

Journal Club

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METHOD ARTICLE

REVISED **Fixing the stimulus-as-fixed-effect fallacy in task fMRI**

[version 2; referees: 1 approved, 2 approved with reservations]

Jacob Westfall^{1*}, Thomas E. Nichols ², Tal Yarkoni^{1*}

¹Department of Psychology, University of Texas, Austin, USA


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- How 'recycling' stimuli may lead to statistical problems




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- ▶ How 'recycling' stimuli may lead to statistical problems
- ▶ That these problems are to be taken serious



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- ▶ How 'recycling' stimuli may lead to statistical problems
- ▶ That these problems are to be taken serious
- ▶ A statistical model (RSM) that accounts for of these problems

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- ▶ Method 2: Get 3 people from MV (Jack, Julia, Pete) and 3 from PA (Mary, Sue, Bob). Let them throw balls 100 times each and do hypothesis testing on the $n = 600$ throws ($H_0 : A = B$)

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- ▶ Important not to examine method 2 and conclude as method 1

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More subjects = better estimates.

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- ▶ As for throwing, under mild assumptions Case 2 will be trivially true, but it is not interesting

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- ▶ A: Yes

We ultimately found that 63/100 of the experiments (...) published test statistics for these experiments [that] are likely inflated (...)

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- ▶ Artificial data analyzed with SPM (false positive rate of test)

		alpha = .05	alpha = .01	alpha = .005	alpha = .001
16	16	0.344	0.190	0.148	0.082
	32	0.274	0.150	0.106	0.046
	64	0.208	0.104	0.080	0.030
32	16	0.478	0.316	0.270	0.200
	32	0.438	0.322	0.262	0.170
	64	0.290	0.186	0.148	0.080
64	16	0.642	0.548	0.514	0.410
	32	0.568	0.442	0.400	0.310
	64	0.452	0.298	0.254	0.184

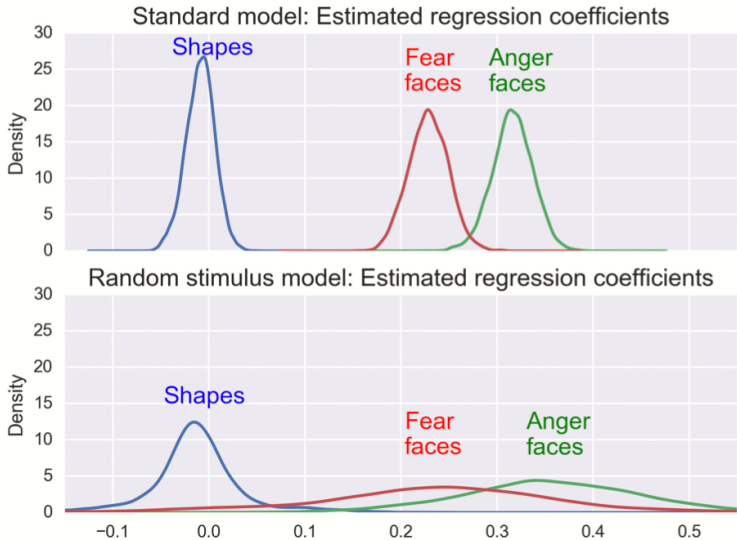
Number of different stimuli

Subjects

FPR

Bad news bears!

- ▶ Is the effect significant?
- ▶ HPC amygdala response to shapes, anger and fear faces



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- ▶ Frame generalizations narrowly (i.e. "the set of stimuli considered from image database X") rather than the entire class the stimuli represents (i.e. "sad and angry faces").
- ▶ Extend GLM to get around this problem

- Recall $Y_{it} = \mu_{it} + \epsilon_{it}$, $\epsilon_{it} \sim \text{AR}(2)$

$$\text{Basic GLM: } \mu_{it} = \beta_0 + \sum_{k=1}^K (\beta_k + p_{ki}) X_{kit}$$

Y_{it} is observed BOLD for subject i at time t , k is experimental condition (i.e. angry, fearful) and X_{kit} is the (canonical) response for stimuli k obtained as

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- ▶ Ensures outliers ($p_{k,37} \gg 0$) not affect estimate of β_k thereby inducing false positives
- ▶ Bayesian perspective: Model subject-specific effects if they are (plausibly) present

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- ▶ If this feature *is* present in data, and our model *does not* capture it, the model will more often find spurious results (false positives, bad estimates, etc.)

