**Abstract**

Intensive longitudinal studies are essential for understanding dynamic processes. Multilevel models are used to analyze longitudinal data and include individual-specific random effects, allowing for individual differences in longitudinal trajectories. Multilevel location scale models (MLSMs) were developed to analyze intensive longitudinal data and extend multilevel models to account for heterogeneity of the within- and between-individual variance by modeling these variances as functions of covariates. When individual-level covariates with missing data are included in multilevel models, individuals missing the individual-level covariate data are excluded from the analysis. This reduction in sample size can result in decreased statistical power and increased risk of biased inference. Multilevel multiple imputation (MMI), a method for filling in missing observations such that important dataset characteristics are preserved, fails to preserve relationships between covariates and the magnitude of the within- and between-individual variances. Using a Monte Carlo data simulation, this paper evaluates the extent to which parameter estimates of a MLSM are impacted when MMI is applied to data generated according to a MLSM and for which the missingness is ignorable. We observed an increase in the standardized bias of the regression coefficients corresponding to variables with missing data, especially for continuous variables with over 10% missing data.

*Keywords:* random intercept model, mixed-effects model, Monte Carlo simulation,

ignorable missingness, intensive longitudinal data

**Multiple Imputation of Missing Covariates in Multilevel Location Scale Models:**

**Simulations and Recommendations**

With the advent of consumer smartphones and physiological sensors, data collection in behavioral research has expanded to include real-time data measured intensively over time (Trull & Ebner-Priemer, 2014). These data have been referred to in the literature as intensive longitudinal data. Given the series of repeated measures collected for each of multiple individuals, these data allow researchers to evaluate trends at the individual and population levels, as well as the variability of responses within the individual about their fitted trajectory. Indeed, intensive longitudinal data have the potential to improve our understanding of dynamic longitudinal processes at a granular level (Hamaker & Wichers, 2017).

Multilevel models were developed to address the dependencies of repeated measures data, and recent developments have made these models highly amenable for the analysis of intensive longitudinal data. Multilevel location scale models (MLSM), a relatively new type of multilevel model, include random coefficients to account for the correlations between scores within subject but also include particular ways to address heterogeneity of the within-subject residual variance and heterogeneity of the between-subject variances of the random coefficients (Hedeker, Mermelstein, & Demirtas, 2008; Hedeker & Nordgren, 2013). In these models, the random ‘location’ part relates to the random coefficients that characterize central tendencies in the response at the individual level, with the variances of the random coefficients possibly being functions of measured covariates. The random ‘scale’ part of the model relates to the level 1 residual variance that can be a function of measured covariates, as well as a random subject effect to permit within-subject variability due to unmeasured influences. These models have been applied to intensive longitudinal data with an aim to characterize both intra- and inter-individual differences in behavioral data, including positive and negative affect, learning, physical activity, reaction time, working memory, and time use (see e.g., Blozis, McTernan, Harring, & Zheng, 2020; Hedeker, Mermelstein, & Demirtas, 2008; Rast & Ferrer, 2018; Rast & Zimprich, 2011; Schuster, Mermelstein, & Hedeker, 2016; Watts, Walters, Hoffman, & Templin, 2016; Williams, Zimprich, & Rast, 2019).

Missing data in a longitudinal study can be complex, with missing data possibly occurring in the outcome variable, time-varying covariates, and individual-level covariates. Multilevel models naturally handle missing response data but not missing data for covariates. The consequence of this is perhaps most problematic for missing covariates measured at the individual level because the individual is dropped entirely from the analyses, whereas missing covariates measured at the occasion level will result in dropped values for only that occasion. So, although a multilevel model accommodates missing response data, such as that due to attrition, there can be significant data loss if individual-level covariates are missing. This presents a significant challenge for intensive longitudinal studies when individual-level covariates are included in a model to study individual differences.

The missing data literature is replete with strategies for addressing missing longitudinal data. For a multilevel model, there are multiple strategies for addressing subject attrition, and these have been considered for multilevel location scale models as well (Lin, Mermelstein, & Hedeker, 2018a; Lin & Xun, 2021). The purpose of this paper is to investigate strategies for missing covariates when fitting a MLSM to intensive longitudinal data. Specially, we investigate multiple imputation (MI) methods for problems where the missingness, that is, whether or not data are missing for an individual, is ignorable such that given the observed data, the data distribution is the same in the response and non-response groups (Laird, 1988). The primary goal is then to use the observed data to impute the missing covariates to avoid the data loss that results from missing individual-level covariates.

