The Concept of Shape in Geography

Author(s): Ronald R. Boyce and W. A. V. Clark

Source: *Geographical Review*, Oct., 1964, Vol. 54, No. 4 (Oct., 1964), pp. 561-572

Published by: Taylor & Francis, Ltd.

Stable URL: https://www.jstor.org/stable/212982

THE CONCEPT OF SHAPE IN GEOGRAPHY

RONALD R. BOYCE AND W. A. V. CLARK

SHAPE has always been of concern in geography. Indeed, many geographical features such as deltas, oxbow lakes, and island chains are named from their distinctive shapes or forms, and shape has been found to be extremely useful for describing towns, trade areas, and political units. Although shape has been primarily used in geography as a descriptive device, it has also been of value as an analytical tool. But it is in the related disciplines, such as the biological sciences and mineralogy, that shape has been productive of theories.[1] However, lack of adequate methods for measuring shape has restricted its widespread use. Two methods of measurement have now been developed and are presented here, together with a review of some of the more pertinent articles concerned with shape or form and an example of the application of shape in urban geography. It is hoped that the study will lead to further applications of shape in solving gogeraphical problems.

Pertinent Geographical Studies Concerned with Shape

Major geographical journals from 1920 to the present were reviewed for articles using shape either in description or in analysis. Most of the articles considered shape only as a descriptive device. It was found that shape was most often used with reference to four aspects of geography: urban form, trade areas, political areas, and physical features.

Urban form. The use of shape for describing urban units is common; for example, the Y-shaped or L-shaped village, the strassendorf, or string village, and the star-shaped, or clustered, village. Hall[2] uses this external classification extensively in his study of the Yamato Basin in Japan, and many other geographers have made considerable use of such a system.[3] Except for the

[1] See, as a classic example, Sir D'Arcy Wentworth Thompson: On Growth and Form (new edit.; New York and London, 1942).

[2] Robert Burnett Hall: The Yamato Basin, Japan, *Annals Assn. of Amer. Geogrs.*, Vol. 22, 1932, pp. 243–292.

[3] As examples of studies using such a classification system, and of general morphological studies, see H. J. Fleure: Some Types of Cities in Temperate Europe, *Geogr. Rev.*, Vol. 10, 1920, pp. 357–374; William T. Chambers: Geographic Areas of Cities, *Econ. Geogr.*, Vol. 7, 1931, pp. 177–188; Mark Jefferson: Distribution of the World's City Folks, *Geogr. Rev.*, Vol. 21, 1931, pp. 446–465; A. Davies: A Study in City Morphology and Historical Geography, *Geography*, Vol. 18, 1933, pp. 25–37; R. E. Dickinson: The Town Plans of East Anglia: A Study in Urban Morphology, *ibid.*, Vol. 19, 1934, pp.

➤ Dr. Boyce is associate professor of geography and research associate in the Bureau of Business and Economic Research, University of Iowa, Iowa City. Dr. Clark is lecturer in geography at the University of Canterbury, New Zealand.

influence of transportation and natural barriers, little analysis has been made of the causes for any given geometrical form. By the same token, little research on the effects of external form, or on the precise measurement of shape, has been attempted. Often shape has been differentiated only in visual terms, and many of the shape features that have been identified in the geographical literature may therefore be more apparent than real.

The internal geographical patterns of cities have also received considerable attention. For example, Hartman discusses the variations in the shape of the central business district from pure geometric forms such as the circle, the star, the diamond, and the cross. He contends that "in spite of the great complexity and irregularity of shapes, . . . definite geometric patterns are discernible."[4] Murphy and Vance have also been concerned with shape and have directed attention to the physical "barriers" that affected the shapes of the nine central business districts they studied.[5] "Elongated industrial areas," "strip commercial development," and "cellular neighborhood units" are other examples of the way in which shape has been used to describe the internal functions of cities.

Trade areas. After the formal identification of trade areas, or hinterlands, discussion of their shapes followed almost immediately. Davis, in 1926, referred to the general pattern of trade areas as doubtless being "roughly circular in outline, but in detail it will present many irregularities which must be determined with accuracy."[6] In 1933 Christaller theorized that the ideal trade-area shape would not be circular but hexagonal.[7] The shape and nature of trade areas have been an integral part of central-place theory. As more empirical data have become available on trade areas, new theory on the nature of their shapes has been developed.

