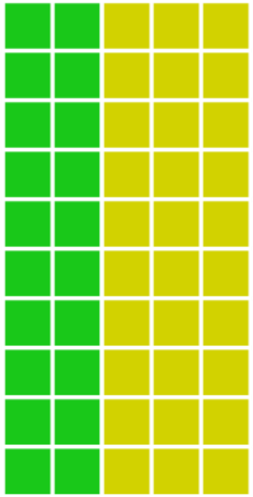
Evaluating Automated Redistricting Algorithms Using Measures of Compactness and Partisan Fairness: A Case Study of 2021 Congressional Redistricting in Virginia

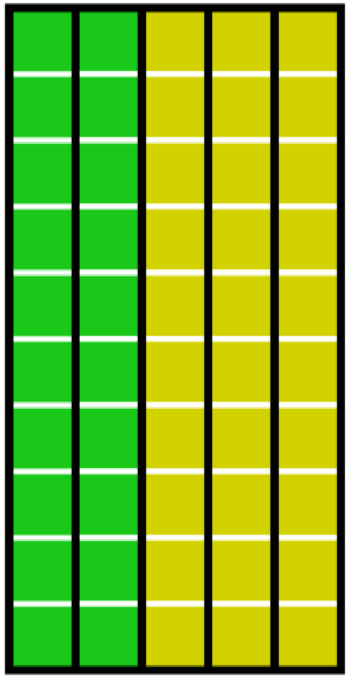
Madeleine Goertz

50 Precincts
60% Yellow
40% Green

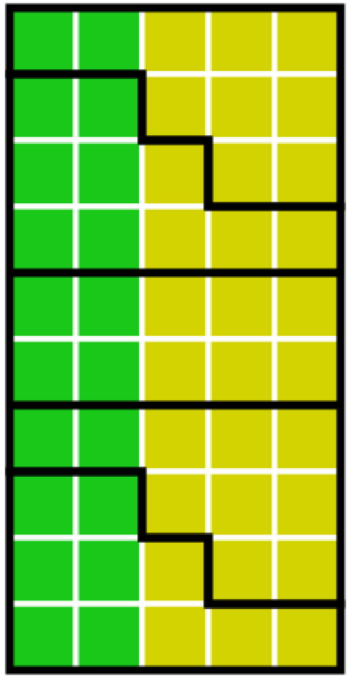


50 Precincts are to be apportioned into 5 districts, 10 precincts each district.

Proportionate Outcomes



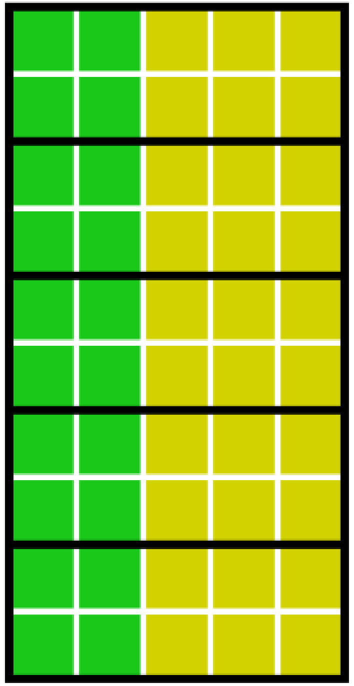
5 DISTRICTS
3 Yellow
2 Green



5 DISTRICTS
3 Yellow
2 Green

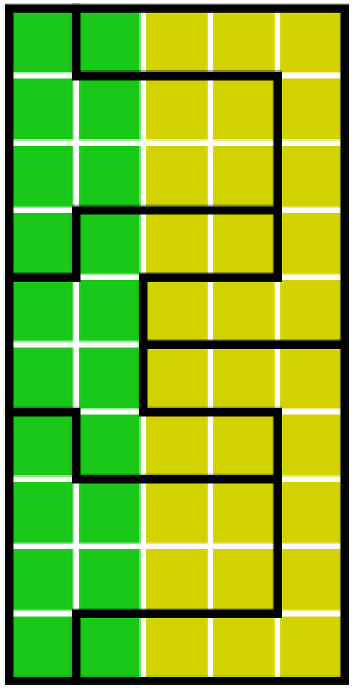
Green and yellow win in proportion to their voting

Disproportionate Outcomes “gerrymandering”



5 DISTRICTS
5 Yellow
0 Green

YELLOW WINS ALL



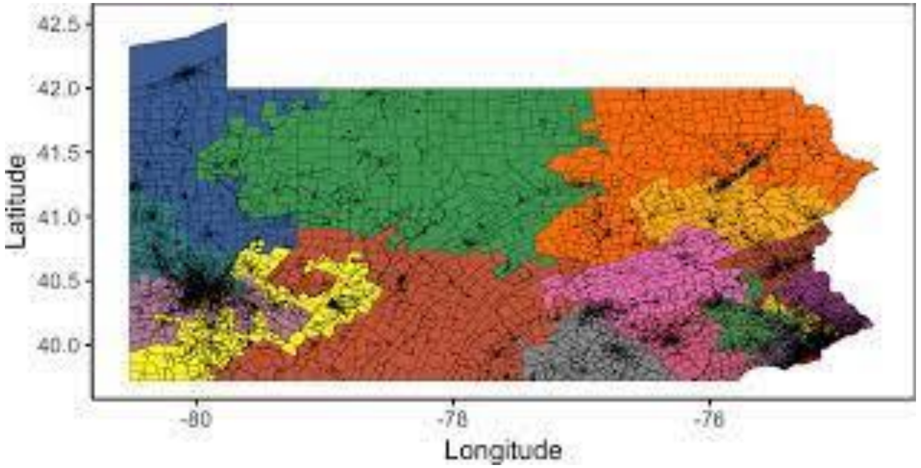
5 DISTRICTS
3 Green
2 Yellow

GREEN WINS MAJORITY

Adapted from M.boli, 2017

Automated Redistricting Algorithms

Population Data &
Geographic Data

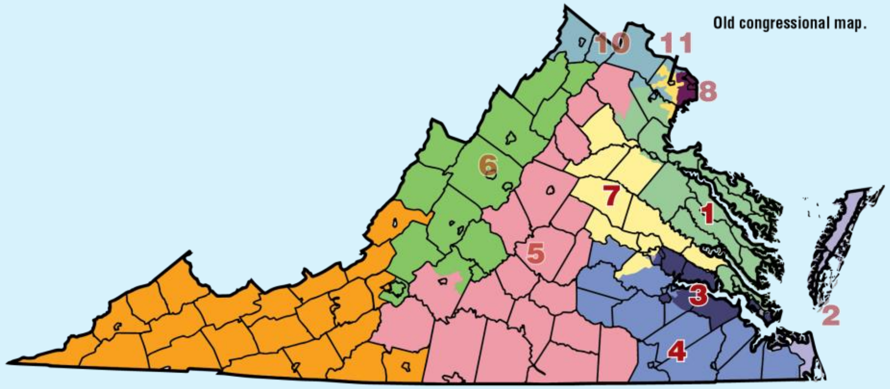


(many maps)

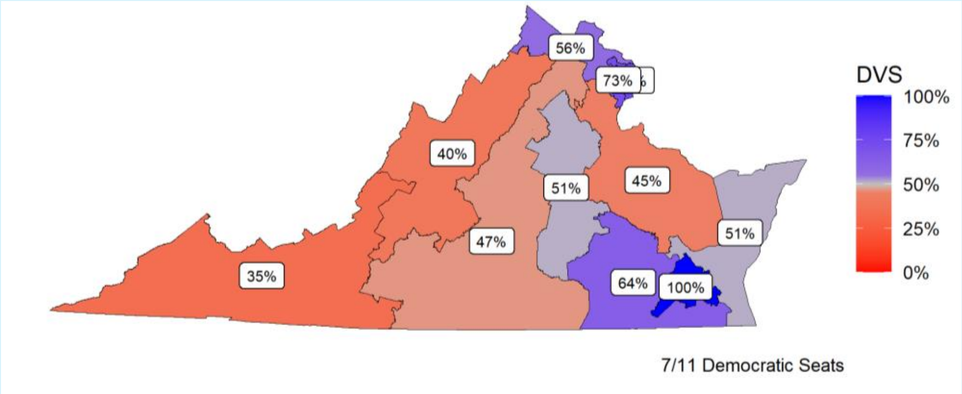
(Fifield et al., 2020)

How do the hypothetical district maps for the Virginia Congressional delegation for the 2020s generated by different automated redistricting algorithms compare based on compactness and partisan fairness measures?

