log-likelihood of the mixed SDE:

$$dX_j(t) = (\alpha - \beta X_j(t))dt + \sigma_j a(X_j(t))dW_j(t)$$

with inverse Gamma distribution for σ_j^2 and fixed parameters α, β in the drift.

Usage

```
likelihoodGamma(a, lambda, U, V, S, SigDelta, K, drift.fixed)
```

Arguments

a	value of the shape of the Gamma distribution.
lambda	value of the scape of the Gamma distribution.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS).
K	number of times of observations.
drift.fixed	values of the fixed effects in the drift.

Value

L	value of -2 x log-likelihood
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References

Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, M. Delattre, V. Genon-Catalot and A. Samson, *ESAIM: Probability and Statistics 2015*, Vol 19, **671 – 688**

Parametric inference for discrete observations of diffusion processes with mixed effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint 2016*, hal-01332630

likelihoodMixtureNormal

Computation of the Log Likelihood in mixtures of Mixed Stochastic Differential Equations

Description

Computation of -2 log-likelihood the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$$

with random effects in the drift α_j, β_j following a mixture of Gaussian distributions.

Usage

```
likelihoodMixtureNormal(mu, omega, sigma, mixt.prop, U, V, S, K, estimphi,
  drift.random)
```

Arguments

mu	mean of the random effects. N x 2 matrix, first (resp. second) column is the mean of α_j (resp. β_j) in each mixture component if α_j (resp. β_j) is random, the fixed effect value otherwise.
omega	standard deviation of the random effects. N x 2 matrix, the components corresponding to a fixed effect should be set to 0.
sigma	value of the diffusion parameter.
mixt.prop	vector of mixture proportions.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
estimphi	matrix of the estimators of the random effects in the drift.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

L	value of -2 x loglikelihood
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References

Mixtures of stochastic differential equations with random effects: application to data clustering, M. Delattre, V. Genon-Catalot and A. Samson, *Journal of Statistical Planning and Inference* 2016, Vol 173, **109–124**

likelihoodNormal	<i>Computation Of The Log Likelihood In Mixed Stochastic Differential Equations with random effects in the drift and fixed effect in the diffusion.</i>
------------------	---

Description

Computation of -2 log-likelihood of the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$$

with Normal distribution of the drift random effects α_j, β_j and fixed effect in the diffusion coefficient.

Usage

```
likelihoodNormal(mu, omega, sigma, U, V, S, SigDelta = 0, K, estimphi,
  drift.random, discrete = 1)
```

Arguments

mu	current value of the mean of the normal distribution.
omega	current value of the standard deviation of the normal distribution.
sigma	current value of the diffusion coefficient.
U	vector of the M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS). Required only if discrete = 1. Defaults to 0.
K	number of times of observations.
estimphi	matrix of estimators of the random effects.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
discrete	1 for discrete observations, 0 otherwise. If discrete = 0, the exact likelihood associated with continuous observations is discretized. If discrete = 1, the likelihood of the Euler scheme of the mixed SDE is computed. Defaults to 1.

Value

L	value of -2 x loglikelihood
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References

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**
 Parametric inference for discrete observations of diffusion processes with mixed effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint*, hal-01332630

likelihoodNormalGamma *Computation Of The Log Likelihood of the Euler scheme in Mixed Stochastic Differential Equations with random effects in the drift and in the diffusion.*

Description

Computation of -2 log-likelihood of the Euler scheme the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t),$$

with Normal conditional distribution of the random effects in the drift $\alpha_j, \beta_j | \sigma_j \sim N(\mu, \sigma_j^2 \Omega)$ and inverse Gamma distribution of the random effect in the diffusion $1/\sigma_j^2 \sim \text{Gamma}(a, \lambda)$.