Imputation models have been implemented using MI software packages, including multilevel MI which utilizes a multilevel imputation model to preserve within-individual residual dependencies (Enders, Mistler, & Keller, 2016; Van Buuren, 2011). Because the imputation model does not include sub-models for the within- and between-individual variance of a MLSM, however, values are imputed under the assumption of homogeneity of the within- and between-individual variance. Thus, multilevel MI, as currently implemented, fails to preserve relationships between covariates and the within- and between-individual variance. This study, therefore, aims to investigate the use of multilevel multiple imputation in the context of a multilevel location scale model when the missingness is ignorable. Given that this strategy is likely the one that practitioners will look to in dealing with missing covariates, our aim was to evaluate how well current multilevel MI performs when applied to data generated by a MLSM, thus providing a framework for future developments.

The rest of this paper is organized as follows. First, we review the MLSM literature and provide equations for a MLSM. Next, we describe missing data in the context of MLSMs and present our hypotheses. After, we document the simulation procedure and present the simulation results. The paper concludes with a discussion of our recommendations for practice, study limitations and future research.

**Multilevel Location Scale Models**

A MLSM is a multilevel model that includes sub-models to account for heterogeneity of the random components at the occasion and subject levels (also see Lin, Mermelstein & Hedeker, 2018, for a three-level MLSM). Although a MLSM is applicable to repeated measures that change as a function of time (Lin, Mermelstein, & Hedeker, 2018b; Williams, Zimprich, & Rast, 2019), we consider the simpler problem in which an individual’s responses are not expected to change systematically with time, but instead, to vary about the individual’s expected response level that is constant across time. For instance, these models have been applied to affect measures with the goal of studying individual differences in the degree of within-person variation in mood, as well as to study between-person heterogeneity in mood levels (Hedeker, Mermelstein, & Demirtas, 2008). With this, the model includes a random subject effect to account for individual differences in response levels. Letting be the occasion- and individual-specific outcome, a multilevel model for the response is

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where is a vector of fixed coefficients corresponding to the vector that contains a “1” for the intercept and includes occasion-level covariates, is a vector of fixed effect coefficients corresponding to the vector that includes individual-level covariates, is an individual-specific random intercept, and is an occasion- and individual-specific residual.

In a MLSM, the variance of the residual at level 1 and the variance of the random effects at level 2 are assumed to follow log-linear models (Hedeker, Mermelstein, & Demirtas, 2008):

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Specifying a log-linear model ensures that the variance is never negative and creates a convenient way to allow the variance to be a function of covariates. Thus, in the log-linear equations, and are vectors of fixed effect coefficients corresponding to the vectors and , respectively, which each contain a “1” for the reference residual and random effect variances ( and ), respectively, along with the values of any occasion- and individual-level covariates. An individual-specific random scale effect allows individuals to deviate from the average residual variance above and beyond the influence of covariates.

MLSMs assume that the random location and scale effects are multivariate normally distributed with mean zero and covariance matrix

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where represents the covariance between the random location and scale effects.

**Missing Data in Multilevel Location Scale Models**

Intensive longitudinal studies are inherently susceptible to missing data, perhaps most common in the forms of dropout and intermittent missingness. Other forms of missingness may result from building missing data into the study design to reduce participant burden. In determining whether the statistical analysis should take the missing data into consideration, one should thoughtfully consider the missing data mechanism, potential causes of missingness, and whether the missing data occur in the outcome variable, occasion-level covariates, and individual-level covariates.

For valid inference from a MLSM, data must be missing completely at random (MCAR) or missing at random (MAR). Data are MCAR if the probability of missing outcome data for an individual does not depend on either observed or missing outcome data, and data are MAR if this probability depends on observed but not missing response data, even after conditioning on covariates. It is well known that inference from a model for which data that are MAR or missing not at random (MNAR) are excluded from the analysis can result in biased inference because the statistical model is fitted to a subset of the hypothetical complete data that is unlikely to be representative of the population (Dempster, Laird, & Rubin, 1977). For problems that involve the exclusion of observations for which the missingness is ignorable, however, such as excluding all data from any participant who is missing subject-level covariates, the issue is a reduction in sample size, and consequently, statistical power. If data are missing for any time-varying covariate, only the data for that occasion (the response and any other covariates included) will be excluded from the analysis, and the individual contributes their available data from other occasions to the analysis. If data are missing from the individual-level covariates, that individual is excluded entirely from the analysis.

