Problem 4.1:

(a)

Solving for equilibria

$$\frac{dN}{dt} = rN[1 - \frac{N^{\theta}}{K}]$$

$$0 = rN[1 - \frac{N^{\theta}}{K}]$$

$$r = 0$$

$$N = 0$$

$$0 = 1 - \frac{N^{\theta}}{K}$$

$$1 = \frac{N^{\theta}}{K}$$

$$\theta\sqrt{1} = \frac{N}{K}$$

$$N = K$$

Equilibria are: r = 0, N = 0, N = K

Evaluating stability

$$\frac{dn}{dt} \approx n\frac{dF}{dN} \mid N = \hat{N}$$

$$F(N) = rN[1 - \frac{N^{\theta}}{K}]$$

$$\frac{dF}{dN} = -r(-1 + (\frac{n}{k})^{\theta} + (\frac{n}{k})^{\theta}\theta)$$

At $\hat{N} = 0$:

$$\frac{dn}{dt} = (\frac{dF}{dN} \mid N = 0)n$$

$$\frac{dn}{dt} = r$$

This equilibrium is unstable.

At $\hat{N} = K$:

$$\frac{dn}{dt} = (\frac{dF}{dN} \mid N = K)n$$

$$\frac{dn}{dt} = -r\theta n$$

This equilibrium is stable and the system wil approach N = K.

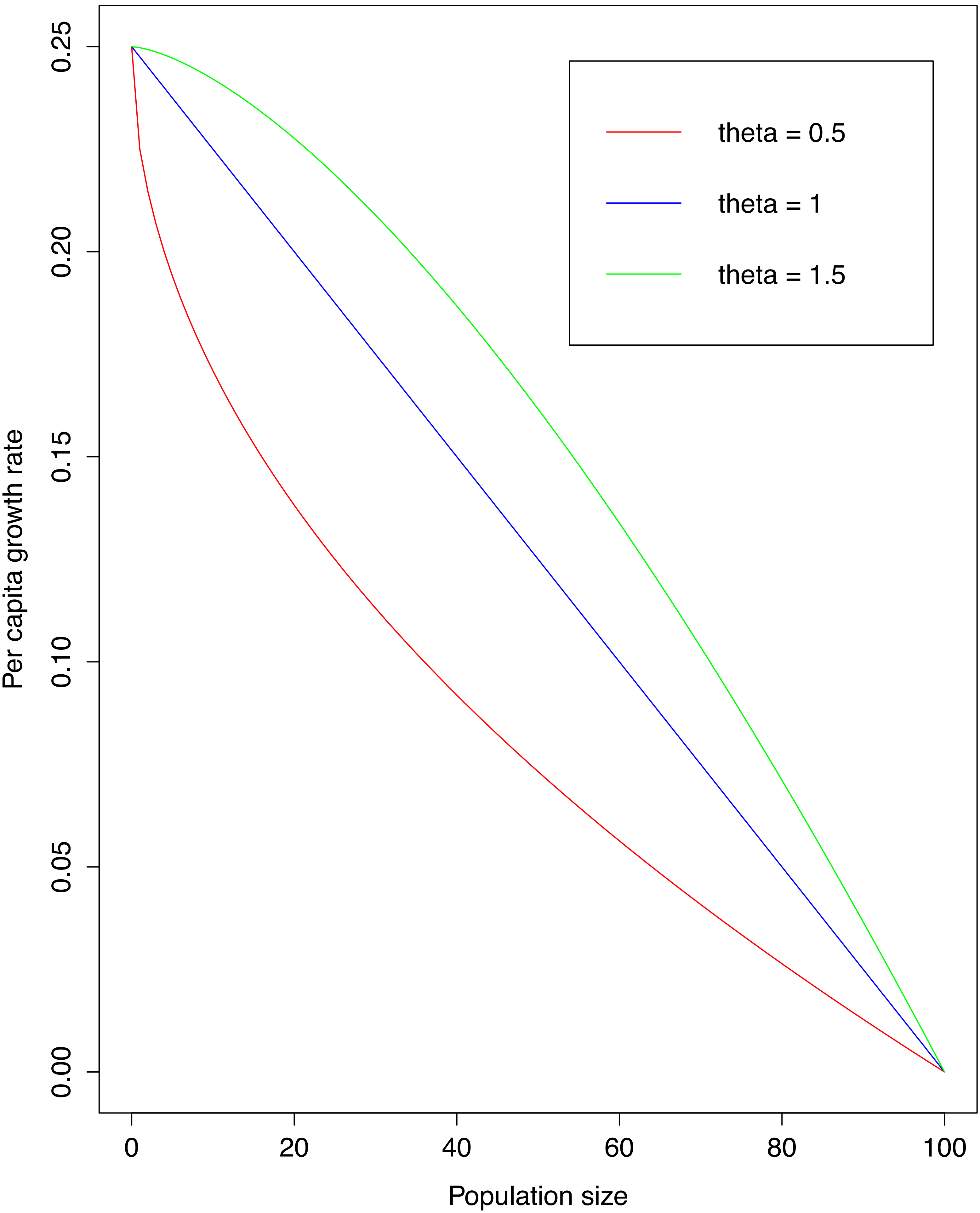
(b)