It has been found that the patterns of trade areas are far more varied and complex than had been postulated. For example, Applebaum and Cohen in 1961 noted that the shape of a shopping-center trade area is parabolically

(*Cont.*) 37–50; H. M. Keating: Village Types and Their Distribution in the Plain of Nottingham, *ibid.*, Vol. 20, 1935, pp. 283–294; and Glenn T. Trewartha: The Unincorporated Hamlet: One Element of the American Settlement Fabric, *Annals Assn. of Amer. Geogrs.*, Vol. 33, 1943, pp. 32–81.

4 George W. Hartman: The Central Business District—A Study in Urban Geography, *Econ. Geogr.*, Vol. 26, 1950, pp. 237–244; reference on p. 244.

5 Raymond E. Murphy, J. E. Vance, Jr., and Bart J. Epstein: Central Business District Studies (Worcester, Mass., 1955). This is a compilation of articles previously published in *Economic Geography*.

6 D. H. Davis: Objectives in a Geographic Field Study of a Community, *Annals Assn. of Amer. Geogrs.*, Vol. 16, 1926, pp. 102–109; reference on p. 106.

7 Central-place theory is clearly discussed, with an extensive bibliography, in Brian J. L. Berry and Allan Pred: Central Place Studies: A Bibliography of Theory and Applications, *Bibliography Ser. No. 1*, Regional Science Research Institute, Philadelphia, 1961.

elongated away from the central business district.[8] This empirical knowledge raises questions as to the underlying causes for any particular shape.

Political areas. Shape has been given particular emphasis in political geography, if only on an elementary and descriptive level. Attention has centered on the shapes of countries and the possible implications of these shapes. As White and Renner have pointed out, most countries are recognized by any schoolchild by their shapes; for example, Italy as a boot, Ceylon as a pear, New Guinea as a roosting turkey, and Mexico as a leg of lamb.[9] Inasmuch as such recognition schemes are far too elementary, they propose a five-way classification. Instead of using pure geometric forms to describe shapes, White and Renner describe them as follows, with examples: (1) *compact,* Rumania; (2) *attenuated* or *elongated,* Chile; (3) *prorupted,* the Soviet Union; (4) *perforated* or *punctured,* the Republic of South Africa; and (5) *fragmented,* Greece. By this method one can discuss any given shape in terms of these particular characteristics and draw inferences as to the effects of that shape. Unfortunately, the method does not provide a neat classification scheme, nor does it permit quantification of shapes.

Physical features. In contrast with most of the foregoing studies, shape has been used in physical geography as a rigorous classification and analytical device. Miller, in his study of drainage basins,[10] developed a method for measuring the shape of any given basin. The drainage-basin shape, C, was expressed as the ratio of the area of the drainage basin, A_b, to the area of a circle having the same perimeter, A_c; that is, $C = A_b/A_c$. Therefore a perfectly circular basin would have an index of 1, and all other drainage-basin shapes would have an index number ranging from 0 to 1.

Shape has been considered sufficiently important for the understanding of drainage-basin characteristics that Chorley[11] has further developed the measurement of shape originally devised by Miller. Chorley, however, uses a lemniscate, or pear shape, which, he argues, is capable of more precise definition, and is more consistent with empirical reality, than a circular-based

---

[8] William Applebaum and Saul B. Cohen: The Dynamics of Store Trading Areas and Market Equilibrium, *Annals Assn. of Amer. Geogrs.,* Vol. 51, 1961, pp. 73–101; reference on p. 81.

[9] C. Langdon White and George T. Renner: College Geography: Natural Environment and Human Society (New York, 1957), pp. 590–599.

[10] Victor C. Miller: A Quantitative Geomorphic Study of Drainage Basin Characteristics in the Clinch Mountain Area, Virginia and Tennessee, *Columbia Univ., Dept. of Geol., Tech. Rept. No. 3,* New York, 1953.