Full information maximum likelihood (FIML) is a state-of-the-art method of estimation that is applicable to problems for which the outcome measures, i.e., endogenous measures, are MCAR or MAR (Schafer & Graham, 2002). Missing data theory does not, however, apply to observed exogenous variables. In the estimation of a MLSM, the response, the residual variance at level 1 and the variances of the random effects at level 2 are conditioned on covariates. Thus, missing data in observed covariates is problematic for estimation of a MLSM. This brings about a need to address the missing covariate values.

Multiple imputation (MI) is an alternative method for dealing with missing data if data are MCAR or MAR[[1]](#footnote-1) (Schafer & Graham, 2002; White & Carlin, 2010). Briefly, MI consists of an imputation phase, an analysis phase, and a pooling phase (Rubin, 1987). In the imputation phase, replacements are drawn for the missing values from the posterior predictive distribution of the missing values given the observed data and the imputation model specified by the analyst. In the analysis phase, the substantive model of interest is fit to each of the imputed datasets. In the final phase, the parameters resulting from the analyses are pooled according to a set of rules (see Rubin, 1987). To avoid biased inference, it is crucial that the imputation model is correctly specified, preserving important characteristics of the data, such as variances, correlations, interactions, transformations, and functional form. Consequently, much research has been devoted to correct specification of the imputation model (see e.g., Collins, Schafer, & Kam, 2001; Demirtas & Hedeker, 2008; Doove, Van Buuren, & Dusseldorp, 2014; Enders, Baraldi, & Cham, 2014; Kaplan & Yavuz, 2019; Lee & Carlin, 2017; Meng, 1994; Seaman, Bartlett, & White, 2012).

Currently, no known software packages implement multilevel multiple imputation that accounts for the special structures of a MLSM, that is, where the within- and between-individual covariance structures depend on observed covariates and unobserved individual-specific deviations from the average within-individual variance. To evaluate the practical value of developing such a software package, we propose a simulation study to help explicate the conditions under which multilevel multiple imputation results in biased parameter estimation when applied to data generated according to a MLSM.

We hypothesize that multilevel multiple imputation under these conditions will result in attenuated relationships between imputed covariates and the within- and between-individual variances. More specifically, we hypothesize that the strengths of these relationships will decrease as the percentage of missing data increases. This is because multilevel multiple imputation will generate imputations from a model that assumes no relationship between covariates and the within- and between-individual variances, and therefore, the increasing loss of this information will attenuate these relationships. In addition, we hypothesize that the imputation model’s assumptions of homogeneous variances at both levels of the model will result in biased estimates of the within- and between-individual variances and the covariance between the random location and scale effects. We expect this bias to increase as the percentage of missing data increases. This is because multilevel multiple imputation ignores heterogeneity of the within- and between-individual variance, generating imputations from a model that assumes zero individual-specific deviations from the average residual variance.

**Simulation Procedure**

***Data Generation***

Using RStudio version 1.4.1717 (RStudio Team, 2021), datasets were simulated to resemble a longitudinal structure according to the following MLSM:

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denotes the time-varying continuous covariate, the covariate denotes a time-varying binary covariate, denotes the time-invariant continuous covariate, and the covariate denotes a time-invariant binary covariate. Note that it is possible to model the variance of a random effect at level 2 as a function of both time-varying and time-invariant covariates (Hedeker & Nordgren, 2013), this study aimed to evaluate only the effects of time-invariant covariates.

We reparametrized the random location and scale effects and in standardized form according to the Cholesky decomposition used in Hedeker, Mermelstein, and Demirtas (2008):

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such that and , where

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, and

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, , and denote the Cholesky elements and are subscripted with an because varies over individuals. and denote the standardized random location and scale effects, respectively, and are assumed to be normally distributed in the population with mean zero and standard deviation one. The correlation between the standardized random location and scale effects is denoted as . In this way, the multilevel location scale model can be rewritten as:

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According to the data generating model, 1000 replications were simulated per combination of experimental conditions. The experimental conditions of the data generation procedure are contained in Table 1. The number of individuals *N* and the number of repeated measures *J* per individual were selected to reflect values observed in a selection of intensive longitudinal studies from the behavioral science literature (see Table 2). In selecting these experimental conditions, we also considered values of and selected in previous two-level location scale model simulations (see Table 3).

