In 1961 the U. S. Supreme Court ruled that federal courts have jurisdiction in cases concerning states' apportionment laws when there is a possible denial of rights guaranteed by the Fourteenth Amendment to the Constitution. Because of this decision, people in some states may for the first time have an opportunity to obtain equality of representation in state and federal legislatures. Reapportionment and redistricting are taking place, but what are the chances for obtaining and maintaining something approaching true equality? The changes may not be sufficient to upset the political power now held by minority political groups in these states, since there is still a possibility of gerrymandering. A scientific method of apportionment which includes automatic changes after every census has been accepted for the apportionment of the U. S. House of Representatives. As a result it seems that the bitter congressional fights and power struggles and the resulting inequalities of representation due to reapportionment have been reduced. But why stop there? Why not do the same for the process of districting?

Presented in this paper is one possible scientific or impartial method of districting that the author believes would free, to a certain degree, the process of districting from power struggles and gerrymandering. The method can apply to either the U. S. House of Representatives or to state legislative chambers in states where apportionment is based on population.

A SCIENTIFIC METHOD OF DISTRICTING

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THE need for redistricting of state legislative chambers and the U. S. House of Representatives is unquestionable. The consequences of inequalities in representation and concentration of political power, created either by failure to redistrict and reapportion[1] or by malredistricting and malreapportionment, are well documented and need not be discussed here (Baker, 1955; Farreley & Hinderaker, 1952; *Law and Contemporary Problems*, 1952; *Congressional Record*, 1956; Dauer & Kelsay, 1955, pp. 572, 574; Shull, 1952).

Many possible methods of impartial districting could be devised to create districts with nearly equal populations. An example of one, along with an excellent rationale for impartial methods, is given by Vickrey (1961). Both the Vickrey method and the one described here eliminate, to a large extent, the element of human discretion. The method presented here, however, yields a unique solution of districts with nearly equal populations by applying a precise rule for compactness.

[1] A distinction is made between "apportionment" and "districting," although the words are sometimes used interchangeably and it is sometimes impossible to have one without the other. "Apportionment" is the assigning of representatives to districts, while "districting" is the process of locating district boundaries.

PRINCIPLES OF DISTRICTING

Presentation of the method of districting is facilitated when the principles of districting are divided into two groups. Group 1 will be used to obtain the first approximation to district lines, and Group 2 the second approximation.

The principles in Group 1 are (1) populations of districts shall be as nearly equal as possible, (2) districts shall be composed of contiguous territory, (3) districts shall be composed of compact territory, and (4) there shall be one, and only one, representative from each district. These principles were recommended by the American Political Science Association Committee on Con-

gressional Reapportionment (1951), and by President Truman in 1951. They are not provisions of the present apportionment law.

The apportionment laws of 1901 and 1911 contained all four principles, while earlier laws contained some of them. (For a list of apportionment laws and their contents see Celler [1952].) The apportionment laws of 1842 and 1862 contained provisions which stated that districts should be composed of contiguous territory, and equal in number to the number of representatives. In 1872, 1880, and 1891 an additional provision was added which stated that districts should contain as nearly equal numbers of inhabitants as is practicable.

Some state constitutions and laws also contain some of these principles of districting. Twenty-six states require contiguous territory, nine specify compactness, and 18 stipulate that districts should have populations as nearly equal as possible (Harvey, 1952, p. 368).

The principles in Group 2 are all those not in Group 1. Some of these are: (1) districts should contain communities of common economic and social interests, and allowance should be made for other ecological and psychological factors; (2) districts' boundaries should follow geographic and topographic features; (3) districting should make allowance for transportation facilities; (4) districting should take into account desires of the general public, the politicians, and political parties; and (5) district lines should follow other political boundaries.

THE FIRST APPROXIMATION

The recommendations of President Truman and the American Political Science Association Committee gave more precise meaning to the first principle of Group 1 listed above. The APSA Committee recommended that a district population should be kept within 10 per cent of the average, and in no event should it exceed 15 per cent. President Truman recommended that the population of each district be within 15 per cent of the state average population per district. In the First Session of the Eighty-second Congress, Representative Celler introduced a bill, HR-2648, that would have restored the Group 1 principles of districting, with the requirement that the population in each district be within 15 per cent of the state average (Celler, 1952, p. 274).