Why Virginia?



Gerrymandered Map



Competitive Elections

(Loughran, 2016; Virginia Division of Legislative Services, 2021)

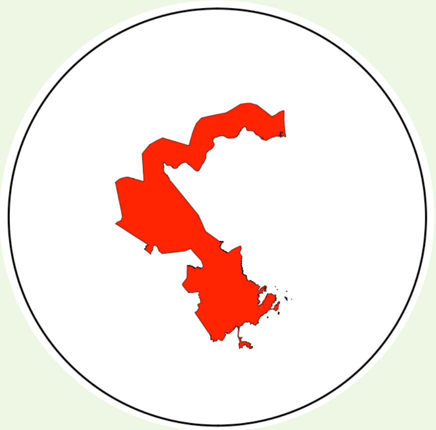
Which Algorithms?

Compact Random Seed Growth (CRSG), 2013

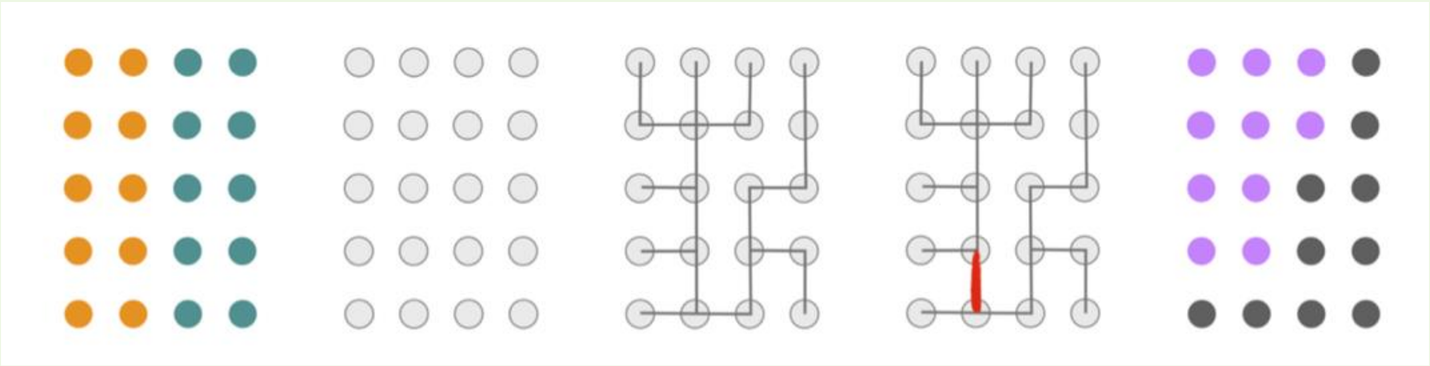
Sequential Monte Carlo (SMC), 2020

(Chen & Rodden, 2013; McCartan & Imai, 2020)

Which Compactness Measures?



Geometry Perspective



Graph Theory
Perspective

(Fisher, 2017; DeFord et al., 2021)

Which Partisan Fairness Measures?

Seats-Votes Curve

Efficiency Gap

Partisan Symmetry

Declination

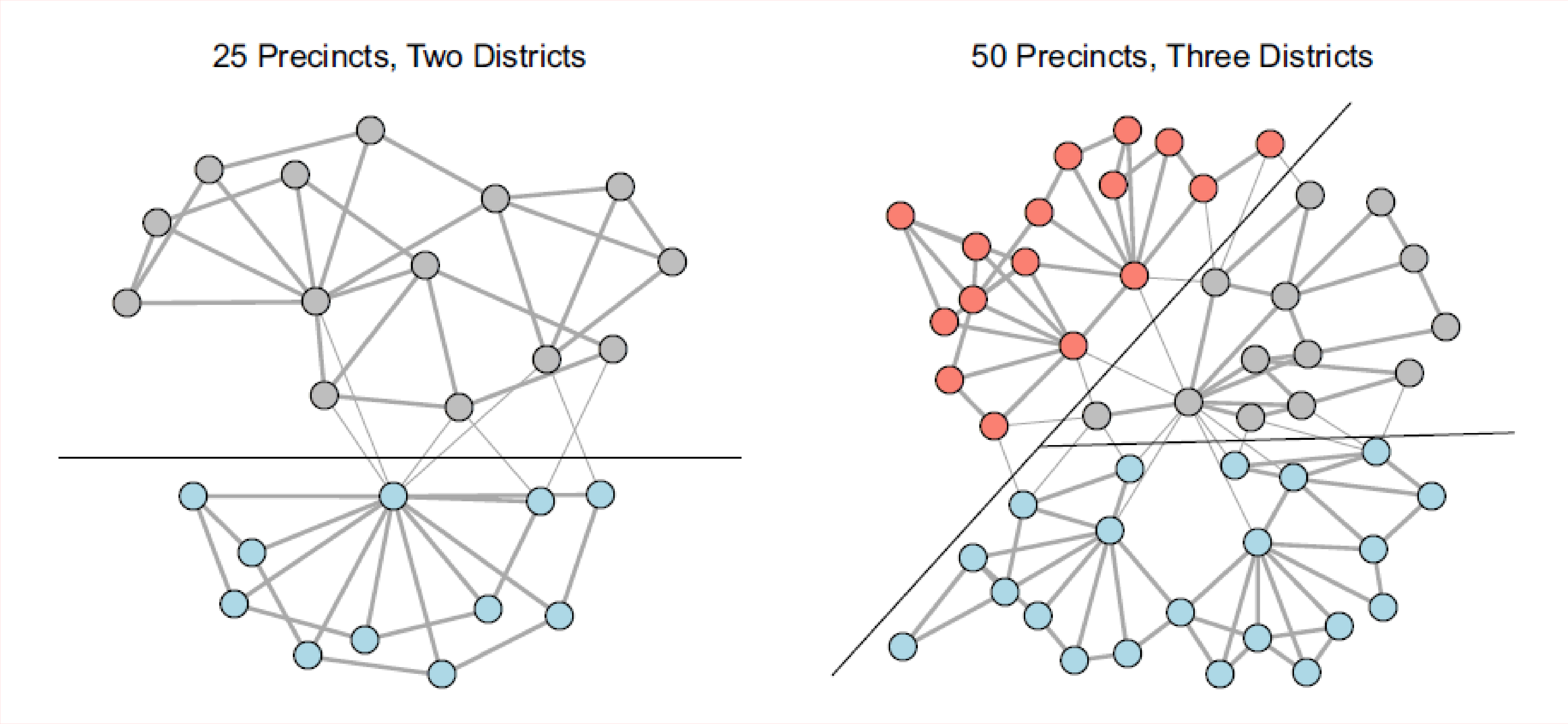
Partisan Bias

Mean-Median Difference

(Tufte, 1973; Katz et al., 2020; Stephanopoulos & McGhee, 2014; Warrington, 2018; McDonald & Best, 2015)