Usage

```
likelihoodNormalGamma(a, lambda, mu, omega, U, V, S, SigDelta, K, drift.random)
```

Arguments

a	current value of the shape of the Gamma distribution.
lambda	current value of the scape of the Gamma distribution.
mu	current value of the mean of the normal distribution.
omega	current value of the standard deviation of the normal distribution.
U	vector of the M sufficient statistics U (see UVS).
V	list of the M sufficient statistics V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
SigDelta	vector of the M constant terms of the individual likelihood (see UVS). Required only if discrete = 1. Defaults to 0.
K	number of times of observations.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

L	value of -2 x loglikelihood
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References

Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, M. Delattre, V. Genon-Catalot and C. Laredo, *Preprint*, hal-01446063.

likelihoodNormalindi	<i>Computation of the individual Log-Likelihoods in Mixed Stochastic Differential Equations</i>
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Description

Computation of -2 log-likelihood of individual j in the mixed SDE with Normal distribution of the drift random effects and fixed effect in the diffusion:

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t).$$

Usage

```
likelihoodNormalindi(mu, omega, sigma, Uj, Vj, Sj, K, estimphij, drift.random)
```

Arguments

mu	vector of mean of the random effects. First (resp. second) value is the mean of α_j (resp. β_j) if α_j (resp. β_j) is random, the fixed effect value otherwise.
omega	standard deviation of the random effects. The components corresponding to a fixed effect should be set to 0.
sigma	value of the diffusion parameter.
Uj	vector of the sufficient statistics U for individual j (see UVS).
Vj	matrix of the sufficient statistics V for individual j (see UVS).
Sj	value of the sufficient statistic S for individual j (see UVS).
K	number of times of observations.
estimphij	vector of estimators of the random effects for individual j .
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

L	value of -2 x loglikelihood
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References

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

Mixture.fit.class-class

S4 class for the estimation results when the random effects in the drift follow mixture of normal distributions

Description

S4 class for the estimation results when the random effects in the drift follow mixture of normal distributions

Slots

model character 'OU' or 'CIR'

drift.random numeric 1, 2, or c(1,2)

gridf matrix of values on which the estimation of the density of the random effects is done

mu array estimated value of the mean of the drift random effects at each iteration of the EM algorithm (Niter x nb.mixt x 2)

omega array estimated value of the standard deviation of the drift random effects at each iteration of the EM algorithm (Niter x nb.mixt x 2)

mixt.prop matrix estimated value of the mixing proportions at each iteration of the EM algorithm (Niter x nb.mixt)

sigma2 numeric estimated value of σ^2

index index of the used trajectories

estimphi matrix of the estimator of the drift random effects

probindi matrix of posterior component probabilities

estimf matrix estimator of the density of the drift random effects

estim.drift.fix numeric 1 if the user asked for the estimation of fixed parameter in the drift

bic numeric bic

aic numeric aic

times vector of observation times, storage of input variable

X matrix of observations, storage of input variable

mixture.sim	<i>Simulation Of A Mixture Of Normal Distributions</i>
-------------	--

Description

Simulation of M random variables from a mixture of Gaussian distributions

Usage

```
mixture.sim(M, param, prob)
```

Arguments

M	number of simulated variables
param	vector of parameters with the means and standard-deviations of the normal distributions
prob	mixture components probabilities

Details

If the distribution is $p1N(\mu1, \sigma1^2) + (1 - p1)N(\mu2, \sigma2^2)$
 param=c($\mu1, \sigma1, \mu2, \sigma2$) and prob=c(p1,1-p1)

Value

Y	vector of simulated variables
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msde.fit	<i>Estimation Of The Random Effects In Mixed Stochastic Differential Equations</i>
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Description

Parametric estimation of the joint density of the random effects in the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t),$$

$j = 1, \dots, M$, where the $(W_j(t))$ are independant Wiener processes and the $(X_j(t))$ are observed without noise. There can be random effects either in the drift (α_j, β_j) or in the diffusion coefficient σ_j or both.

Usage

```
msde.fit(times, X, model = c("OU", "CIR"), drift.random = c(1, 2),
  drift.fixed = NULL, diffusion.random = 0, diffusion.fixed = NULL,
  mixture = 0, nb.mixt = 1, Niter = 10, discrete = 1)
```