Obviously, inequities would be reduced if the population in each district were within 15 per cent of the state average. But since there is a large number of arrangements of districts that could meet this requirement, the drawing of district lines still would be subject to gerrymandering. By giving precise meaning to compactness, it is possible to draw impartial district lines.

The word "compact" means closely united or arranged so as to economize space. A circle, then, would have the greatest compactness. But obviously circular districts would not be feasible. If one or several districts were formed into circles, the remaining districts would be odd-shaped, since circles do not fit together neatly. A few circular districts would mean a sacrifice of compactness in other districts.

A square district would be compact. Squares are more compact than other rectangles, and they fit together side by side without any waste of space. But upon closer examination it would be impossible to have *all* square districts. The shape of each state is fixed and its outside boundaries are irregular. Even with straight outside boundaries it would be impossible to have all districts square and have the population per district equal. Forming as many square districts as possible could leave nonsquare residual districts, as illustrated in Figure 1.

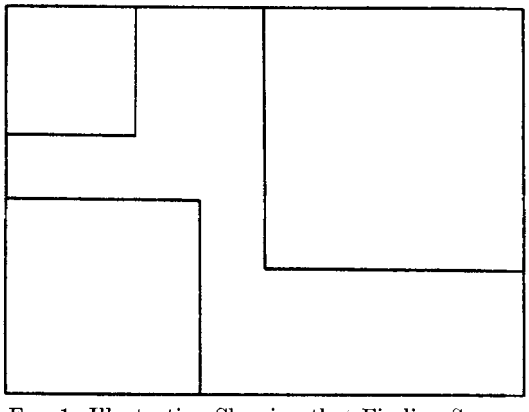

FIG. 1. Illustration Showing that Finding Square Districts with Equal Populations Can Force Nonsquare Residual Districts.

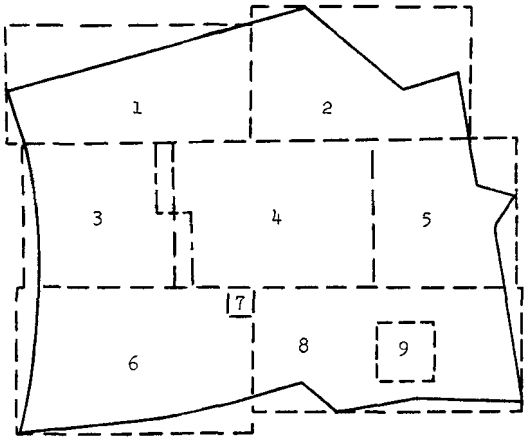

FIG. 2. Illustration Showing Compactness in
State boundary
– – – District boundary
— — — — A jogged district line
———Districts of a Hypothetical State.

The next best degree of compactness would be rectangles, including squares. Nothing can be done about the irregular outside boundaries of states, but if lines between districts were straight, and districts wholly within the state were rectangles, then districts could be made compact. (Rather than having true rectangles, it might be more practicable to have district lines follow longitude and latitude lines.) This is illustrated in Figure 2. District boundaries on irregular state lines are assumed to be straight, so that all districts are rectangles.

Wherever possible, the lines between districts would not contain jogs or be irregular. Figure 2 illustrates that a straight line between districts 3 and 4 would make them more compact than would the jogged line. Lines between districts would run only in two directions—for example, east and west and north and south—otherwise there would be a sacrifice in compactness.

In order to get maximum compactness of *all* districts, it is necessary to have a simultaneous solution. We could not start, say, in the northwest corner of the state and find an area with a population equal to the state average per district, then proceed to the next area which has the greatest compactness. Just as with forming circular or square districts, this procedure probably would leave residual districts irregular in shape, such as an "L" shape. However, the aim is to get districts as nearly square as possible, i.e., to have a minimum difference between length and width.

The solution giving the greatest compactness to all districts, considered together or simultaneously, would be an arrangement that minimizes the sum of the difference between the length and width of all districts. This is expressed as:

$$\text{Min} \sum_{i=1}^{n} L_i - W_i \qquad (1)$$

where n is the number of representatives or the number of districts, L is the maximum length of any district, and W is the maximum width of any district.