Literature Review

Redistricting as Graph Cutting



(Fifield et al., 2020)

Compact Random Seed Growth (CRSG)

1. Each precinct is one district.

Repeat until desired number of districts formed:

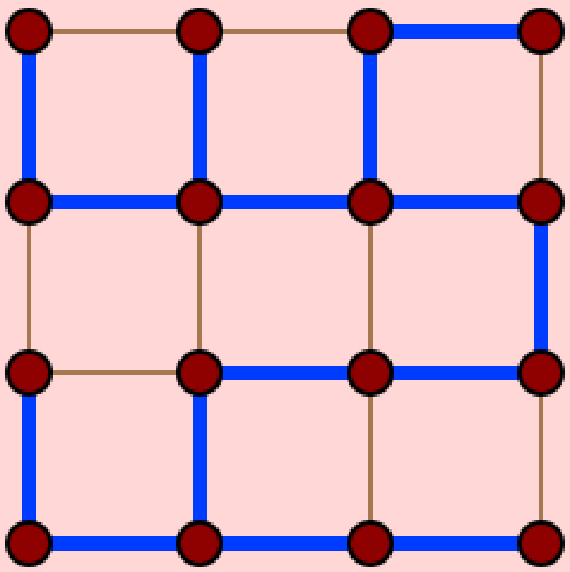
2. Random district: merge with closest neighbor.

Repeat until desired population parity reached:

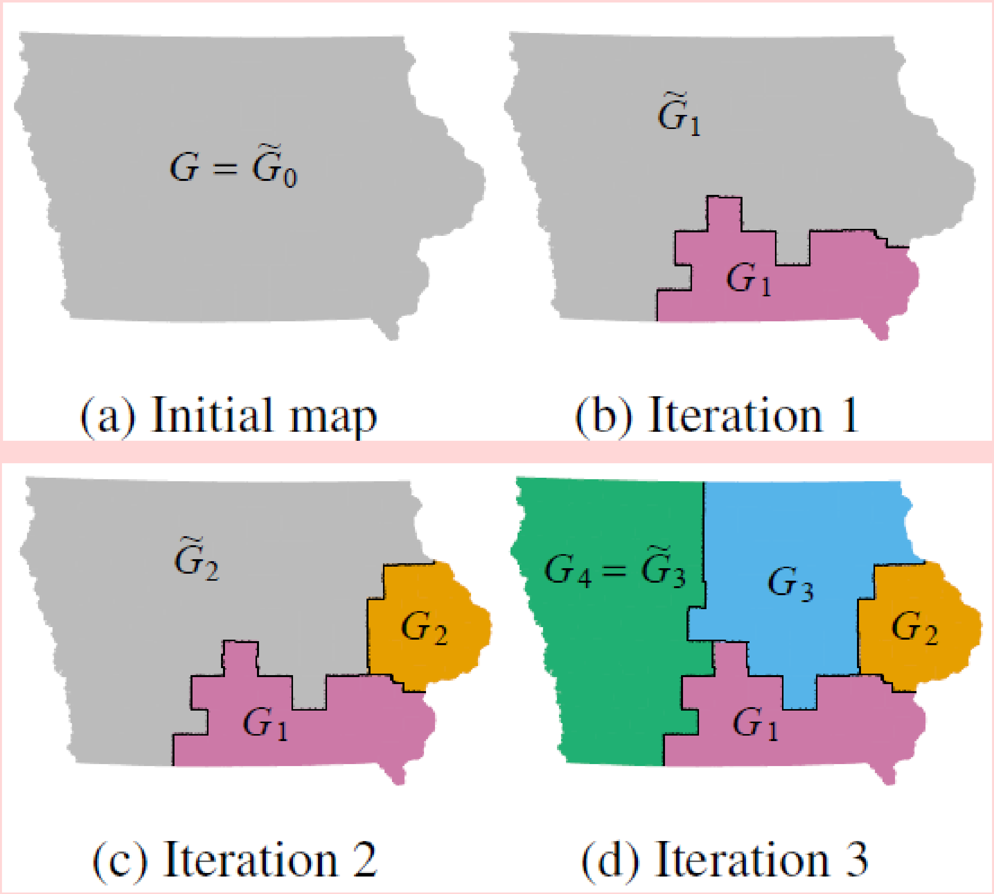
3. Reassign one precinct from most-populous district to less populous district.

(Chen & Rodden, 2013)

Sequential Monte Carlo (SMC)



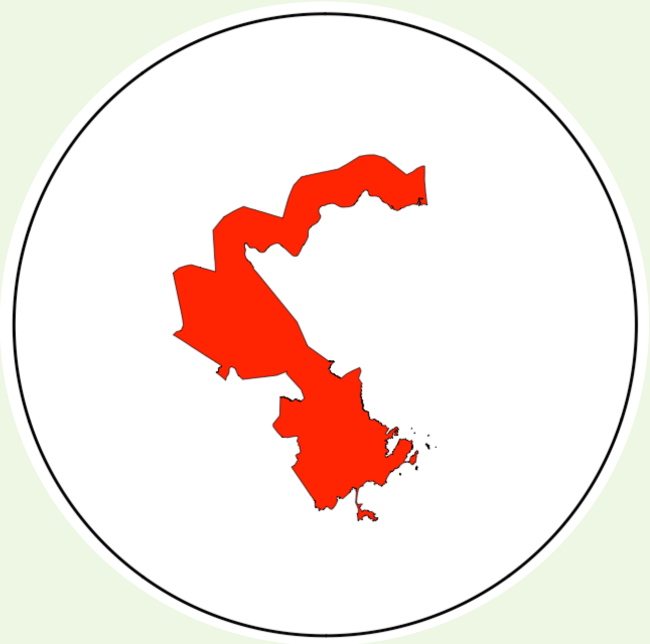
A Spanning Tree



Sample Run of SMC

(Eppstein, 2007; McCartan & Imai, 2020)

Polsby-Popper Score

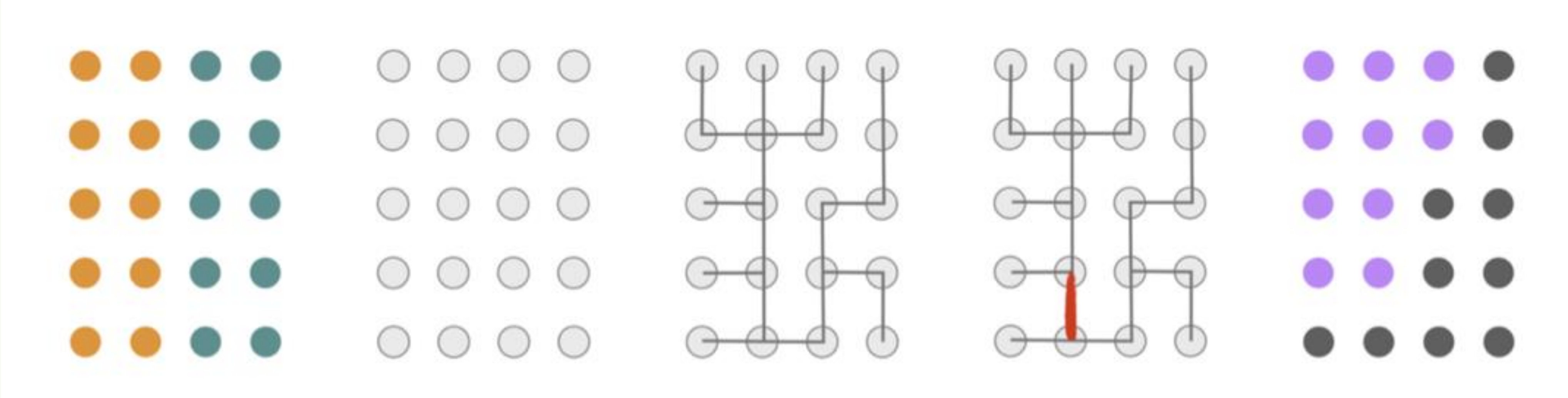


$$PP(d) = \frac{4\pi A(d)}{P(d)^2}$$

$$PP(d) \propto compactness$$

(Fischer, 2017; Polsby & Popper, 1991)

