Magnetotellurics Overview

Compiled by M Tan

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1 Introduction

Magnetotellurics (MT): Passive EM exploration method that measures orthogonal components of EM field on earth's surface.

Source field: Naturally generated by variations in earth's magnetic field.
$\rightarrow$ this induces currents into earth
$\longrightarrow$ and can be measured at surface to find info about subsurface resistivity structures

Magnetosphere: Due to interactions between the earth's EM field and solar winds.
$\rightarrow$ Solar wind varies in terms of *density, velocity, intensity.* As a result, this produces time-varying EM fields.

For frequencies:
f > 1 Hz: significant EM source is lightning discharges
1 Hz > f > 0.01 Hz: ionospheric resonances
0.1 Hz > f > 0.00001 Hz: EM source is magnetosphere

2 Physics

2.1 Notation & Definitions

σ: electrical conductivity – measure of a material's ability to conduct an electric current.
ϵ: dielectric permitivity
μ: magnetic permeability

Wave equation: Second-order linear PDE. Describes waves or standing waves.

$$\frac{d^2u}{dt^2} = c^2(\frac{d^2u}{dx_1^2} + \frac{d^2u}{dx_2^2} + ... + \frac{d^2u}{dx_n^2}) \qquad \text{Scalar wave equation}$$
$$\frac{d^2u}{dt^2} = c^2\frac{d^2u}{dx^2} \qquad \text{1-D equation} \tag{1}$$

Where c is a fixed, non-negative real coefficient.

Other equations:

$$\ddot{u} = c^2 \nabla^2 u$$
$$\frac{1}{c^2}\frac{d^2}{dt^2} - \nabla^2$$

Plane wave: Describes a 3D waveform. It is unphysical, meaning it has no bounds and has infinite energy. The generalized plane wave equation is:

$$\Psi = A \sin(\vec{k} \cdot \vec{r} - \omega t)$$

Where:

$$\vec{k} \text{ is any direction}$$
$$\vec{k} \cdot \vec{r} = (k_x, k_y, k_z) \cdot (x, y, z)$$
$$= xk_x, yk_y, zk_z = kr\cos\theta = ks$$
$$\text{(s is components of } \vec{r} \text{ along } \vec{k})$$

More about plane waves can be found here: https://en.wikipedia.org/wiki/PlaneWave

Plane wave propagation through the earth is at the core of the magnetotelluric problem.

2.2 Governing Equations:

$$\nabla \times E_x = -i\omega\mu B_y \qquad \text{Faraday's Law}$$
$$\nabla \times B_y = (\sigma + i\omega\epsilon)E_x \qquad \text{Ampere's Law}$$

Knowning that E and B are divergence free, we can combine the equations to write Helmhotz equations:

$$\nabla^2 E_x + k^2 E_x = 0 \qquad \text{Helmhotz equation for E field}$$
$$\nabla^2 B_y + k^2 B_y = 0 \qquad \text{Helmhotz equation for B field}$$
$$\sqrt{\omega^2\mu - i\omega\mu\sigma} = k \qquad \text{wavenumber}$$

In the ground, we can usually assume the displacement current is 0. This means:

$$\sigma \gg \omega\epsilon \tag{2}$$
$$k_{ground} \simeq (1 - i)\sqrt{\frac{\omega\mu\sigma}{2}} \tag{3}$$
$$k_{air} \simeq \omega\sqrt{\mu_0\epsilon_0} \tag{4}$$

The Helmhotz equations have the solutions in the form of:

$$E_x(z) = Ue^{ikz} + De^{-ikz}$$
$$B_y(z) = \frac{1}{-i\omega\mu}(\nabla \times E_x)_y = \frac{k}{\omega\mu}(De^{-ikz} - Ue^{ikz}) = \frac{1}{Z}(De^{-ikz} - Ue^{ikz})$$

where:

U and D are complex amplitudes of the Up and Down components of the field

$Z = \frac{\omega\mu}{k}$ is the intrinsic impedance of the space

We can write the solution for the j-th layer in matrix form:

$$\begin{pmatrix} E_{x,j} \\ B_{y,j} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -\frac{1}{Z_j} & \frac{1}{Z_j} \end{pmatrix} \begin{pmatrix} U_j \\ D_j \end{pmatrix} = P_j \begin{pmatrix} U_j \\ D_j \end{pmatrix} \tag{5}$$

