EE645 Final Report

Madeline Hayes

Table of Contents:

Part 1: Important Checks

1. Checks for area scattering phase function symmetry:

$$\Gamma_d^{\pm}(\Omega' \rightarrow \Omega) = \pm\frac{1}{2\pi}\int_0^1 d\mu_L \int_0^{2\pi} d\varphi_L\, g_L(\mu_L, \varphi_L)(\Omega \bullet \Omega_L)(\Omega' \bullet \Omega_L)$$

$$\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(\Omega \rightarrow \Omega') = \Gamma_d(-\Omega' \rightarrow -\Omega)$$

For the case of ρ=τ, additional symmetry:

$$\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(-\Omega' \rightarrow \Omega)$$

Case 1: Leaf normal inclination: **Planophile** $\frac{2}{\pi}(1 + \cos 2\theta_L)$
Leaf normal azimuth: Uniform 1.0
θ' = 135 φ'= 90
θ = 60 φ= 180
ρ=0.35 τ=0.04
$\mathbf{\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(\Omega \rightarrow \Omega') = \Gamma_d(-\Omega' \rightarrow -\Omega) = 0.101698}$

ρ=0.4 τ=0.4
$\mathbf{\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(-\Omega' \rightarrow \Omega) = 0.1153948}$

Case 2: Leaf normal inclination: **Planophile** $\frac{2}{\pi}(1 + \cos 2\theta_L)$
Leaf normal azimuth: Uniform 1.0
θ' = 135 φ'= 90
θ = 60 φ= 180
ρ=0.025 τ=0.04
$\mathbf{\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(\Omega \rightarrow \Omega') = \Gamma_d(-\Omega' \rightarrow -\Omega) = 0.007387091}$

$\rho=0.4$ $\tau=0.4$

$\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(-\Omega' \rightarrow \Omega) = 0.01153948$

Case 3: Leaf normal inclination: **Plagiophile** $\frac{2}{\pi}(1 - \cos 4\theta_L)$

Leaf normal azimuth: Uniform 1.0

$\theta' = 135$ $\varphi' = 90$

$\theta = 60$ $\varphi = 180$

$\rho=0.35$ $\tau=0.4$

$\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(\Omega \rightarrow \Omega') = \Gamma_d(-\Omega' \rightarrow -\Omega) = 0.08530617$

$\rho=0.4$ $\tau=0.4$

$\Gamma_d(\Omega' \rightarrow \Omega) = \Gamma_d(-\Omega' \rightarrow \Omega) = 0.09570729$

2. Checks for area scattering phase function:

Leaf normal inclination: **Spherical** $\sin(\theta_L)$

Leaf normal azimuth: Uniform 1.0

$\theta' = 135$ $\varphi' = 90$

$\theta = 60$ $\varphi = 180$

$\rho=0.35$ $\tau=0.4$

$\Gamma_d(\Omega' \rightarrow \Omega) = 0.0817$

Calculate $\Gamma_d(\Omega' \rightarrow \Omega)$ using the analytical function:

$\Gamma_d(\Omega' \rightarrow \Omega) = \frac{\omega}{\pi}(\sin\beta - \beta\cos\beta) + \frac{\tau}{\pi}\cos\beta$ $\beta = \arccos(\Omega' \bullet \Omega)$

$\Gamma_d(\Omega' \rightarrow \Omega) = 0.0838$

3. Check scattering phase function for normalization:

$$\frac{1}{4\pi}\int_{4\pi} d\Omega P_d(\Omega' \rightarrow \Omega) = \frac{1}{\pi}\int_{4\pi} d\Omega \frac{\Gamma_d(\Omega' \rightarrow \Omega)}{\omega_{L,d} G(\Omega')} = 1$$