Edge-Cut Compactness

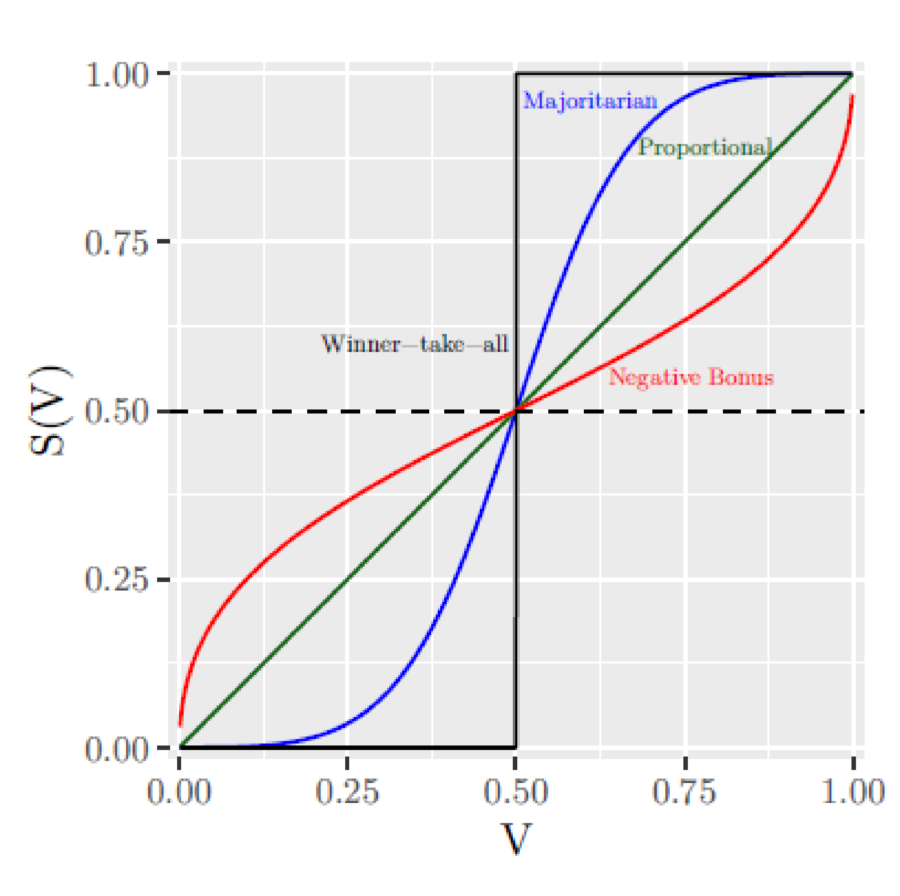


$$ECC = 1 - \frac{n}{N}$$

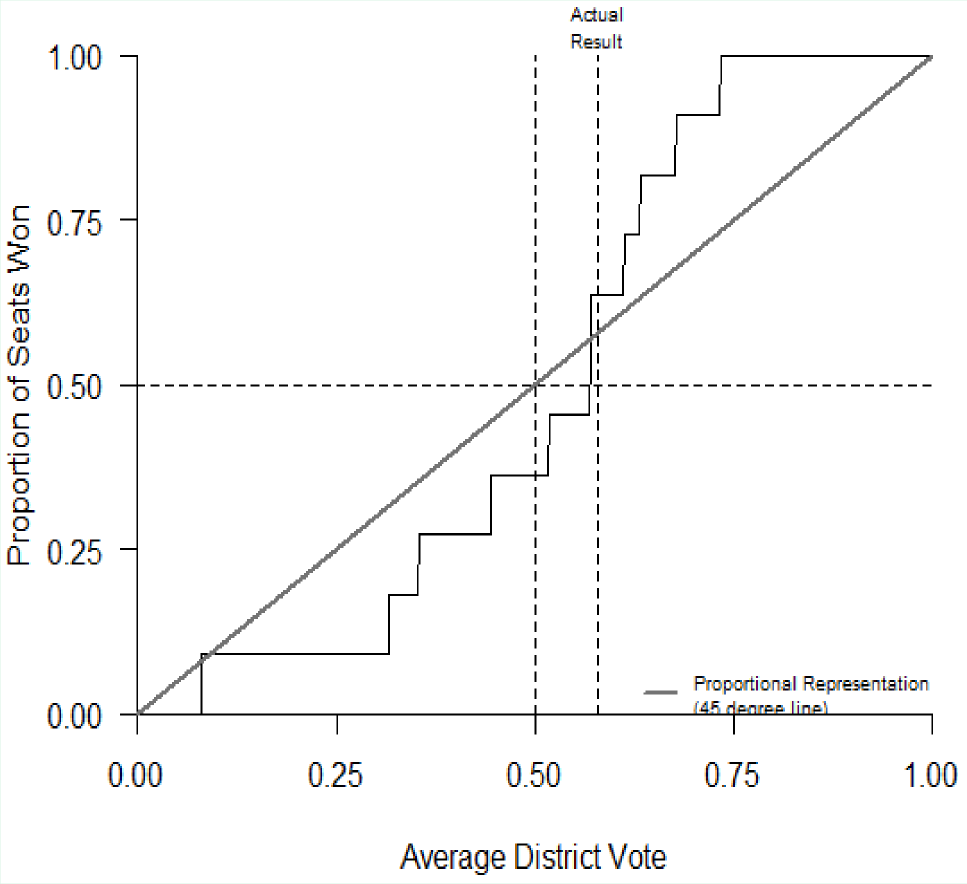
$$ECC \propto compactness$$

(DeFord et. al, 2021; Dube & Clark, 2016)