Arguments

times	vector of observation times
X	matrix of the M trajectories (each row is a trajectory with as much columns as observations)
model	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross)
drift.random	random effects in the drift: 0 if only fixed effects, 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects. Default to c(1,2)
drift.fixed	NULL if the fixed effect(s) in the drift is (are) estimated, value of the fixed effect(s) otherwise. Default to NULL
diffusion.random	1 if σ is random, 0 otherwise. Default to 0
diffusion.fixed	NULL if σ is estimated (if fixed), value of σ otherwise. Default to NULL
mixture	1 if the random effects in the drift follow a mixture distribution, 0 otherwise. Default to 0
nb.mixt	number of mixture components for the distribution of the random effects in the drift. Default to 1 (no mixture)
Niter	number of iterations for the EM algorithm if the random effects in the drift follow a mixture distribution. Default to 10
discrete	1 for using a contrast based on discrete observations, 0 otherwise. Default to 1

Details

Estimation of the random effects density from M independent trajectories of the SDE:

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t),$$

$j = 1, \dots, M$, where the $(W_j(t))$ are independant Wiener processes and the $(X_j(t))$ are observed without noise.

Specification of the random effects:

The drift includes no, one or two random effects:

1. if drift.random = 0: $\alpha_j \equiv \alpha$ and $\beta_j \equiv \beta$ are fixed
2. if drift.random = 1: $\beta_j \equiv \beta$ is fixed and α_j is random
3. if drift.random = 2: $\alpha_j \equiv \alpha$ is fixed and β_j is random
4. if drift.random = c(1,2): α_j and β_j are random

The diffusion includes either a fixed effect or a random effect:

1. if diffusion.random = 0: $\sigma_j \equiv \sigma$ is fixed
2. if diffusion.random = 1: σ_j is random

Distribution of the random effects

If there is no random effect in the diffusion (diffusion.random = 0), there is at least on random effect in the drift that follows

1. a Gaussian distribution (nb.mixt=1): $\alpha_j \sim N(\mu, \Omega)$ or $\beta_j \sim N(\mu, \Omega)$ or $(\alpha_j, \beta_j) \sim N(\mu, \Omega)$,
2. or a mixture of Gaussian distributions (nb.mixt=K, K>1): $\alpha_j \sim \sum_{k=1}^K p_k N(\mu_k, \Omega_k)$ or $\beta_j \sim \sum_{k=1}^K p_k N(\mu_k, \Omega_k)$ or $(\alpha_j, \beta_j) \sim \sum_{k=1}^K p_k N(\mu_k, \Omega_k)$, where $\sum_{k=1}^K p_k = 1$.

If there is one random effect in the diffusion (diffusion.random = 1), $1/\sigma_j^2 \sim \Gamma(a, \lambda)$, and the coefficients in the drift are conditionally Gaussian: $\alpha_j|\sigma_j \sim N(\mu, \sigma_j^2 \Omega)$ or $\beta_j|\sigma_j \sim N(\mu, \sigma_j^2 \Omega)$ or $(\alpha_j, \beta_j)|\sigma_j \sim N(\mu, \sigma_j^2 \Omega)$, or they are fixed $\alpha_j \equiv \alpha, \beta_j \equiv \beta$.

SDEs

Two diffusions are implemented:

1. the Ornstein-Uhlenbeck model (OU) $a(X_j(t)) = 1$
2. the Cox-Ingersoll-Ross model (CIR) $a(X_j(t)) = \sqrt{X_j(t)}$

Estimation

- If discrete = 0, the estimation is based on the exact likelihood associated with continuous observations ([1],[3]). This is only possible if diffusion.random = 0.
- If discrete = 1, the likelihood of the Euler scheme of the mixed SDE is computed.
- If nb.mixt > 1, an EM algorithm is implemented and the number of iterations of the algorithm must be specified with Niter.

Value

index	is the vector of subscript in 1,...,M used for the estimation. most of the time index=1:M, except for the CIR that requires positive trajectories.
estimphi	matrix of estimators of the drift random effects $\hat{\alpha}_j$, or $\hat{\beta}_j$ or $(\hat{\alpha}_j, \hat{\beta}_j)$
estimps2	vector of estimators of the squared diffusion random effects $\hat{\sigma}_j^2$
gridf	grid of values for the plots of the random effects distribution in the drift, matrix form
gridg	grid of values for the plots of the random effects distribution in the diffusion, matrix form
estimf	estimator of the density of α_j, β_j or (α_j, β_j) . Matrix form.
estimg	estimator of the density of σ_j^2 . Matrix form.
mu	estimator of the mean of the random effects normal density
omega	estimator of the standard deviation of the random effects normal density
a	estimated value of the shape of the Gamma distribution
lambda	estimated value of the scale of the Gamma distribution
sigma2	value of the diffusion coefficient if it is fixed
bic	BIC criterium
aic	AIC criterium
model	initial choice
drift.random	initial choice

diffusion.random initial choice

drift.fixed initial choice

estim.drift.fix 1 if the fixed effects in the drift are estimated, 0 otherwise.