[11] Richard J. Chorley, Donald E. G. Malm, and Henry A. Pogorzelski: A New Standard for Estimating Drainage Basin Shape, *Amer. Journ. of Sci.,* Vol. 255, 1957, pp. 138–141. See also Richard J. Chorley and L. S. D. Morley: A Simplified Approximation for the Hypsometric Integral, *Journ. of Geol.,* Vol. 67, 1959, pp. 566–571.

ideal shape. Neither Miller's nor Chorley's method provides a sufficiently accurate way of measuring the shape of a drainage basin.[12] Both methods are based on computed areas as determined from a hypothetical circumference. The essential difficulty is that area rather than outer configuration is used as the index of shape. There can be many shapes with approximately the same area.

The Use of Shape in Related Disciplines

Undoubtedly the classic work on shape and form in any field is D'Arcy Thompson's "On Growth and Form."[13] This work shows the value of shape in enabling a greater understanding of biological functions. Although Thompson makes an excellent case for the study of shape in the biological sciences, he draws on geography as one of the prime proofs of the general importance of shape. For example, he states that "after the fundamental advance had been made which taught us that the world was round, Newton shewed [*sic*] that the forces that work upon it must lead to its being imperfectly spherical, and in the course of time its oblate spheroidal shape was actually verified. But now, in turn, it has been shewn [*sic*] that its form is still more complicated, and the next step is to seek for the forces that have deformed the oblate spheroid."[14] The importance of shape in leading to relevant research questions, and in formulating theory, is well demonstrated throughout the book.

One of the examples Thompson discusses concerns the differing shapes of eggs. Careful examination of the sizes and shapes of eggs led to increased knowledge of the anatomy of the animals. Thompson states this idea explicitly: "To enquire how an elastic sphere or spheroid will be deformed in passing down a peristaltic tube is an ill-defined and indeterminate problem; but we can study the effect produced in the shape of any particular egg, and so infer the forces which have been in action."[15] The mathematics used in Thompson's monumental work are complicated, since he is often concerned with three-dimensional shape.

Mineralogy is another discipline that has profited greatly from its concern with shape. Minerals are often identified primarily by their geometrical forms. Questions as to why minerals have specific shapes led to considerable

---

[12] Further statements on the possible importance of shape for the understanding of drainage basins are found in Mark A. Melton: Correlation Structure of Morphometric Properties of Drainage Systems and Their Controlling Agents, *Journ. of Geol.*, Vol. 66, 1958, pp. 442–460.

[13] Thompson, *op. cit.* [see footnote 1 above].

[14] *Ibid.*, pp. 1029–1030.

[15] *Ibid.*, p. 941.

progress in knowledge of the nature of their underlying physical and chemical structure, which causes the shapes. Hence the study of shape is a two-way process, suggesting research hypotheses and providing answers.

Shape is used to distinguish such everyday things as snowflakes, spider webs, and mushrooms. That shape has not been used more extensively in geography may result from the earlier inability to measure it precisely. Many of the terms and measurements that have been used to identify shape have been inadequate. Moreover, geographers have been primarily concerned with an ideographic, rather than a nomothetic, approach to geographical problems.

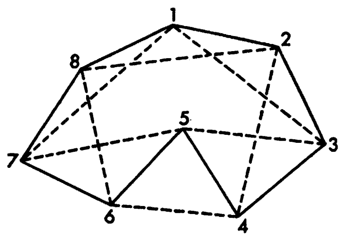
Development of Shape Indices

Two methods of measuring shape have been developed recently by geographers.[16] William Bunge, in his book "Theoretical Geography," has devised a system of classifying shapes based on the measurement of distances between selected vertices of a polygon. The present authors have developed a shape index based on the measurement of radial distances from a selected point to the circumference of a geometric form. Both methods accomplish the same objective; that is, they measure shape in such a way that shapes can be ranked, or can be compared with specific geometric forms. However, because of the different procedures, each method has advantages for particular purposes.