Seats-Votes Curves, Partisan Symmetry & Bias



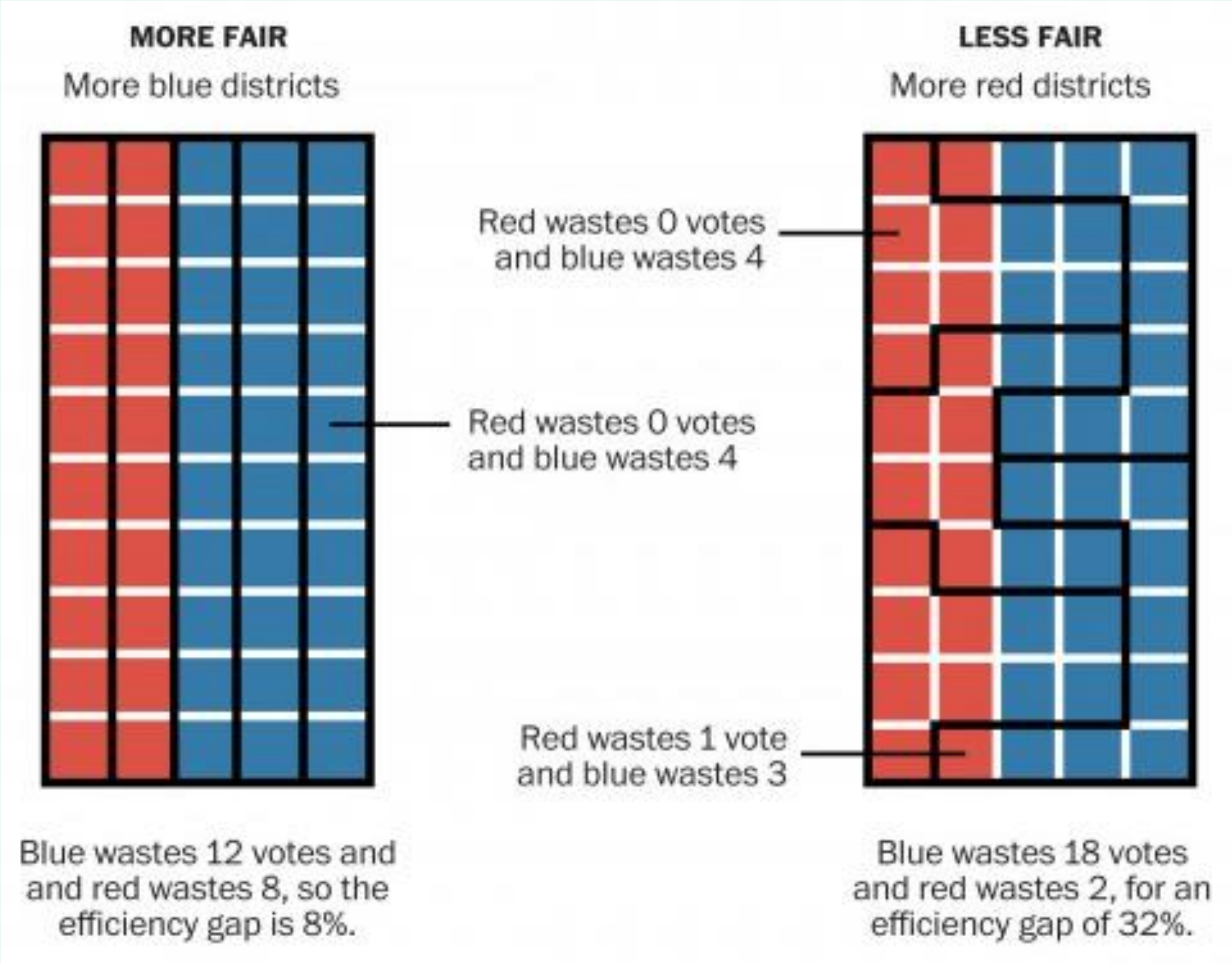
Hypothetical Curve



Curve generated by UPS

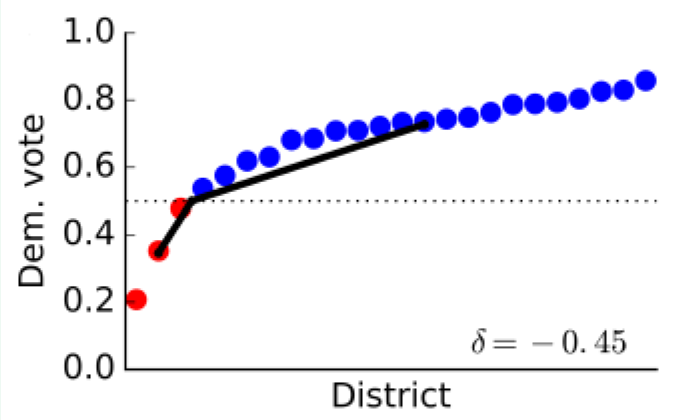
(Katz et. al, 2020; Tufte, 1973)

Efficiency Gap

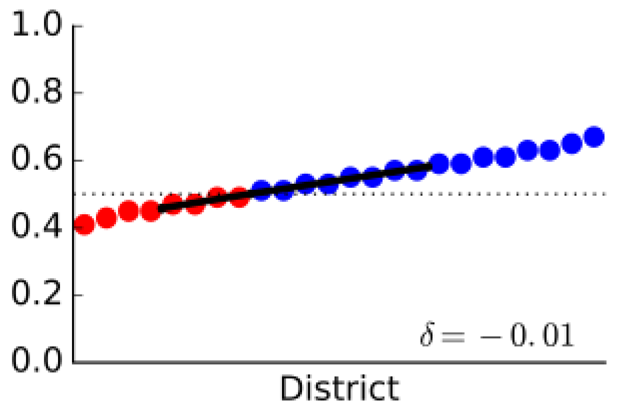


(McGlone, 2017; Stephanopoulos & McGhee, 2014)

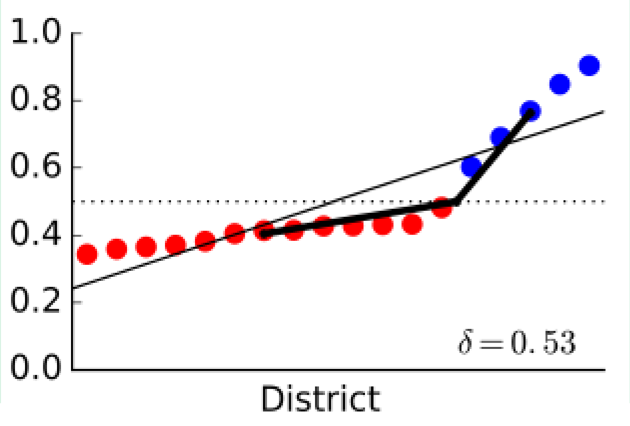
Declination



Favoring Dem. ($\delta < 0$)