estim.diffusion.fixed 1 if the fixed effect in the diffusion is estimated, 0 otherwise.

discrete initial choice

times initial choice

X initial choice

For mixture distributions in the drift:

mu estimated value of the mean at each iteration of the algorithm. Niter x N x 2 array.

omega estimated value of the standard deviation at each iteration of the algorithm. Niter x N x 2 array.

mixt.prop estimated value of the mixture proportions at each iteration of the algorithm. Niter x N matrix.

probindi posterior component probabilities. M x N matrix.

References

See

- [1] Maximum Likelihood Estimation for Stochastic Differential Equations with Random Effects, Delattre, M., Genon-Catalot, V. and Samson, A. *Scandinavian Journal of Statistics* 40(2) 2012 **322-343**
- [2] Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, Delattre, M., Genon-Catalot, V. and Samson, A. *ESAIM:PS* 19 2015 **671-688**
- [3] Mixtures of stochastic differential equations with random effects: application to data clustering, Delattre, M., Genon-Catalot, V. and Samson, A. *Journal of Statistical Planning and Inference* 173 2016 **109-124**
- [4] Parametric inference for discrete observations of diffusion processes with mixed effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01332630* 2016
- [5] Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01446063* 2017

msde.pred

*Prediction Of Mixed Stochastic Differential Equations Trajectories***Description**

This function proposes to keep two thirds of the data (randomly chosen) to do the parametric estimation of the density of the parameters and of the fixed parameters of model $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t)$ using the same method as in the estimation function `msde.fit` and then to predict new trajectories from the estimated model. The plot reflect the adequation between the last third of the data and the simulated one.

Usage

```
msde.pred(times, X, model = c("OU", "CIR"), drift.random,
  drift.fixed = NULL, diffusion.random = 0, diffusion.fixed = NULL,
  mixture = 0, nb.mixt = 1, Niter = 10, discrete = 1,
  plot.pred = TRUE, level = 0.05, newwindow = FALSE)
```

Arguments

<code>times</code>	vector of observation times
<code>X</code>	matrix of the <code>M</code> trajectories (each row is a trajectory with as much columns as observations)
<code>model</code>	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross)
<code>drift.random</code>	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects
<code>drift.fixed</code>	default NULL, fixed effect in the drift: value of the fixed effect when there is only one random effect and it is not estimated, NULL otherwise
<code>diffusion.random</code>	default 0, 1 if one random effect in the diffusion, 0 if there is no random effect in the diffusion
<code>diffusion.fixed</code>	default NULL, fixed effect in the diffusion: value of the fixed effect when there is no random effect in the diffusion and it is not estimated, NULL otherwise
<code>mixture</code>	1 if the random effects in the drift follow a mixture distribution, 0 otherwise. Default to 0.
<code>nb.mixt</code>	default 1, number of mixture components for the distribution of the random effects in the drift
<code>Niter</code>	default 10, number of iterations for the EM algorithm if <code>mixture = 1</code>
<code>discrete</code>	default 1, 1 for discrete observations, 0 otherwise. If <code>discrete = 0</code> , and <code>diffusion.random = 0</code> , the exact likelihood associated with continuous observations is discretized. If <code>discrete = 1</code> , the likelihood of the Euler scheme of the mixed SDE is computed.
<code>plot.pred</code>	logical(1), if TRUE, the results are depicted grafically
<code>level</code>	alpha for the predicion intervals, default 0.05
<code>newwindow</code>	logical(1), if TRUE, a new window is opened for the plot

