

Problem 7.1

$$\frac{dN_1}{dt} = r_1 N_1 (1 - a_{11} N_1 - a_{12} * (N_2 - m N_2) - m N_1)$$

$$\frac{dN_2}{dt} = r_2 N_2 (1 - a_{21} * (N_1 - m N_1) - a_{22} N_2) - m N_2$$

Problem 7.2

(a)

$$\frac{dp_1}{dt} = m_1 p_1 (1 - p_1) - e p_1$$

$$0 = m_1 p_1 (1 - p_1) - e p_1$$

$$\frac{-e + m_1}{m_1} = p_1$$

If $m_1 > e$ then this equation will be positive. This means that the rate of colonization must be greater than the rate of extinction.

(b)

$$\frac{dp_2}{dt} = m_2 p_2 (1 - p_1 - p_2) - m_1 p_1 p_2 - e p_2$$

$$\frac{dp_2}{dt} = m_2 p_2 (1 - (\frac{-e + m_1}{m_1}) - p_2) - m_1 (\frac{-e + m_1}{m_1}) p_2 - e p_2$$

$$0 = m_2 p_2 (1 - (\frac{-e + m_1}{m_1}) - p_2) - m_1 (\frac{-e + m_1}{m_1}) p_2 - e p_2$$

$$\frac{-m_1 + m_2 - \frac{(-e + m_1) * m_2}{m_1}}{m_2} = p_2$$

If both species are going to survive, then m_2 must be greater than m_1 .

(c)

Extinction rates will have a greater effect on species one, since there are many more terms that affect the equilibrium for species two, so species one will go extinct first.

(d)

As the extinction rate goes up, the equilibrium level of species two will go down. This makes sense, because a greater number of populations are dying out.

Problem 7.4

(a)

Lotka volterra equations require knowing the growth rate over both populations, the carrying capacities and the rate of competition. Working in the laboratory, you are more likely to be able to work with a species with a shorter generation time, and will be able to more easily measure the population size at regular intervals. This means that getting measurements of these parameters will be easier.

(b)

In the laboratory, you can control the conditions so that measuring the numbers of two species grown together truly reflects the effect of one species on the other. In the field, it will be more difficult to control for the fact that there are other species potentially involved.

(c)

Some of the advantages of looking for overlap of resources in the field, is that you will be better able to examine the actualized overlap of all the resources present in the natural habitat, rather than partitioning of whatever resources are given in the laboratory.

(d)

Field manipulations can allow you to examine the impact of adding or removing individuals from one species on the population size of the other species. You can also examine how the addition or removal of resources affects populations of either species.

Problem 3

```
library(deSolve)

## write competition function
comp <- function(t, y, p){
  N1 <- y[1]
  N2 <- y[2]
  with(as.list(p), {
    dN1.dt <- (r1 * N1 / K1) * (K1 - N1 - a12 * N2)
    dN2.dt <- (r2 * N2 / K2) * (K2 - N2 - a21 * N1)

    return(list(c(dN1.dt, dN2.dt)))
  })
}

## Define parameters

p <- c('r1' = 0.1, 'r2' = 0.6,
      'K1' = 2, 'K2' = 1,
```

```

    'a12' = 0.15, 'a21' = 0.3)

y0 <- c('N1' = 0.1, 'N2' = 0.1)

t <- 1:20 ## First run for 20 generations

## run simulations
sim <- ode(y = y0, times = t, func = comp, parms = p, method = 'lsoda')

## save data frame
sim <- as.data.frame(sim)

# plot N1
plot(N1 ~ time, type = 'l', col = "blue", bty = 'l', data = sim, ylim = c(0,2),
     xlab = "Time", ylab = "Population size")

# add N2
points(N2 ~ time, type = 'l', col = 'red', lty = 2, data = sim)

t2 <- 1:100 ## change time to 100 generations

## run simulation again
sim2 <- ode(y = y0, times = t2, func = comp, parms = p, method = 'lsoda')

sim2 <- as.data.frame(sim2)

# plot sim2
plot(N1 ~ time, type = 'l', col = "blue", bty = 'l', data = sim2, ylim = c(0,2),
     xlab = "Time", ylab = "Population size")

points(N2 ~ time, type = 'l', col = 'red', lty = 2, data = sim2)

```

Based on the first simulation, run for 20 generations, you would say that species 2 appears to outcompete species 1, although the population of species 1 begins to increase towards the end. Based on 100 generations you would conclude that species 1 is clearly outcompeting species 2. This highlights the importance of running experiments for a sufficient amount of time. 20 generations may not be long enough for the populations to reach their equilibrium values, so you will be unable to predict the long term dynamics.