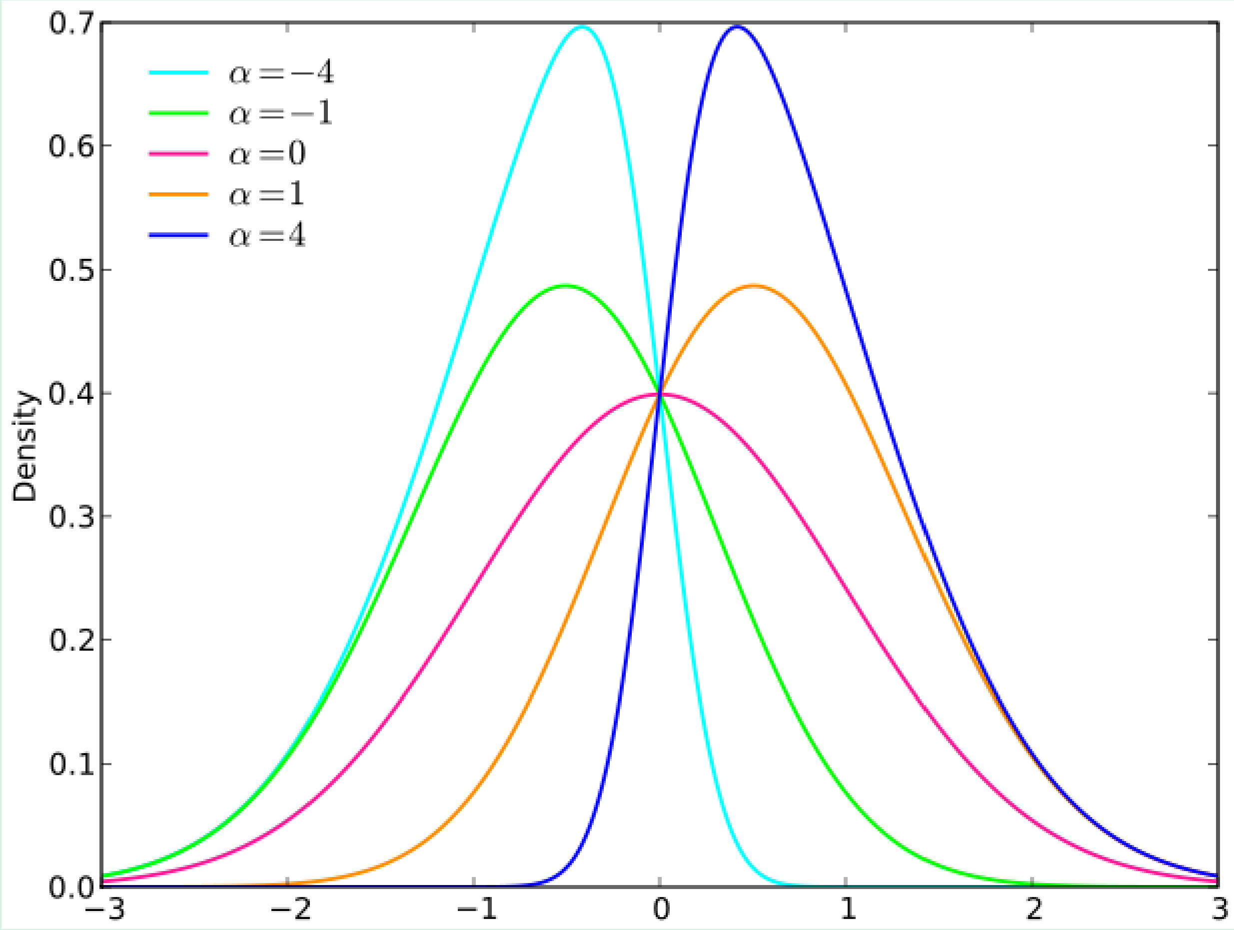
Fair ($\delta \approx 0$)



Favoring Rep. ($\delta > 0$)

(Warrington, 2018)

Mean-Median Difference



$$MMD = \mu - m$$

$$\left|\frac{1}{MMD}\right| \propto fairness$$

(Skbkekas, 2009; McDonald & Best, 2015)

Method

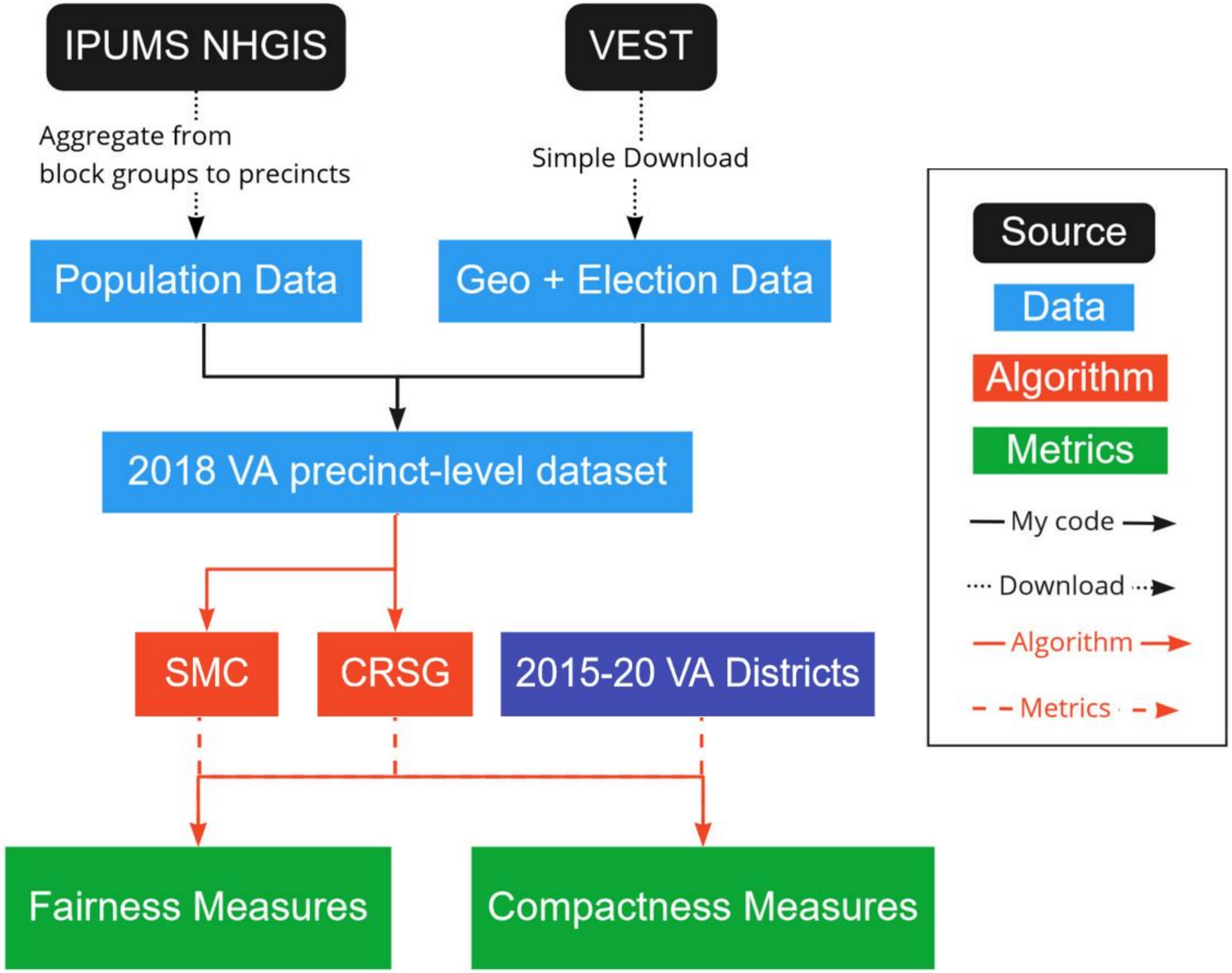
Experimental Research

Isolate effect of algorithms on fairness.