Gamma_d_check(=1.0?): 1.005

4. Check G function for normalization:

$$\frac{1}{2\pi}\int_{2\pi} d\Omega' G(\Omega') = 0.5$$

Gfun_check(=0.5?): 0.5005

Part 2: Figures

1. Leaf Normal Inclination Distribution Function

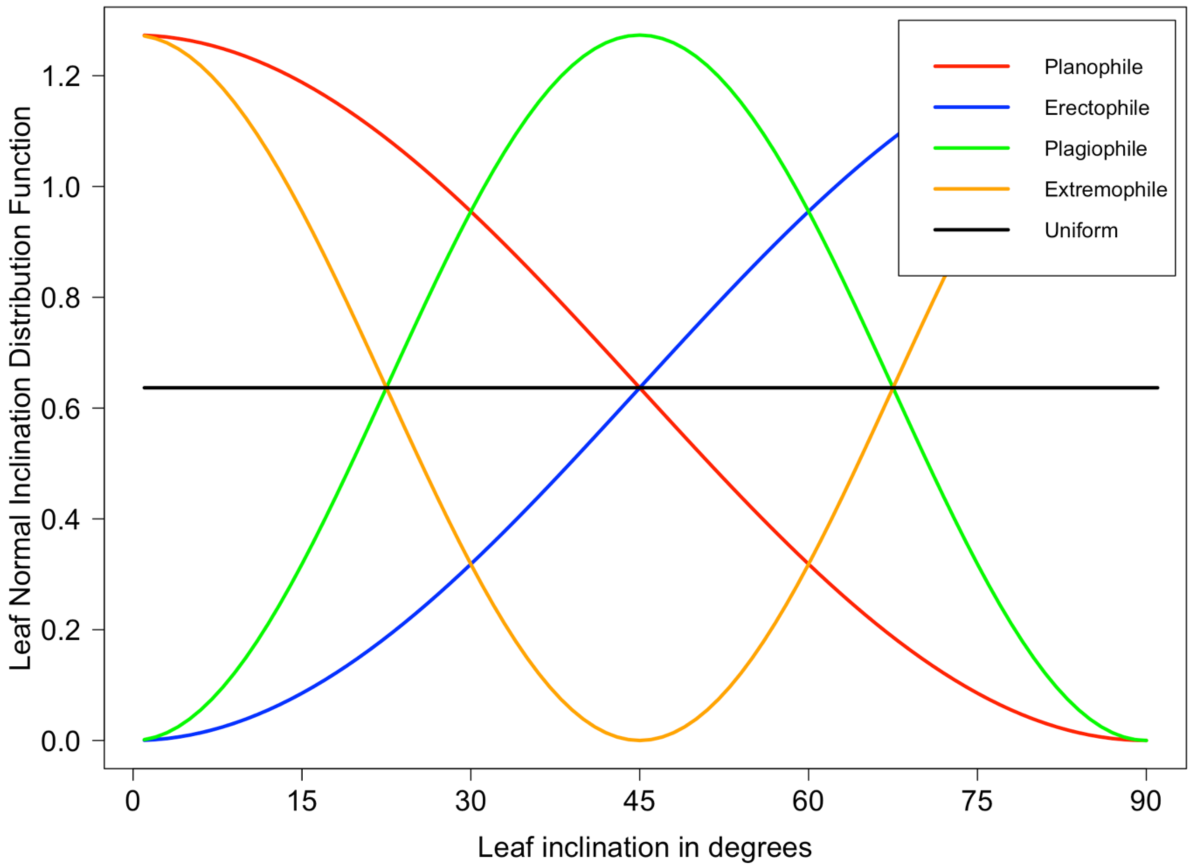

Figure 1. Widely used distribution functions for leaf normal inclination: (1) **Planophile –** mostly horizontal leaves, (2) **Erectophile –** mostly erect leaves, (3) **Plagiophile –** mostly leaves at 45 degrees, (4) **Extremophile –** mostly horizontal and vertical leaves, (5) **Uniform –** all inclinations equally probable. The integration of each leaf normal inclination distribution function is equal to 1.