Graph per capita growth rate vs. N for different values of theta, using R.

```
> growth <- function(r,N,K,theta){
> r*(1-(N/K)^(theta))
> }  ## write function to calculate the growth rate
> per.cap1 <- growth(r = 0.25, N = 0:100, K = 100, theta = 0.5) ## theta < 1
>
> pop_size <- 0:100 ## vector of N
>
> plot(per.cap1~pop_size, xlab = "Population size", ylab = "Per capita growth
rate", type = "l", col = "red") ## plot growth rate v. pop size
>
> per.cap2 <- growth(r = 0.25, N = 0:100, K = 100, theta = 1) ## theta = 1
> points(per.cap2~pop_size, col = 'blue', type = "l") ## plot per.cap2
>
> per.cap3 <- growth(r = 0.25, N = 0:100, K = 100, theta = 1.5) ## plot per.cap3
> points(per.cap3~pop_size, col = 'green', type = "l") ## plot per.cap3
```

(c)

Including theta in this model affects the relationship of per capita growth rate and population size. When theta = 1, this model becomes the normal logist growth model and per capita growth rate decreases linearly. If theta < 1, then the growth rate decreases faster than the linear decline and if theta > 1 the growth rate decreases slower than the linear decline. This could be useful for populations for which density dependence is very strong if resources are very limited. Additionally it could be useful for populations that do show density dependence, but not until the population gets quite large (people?).



theta = 0.5
theta = 1
theta = 1.5
Per capita growth rate
Population size
0.00
0.05
0.10
0.15
0.20
0.25
0
20
40
60
80
100

Problem 4.3

(a)

$$\frac{dN}{dt} = rN(N-a)[1-\frac{N}{K}]$$

$$0 = rN(N-a)[1-\frac{N}{K}]$$

$$r = 0$$

$$N = 0$$

$$N = a$$

$$0 = 1-\frac{N}{K}$$

$$1 = \frac{N}{K}$$

$$K = N$$

Equilibra are:

r = 0

N = 0

N = a

N = K

(b) Evaluating stability

$$\frac{dn}{dt} \approx n\frac{dF}{dN} \mid N = \hat{N}$$

$$F(N) = rN(N-a)[1-\frac{N}{K}]$$

$$\frac{dF}{dN} = \frac{(-a(K-2N)+N(-2K+3N))r}{K}$$

At $\hat{N} = 0$:

$$\frac{dn}{dt} = (\frac{dF}{dN} \mid N = 0)n$$

$$\frac{dn}{dt} = -ar$$

The equilbrium is stable, if the system starts nearby it will approach 0.

At $\hat{N} = a$:

$$\frac{dn}{dt} = (\frac{dF}{dN} \mid N = a)n$$
$$\frac{dn}{dt} = -(\frac{(a(3a - 2K) + a(-2a + K))r}{K})n$$

This equilbrium is also stable and the system will approach a if starting nearby.

At $\hat{N} = K$:

$$\frac{dn}{dt} = (\frac{dF}{dN} \mid N = K)n$$
$$\frac{dn}{dt} = (\frac{-(-aK + K^2)r}{K})n$$

This equilibrium will also be stable.