In the Bunge method edge lengths of a polygon are measured, in edge units, between pairs of vertices selected by skipping, or "lagging." Given an eight-vertex polygon, three initial measurements result—lag 1, lag 2, lag 3 (Fig. 1). For lag 1 measurements, every other vertex is skipped (that is, distances are measured between vertices 1 and 3, 2 and 4, 3 and 5, 4 and 6, 5 and 7, 6 and 8, 7 and 1, and 8 and 2), and the results are added; for lag 2 measurements, two vertices are skipped; for lag 3 measurements, three vertices. The individual measurements are also squared and summed, creating lag 1^2, lag 2^2, and lag 3^2 measurements. These six distance measurements describe any geometric form when eight vertices are used. Of course, the greater the number of vertices used, the greater the accuracy of the indices. When more than eight vertices are used, more units of shape information result. For example, when sixteen vertices are used, fourteen shape sums result—lags 1 through 7 and lags 1 through 7 squared.

[16] William Bunge: Theoretical Geography, *Lund Studies in Geography*, Ser. C, General and Mathematical Geography, No. 1, 1962; and Ronald R. Boyce and W. A. V. Clark: Selected Spatial Variables and Central Business District Sales, *Papers and Proc. Regional Science Assn.*, Vol. 9 (Ninth Annual Meeting, 1962) (forthcoming).

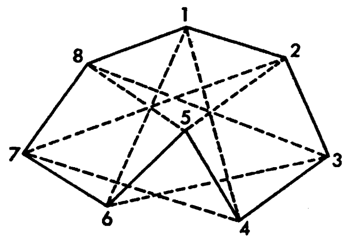
Lag 1 Connections



Lag 1 Measurements

Connections	Edge Distance	Distance^2
1,3	1.78	3.17
2,4	1.61	2.59
3,5	1.33	1.77
4,6	1.22	1.49
5,7	1.61	2.59
6,8	1.28	1.64
7,1	1.94	3.76
8,2	1.89	3.57
Shape sums	12.66	20.58

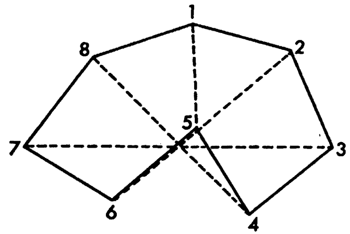
Lag 2 Connections



Lag 2 Measurements

Connections	Edge Distance	Distance^2
1,4	1.89	3.57
2,5	1.11	1.23
3,6	2.06	4.24
4,7	2.11	4.45
5,8	1.11	1.23
6,1	1.83	3.35
7,2	2.67	7.13
8,3	2.39	5.71
	15.17	30.91

Lag 3 Connections



Lag 3 Measurements

Connections	Edge Distance	Distance^2
1,5	1.00	1.00
2,6	2.22	4.93
3,7	2.33	5.43
4,8	2.11	4.45
5,1	1.00	1.00
6,2	2.22	4.93
7,3	2.33	5.43
8,4	2.11	4.45
	15.32	31.62

FIG. 1—The vertex-lag method of measuring shape, after Bunge (see text footnote 16 for reference). The six shape sums equal the shape index.

EXAMPLE: GEOMETRIC SHAPE

Square

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	Σ
r_i	9	10	13	10	9	10	13	10	9	10	13	10	9	10	13	10	168
$\frac{r_i}{\Sigma_1^{16}} \cdot 100$	5.36	5.95	7.74	5.95	5.36	5.95	7.74	5.95	5.36	5.95	7.74	5.95	5.36	5.95	7.74	5.95	100
$\left\|\left(\frac{r_i}{\Sigma_1^{n16}} \cdot 100-6.25\right)\right\|$	.89	.30	1.49	.30	.89	.30	1.49	.30	.89	.30	1.49	.30	.89	.30	1.49	.30	11.92