Value

res	is the vector of subscript in $1, \dots, M$ where the estimation of ϕ has been done, most of the time $index = 1 : M$
Xpred	is the vector of subscript in $1, \dots, M$ where the estimation of ϕ has been done, most of the time $index = 1 : M$
indexpred	matrix of estimators of $\phi = \alpha, \text{ or } \beta, \text{ or } (\alpha, \beta)$ from the efficient statistics (see UVS), matrix of two lines if <code>drift.random=c(1,2)</code> , numerical type otherwise
phipred	matrix of estimators of $\psi^2 = \sigma^2$ from the efficient statistics (see UVS), matrix of one line

References

See Maximum Likelihood Estimation for Stochastic Differential Equations with Random Effects, Delattre, M., Genon-Catalot, V. and Samson, A. *Scandinavian Journal of Statistics* 40(2) 2012 **322-343**

Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, Delattre, M., Genon-Catalot, V. and Samson, A. *ESAIM:PS* 19 2015 **671-688**

Mixtures of stochastic differential equations with random effects: application to data clustering, Delattre, M., Genon-Catalot, V. and Samson, A. *Journal of Statistical Planning and Inference* 173 2016 **109-124**

Parametric inference for discrete observations of diffusion processes with mixed effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01332630* 2016

Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01446063* 2017

Examples

```
## Not run:
# Example 1: one random effect in the drift and one random effect in the diffusion coefficient.

## End(Not run)
```

msde.sim

Simulation Of A Mixed Stochastic Differential Equation

Description

Simulation of M independent trajectories of a mixed stochastic differential equation (SDE) with linear drift

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t), j = 1, \dots, M.$$

There may be two random effects (α_j, β_j) in the drift and one random effect σ_j in the diffusion coefficient.

Usage

```
msde.sim(M, T, N = 100, model, drift.random, diffusion.random, mixture = 0,
  drift.param, diffusion.param, nb.mixt = 1, mixt.prop = 1, t0 = 0,
  X0 = 0.01, delta = T/N, op.plot = 0, add.plot = FALSE)
```

Arguments

M	number of trajectories.
T	horizon of simulation.
N	number of simulation steps, default $T \times 100$.
model	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross).
drift.random	random effects in the drift: 0 if no random effect, 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
diffusion.random	random effect in the diffusion coefficient: 0 if no random effect, 1 if one multiplicative random effect.
mixture	1 if the random effects in the drift follow a mixture of Normal distributions, 0 otherwise. Default to 0.
drift.param	fixed effects in the drift: value of the fixed effect when there is only one random effect, 0 otherwise. If drift.random=2, fixed can be 0 but β has to be a non negative random variable for the estimation. vector (not mixture) or matrix (mixture) of parameters of the distribution of the random effects in the drift.
diffusion.param	diffusion parameter if the diffusion coefficient is fixed, vector of parameters of the distribution of the diffusion random effect otherwise.
nb.mixt	number of mixture components if the drift random effects follow a mixture distribution, default nb.mixt=1.
mixt.prop	vector of mixture proportions if the drift random effects follow a mixture distribution, default mixt.prop=1.
t0	time origin, default 0.
X0	initial value of the process, default $X_0=0.001$.
delta	time step of the simulation (T/N).
op.plot	1 if a plot of the trajectories is required, default 0.
add.plot	1 for add trajectories to an existing plot

Details

Simulation of M independent trajectories of the SDE (the Brownian motions W_j are independent), with linear drift. There may be one or two random effects in the drift:

If drift.random = 0, α and β are fixed effects ($\alpha_j \equiv \alpha$ and $\beta_j \equiv \beta$)

If drift.random = 1, β is a fixed effect ($\beta_j \equiv \beta$), and the drift function is written $(\alpha_j - \beta_j X_j(t))$

If drift.random = 2, α is a fixed effect ($\alpha_j \equiv \alpha$), and the drift function is written $(\alpha - \beta_j X_j(t))$

If drift.random = c(1,2), both effects are random, and the drift function is written $(\alpha_j - \beta_j X_j(t))$

Two diffusions are implemented:

Ornstein-Uhlenbeck model (OU): $a(X_j(t)) = 1$

Cox-Ingersoll-Ross model (CIR): $a(X_j(t)) = \sqrt{X_j(t)}$

There may be either a fixed or a random effect in the diffusion coefficient:

If `diffusion.random = 0`, σ is a fixed effect ($\sigma_j \equiv \sigma$). In that case, the random effects in the drift follow a Normal distribution or a mixture of Normal distributions.