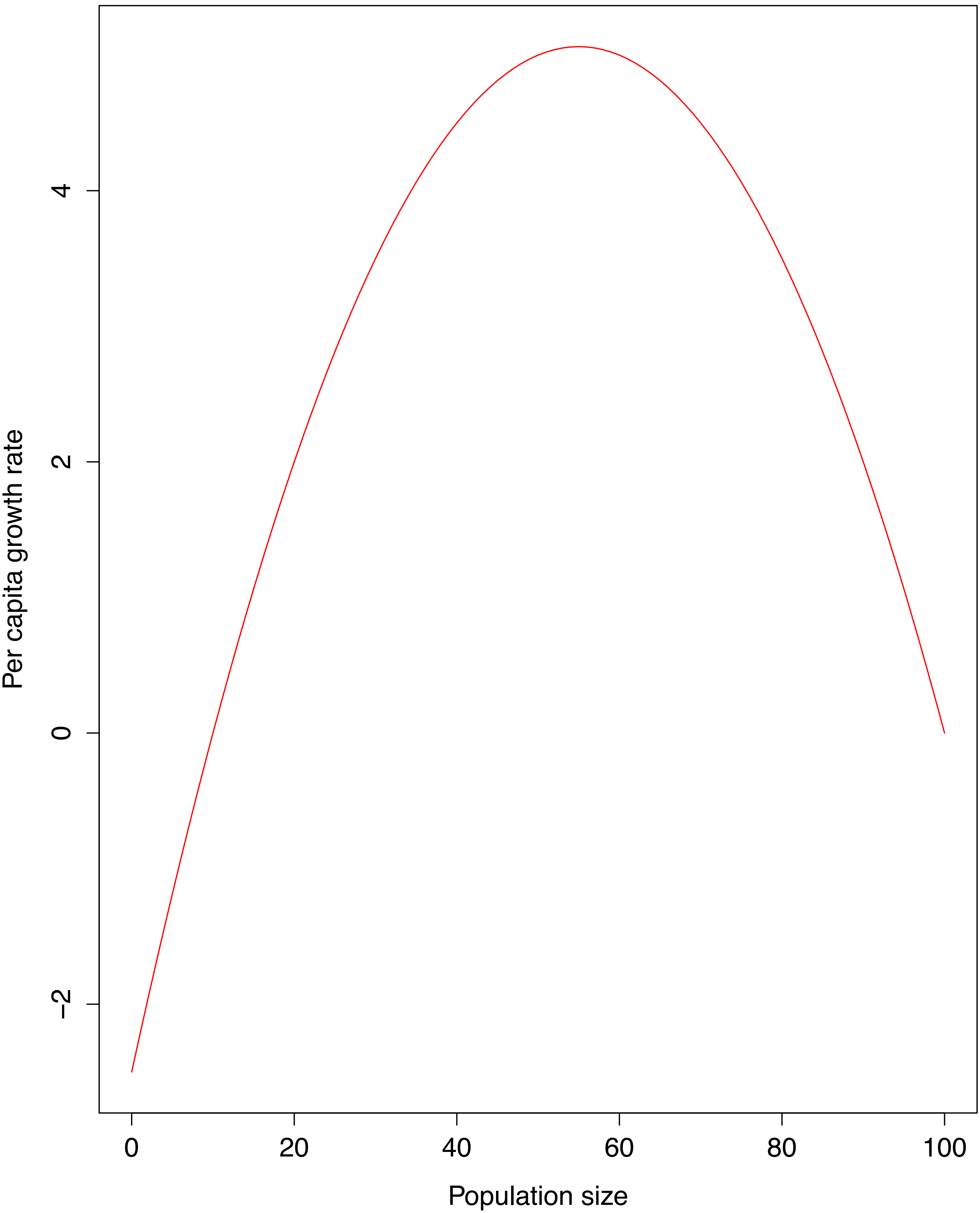
(c) Graph per capita growth rate v. pop size

```
>growth.a <- function(r,N,K,a){
> r*(N-a)*(1-(N/K))
> } ## write function to calculate growth rate
>
> a.per.cap <- growth.a(r = 0.25, N = 0:100, K = 100, a = 10)
>
> plot(a.per.cap1~pop, xlab = "Population size", ylab = "Per capita growth
rate", type = "l", col = "red") ## plot per cap growth rate v. pop size
```

(d) This model requires a threshold population size in order for the population to be able to grow. This means if the population is smaller than a size `a` then it will decline. This is in contrast to the simple logistic model where the population has the largest per capita growth rate when the population is small.