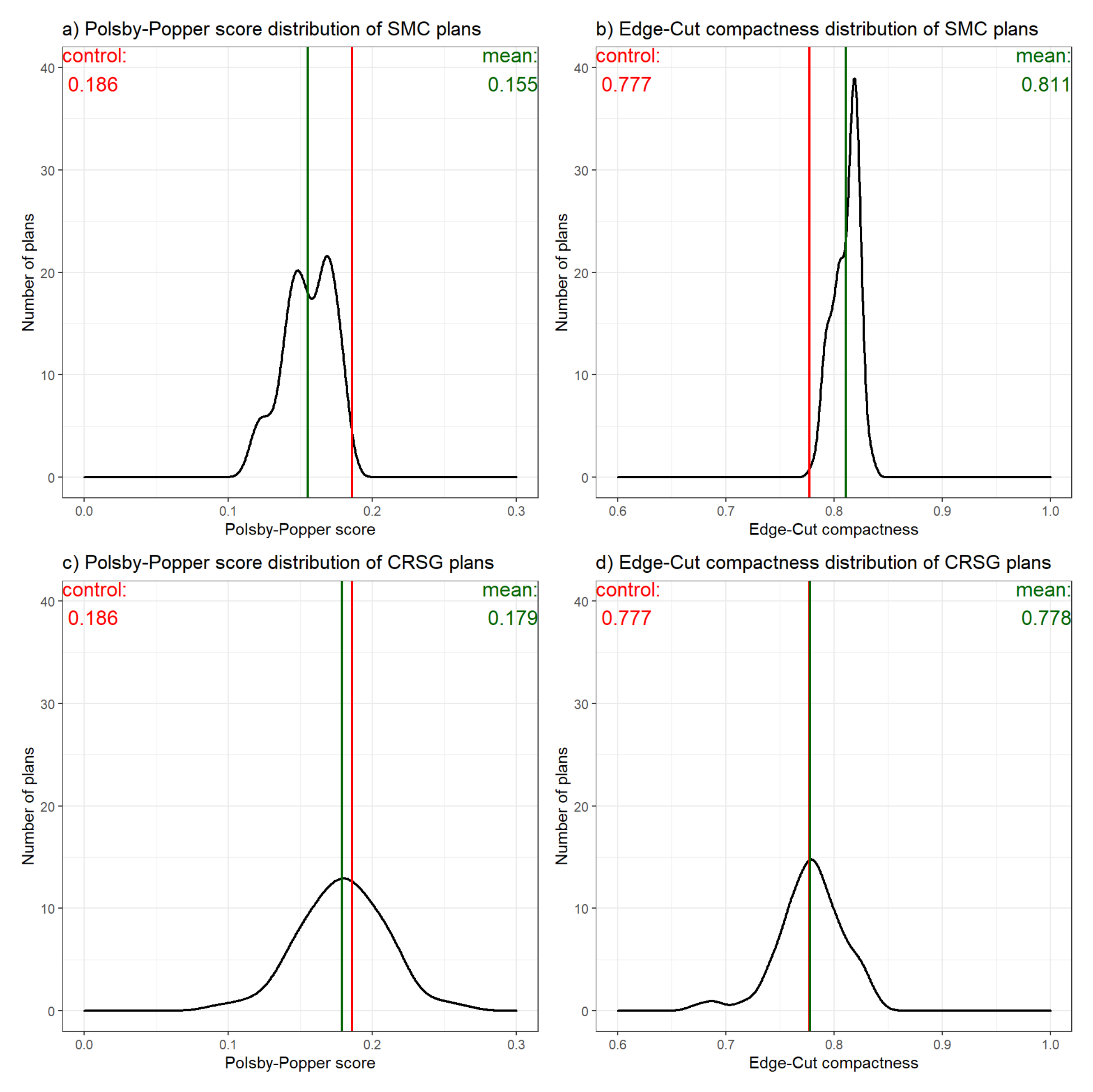
Explanatory variable: algorithm choice.

Response variables: compactness, fairness measures.

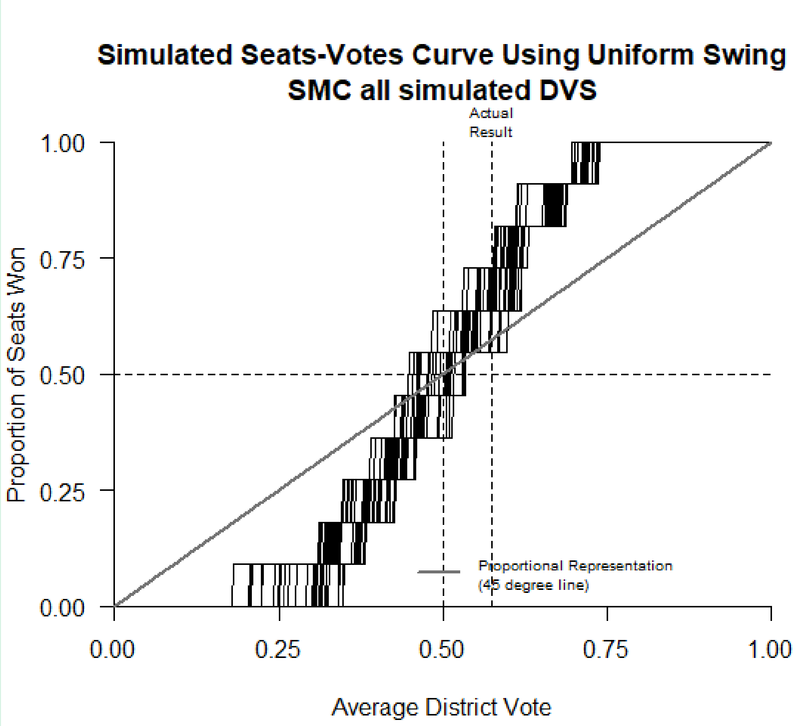
Control: 2015-2020 district map.

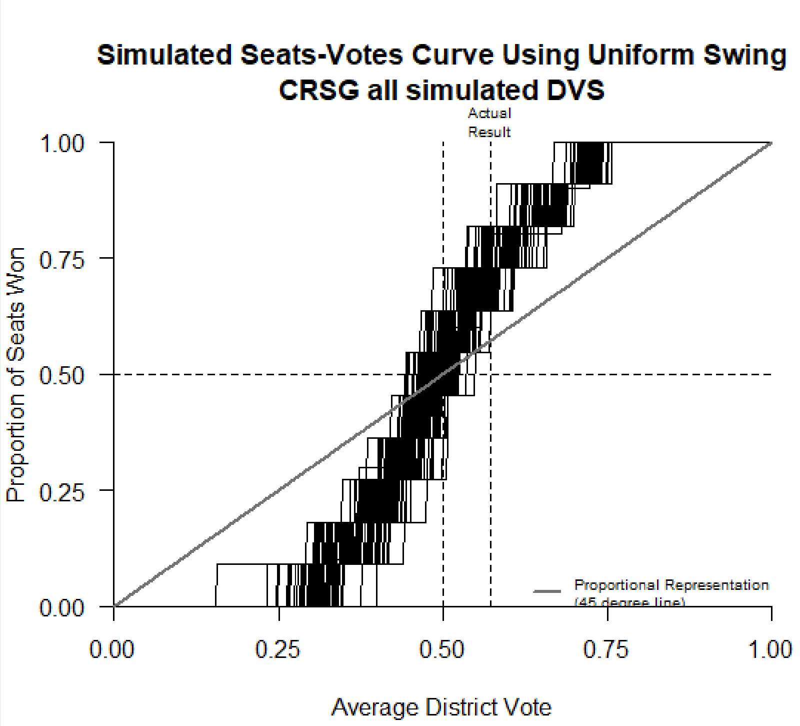

IPUMS NHGIS
VEST
Aggregate from
block groups to precincts
Simple Download
Population Data
Geo + Election Data
2018 VA precinct-level dataset
SMC
CRSG
2015-20 VA Districts
Fairness Measures
Compactness Measures
Source
Data
Algorithm
Metrics
My code
Download
Algorithm
Metrics