Another possible formulation of compactness would minimize the sum of the lengths divided by the widths. If all districts were square the sum would be n, the number of districts. It might be argued that this method would not penalize large districts in the final solution, since relative measures are used. That is, a district two miles by six miles in a heavily populated city would lack compactness to the same degree as a district measuring 200 miles by 600 miles. Size, however, does have something to do with compactness. Compactness, as mentioned above, means closely united or arranged so as to economize space. Thus a small district would be more compact than a large one. The measure of length minus the width would mean that a district two by six miles would be more compact than a district 200 by 600 miles.

In the final solution some districts may not be entirely rectangular (disregarding irregular outside boundaries) because of population concentrations. In Figure 2 two lines of district 6 have a jog because there were enough people in the city, at 7, to form an entire district. Also, there may be island solutions such as district 9, which is surrounded by district 8. The possibilities that these solutions are more compact than others will become apparent when the operational procedure of finding district lines is discussed.

Procedure for drawing district lines

The bases for districting, as used here, are the requirements of equality of population,

contiguity, compactness, and singleness of districts. The requirement equality of population will be met if population in each district is within 10 per cent of the state average population per district. Districts will be as compact as possible when the sum of the lengths minus the widths of all districts is smaller than it would be under any other arrangement.

The operational steps for districting are as follows:

1. Locate cities and urban areas with populations greater than the average population per district. It is quite possible that parts of these areas will be whole districts in the final solution—the area with the greatest population density would be the smallest possible district.

2. Place a grid over or under a map of the state so that the entire state is contained within the grid. A convenient size for the cells in the grid would be the size of the smallest possible district, since this may be a district in the final solution. For example, if 100 square miles of the most densely populated area contains a population equal to that of the average district, then cell size of the grid would be 100 square miles, scaled down.

3. Estimate the population in each cell as accurately as possible. Census data are reported for all counties, for census county divisions in some states, and for census tracts in some areas. Using just this reported data would give fairly accurate estimates of populations in each cell. The cities could be located in their appropriate cells and the rural population could be assigned to cells by assuming an equal rural population density throughout the county, census county division, or census tract. The accuracy of the estimates would probably be sufficient, particularly in states with a small number of districts. In most cases, the difference in population per district in the final solution would be so small that any error in measurement probably would still leave the estimates within the 10 per cent tolerance limit.

A more accurate estimate of population for each cell could be obtained by using unpublished census data, such as data by enumeration districts; air photos to estimate rural population densities; sampling techniques to estimate city population densities; and voting precinct records to estimate both rural and city population densities.

4. Arrange contiguous cells into rectangular districts with populations nearly equal to state average population per district. There are many possible arrangements, and the best arrangement is found empirically. The lack of a precise method, however, in no way reflects on the end result. An arbitrary arrangement, with its sum of the lengths minus the widths, can be found and other arrangements tested against it. When a better arrangement is found, reject the previous one, until the best is obtained. This procedure could be programmed on large computers, but for states with a small number of districts the solutions easily can be found by hand. Many possible arrangements are rejected at sight, such as districts running the length of the state; others can be eliminated by making rough approximations of the placement of district lines.

Example

Districting for the U. S. House of Representatives in Colorado was chosen to illustrate the above procedure because Colorado has only four districts, only one city with a population greater than the average population per district, and is nearly rectangular in shape. The final solution was easily obtainable. The work was done entirely from maps and data published in the 1960 population census (Bureau of the Census, 1960a, 1960b); therefore, there is some margin of error that could be corrected with a more detailed study.

Denver is the only city or urban area with a population above the average population per district for the state. Within the city (and county) limits there are 493,887 people in 71 square miles. Assuming a fairly even population density of 6,936 people per square mile, the area containing the average population per district of 438,487 would be between 63 and 64 square miles.

Sixty-four square miles (8 by 8) was used as the unit size of the cells in the grid. The grid was centered so that most of Denver was

in one cell, and so that the lines ran in north-south and east-west directions. Population for each cell was estimated by: (1) locating each city and entering its population into the appropriate cells; (2) finding the rural population in each census county division, by subtracting the total recorded city population from the total population of the area; (3) assigning the rural population to appropriate cells by assuming the rural population density of each census county division to be evenly distributed throughout; and (4) adding the city and rural populations to get the total population in each cell.