Problem 2:

(a)

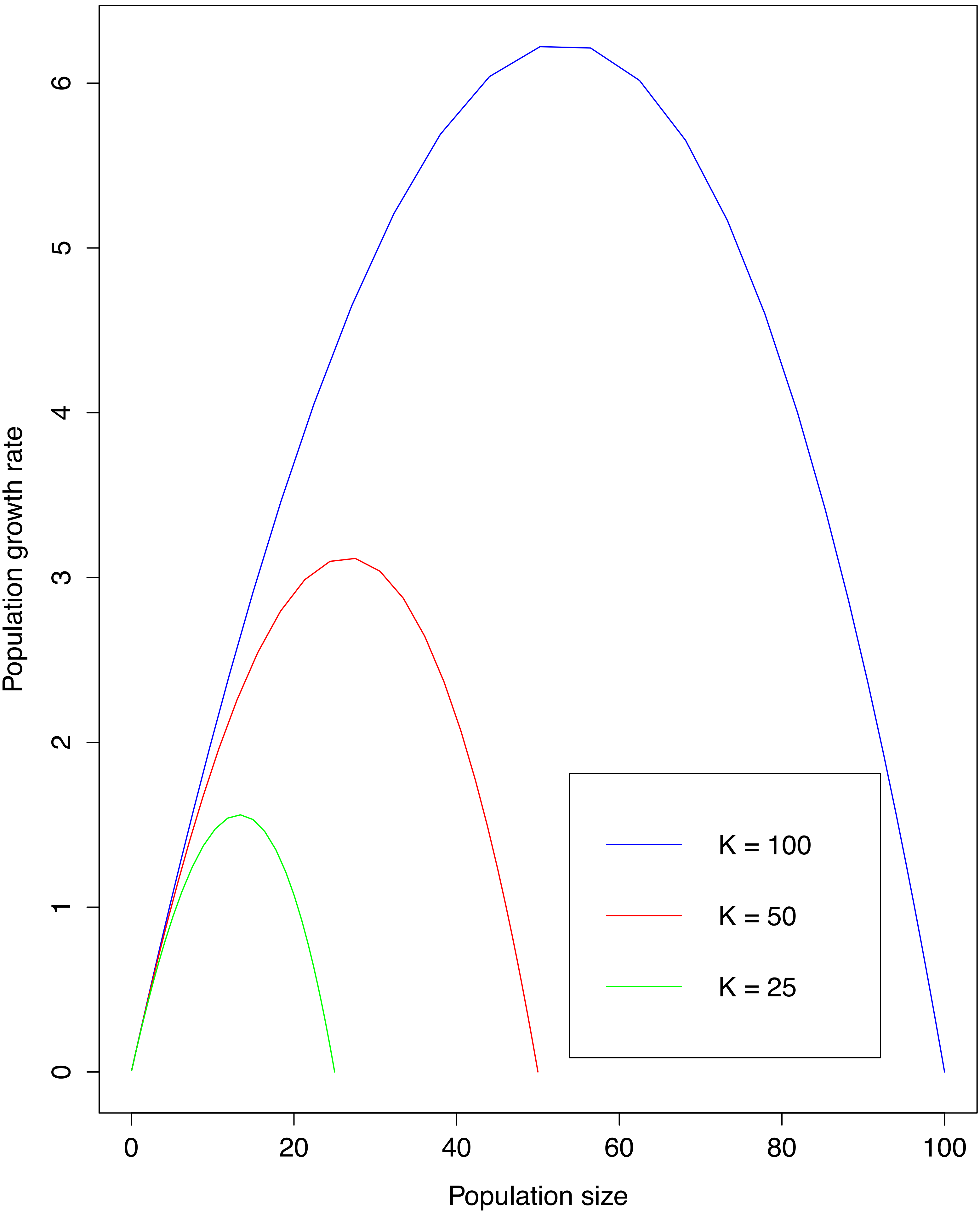

Per capita growth rate
Population size
4
2
0
−2
0
20
40
60
80
100

```
> log.growth <- function(t, y, p){
+   N <- y[1]
+   with(as.list(p), {
+     dN.dt <- r * N * (1-(N/K))
+     return(list(dN.dt))
+   })
+ }  ## logistic growth function
>
> p <- c('r' = 0.25, 'K' = 100) ## parameters, growth rate and carrying capacity
>
> y0 <- c('N' = runif(1, min = 0.01, max = 0.1)) ##randomly generated starting
pop size, between 0.01 and 0.1
>
> t <- 1:100 ## number of generations
>
> sim <- ode(y = y0, times = t, func = log.growth, parms = p, method = 'lsoda')
## simulate model
>
>sim <- as.data.frame(sim) ## save simulation to dataframe
>
> plot(N ~ time, data = sim, type = 'l', col = 'green', xlab = "Time", ylab =
"Population size") ##Plot model
```

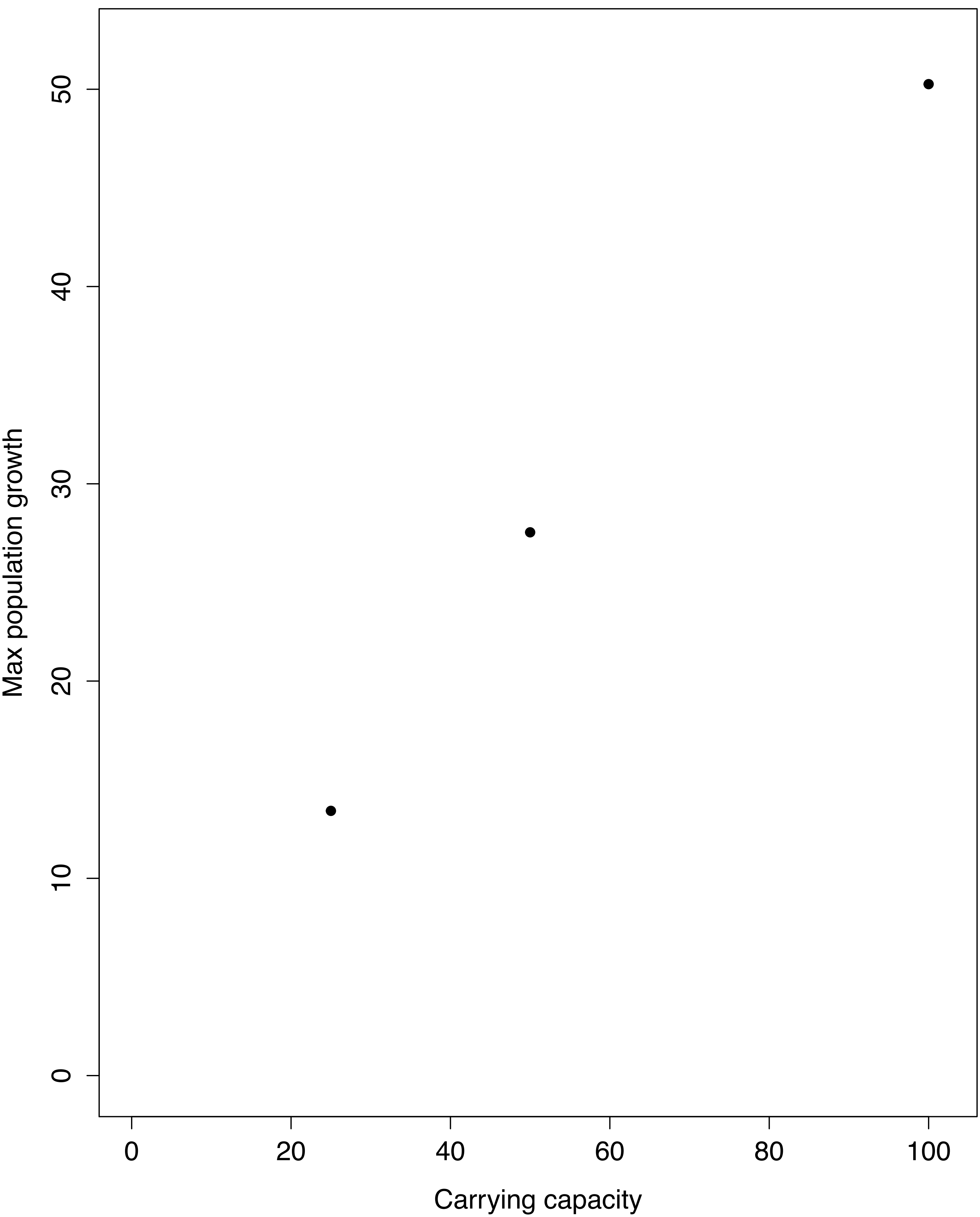
(b)

```
> p2 <- c('r' = 0.25, 'K' = 50) ## params for k = 50
> p3 <- c('r' = 0.25, 'K' = 25) ## params for k = 25
>
> sim2 <- ode(y = y0, times = t, func = log.growth, parms = p2, method =
'lsoda') ## simulate with k = 50
> sim3 <- ode(y = y0, times = t, func = log.growth, parms = p3, method =
'lsoda') ## simulate with k = 25
>
> sim2 <- as.data.frame(sim2)
> sim3 <- as.data.frame(sim3) ## save simulations to dataframes
>
> growth1 <- diff(sim$N, differences = 1) ##calculate population growth rate for
sim1
> N <- sim$N[2:100] ## N for sim1
> plot(growth1~N, type = "l", col = "blue", xlab = "Population size", ylab =
"Population growth rate") ##Plot growth rate v. pop size for sim1
>
> growth2 <- diff(sim2$N, difference = 1) ## pop growth rate sim2
> N2 <- sim2$N[2:100] ## pop size sim2
> points(growth2~N2, type = "l", col = "red") ## add pop growth rate v. pop size
for sim2 to plot
>
## same steps for sim3
> growth3 <- diff(sim3$N, difference = 1)
> N3 <- sim3$N[2:100]
> points(growth3~N3, type = "l", col = "green")
>
> legend("bottomright", legend = c('K = 100', 'K = 50', 'K = 25'), col =
c("blue","red", "green"), lty =1, inset = c(0.11,0.05)) ## add legend
```