- Full model specification: $Y_{it} \sim \mathcal{N}(\mu_{it}, \sigma_Y)$ where

$$\mu_{it} = \alpha_1 Y_{i,t-1} + \alpha_2 Y_{i,t-2} + \beta_0 + \sum_{k=1}^K (\beta_k + p_{ki}) X_{kit} + \sum_{j=1}^J s_j X_{ijt}$$

$$\beta_k \sim \mathcal{N}(0, 10^6), \quad p_{ki} \sim \mathcal{N}(0, \sigma_k), \quad s_j \sim \mathcal{N}(0, \sigma_s)$$

$$\sigma_k, \sigma_s, \sigma_Y \sim \text{Cauchy}^+(10), \quad \alpha_1, \alpha_2 \sim \text{Cauchy}(0, 1)$$

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- ▶ *"This model cannot be fit using standard mixed modeling statistical packages, such as lme4 in R or SAS PROC MIXED (...) For the models in this paper, we used the PyMC3 Python package, which is built on the Theano deep learning package and implements the state-of-the-art No U-Turn MCMC Sampler"*

- If we generate $z \sim \mathcal{G}(\alpha = \frac{1}{2}, \beta = \frac{\sigma^2}{2})$, $x \sim \mathcal{N}(0, \sigma^{-2} = z)$ then:

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- ▶ Therefore, we can replace all the Cauchy distributions with $\text{Cauchy}(x; 0, \sigma) = \int \mathcal{N}(x; 0, \sigma^{-2} = z) \mathcal{G}(z; \alpha = \frac{1}{2}, \beta = \frac{\sigma^2}{2}) dz$

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we collect all parameters $\boldsymbol{\theta} = [\alpha_1 \quad \alpha_2 \quad \beta_0 \quad \beta_k \quad \dots]^T$

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- ▶ (i) the prior distribution of $\boldsymbol{\theta}$ is a multivariate normal
- ▶ (ii) $\boldsymbol{\theta}$ enters linearly in the likelihood
- ▶ (iii) conditional on $\boldsymbol{\theta}$, all other parameters become independent

- ▶ We can easily derive the marginal of θ . Let's for simplicity just focus on the marginal of s_j . It can be found by noting

$$\mu_{it} = \{\text{other terms}\} + \sum_{j=1}^J s_j x_{ijt} \text{ or:}$$

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- ▶ This means that

$$\begin{aligned} \text{vec}(\boldsymbol{\mu}) &= \text{vec}\left(\sum_{i=1}^N \mathbf{e}_i \mathbf{s}^T \mathbf{x}_{i,\cdot,\cdot}\right) = \sum_{i=1}^N \text{vec}(\mathbf{e}_i \mathbf{s}^T \mathbf{x}_{i,\cdot,\cdot}) \\ &= \sum_i (\mathbf{x}_{i,\cdot,\cdot} \otimes \mathbf{e}_i) \text{vec}(\mathbf{s}^T) = \left[\sum_i (\mathbf{x}_{i,\cdot,\cdot} \otimes \mathbf{e}_i) \right] \mathbf{s} \end{aligned}$$

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- Therefore

$$\text{vec}(\boldsymbol{\mu}) = \hat{\mathbf{X}} \boldsymbol{\theta} \Rightarrow \text{vec}(\mathbf{Y} - \boldsymbol{\mu}) \sim \mathcal{N}(\text{vec}(\mathbf{Y}) - \hat{\mathbf{X}} \boldsymbol{\theta}, \sigma_Y)$$

From which the posterior of θ as a multivariate normal trivially follows

- Marginals can easily be derived. Take as an example σ_Y and note likelihood has this form

$$p(\cdots, \sigma_Y, \cdots) = \{\text{Other stuff}\} \times \prod_{i,t} \mathcal{N}(Y_{it} | \mu_{it}, \sigma_Y) \mathcal{N}(\sigma_Y; 0, \tau)$$

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- Writing out this expression we see σ_Y enters as:

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 - ▶ Sample θ from a single, multivariate normal distribution
 - ▶ Sample variances $(\sigma_Y, \sigma_s, \sigma_k)$ from GiG distributions
 - ▶ Sample the latent parameters (z_Y, z_s, z_k, \dots) from their marginals (Gamma)

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- ▶ If stimuli from the same class *cannot* be assumed to induce similar response, use a model which captures this feature to avoid many false positives
- ▶ Get code from `citron.stanford.edu` at:
`/mnt/mapricot/musk2/home/herlau/shared/bayes_rsm/`