Rows and columns of the grid were totaled, and their totals were accumulated. Starting with the accumulation of row totals, solutions were found that appeared to be compact arrangements of districts. Many possible solutions were rejected, since rough approximations indicated that the sum of the lengths minus the widths was much greater than the solution being used as the guide. Rough placement of district lines was made by looking at row and column totals, and partial row and column totals for certain groups of columns and rows, respectively. When rough approximations of different solutions indicated nearly equal compactness, more precise estimates were made.

The best solution starting with row accumulations divided Denver into about four equal parts. Two lines passing through Denver—one in a north-south direction and the other east to west—divide the state into four fairly compact districts with nearly equal populations.

Two solutions were found to have greater compactness than the one described above.

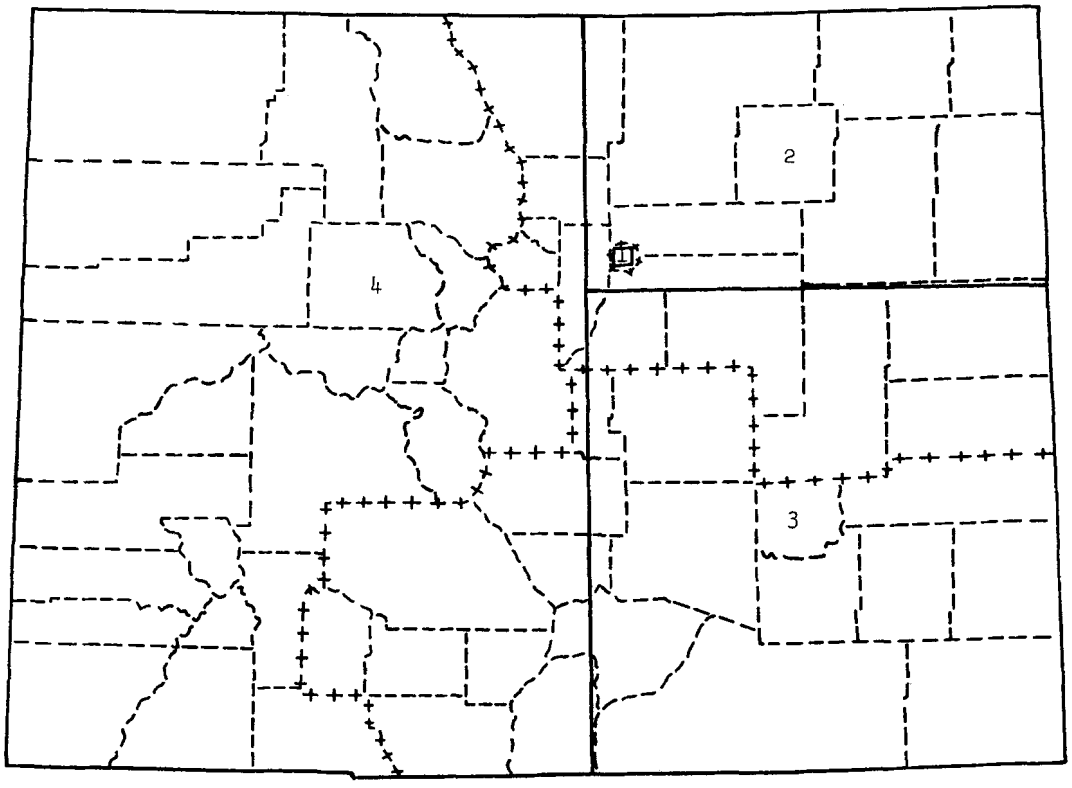

Fig. 3. Map Showing the First Approximation of the Optimum Arrangement of Congressional Districts and Existing Congressional Districts in Colorado.

– – – – County boundaries

++++ Present congressional district line

———— First approximation of the optimum congressional district line

In one, 64 square miles of Denver was one district, and another district included Boulder City and all of the urban areas surrounding Denver. The remaining two districts were divided by a north-south line running from border to border and located just east of Denver. Thus the western district contained two island districts. The suburban district was contained within the western district and the Denver district within the suburban district.