(c)



Population growth rate
Population size
0
1
2
3
4
5
6
0
20
40
60
80
100
K = 100
K = 50
K = 25



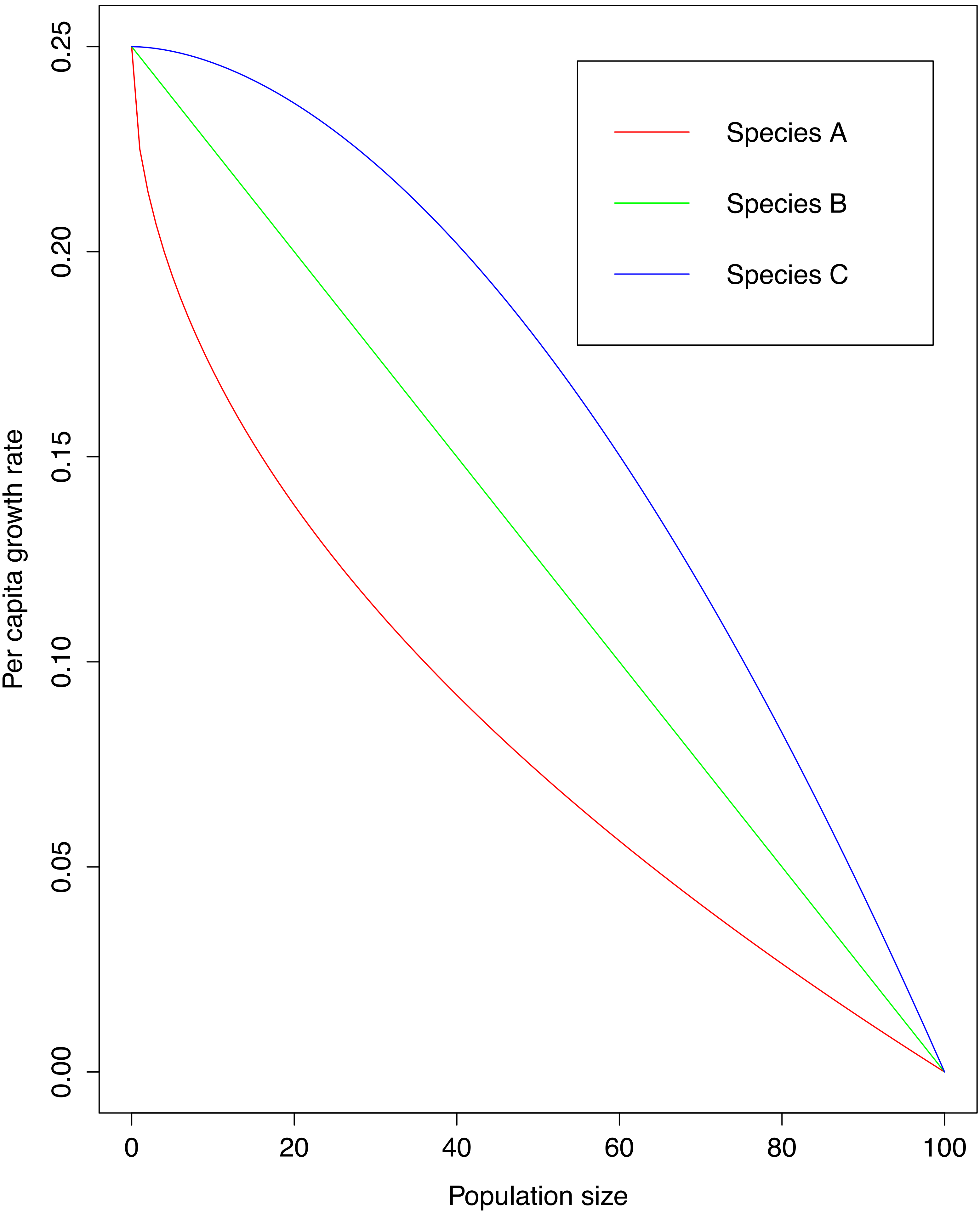
Max population growth
0
10
20
30
40
50
0
20
40
60
80
100
Carrying capacity

```
> max(growth1) ## find max growth rate sim1
> cbind.data.frame(N, growth1) ##popsize at max growth = 50.26
>
> max(growth2) ## find max growth rate sim2
> cbind.data.frame(N2, growth2) ## popsize at max growth = 27.54
>
> max(growth3) ## find max growth rate sim3
> cbind.data.frame(N3, growth3) ## popsize at max growth = 13.42
>
> Ks <- c(25,50,100) ## vector of carrying capacities
> size.max.growth <- c(13.42, 27.54, 50.26) ## vector of pop size at max growth
>
> plot(size.max.growth~Ks, ylim = c(0,52), xlim = c(0,102), xlab = "Carrying
capacity", ylab = "Max population growth", pch = 19) ## plot size at max growth
v. K
```

Problem 3

```
> per.cap.theta <- function(r,N,K,theta){
> r*(1-(N/K)^(theta))
> }  ## write function for per capita growth
>
> per.cap.A <- per.cap.theta(r = 0.25, N = 0:100, K = 100, theta = 0.5) ##
calculate per capita growth species A
> per.cap.B <- per.cap.theta(r = 0.25, N = 0:100, K = 100, theta = 1) ## species
B
> per.cap.C <- per.cap.theta(r = 0.25, N = 0:100, K = 100, theta = 1.8) ##
species C
> pop <- 0:100
> plot(per.cap.A ~ pop, type = 'l', xlab = "Population size", ylab = "Per capita
growth rate", col = "red")
> points(per.cap.B ~ pop, type = 'l',col = "green")
> points(per.cap.C ~ pop, type = 'l',  col = "blue")
> legend("topright", legend = c('Species A', 'Species B', 'Species C'), col =
c("red","green", "blue"), lty =1, inset = c(0.05,0.05))
```

Species three will be maintained at the highest frequency, because its per capita growth rate decreases the slowest.



Species A
Species B
Species C
Per capita growth rate
0.00
0.05
0.10
0.15
0.20
0.25
0
20
40
60
80
100
Population size