2. G Function

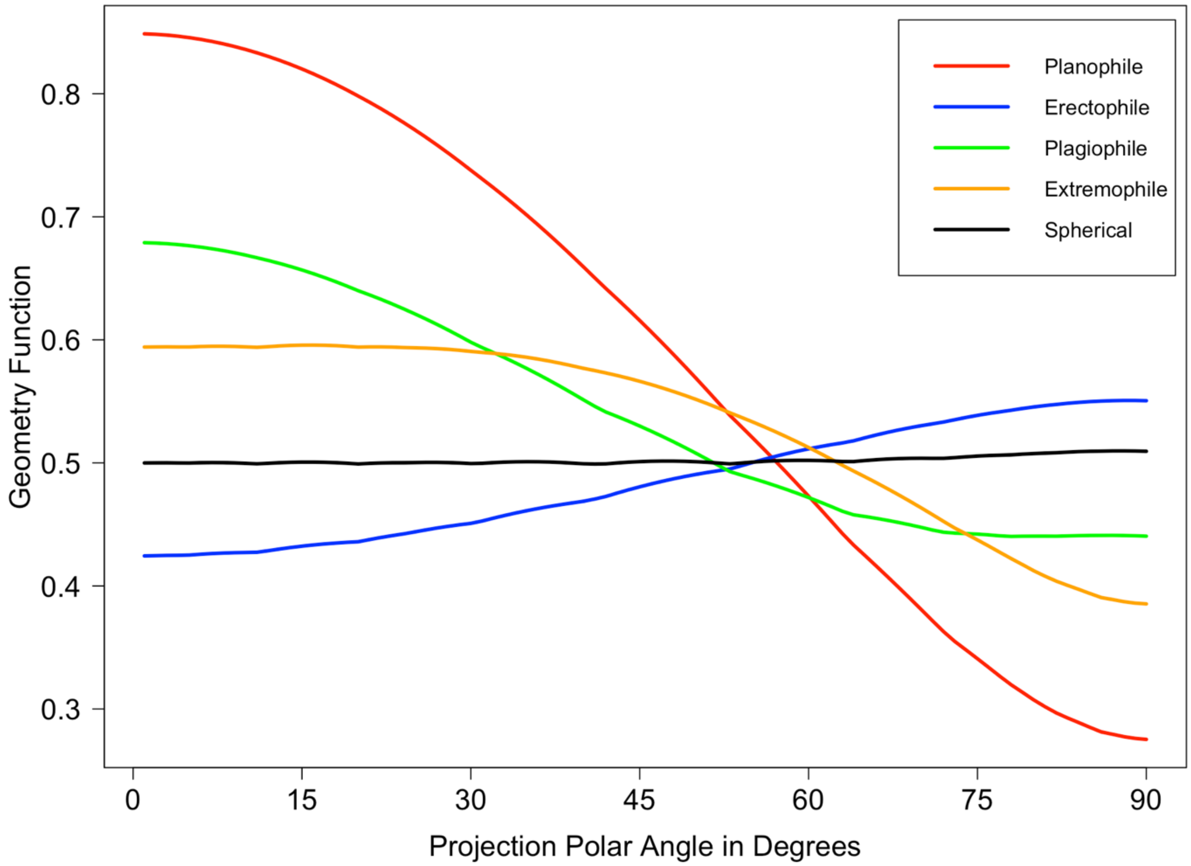

Figure 2. The G Function (projection zenith angle θ') for 5 widely used leaf normal inclination distribution functions. The geometry factor is a function of the direction of photon travel for non-uniformly distributed leaf normals. The **planophile** distribution (mostly horizontal leaves) has the highest probability of intercepting photons incident from directions close to the vertical at 0°, while an **erectophile** distribution (mostly vertical leaves) has the highest probability of intercepting photons incident from 90° of zenith angle. The transport problem is in the rotationally invariant form for the **spherical** distribution (G = 0.5).