***[Table 1 near here]***

***[Table 2 near here]***

***[Table 3 near here]***

The magnitudes selected for , , and are consistent with the magnitudes observed in intensive longitudinal studies from the behavioral science literature (see Hedeker, Mermelstein, & Demirtas, 2008; Hedeker, Mermelstein, & Demirtas, 2012; Leckie, 2014; Leckie, French, Charlton, & Browne, 2014; Rast, Hofer, & Sparks, 2012; Rast & Zimprich, 2011). The remaining multilevel location scale model parameters are fixed at values consistent with those observed in intensive longitudinal studies from the behavioral science literature. The population values selected are summarized in Table 4.

***[Table 4 near here]***

***Data Amputation***

The ampute function from the R-package mice was used to generate missing values in 10%, 25%, and 50% of the rows under MCAR and MAR missingness in the individual-level covariates and (Schouten, Lugtig, & Vink, 2018; Van Buuren & Groothuis-Oudshoorn, 2011).

*MCAR Amputation*

For each missingness percentage (10%, 25%, and 50%), three missing data patterns were generated: one third of the incomplete cases has missing values in , one third of the incomplete cases has missing values in and one third has missing values in both and . The ampute function was used to induce MCAR missingness by assigning each individual a probability of being incomplete.

*MAR Amputation*

The generation of MAR missingness required an adaptation of the amputation methodology, which was originally developed for cross-sectional data, because the probability of the individual-level covariates and being incomplete depends on observed data in time-varying variables , and, as well as observed data in the individual-level covariate for pattern 1 and observed data in the individual-level covariate for pattern 2. For cross-sectional data, the amputation methodology combines the values of multiple observed variables into one weighted sum score per case. The weighted sum scores are used to allocate probabilities of being incomplete (Schouten, Lugtig & Vink, 2018). In order to obtain weighted sum scores in a longitudinal data setting, time-varying covariates can be aggregated per individual. Thus, the values of the individual-level covariates and the aggregated variables are combined into one weighted sum score per case. This study uses the average as an aggregation function for the continuous variables and and the mode[[2]](#footnote-2) as an aggregation function for the binary variable . Furthermore, a RIGHT type of missingness is generated, which assigns higher probability of being incomplete to cases with higher weighted sum scores (Schouten, Lugtig & Vink, 2018).

***Multilevel Multiple Imputation***

After inducing missingness in the individual-level covariates and , multilevel multiple imputation via the mice function from the R-package mice was used to impute the missing values (Van Buuren & Groothuis-Oudshoorn, 2011). , , , and were selected as predictors in the imputation model for , and , , , and were selected as predictors in the imputation model for . The imputation method 2lonly.pmm was used to aggregate the time-varying predictors , , and and impute the binary covariate by predictive mean matching. The imputation method 2lonly.norm was used to aggregate the time-varying predictors , , and and impute the continuous covariate by the normal model. 20 imputed sets were generated with 10 iterations each.

***Maximum Likelihood Estimation***

Maximum likelihood estimation was carried out using SAS version 9.4 and the nonlinear multilevel model procedure PROC NLMIXED using adaptive Gaussian quadrature (SAS Institute Inc., 2018). We accessed SAS via a Unix server, allowing for models to be fitted in parallel by experimental condition. The population values selected for various experimental conditions and starting values used to fit each model for all replications and experimental conditions are summarized in Table 4. Given the sensitivity of PROC NLMIXED to poorly defined starting values, it is worth noting that the starting values correspond more accurately to one value selected for the variable parameters , , and than the other.

The following MLSM was fitted to each replication and experimental condition:

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denotes the time-varying continuous covariate centered at the within-individual mean , the covariate denotes a time-varying binary covariate, denotes the time-invariant continuous covariate centered at the grand mean , and the covariate denotes a time-invariant binary covariate.

***Convergence Exclusion Criteria***

Parameter estimates with gradient values greater than 0.01 or less than -0.01 and with standard errors equal to 0 or missing were removed before pooling the parameter estimates.

***Pooling of Estimates***

Pooling of the parameter estimates according to Rubin’s 1987 pooling rules was carried out using SAS version 9.4 and the procedure PROC MIANALYZE (SAS Institute Inc., 2018). Due to exclusion of non-convergent parameter estimates, it was possible to obtain pooled estimates with zero between-imputation variance. Such estimates were removed from the dataset of pooled parameter estimates used to summarize the simulation results.