The transfert of the U and D components inside a layer can be written as:

$$\begin{pmatrix} U'_j \\ D'_j \end{pmatrix} = \begin{pmatrix} e^{ikh_j} & 0 \\ 0 & e^{-ikh_j} \end{pmatrix} \begin{pmatrix} U_j \\ D_j \end{pmatrix} = T_j \begin{pmatrix} U_j \\ D_j \end{pmatrix} \tag{6}$$

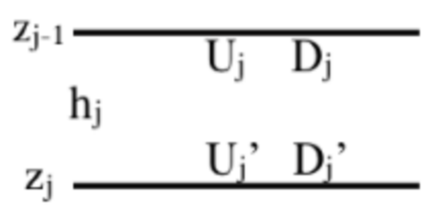

Figure 1: Transfert of Up and Down components inside a layer.

We can find the iterative relation between the fields in consecutive layers. This is because there is continuity of the tangential E_x and B_y field at the interfaces.

$$\begin{pmatrix} E_{x,j} \\ B_{y,j} \end{pmatrix} = P_j T_j P_J^{-1} \begin{pmatrix} E_{x,j+1} \\ B_{y,j+1} \end{pmatrix} \tag{7}$$

The last thing we must take into account is the boundary condition in the last layer. Since there is no reflection from another interface below the last layer, we can set the Up and Down components of the last layer, n, as:

$$\begin{pmatrix} U_n \\ D_n \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

Now, the forward can be solved. First, we use the matrix P_n to calculate $E_{x,n}$ and $B_{y,n}$. We then propagate the field iteratively up to the top layer using the matrix $P_j T_j P_J^{-1}$.

3 Measurement in the field

3.1 Depth of investigation

Taking the amplitude of the incident component of the E wave:

$$E_x(z) = De^{Im(k)z} \tag{9}$$

We can find the skin depth, δ. This is defined as the depth where the signal has decayed to $\frac{1}{e} \simeq$ 36 %.

We can see that skin depth is dependent on both the frequency and conductivity of the Earth material:

$$e^{iIm(k)\delta} = \frac{1}{e}\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \simeq \frac{500}{\sqrt{\sigma f}} \tag{10}$$

From equation 9, we find:

- $\uparrow$ conductive material = $\uparrow$ decay. MT can see very deep in resistive environments.
- $\downarrow$ frequency = $\downarrow$ decay. Lowest frequencies sample the deepest structures. High frequencies sample shallower structures.

3.2 Apparent Impedance

In the field, the source for MT is unknown. We avoid problems with this by measuring the amplitude of the E and B fields. This gives us apparent impedance, which is a complex number with a norm and angle:

$$\hat{Z}_{xy} = \frac{E_x}{B_y} \tag{11}$$

Using the same types of relationship to link E_x and B_y, we can write E_y and B_x in a matrix system:

$$E \quad = \quad \hat{Z} \quad B \tag{12}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} \tag{13}$$

For a 3D earth, there is no symmetry in the matrix and all components of the impedance are non-zero – this is the 3D tensor equation.

3.3 Tipper

Tipper: Ratio of the magnetic fields.

$$B_z \;=\; T \quad B \tag{14}$$

$$B_z \;= \begin{pmatrix} T_{zx} & T_{zy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} \tag{15}$$

3.4 Apparent Resistivity

Electrical resistivity: Fundamental property of a material that measures how strongly it resists electric current.

The apparent resistivity is obtained through the amplitude of the apparent impedance, $\hat{Z}_{xy}$.

3.5 Phase

The phase is obtained through the angle of the apparent impedance, $\hat{Z}_{xy}$.

$$\Theta = \tan^{-1} \frac{Im(Z_{xy})}{Re(Z_{xy})} \tag{16}$$

3.6 Set-Up

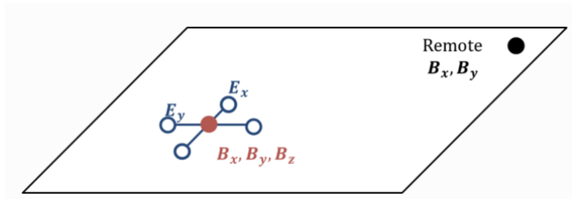

Figure 2: Set-up.

4 References

1. em.geosci: https://em.geosci.xyz/content/GeophysicalSurveys/mt/index.html
2. Wikipedia Plane Wave: https://en.wikipedia.org/wiki/PlaneWave