3. Normalized Scattering Phase Function

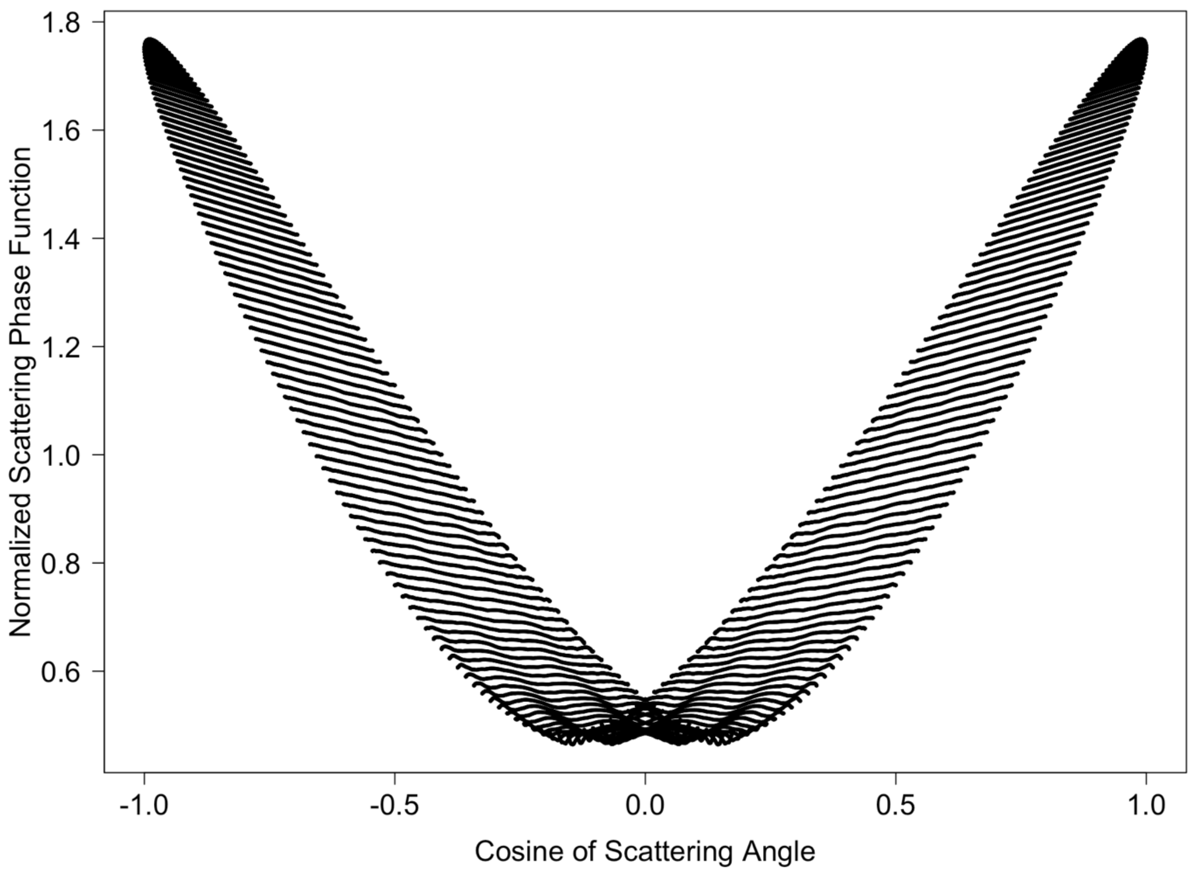

Figure 3. The normalized scattering phase function $\mathbf{\Gamma_d(\Omega' \rightarrow \Omega)}$ for **planophile** leaf normal inclination distribution. Leaf transmittance (τ) and reflectance (ρ) are both equal to 0.5, and Ω' is fixed at θ'=170° and ϕ=0°. Each dot is the value of the phase function for a discrete value of $\Omega_{ij} = (\mu_i, \varphi_j)$ in which μ_i = -1:1 and φ_j = 0:2π. The results indicate that the phase function is azimuthally dependent **(i.e., non-rotationally invariant)**, it depends on the coordinates Ω' and Ω (incident and outward directions) and not just the scattering angle [arccos ($\Omega \bullet \Omega$')].

4. Bi-Hemispherical Reflectance

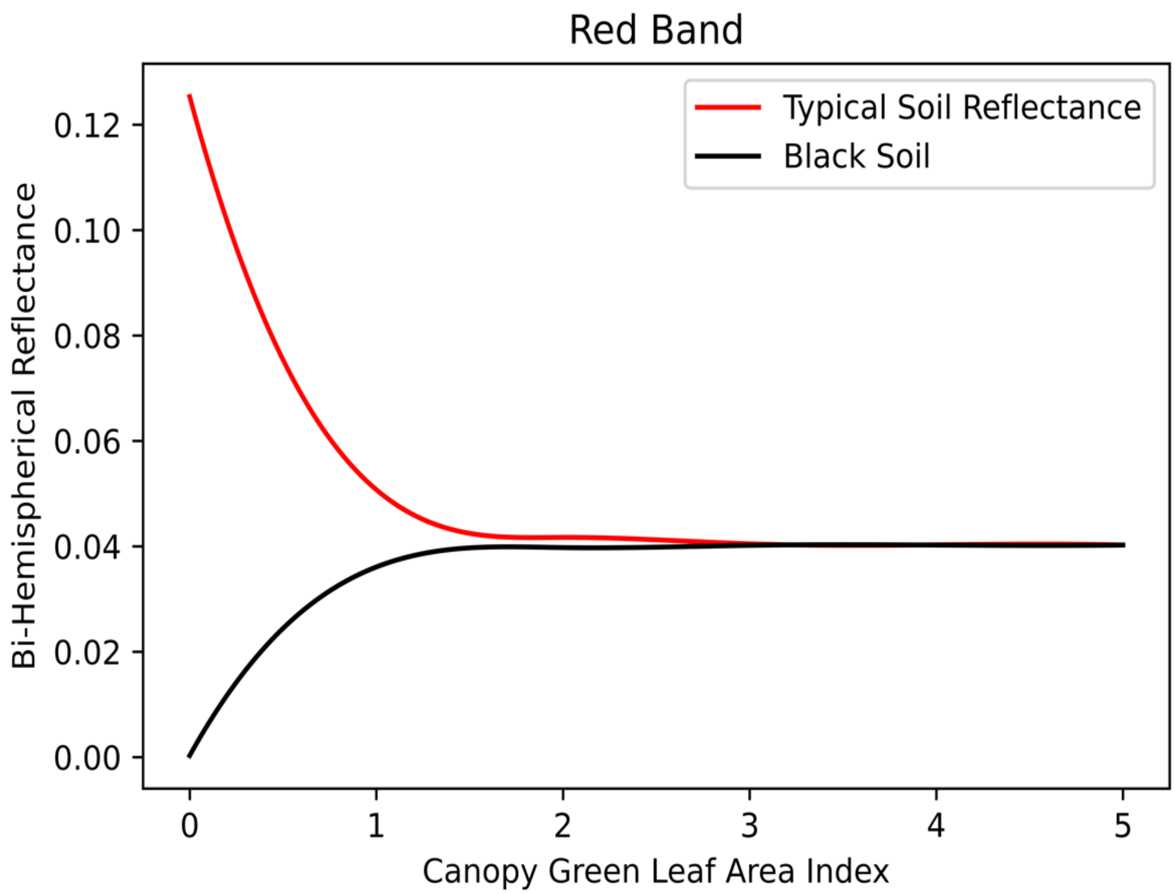

Figure 4a. Bi-Hemispherical Reflectance (BHR) at red wavelengths as a function of Leaf Area Index (LAI). The polar angle of the sun is 150º and the azimuth angle of the sun is 0º. The fraction of direct solar radiation is 0.7. The leaf normal distribution is planophile. Leaf reflectance (ρ) is 0.075 and leaf transmittance (τ) is 0.035. Ground reflectance for typical soil (0.125) and black soil (0) is shown. For red wavelengths, when LAI is small, typical soil will reflect most of the incoming flux. The effect of soil reflectance on BHR becomes insignificant for higher LAI, and BHR will eventually saturate.