FORMULA

$$\Sigma_1^n \left|\left(\frac{r_i}{\Sigma_1^n} \cdot 100 - \frac{100}{n}\right)\right|$$

r = radial
n = number of radials

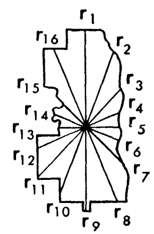
Rectangle

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	Σ
r_i	4.0	4.4	5.6	8.7	8.0	8.7	5.6	4.4	4.0	4.4	5.6	8.7	8.0	8.7	5.6	4.4	98.8
$\frac{r_i}{\Sigma_1^{16}} \cdot 100$	4.04	4.44	5.65	8.78	8.08	8.79	5.65	4.44	4.04	4.44	5.65	8.79	8.08	8.79	5.65	4.44	100
$\left\|\left(\frac{r_i}{\Sigma_1^{16}} \cdot 100-6.25\right)\right\|$	2.21	1.81	.60	2.53	1.83	2.54	.60	1.81	2.21	1.81	.60	2.53	.83	2.54	.60	1.81	27.86

EXAMPLE: MILWAUKEE

Actual shape from geometric center

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	Σ
r_i	15.7	12.7	7.2	7.2	6.0	6.4	11.5	15.4	14.2	9.0	11.0	9.5	8.0	8.0	12.2	10.6	164.6
$\frac{r_i}{\Sigma_1^{16}} \cdot 100$	9.42	7.62	4.32	4.36	3.60	3.84	6.90	9.24	8.52	5.40	6.60	5.70	4.80	4.86	7.41	6.43	98.75
$\left\|\left(\frac{r_i}{\Sigma_1^{16}} \cdot 100-6.25\right)\right\|$	3.17	1.37	1.93	1.93	2.65	2.41	.65	2.99	2.27	.85	.35	.55	1.45	1.45	1.06	.11	25.19



Actual shape from CBD

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	Σ
r_i	11.7	4.0	.8	.5	.5	.5	.5	8.0	13.8	13.9	11.3	15.0	11.1	15.0	15.0	15.4	14.0
$\frac{r_i}{\Sigma_1^{16}} \cdot 100$	8.31	2.84	.57	.36	.36	.36	.36	5.68	9.80	9.87	8.02	10.65	7.88	10.65	10.93	12.75	99.42
$\left\|\left(\frac{r_i}{\Sigma_1^{16}} \cdot 100-6.25\right)\right\|$	2.06	3.41	5.68	5.89	5.89	5.89	5.89	.57	3.55	3.62	1.77	4.40	1.63	4.40	4.68	6.53	65.86

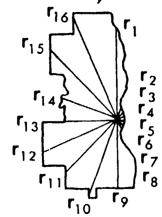


FIG. 2—The radial-distance method of measuring shape: the shape index.

Bunge presents his technique as an analytical device and suggests applications of shape to geographical problems. He demonstrates the method by determining the shapes of ninety-seven Mexican communities, and grouping them into shape types, which, he points out, fit visual notions reasonably well. As he suggests, however, more than eight vertices are required to measure some of these communities satisfactorily.

The method for measuring shape developed by the present writers is presented diagrammatically in Figure 2. In most succinct mathematical form the method can be expressed as follows:

$$\sum_1^n \left| \left(\frac{r_i}{\sum_1^n} \cdot 100 - \frac{100}{n} \right) \right|$$

where r = the radials extending outward from a central node, and n = the number of radials used. The shape index for a circle is 0. All other geometric forms have shape indices greater than 0.

For any given shape determine the center of gravity.[17] Measure the distance from this center to the outside edges of the shape along equally spaced radials. Compute the percentage of each radial distance with respect to the sum of all radials, and subtract each percentage from the percentage each radial would be expected to have as based on a circle; in the example given in Figure 2, using sixteen radials, a constant of 6.25 is appropriate. The signs should then be ignored, and the cells summed to determine the shape index.

It can be seen that a square has a shape index of approximately 12, and a rectangle approximately 28. All shapes form a continuum (Fig. 3), ranging from 0 for a circle to 175 as measured from the center of a straight line. Thus any shape can be ranked relative to others and, in addition, can be classified according to the geometric shape it most nearly approximates.