If `diffusion.random = 1`, σ_j is a random effect. In that case, σ_j^2 follow an inverse Gamma distribution with parameters `diffusion.param=c(shape,scale)`, and conditional on σ_j , the random effects in the drift follow a Normal distribution with mean `mu` and variance $\Omega * \sigma_j^2$

Value

`x` matrix (M x (N+1)) of the M trajectories.
`phi` vector (or matrix) of the M simulated random effects.

References

This function `mixedsde.sim` is based on the package `sde`, function `sde.sim`. See Simulation and Inference for stochastic differential equation, S.Iacus, *Springer Series in Statistics 2008* Chapter 2

See Also

<http://cran.r-project.org/package=sde>

out	<i>Transfers the class object to a list</i>
-----	---

Description

Method for the S4 classes

Usage

`out(x)`

Arguments

`x` `Fit.class` or `Mixture.fit.class` class

`plot,Fit.class,ANY-method`*Plot method for the estimation class object*

Description

Plot method for the S4 class Fit.class

Usage

```
## S4 method for signature 'Fit.class,ANY'  
plot(x, newwindow = FALSE, ...)
```

Arguments

x	Fit.class class
newwindow	logical(1), if TRUE, a new window is opened for the plot
...	optional plot parameters

`plot,Mixture.fit.class,ANY-method`*Plot method for the mixture estimation class object*

Description

Plot method for the S4 class Mixture.fit.class

Usage

```
## S4 method for signature 'Mixture.fit.class,ANY'  
plot(x, newwindow = FALSE, ...)
```

Arguments

x	Mixture.fit.class class
newwindow	logical(1), if TRUE, a new window is opened for the plot
...	optional plot parameters

probind

*Computation of the component probabilities***Description**

Computation of the individual component probabilities in the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$$

with random effects in the drift following a mixture of Gaussian distributions, and fixed effect in the diffusion.

Usage

```
probind(mu, omega, mixt.prop, sigma, U, V, S, K, estimphi, drift.random)
```

Arguments

mu	mean of the random effects. N x 2 matrix, first (resp. second) column is the mean of α_j (resp. β_j) in each mixture component if α_j (resp. β_j) is random, the fixed effect value otherwise.
omega	standard deviation of the random effects. N x 2 matrix, the components corresponding to a fixed effect should be set to 0.
mixt.prop	vector of mixture proportions.
sigma	value of the diffusion parameter.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
estimphi	matrix of estimators of the fixed/random effects. 2 x M matrix.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

probindi M x N matrix of individual component probabilities.

Mixtures of stochastic differential equations with random effects: application to data clustering, M. Delattre, V. Genon-Catalot and A. Samson, *Journal of Statistical Planning and Inference* 2016, Vol 173, **109–124**

Q_EM

Computation of the E-step of the EM algorithm for mixtures of stochastic differential equations with random effects

Description

Computation of the E-step of the EM algorithm for parameter estimation in the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$$

with random effects in the drift following a mixture of Gaussian distributions, and a fixed effect in the diffusion.

Usage

```
Q_EM(mu, omega, sigma, probindi, U, V, S, K, estimphi, drift.random)
```

Arguments

mu	mean of the random effects. N x 2 matrix, first (resp. second) column is the mean of α_j (resp. β_j) in each mixture component if α_j (resp. β_j) is random, the fixed effect value otherwise.
omega	standard deviation of the random effects. N x 2 matrix, the components corresponding to a fixed effect should be set to 0.
sigma	value of the diffusion parameter.
probindi	M x N matrix of individual component probabilities.
U	matrix of M sufficient statistics U (see UVS).
V	list of the M sufficient statistics matrix V (see UVS).
S	vector of the M sufficient statistics S (see UVS).
K	number of times of observations.
estimphi	matrix of estimators of the fixed/random effects. 2 x M matrix.
drift.random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

Value

Q value of the E-step.