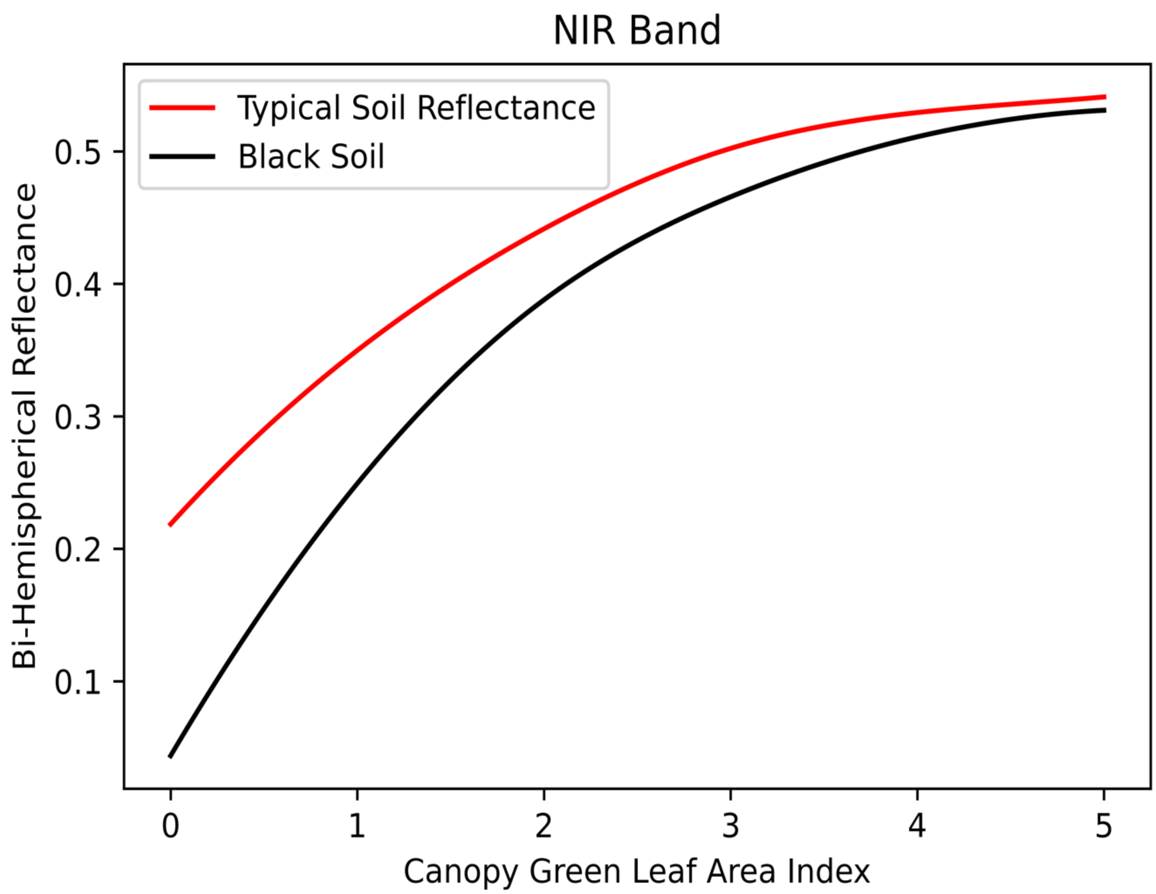

Figure 4b. Bi-Hemispherical Reflectance (BHR) at NIR wavelengths as a function of Leaf Area Index (LAI). The polar angle of the sun is 150º and the azimuth angle of the sun is 0º. The fraction of direct solar radiation is 0.7. The leaf normal distribution is planophile. Leaf reflectance (ρ) is 0.475 and leaf transmittance (τ) is 0.45. Ground reflectance for typical soil (0.2) and black soil (0) is shown. For NIR wavelengths, bi-hemispherical reflectance will increase with LAI. When LAI is small, soil reflectance is the dominant factor. The effect of soil reflectance on BHR becomes insignificant for higher LAI, and BHR will eventually saturate.