A shape curve can also be shown for any solid shape (Fig. 4). By plotting the radial percentage numbers along the horizontal axis from 1 to n, and the percentage distance for each radial along the vertical axis, a straight line is found to be the "shape curve" for a circle. The shape curve for a square is a sine curve having four nodes above the line and four below. Note, in Figure 4, the similarity of Dallas to a square both in shape curve and in index num-

[17] Although it is not necessary to measure shapes from the center of gravity, this can be accomplished in at least two ways: mechanical and mathematical. The writers found the center of gravity mechanically by tracing each shape on acetate, cutting it out, and balancing it on a pencil eraser. For a mathematical discussion of the center of gravity see Stig Nordbeck: Location of Areal Data for Computer Processing, *Lund Studies in Geography*, Ser. C, General and Mathematical Geography, No. 2, 1962.

ber. Milwaukee, on the other hand, as measured from the geometric center, is close in shape curve to a rectangle, though it has an index number of 25.

The Bunge vertex-lag method has several advantages. It appears to measure "boomerang" shapes with greater accuracy than the radial-distance method; it measures multidimensional shapes with more facility; and no

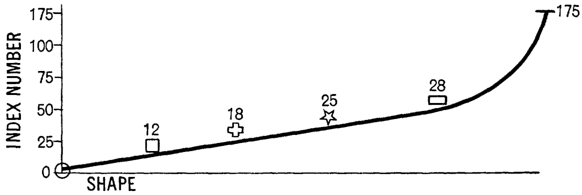


FIG. 3—Index numbers for geometric shapes form a continuum.

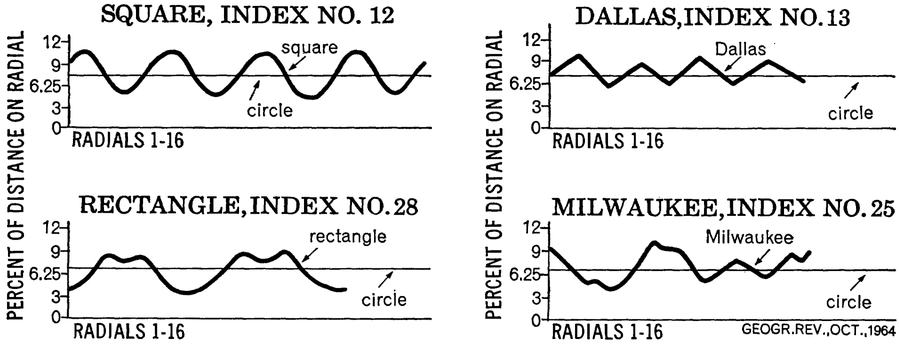


FIG. 4—Shape curves, using Dallas as an illustration of a square and Milwaukee as an illustration of a rectangle.

center of gravity, or starting point, need be predetermined. In contrast with the radial-distance method, "distances" are measured from one vertex to another, not from a central point to an edge.

The radial-distance method also has several advantages. Radial distances can be measured in any measurement unit; the method is simpler to calculate, and easier to interpret, than the vertex-lag method; and it is more closely geared to movement (that is, the nodal region). A shape can be measured from any vantage point. The shape of a metropolis, for example, can be measured from the central business district, from the center of gravity of the area mass, or from any other desired point (Fig. 2). Moreover, different intensities can be assigned to different radials.

Both methods provide only approximations of shapes. The degree of

TABLE I—SMA POPULATION, CBD SALES, AND RESIDENTIAL-AREA SHAPE FOR 37 SELECTED METROPOLISES

METROPOLIS	CODE	SMA POPULATION 1958 (1,000)[a]	% SMA SALES IN CBD, 1958[b]	SHAPE INDEX OF RESIDENTIAL AREA[c]
Atlanta	4	957	26.4	16.4
Baltimore	6	1,645	11.6	28.8
Birmingham	7	615	26.8	34.0
Boston	8	2,535	12.6	43.1
Buffalo	9	1,261	13.2	79.2
Chicago	12	5,894	9.2	79.2
Cincinnati	13	1,028	20.6	40.4
Cleveland	14	1,718	16.3	78.8
Columbus	15	643	26.1	34.1
Dallas	17	1,008	11.5	32.5
Dayton	18	655	21.4	21.2
Denver	19	860	16.3	26.3
Detroit	21	3,600	7.6	84.6
Fort Worth	28	533	33.5	20.2
Houston	31	1,149	23.5	23.7
Indianapolis	32	662	30.0	30.6
Los Angeles	36	6,220	4.2	25.8
Louisville	37	689	26.6	21.5
Memphis	38	590	16.9	59.8
Miami	39	833	8.5	66.4
Milwaukee	40	1,141	11.6	50.2
New Haven	45	300	22.3	30.4
New Orleans	46	825	28.4	21.9
Philadelphia	50	4,165	13.0	37.4
Phoenix	51	588	18.4	38.8
Pittsburgh	52	2,359	12.8	27.0
Portland (Oreg.-Wash.)	53	793	17.3	27.4
Providence	54	800	12.7	30.0
Rochester	57	564	27.9	37.3
St. Louis	62	1,975	10.1	37.3
San Antonio	64	646	26.4	15.1
San Jose	65	571	15.1	56.2
Seattle	67	1,047	16.2	35.2
Syracuse	70	543	20.6	31.0
Washington	75	1,868	16.1	16.2
Waterbury	76	175	33.2	20.4
Worcester	79	319	25.9	22.0