**Simulation Results**

Standardized bias and coverage rates were the criteria used to evaluate the performance of multilevel multiple imputation. The following sections describe results for each evaluation criterion. We exclude results for conditions where the number of replications with converged solutions fell below 500 (see Figure 1 for convergence rates across all conditions). Figure 1 depicts fewer converged solutions for MAR than MCAR missing data under conditions of 25% and 50% missing data.

***[Figure 1 near here]***

For concision, the body of this paper references plots for the condition where = 0.2 (implying ), = 1.1, and = 0.3. Plots for all combinations of magnitudes specified for , , and can be found in the Supplemental Materials.

***Standardized Bias***

For the parameter in , let be the pooled estimator of the true value of from the simulation replication in . The raw bias (where represents the expectation of across replications) is a measure of the accuracy of the pooled estimator . An estimator is considered biased[[3]](#footnote-3) when , however the impact of the raw bias on interval estimates and statistical tests depends on the accuracy of relative to the overall uncertainty of (Collins, Schafer, & Kam, 2001).

The standardized bias is a measure of the accuracy of relative to the precision of , quantified by , the standard deviation of across replications. A standardized bias[[4]](#footnote-4) of 50% indicates that the estimate on average falls half of a standard deviation below the true value (Barnes, Lindborg, & Seaman, 2006; Collins, Schafer, & Kam, 2001).

Figure 2 depicts the precision of for the fixed effects in . The precision of appears to vary across the fixed effects in and as a function of the various combinations of and , but does not appear to vary significantly across the percentage of missing data or the missing data mechanism.

***[Figure 2 near here]***

Figure 3 depicts the standardized bias for , and the fixed effects in the model of the average. The standardized bias of and does not increase as the percentage of missing data increases. For the MAR conditions and across all combinations of magnitudes specified for , , and , the standardized bias of and typically increases as the percentage of missing data increases, but not for the MCAR conditions.

***[Figure 3 near here]***

Figure 4 depicts the standardized bias for , and , the fixed effects in the model of the within-individual variance. The standardized biased of , and does not increase as the percentage of missing data increases. For the MCAR and MAR conditions and across all combinations of magnitudes specified for , , and , the standardized bias of and typically increases as the percentage of missing data increases.

***[Figure 4 near here]***

Figure 5 depicts the standardized bias for , and , the fixed effects in the model of the between-individual variance. It was hypothesized that the bias of the variance of the random location effects () would increase as the percentage of missing data increased. Contrary to what was hypothesized, notable standardized bias is typically observed for under the best-case condition of 0% missing data, and from there, the standardized bias typically decreases as the percentage of missing data increases. For the MCAR and MAR conditions and across all combinations of magnitudes specified for , , and , the standardized bias of and typically increases as the percentage of missing data increases.

***[Figure 5 near here]***

Figure 6 depicts the standardized bias for and , the variance of the random scale effects and the covariance between the random location and scale effects, respectively. It was hypothesized that the bias of and would increase as the percentage of missing data increased. For the MAR conditions and across all combinations of magnitudes specified for , , and , the standardized bias of typically increases as the percentage of missing data increases. For the MCAR conditions where and , the standardized bias of typically increases as the percentage of missing data increases, but the same is not true not where and or where . Contrary to what was hypothesized for , notable standardized bias is typically observed under the best-case condition of 0% missing data, and from there, the standardized bias decreases as the percentage of missing data increases.

***[Figure 6 near here]***

In summary (also see Table 5 for a visual summary of these results), these results demonstrate that the standardized bias increases for , and frequently under MCAR and MAR conditions and across the combinations of magnitudes specified for , , and . The standardized bias increases for , and under the MAR condition and across the combinations of magnitudes specified for , , and . That is, standardized bias increases as the percentage of missing data increases for the coefficients corresponding to the individual-level variables with missing data.

The effect of increasing percentages of missing data is more pronounced for the parameters corresponding to the continuous variable (, , and than for the parameters corresponding to the binary variable (, , and ). These results also demonstrate that, contrary to what was hypothesized, the standardized bias of and does not increase as the percentage of missing data increases. Rather, notable standardized bias is observed under the 0% missing data condition and decreases as the percentage of missing data increases.