5. Bi-Hemispherical Transmittance

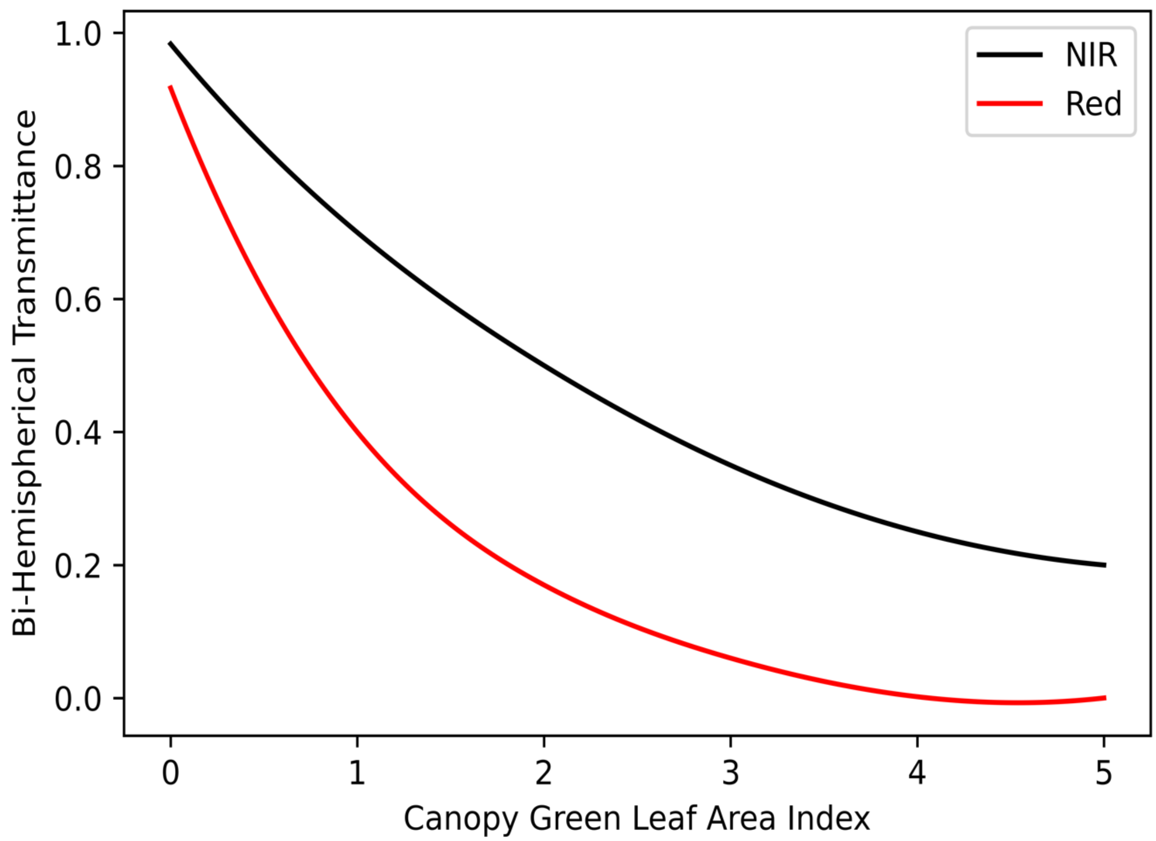

Figure 5. Bi-Hemispherical Transmittance (BHT) at red and NIR wavelengths as a function of Leaf Area Index (LAI). The polar angle of the sun is 150º and the azimuth angle of the sun is 0º. The fraction of direct solar radiation is 0.7. The leaf normal distribution is planophile. Leaf reflectance (ρ) is 0.475 and leaf transmittance (τ) is 0.45 for NIR wavelengths. Leaf reflectance (ρ) is 0.075 and leaf transmittance (τ) is 0.035 for red wavelengths. Bi-hemispherical transmittance is high for both wavelengths when LAI is small. Transmittance will decrease faster as LAI increases for red wavelengths and will be near zero when LAI is 5. For NIR wavelengths, as LAI approaches 5, 20% of the radiation will still pass the canopy.