The final or best solution is illustrated in Figure 3. Part of Denver comprises a district which is an island within the northeast district, number 2 on the map. Boulder City is in the western district and Carson City is in the northeast district. The sum of the lengths minus the widths of all districts in this arrangement was smaller than any other arrangement. The approximate district sizes and population are:

District	*Approximate miles*	*Approximate population*
1	8 × 8	440,000
2	165 × 98	440,000
3	180 × 170	436,000
4	277 × 215	437,000

THE SECOND APPROXIMATION

The principles of districting in Group 2 are not easily adapted for mathematical formulation. Their meanings are not precise and are not the same for all people. For example, different people are apt to define communities of common interest differently and consequently find different dividing lines.

The straight-line solution given as the first approximation can be used as a guide to obtaining the final solution. That is, district lines can follow the geographic features, the political boundaries, or the lines separating communities of common interest that are situated closest to the linear district lines and that result in the least sacrifice in population equality.

Having district lines follow the boundaries of census tracts or census county divisions located near the linear lines would be an excellent way of accounting for most of the districting principles of Group 2. Census tracts and census county divisions have boundaries that normally follow physical features of the land and are "generally designed to achieve some uniformity of population characteristics, economic status, and living conditions." (Bureau of the Census, 1960, p. xxviii). In areas where census tract or census county division data are not available precinct lines or county lines can be followed, or an attempt can be made to find new lines that account for the principles in Group 2. (Data on state economic areas might be helpful. See Bogue & Beale [1961].)

CONCLUSIONS

The first approximation to district lines produces districts that contain nearly equal populations, and that are compact. After the second and final approximation the district lines follow geographic features and community ecological division lines, but there is a small sacrifice in compactness as defined here. The final districts may or may not have the same degree of population equality. It is more likely, however, that there would be some loss of equality by having the lines follow census tract lines. By comparing the first approximation with the second, it is possible to see what sacrifices, if any, are being made in representative equality in order to gain other democratic and representative principles.

There would be many barriers to the adoption of a scientific method, but they might not be more severe than the barriers to any redistricting (Baker, 1955, pp. 64–66; 1960, pp. 37–42; and Harvey, 1955, pp. 370–375). The chances of adoption probably would be greater for districting the membership of the U. S. House of Representatives than for state legislatures, if the districting were controlled by Congress.

Schmeckebier (1941, p. 143) suggests that there would be less danger of gerrymandering in Washington due to the great amount of publicity afforded congressional activities, although he proposes that Congress redistrict only when states fail to meet Federal requirements. For views on the constitutional authority and congressional power to redistrict see Schmeckebier (1941, pp. 131–149); Tabor (1956); Celler (1952, p. 273); Todd (1952, pp. 318–319); and Paschal (1952).

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(Manuscript received August 20, 1963)

Laws are coldly reasoned out and established upon what the lawmakers believe to be a basis of right. But customs are not. Customs are not enacted, they grow gradually up, imperceptibly and unconsciously, like an oak from its seed. In the fullness of their strength, they can stand up straight in front of a world of argument and reasoning, and yield not an inch. We do not know how or when it became custom for women to wear long hair, we only know that in this country it *is* custom, and that settles it. Maybe it is right, maybe it is wrong—that has nothing to do with the matter; customs do not concern themselves with right or wrong or reason. But they have to be obeyed; one may reason all around them until he is tired, but he must not transgress them, it is sternly forbidden. Women may shave their heads elsewhere, but here they must refrain or take the consequences. Laws are sand, customs are rock. Laws can be evaded and punishment escaped, but an openly transgressed custom brings sure punishment. The penalty may be unfair, unrighteous, illogical, and a cruelty; no matter, it will be inflicted just the same. Certainly, then, there can be but one wise thing for a visiting stranger to do—find out what the country's customs are, and refrain from offending against them. . . . Custom is custom; it is built of brass, boiler iron, granite; facts, reasonings, arguments have no more effect upon it than the idle winds have upon Gibralter.

MARK TWAIN, *Letters from the Earth*