References

Mixtures of stochastic differential equations with random effects: application to data clustering, M. Delattre, V. Genon-Catalot and A. Samson, *Journal of Statistical Planning and Inference* 2016, Vol 173, **109–124**

summary,Fit.class-method

Short summary of the results of class object Fit.class

Description

Method for the S4 class Fit.class

Usage

```
## S4 method for signature 'Fit.class'  
summary(object)
```

Arguments

object Fit.class class

summary,Mixture.fit.class-method

Short summary of the results of class object Mixture.fit.class

Description

Method for the S4 class Mixture.fit.class

Usage

```
## S4 method for signature 'Mixture.fit.class'  
summary(object)
```

Arguments

object Mixture.fit.class class

Description

Computation of the sufficient statistics of the (approximate) likelihood of the mixed SDE

$$dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t).$$

Usage

UVS(X, model, times)

Arguments

X matrix of the M trajectories.
 model name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross).
 times times vector of observation times.

Details

Computation of the sufficient statistics of the (approximate) likelihood of the mixed SDE $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma_j a(X_j(t))dW_j(t) = (\alpha_j, \beta_j)b(X_j(t))dt + \sigma_j a(X_j(t))dW_j(t)$ with $b(x) = (1, -x)^t$:

$$U : U(Tend) = \int_0^{Tend} b(X(s))/a^2(X(s))dX(s)$$

$$V : V(Tend) = \int_0^{Tend} b(X(s))^2/a^2(X(s))ds$$

$$S : S(X(t_1), ..., X(t_n)) = 1/delta \sum_{j=1}^n (X(t_j) - X(t_{j-1}))^2/a^2(X(t_{j-1}))$$

$$SigDelta: SigDelta(X(t_1), ..., X(t_n)) = nlog(delta) + \sum_{j=1}^n log(a(X(t_j)))$$

Value

U vector of the M statistics U(Tend)
 V list of the M matrices V(Tend)
 S vector of the M quadratic variations S(X(t_1),...,X(t_n))
 SigDelta vector of the M constant contributions to the Euler scheme approximation to the likelihood SigDelta(X(t_1),...,X(t_n))

References

See

Maximum Likelihood Estimation for Stochastic Differential Equations with Random Effects, Delattre, M., Genon-Catalot, V. and Samson, A. *Scandinavian Journal of Statistics* 40(2) 2012 **322-343**

Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient, Delattre, M., Genon-Catalot, V. and Samson, A. *ESAIM:PS* 19 2015 **671-688**

Mixtures of stochastic differential equations with random effects: application to data clustering, Delattre, M., Genon-Catalot, V. and Samson, A. *Journal of Statistical Planning and Inference* 173 2016 **109-124**

Parametric inference for discrete observations of diffusion processes with mixed effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01332630* 2016

Estimation of the joint distribution of random effects for a discretely observed diffusion with random effects, Delattre, M., Genon-Catalot, V. and Laredo, C. *hal-01446063* 2017

valid	##### <i>VALIDATION Validation of the chosen model.</i>
-------	--

Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the right the corresponding qq-plot for each time.

Usage

```
valid(x, ...)
```

Arguments

x	Fit.class or Mixture.fit.class class
...	other optional parameters

valid,Fit.class-method	<i>Validation of the chosen model.</i>
------------------------	--

Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the right the corresponding qq-plot for each time.

Usage

```
## S4 method for signature 'Fit.class'
valid(x, Mrep = 100, newwindow = FALSE,
      plot.valid = TRUE, numj, ...)
```


Arguments

x	Fit.class class
Mrep	number of trajectories to be drawn
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.valid	logical(1), if TRUE, the results are depicted grafically
numj	optional number of series to be validated
...	optional plot parameters

valid,Mixture.fit.class-method

Validation of the chosen model for the mixture class.

Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the left the corresponding qq-plot for each time.

Usage

```
## S4 method for signature 'Mixture.fit.class'
valid(x, Mrep = 100, newwindow = FALSE,
      plot.valid = TRUE, numj, ...)
```

Arguments

x	Mixture.fit.class class
Mrep	number of trajectories to be drawn
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.valid	logical(1), if TRUE, the results are depicted grafically
numj	optional number of series to be validated
...	optional plot parameters

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