[a]Census of Population, 1960: Number of Inhabitants, United States Summary, *Final Report PC(1)–1A*, U. S. Bureau of the Census, 1961. SMA 1958 population obtained by interpolation of the 1950 and 1960 SMA populations.

[b]1958 Census of Business: Central Business District Statistics, *BC58–CBD*, Vol. 7, U. S. Bureau of the Census, 1961.

[c]Basic data from Sanborn Buying Power Maps [see text footnote 21 below]. Index method is shown by Figures 2 and 3.

accuracy depends on the number of radials or the number of vertices.[18] On the other hand, neither method will accurately measure the shapes of bulbed elongations, doughnut shapes, or extremely fragmented patterns.

[18] If desired, randomly created radii can be used. However, it is important to realize that this may not give systematic coverage of the shape circumference. More radii will be required under a random system than under a systematically stratified system.

An Example of Shape in Urban Geography

There is good reason to believe that the shape of the built-up area of a metropolis affects the importance of its central business district (CBD). A circular-shaped residential pattern would seem to be more advantageous to the *total* population for reaching the CBD than an elongated shape, since a circular shape is likely to have more radial streets focusing on the CBD. On the other hand, in an extremely elongated metropolis a critical limit might be reached in which a part of the population would reside so far from the CBD as to encourage outlying development of CBD-type functions.[19] The hypothesis that the shape of the residential area of a metropolis affects the importance of the CBD is tested as follows:

The percentage of standard metropolitan area (SMA) sales in the CBD, as determined from the 1958 Census of Business CBD bulletins,[20] is used as the Y, or dependent, variable, representing the importance of any given CBD. The residential-area shape of the metropolis, as determined from the Sanborn Map Company's Buying Power Maps,[21] is used as the X, or independent, variable. Because of limitations of data, the analysis is restricted to thirty-seven cases. The CBD was used as the focal point from which the shape index was determined.[22] The basic data are given in Table I.

Figure 5 demonstrates the regression relationship. It is immediately clear that as the shape index increases (that is, as any given metropolitan shape becomes less like a circle), the percentage of sales in the CBD decreases. For example, an "average" metropolis having a shape index of 80, an extremely elongated shape, would be expected to have only 11 percent of SMA sales in the CBD. An "average" metropolis having a shape index of 10, approximating the shape of a square, would be expected to have about 30 percent of SMA sales in the CBD. It will be noted that Los Angeles (Number 36) does not conform to the general distribution of the data. In view of the many peculiarities of Los Angeles, particularly the large amount of space occupied per capita, such deviancy might be expected.

The correlation coefficient between sales and shape is −0.49. Shape, there-

---

[19] See, for example, Richard Lawrence Nelson and Frederick T. Aschman: Real Estate and City Planning (Englewood Cliffs, N. J., 1957), p. 156.

[20] 1958 Census of Business: Central Business District Statistics, *BC 58–CBD,* U. S. Bureau of the Census, 1961.

[21] Published by the Sanborn Map Company of Pelham, N. Y. They are now available for about a hundred metropolises.