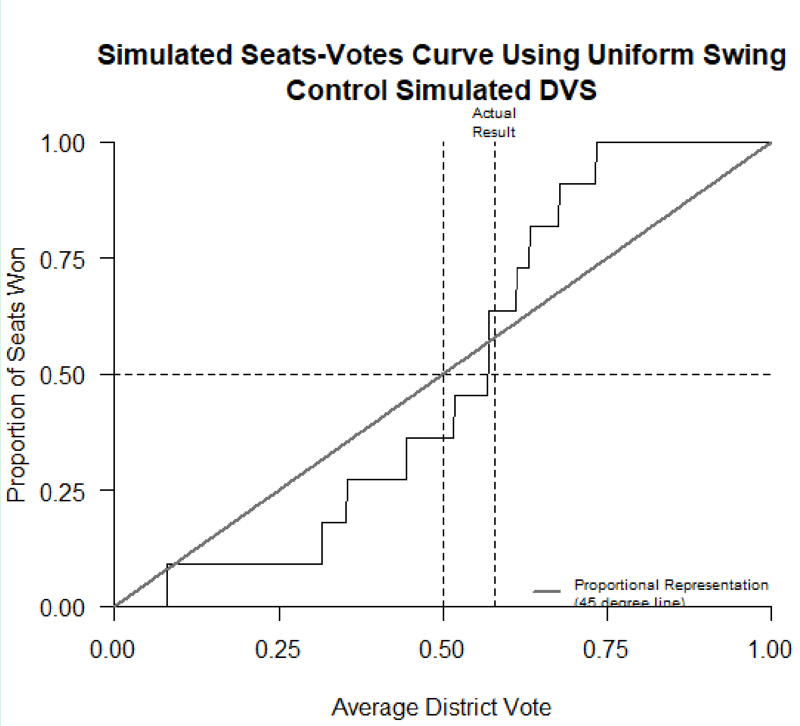
Results

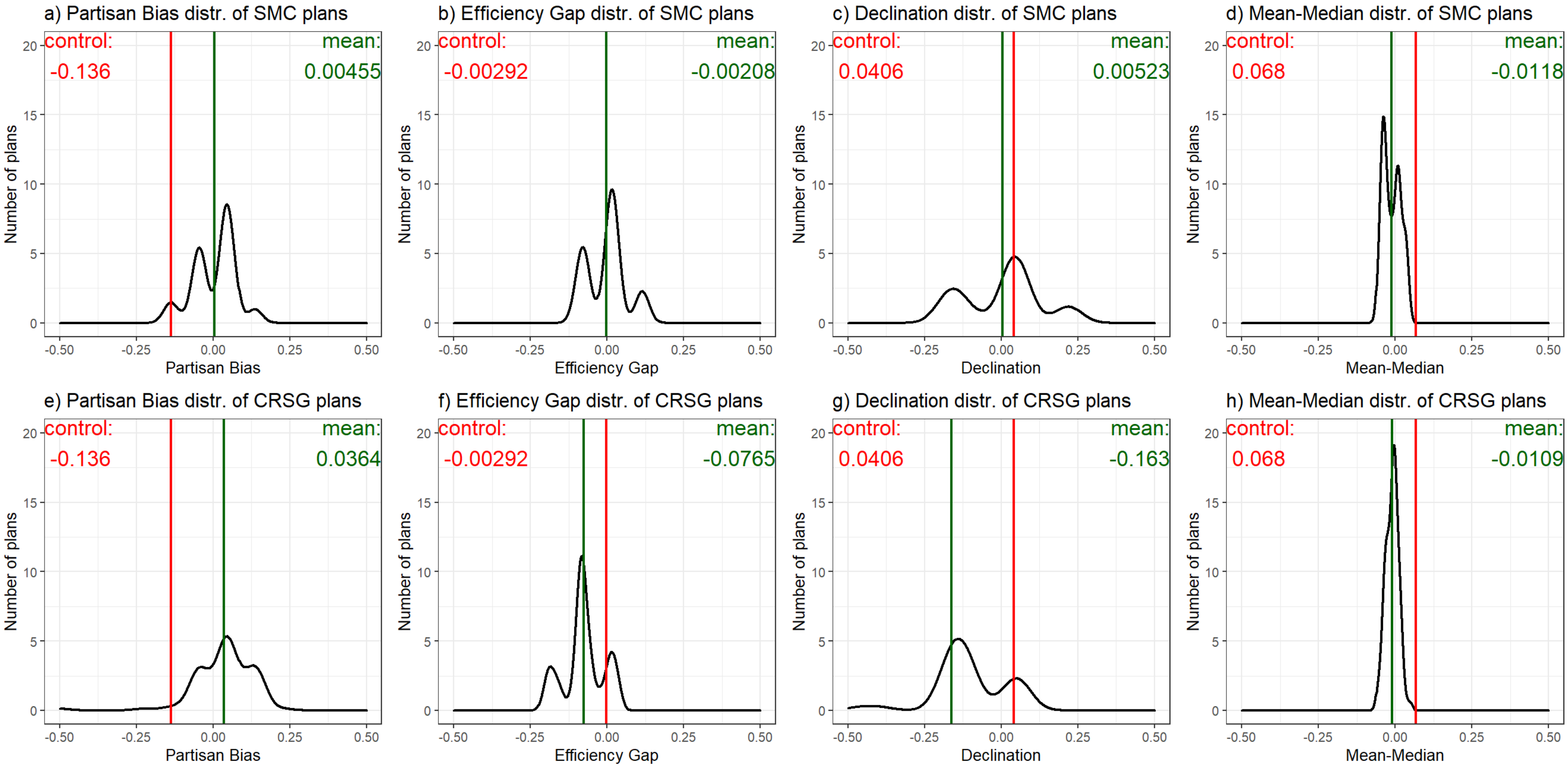


a) Polsby-Popper score distribution of SMC plans
control:
0.186
mean:
0.155
Number of plans
Polsby-Popper score
b) Edge-Cut compactness distribution of SMC plans
control:
0.777
mean:
0.811
Number of plans
Edge-Cut compactness
c) Polsby-Popper score distribution of CRSG plans
control:
0.186
mean:
0.179
Number of plans
Polsby-Popper score
d) Edge-Cut compactness distribution of CRSG plans
control:
0.777
mean:
0.778
Number of plans
Edge-Cut compactness


Simulated Seats-Votes Curve Using Uniform Swing
SMC all simulated DVS
Actual
Result
Proportion of Seats Won
1.00
0.75
0.50
0.25
0.00
Proportional Representation
(45 degree line)
0.00
0.25
0.50
0.75
1.00
Average District Vote


Simulated Seats-Votes Curve Using Uniform Swing
CRSG all simulated DVS
Actual
Result
Proportion of Seats Won
1.00
0.75
0.50
0.25
0.00
Proportional Representation
0.00
0.25
0.50
0.75
1.00
Average District Vote


Simulated Seats-Votes Curve Using Uniform Swing
Control Simulated DVS
Actual
Result
Proportion of Seats Won
1.00
0.75
0.50
0.25
0.00
Proportional Representation
0.00
0.25
0.50
0.75
1.00
Average District Vote



a) Partisan Bias distr. of SMC plans
control: -0.136
mean: 0.00455
b) Efficiency Gap distr. of SMC plans
control: -0.00292
mean: -0.00208
c) Declination distr. of SMC plans
control: 0.0406
mean: 0.00523
d) Mean-Median distr. of SMC plans
control: 0.068
mean: -0.0118
e) Partisan Bias distr. of CRSG plans
control: -0.136
mean: 0.0364
f) Efficiency Gap distr. of CRSG plans
control: -0.00292
mean: -0.0765
g) Declination distr. of CRSG plans
control: 0.0406
mean: -0.163
h) Mean-Median distr. of CRSG plans
control: 0.068
mean: -0.0109
Number of plans
Partisan Bias
Efficiency Gap
Declination
Mean-Median

Conclusion

How do the hypothetical district maps for the Virginia Congressional delegation for the 2020s generated by different automated redistricting algorithms compare based on compactness and partisan fairness measures?

SMC > CRSG > Control

Limitations

Only one state, one election, two algorithms.

Only focused on major parties.

Should observe the Voting Rights Act.

State legislative districts.

Implications

Algorithms are viable tool for redist. commissions.

Necessary under The For The People Act.

Cost-savings for redist. commissions.

Trust in electoral system.

Bibliography

tinyurl.com/redist-sources