6. Bi-Hemispherical Reflectance, Transmittance, and Canopy Absorptance

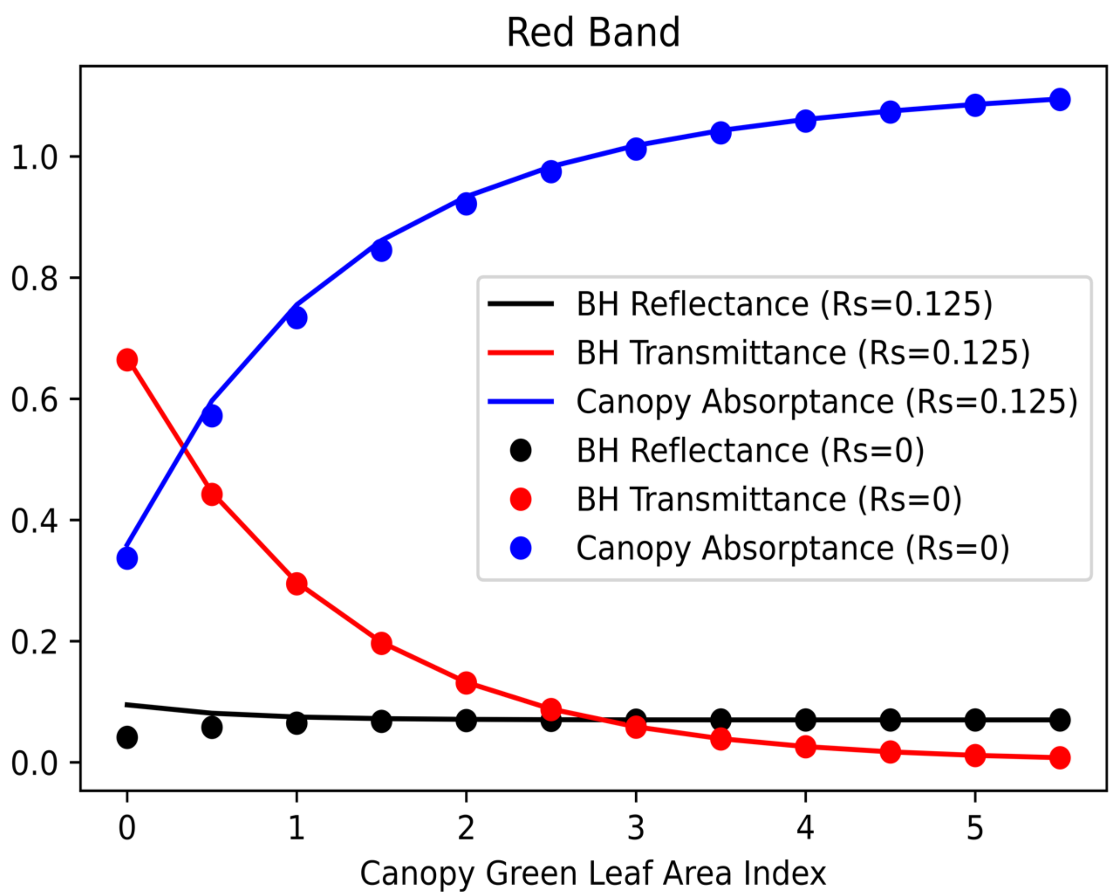

Figure 6. Bi-Hemispherical Reflectance (BHR), Bi-Hemispherical Transmittance (BHT), and Canopy Absorptance at red wavelengths as a function of Leaf Area Index (LAI). The polar angle of the sun is 150º and the azimuth angle of the sun is 0º. The fraction of direct solar radiation is 0.7. The leaf normal distribution is planophile. Leaf reflectance (ρ) is 0.075 and leaf transmittance (τ) is 0.035. Ground reflectance for typical soil (0.125) and black soil (0) is shown. Bi-hemispherical transmittance is the same regardless of soil reflectance, and canopy absorptance is very high in the red band for higher LAI.