***[Table 5 near here]***

***Coverage***

The coverage of is the number of times the true value falls within range of the confidence interval for the pooled estimator divided by the total number of replications with converged solutions and multiplied by 100. A procedure is considered to be working well when the coverage[[5]](#footnote-5) is approximately 95% for a 95% nominal coverage probability. Coverage below the 95% nominal coverage probability increases the risk of a Type I error, and coverage above the 95% nominal coverage probability increases the risk of a Type II error.

Figure 7 depicts the coverage for , and the fixed effects in the model of the average. The coverage of and hovers around 95%. For the MAR conditions, the coverage of typically deviates substantially from 95% as the percentage of missing data increases, but not for the MCAR conditions. No trend in coverage across increasing percentages of missing data was observed for .

***[Figure 7 near here]***

Figure 8 depicts the coverage for , and , the fixed effects in the model of the within-individual variance. The coverage of and hovers around 95%. For the MCAR and MAR conditions, the coverage of both and typically deviates substantially from 95% as the percentage of missing data increases.

***[Figure 8 near here]***

Figure 9 depicts the coverage for , and , the fixed effects in the model of the between-individual variance. For the MCAR and MAR conditions, sub-95% coverage rates are typically observed for under the best-case condition of 0% missing data, and from there, the coverage typically increases as the percentage of missing data increases. Coverage performance was inconclusive for . That is, results differed across the various combinations of and . For the MCAR and MAR conditions, the coverage of typically deviates substantially from 95% as the percentage of missing data increases.

***[Figure 9 near here]***

Figure 10 depicts the coverage for and , the variance of the random scale effects and the covariance between the random location and scale effects, respectively. For the MCAR and MAR conditions, sub-95% coverage rates are typically observed for under the best-case condition of 0% missing data, and from there, the coverage typically increases as the percentage of missing data increases. The coverage of hovers around 95%.

***[Figure 10 near here]***

In summary (also see Table 6 for a visual summary of these results), these results demonstrate that the coverage deviates substantially from 95% for , and across the combinations of magnitudes specified for , , and , and that the coverage deviates substantially from 95% for under the MAR condition. The coverage hovers around 95% for , and . The coverage of and does not deviate substantially from 95% as the percentage of missing data increases. Rather, for both and , sub-95% coverage rates are observed under the 0% missing data condition and the coverage improves as the percentage of missing data increases.

***[Table 6 near here]***

**Discussion**

While MLSMs naturally handle longitudinal data that are MCAR or MAR, a model that includes time-varying covariates will result in the exclusion of data at the corresponding times of measurement, and a model that includes time-invariant covariates will result in the exclusion of all data for any individual who is missing one or more of these covariates. The consequences of this are a reduction in statistical power (especially for the latter situation). Because no current software packages implement MSLM multiple imputation, we designed a simulation study to evaluate the practical value of developing such software.

We designed our simulation study to cover a broad range of realistic behavioral science scenarios. However, researchers may wish to evaluate the consequences of misapplication of multilevel multiple imputation in their own research. To this end, we have created an Open Science Framework repository for the R and SAS scripts we wrote to simulate, ampute, and impute the data and fit the MLSMs (<https://osf.io/uqdg5/?view_only=4bb97ffdd542422a9513ee5d8100a60f>).

***Recommendations for Practice***

This study shows that the application of multilevel multiple imputation to data that were generated according to a MLSM affects the standardized bias of all coefficients of individual-level variables with missing data and the coverage of most of these same coefficients. This effect was more commonly observed for the parameters corresponding to the individual-level variables in the models of the within- and between-individual variance. This effect is also more pronounced for the coefficients corresponding to the continuous variable than for the coefficients corresponding to the binary variable. The effects on coverage are most pronounced for the coefficient corresponding to the continuous variable in the model of the within-individual variance. In general, the standardized bias and coverage effects are minimal for 10% missing data. As such, we recommend against the application of multilevel multiple imputation to data suspected to have been generated according to a MLSM where greater than 10% of the data are missing, especially for the imputation of continuous variables.

***Limitations***

Our amputation procedure may fail to capture the full range of realistic missing data scenarios. The following modifications to our data amputation procedure may influence the results outlined in this simulation study.

The amputation procedure as implemented in the current study aggregates time-varying covariates by way of averaging for the continuous covariate and the mode for the binary covariate. Consequently, information about the within-individual variance of the time-varying covariates is lost. As an alternative, one could consider other types of aggregation functions, such as the variance, the range or the increase/decrease between the first and last occasion. Additionally, one could combine the output of multiple aggregation function into one weighted sum score.