[22] A more detailed presentation of relation of shape to other spatial variables is given in Boyce and Clark, *op. cit.* [see footnote 16 above]. In this case, however, the shape of the "urbanized area" of the census was used as the unit shape instead of the residential-area shape as determined from Sanborn Buying Power Maps.

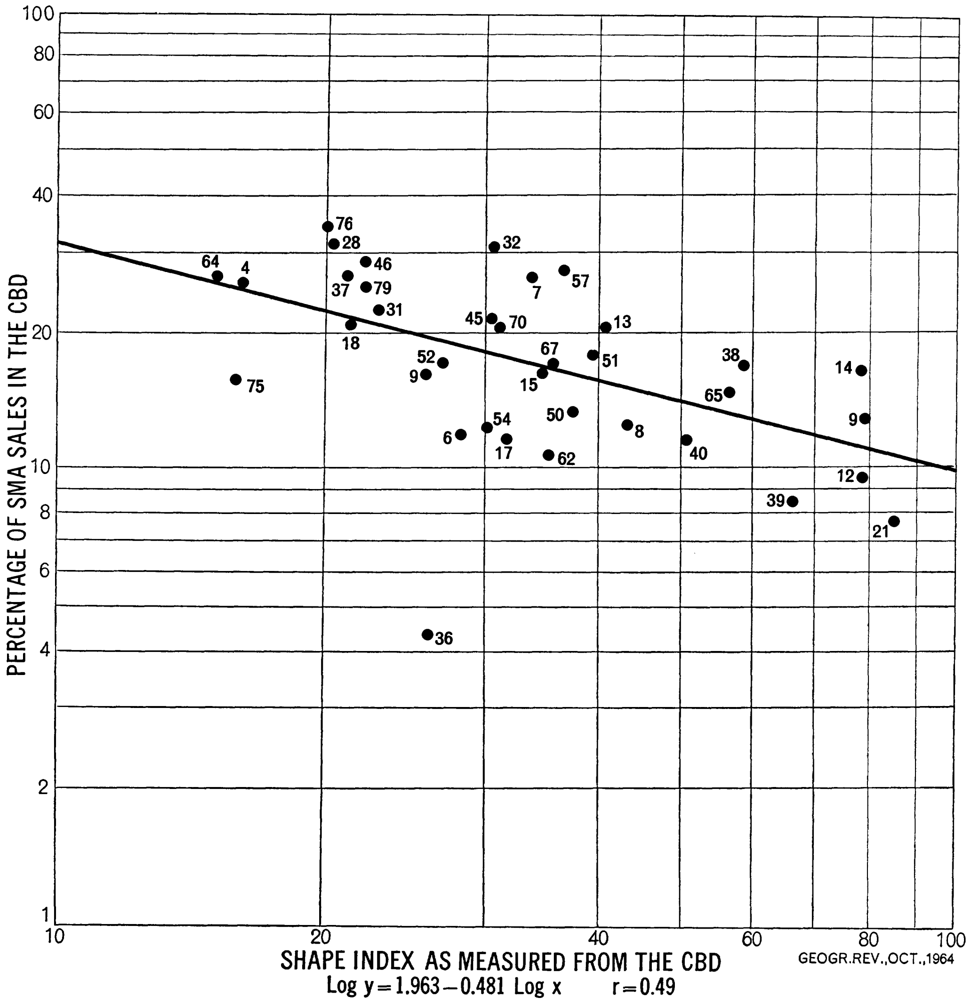


FIG. 5—Percentage of standard metropolitan area (SMA) sales in the central business district (CBD) versus residential-area shape. Figures are code numbers for the cities listed in Table I. Data are for 1958.

fore, relates to about one-quarter (r^2) of the variation in the percentage of SMA sales in any given CBD. From what is known about variables affecting the amount of sales in the CBD, this is a highly significant finding.[23] Nevertheless, the conclusions regarding the importance of shape presented here are meant to be indicative only. Additional study seems not only warranted but necessary to verify further the importance of shape in urban geography.

[23] Boyce and Clark, *op. cit.* [see footnote 16 above]. For an earlier study demonstrating the effect of population on central business district sales see Edgar M. Horwood and Ronald R. Boyce: Studies of the Central Business District and Urban Freeway Development (Seattle, 1959).