Any procedure that generates MAR missingness depends on the correlation between the observed and incomplete variables. Schouten & Vink (2021) have shown that the smaller the correlation, the weaker the implemented MAR mechanism. In this study, the influence of the time-varying covariates on the amputation of the individual-level covariates flows indirectly through the outcome variable by means of the simulation parameters. The generated MAR mechanism could be strengthened by directly controlling the correlation between variables.

***Future Research***

We hope our simulation study encourages the development of methods for addressing missing individual-level covariates in a MLSM context. Potential avenues of development may include the development of MLSM multiple imputation and fully Bayesian methods for fitting the substantive model of interest and a model of the missing data simultaneously (Ibrahim, Chen, Lipsitz, & Herring, 2005). Especially when significant percentages of observations are missing for continuous individual-level variables, these methods are needed.

An unintended finding of this simulation study was the poor performance of estimates of the random location and scale effect variances. Future simulation studies should be designed to explicate the conditions under which poor performance is observed.

**Article Information**

***Declaration of Interest Statement***

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

***Data Availability Statement***

The Monte Carlo simulations involved in this study are too large to archive or transfer. Instead, we have created an Open Science Framework repository for the R scripts we wrote to simulate, ampute, and impute the data (<https://osf.io/uqdg5/?view_only=4bb97ffdd542422a9513ee5d8100a60f>).

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*Table 1. Summary of experimental conditions.*

|  |  |
| --- | --- |
| Data generation conditions |  |
| Number of individuals | 50, 200, 500 |
| Number of repeated measures per individual | 10, 30 |
| Magnitude of | 0.2, 0.9 |
| Magnitude of | 0.3, 1.1 |
| Magnitude of | 0, 0.3 |
| Data amputation conditions |  |
| Missingness mechanism | MCAR, MAR |
| Percent missing | 0, 10, 25, 50 |

*Table 2. Summary of the number of individuals and repeated measures per individual for a selection of intensive longitudinal studies in the behavioral science literature.*

|  |  |  |
| --- | --- | --- |
| Study |  |  |
| Ebner-Priemer, Welch, Grossman, Reisch, Linehan, & Bohus, 2007 | 100 | Prompted every 10-20 minutes for 24 hours |
| Epstein, Willner-Reid, Vahabzadeh, Mezghanni, Lin, & Preston, 2009 | 114 | 5 prompts per day for 5 weeks, then 2 prompts per day for 20 weeks |
| Granholm, Loh, & Swendsen, 2008 | 54 | 4 prompts per day for 1 week |
| Hedeker, Mermelstein, & Demirtas, 2008; Hedeker, Mermelstein, & Demirtas, 2012 | 461 | Approximate average of 30 prompts per individual |
| le Grange, Gorin, Dymek, & Stone, 2002 | 41 | 6 assessments per individual across 2 weeks |
| Rast, Hofer, & Sparks (2012) | 178 | 7 assessments per individual across 10 to 14 days |
| Rast and Zimprich (2011) | 364 | 60 reaction time tasks over 90 seconds |
| Schuster, Mermelstein, & Hedeker, 2016 | 287 | 5-7 assessments per individual across 7 days |
| Stiglmayr, Gratwohl, Linehan, Fahrenberg, and Bohus, 2005 | 103 | Prompted every hour for 2 consecutive days |
| Trull et al., 2008 | 26 | 6 prompts daily for one month |

*Table 3. Summary of the number of individuals and repeated measures per individual selected in previous two-level location scale model simulations.*

|  |  |  |  |
| --- | --- | --- | --- |
| Study |  |  | Replications |
| Hedeker, Mermelstein, & Demirtas, 2008 | 20 | 5 | - |
| Leckie, 2014 | 250 | 10 | 1,000 |
| Leckie, French, Charlton, & Browne, 2014 | 50 | 25 | 1,000 |
| Lin & Xun, 2021 | 60 | 25 | 100 |
| Walters, Hoffman, & Templin, 2018 | 25, 50, 100, 200 | 10, 30, 50 | 20,000 |

*Table 4. Summary of population values and starting values by parameter.*

|  |  |  |
| --- | --- | --- |
|  | Population  Values | Starting  Values |
| Fixed Parameters |  |  |
|  | 0 | -0.0637 |
|  | 0.2 | 0.1002 |
|  | 0.1 | 0.1426 |
|  | 0.1 | 0.1015 |
|  | 0.1 | 0.1188 |
|  | 0.2 | 0.0543 |
|  | 0.1 | 0.1352 |
|  | 0.2 | 0.3595 |
|  | 0.1 | 0.0979 |
|  | 0.2 | 0.1403 |
|  | 0.2 | 0.0847 |
|  | 0.1 | 0.0972 |
| Variable Parameters |  |  |
|  | 0.2, 0.9 | 0.1936 |
|  | 0.3, 1.1 | 0 |
|  | 0, 0.3 | 0 |

*Table 5. Summary of whether the standardized bias typically increases as a function of increasing percentages of missing data across MCAR and MAR conditions for all combinations of magnitudes specified for , , and .*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Does the standardized bias tend to increase as a function of increasing percentages of missing data?* | |  |  |  |  |  |  |  |  |  |
| 0.2\_0.3\_0 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | Yes |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.2\_0.3\_0.3 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.2\_1.1\_0 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | Yes |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.2\_1.1\_0.3 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.9\_0.3\_0 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.9\_0.3\_0.3 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.9\_1.1\_0 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |
| 0.9\_1.1\_0.3 | MCAR | No | No | Yes | Yes | No | Yes | Yes | No | No |
| MAR | Yes | Yes | Yes | Yes | No | Yes | Yes | No | Yes |

*Table 6. Summary of whether the coverage deviates substantially from the nominal coverage probability of 95% as a function of increasing percentages of missing data across MCAR and MAR conditions for all combinations of magnitudes specified for , , and .*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Does the coverage become problematic as a function of increasing percentages of missing data?* | |  |  |  |  |  |  |  |  |  |
| 0.2\_0.3\_0 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.2\_0.3\_0.3 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.2\_1.1\_0 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.2\_1.1\_0.3 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | No | Yes | No | - | Yes | No | No |
| 0.9\_0.3\_0 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.9\_0.3\_0.3 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.9\_1.1\_0 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |
| 0.9\_1.1\_0.3 | MCAR | No | No | Yes | Yes | No | - | Yes | No | No |
|  | MAR | Yes | No | Yes | Yes | No | - | Yes | No | No |

Entries for were left blank because results were not definitive.

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Chart

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Chart

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Chart

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*Figure 1. Number of replications with converged solutions across the magnitudes of , , and , the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism. The horizontal dotted line at 500 represents the cutoff below which results are excluded from subsequent discussions and presentation of results.*

*Figure 2. Boxplots depicting the precision of*  *for the fixed effects in across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 3. Standardized bias (%) for the fixed effects in the model of the average across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 4. Standardized bias (%) for the fixed effects in the model of the within-individual variance across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 5. Standardized bias (%) for the fixed effects in the model of the between-individual variance across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 6. Standardized bias (%) for the variance of the random scale effects and the covariance between the location and scale effects across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 7. Coverage (%) for the fixed effects in the model of the average across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 8. Coverage for the fixed effects in the model of the within-individual variance across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 9. Coverage for the fixed effects in the model of the between-individual variance across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

*Figure 10. Coverage for the variance of the random scale effects and the covariance between the location and scale effects across the number of individuals , the number of repeated measures per individual , the percentage of missing data, and the missingness mechanism, where*  = 0.2, = 1.1, *and* = 0.3*.*

1. Both FIML and MI may result in bias when applied to data that are MNAR. For information on MNAR missing data handling in the context of MLSMs, see Lin, Mermelstein, and Hedeker (2018) and Lin and Xun (2021). [↑](#footnote-ref-1)
2. In the case of multiple modes, we applied a function that selects the first mode in the vector of modes. [↑](#footnote-ref-2)
3. Previous work by Enders & Banalos (2001) and Grund, Ludtke, & Robitzsch (2018) defined bias (%) as suboptimal if it exceeds 10%. [↑](#footnote-ref-3)
4. Previous work by Demirtas (2004) and Grund, Ludtke, & Robitzsch (2018) define standardized bias (%) as suboptimal if it exceeds 40-50% in a positive or negative direction. [↑](#footnote-ref-4)
5. Grund, Ludtke, & Robitzsch (2018) define coverage as suboptimal if below 90% or "very close" to 100%. [↑](#footnote-ref-5)