7. Hemispherical-Directional Reflectance Factor (HDRF)

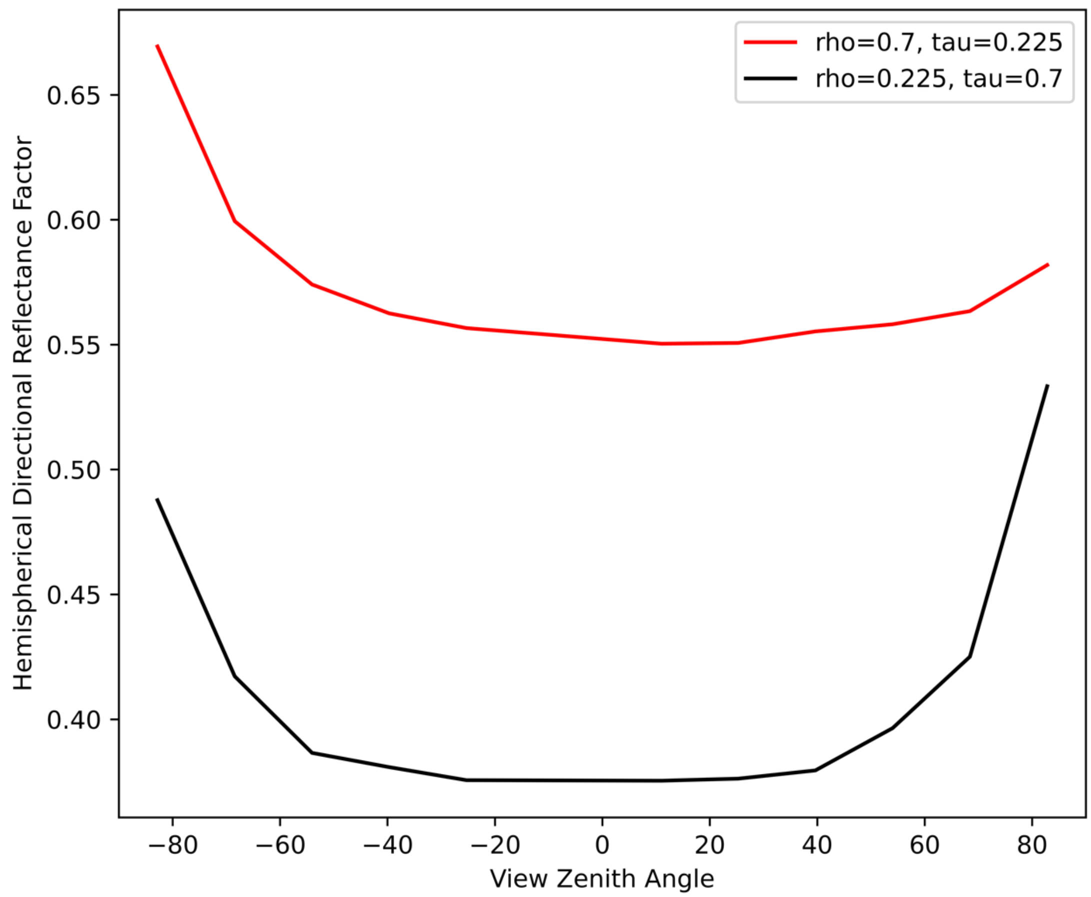

Figure 7. Hemispherical Directional Reflectance Factor (HDF) in the principal plane. $\varphi_{view} = \varphi_0 + \pi$ for backscattering and $\varphi_{view} = \varphi_0$ for forward scattering. Solar radiation is from (θ_{sza}=150º, φ_{sza}=0º). Total LAI is 3.0 and soil reflectance is 0.2. For the red line, leaf reflectance is 0.7 and leaf transmittance is 0.225. The black line is the opposite. Viewing zenith angle will have the strongest effect on backward and forwarding scattering near -90º and 90º.

Part 3. Field Validation

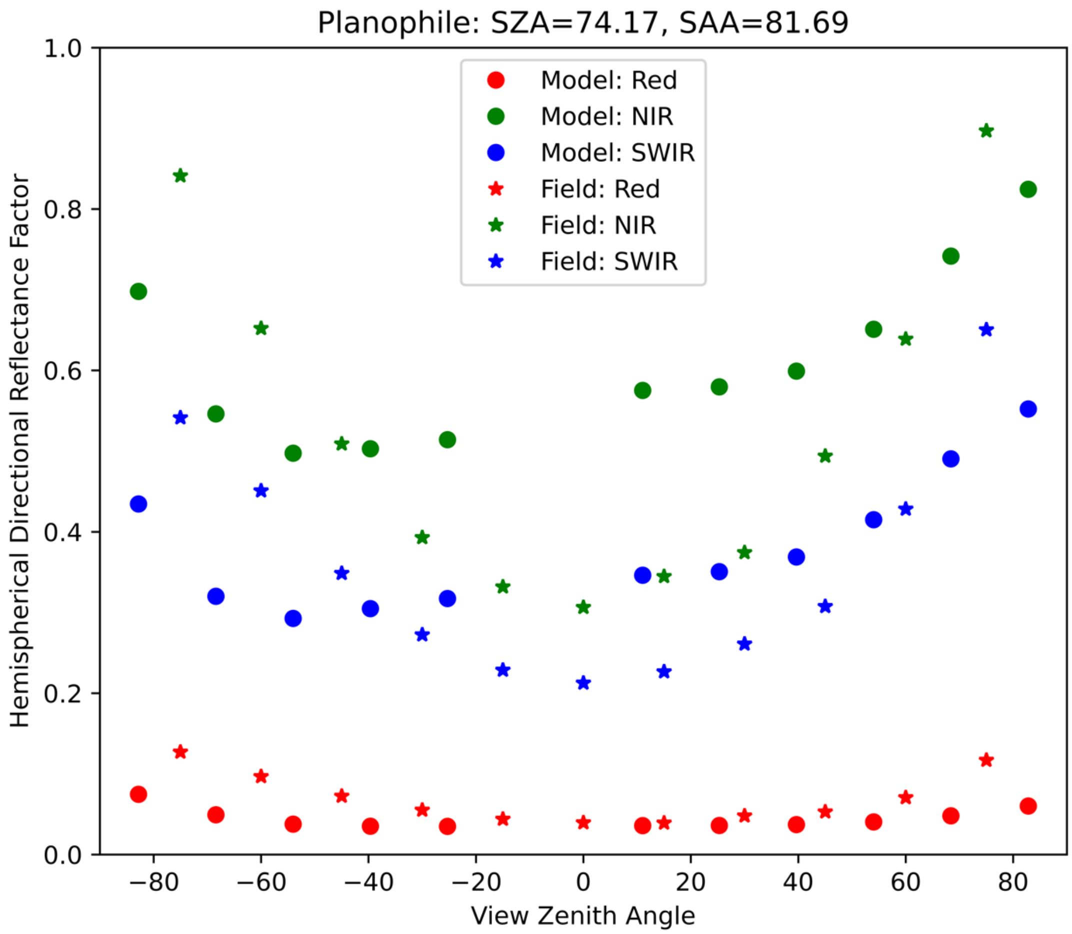

Figure 8. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 74.17º and Solar Azimuth Angle (SAA) is 81.69º. Modelled HDRF is shown for the planophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

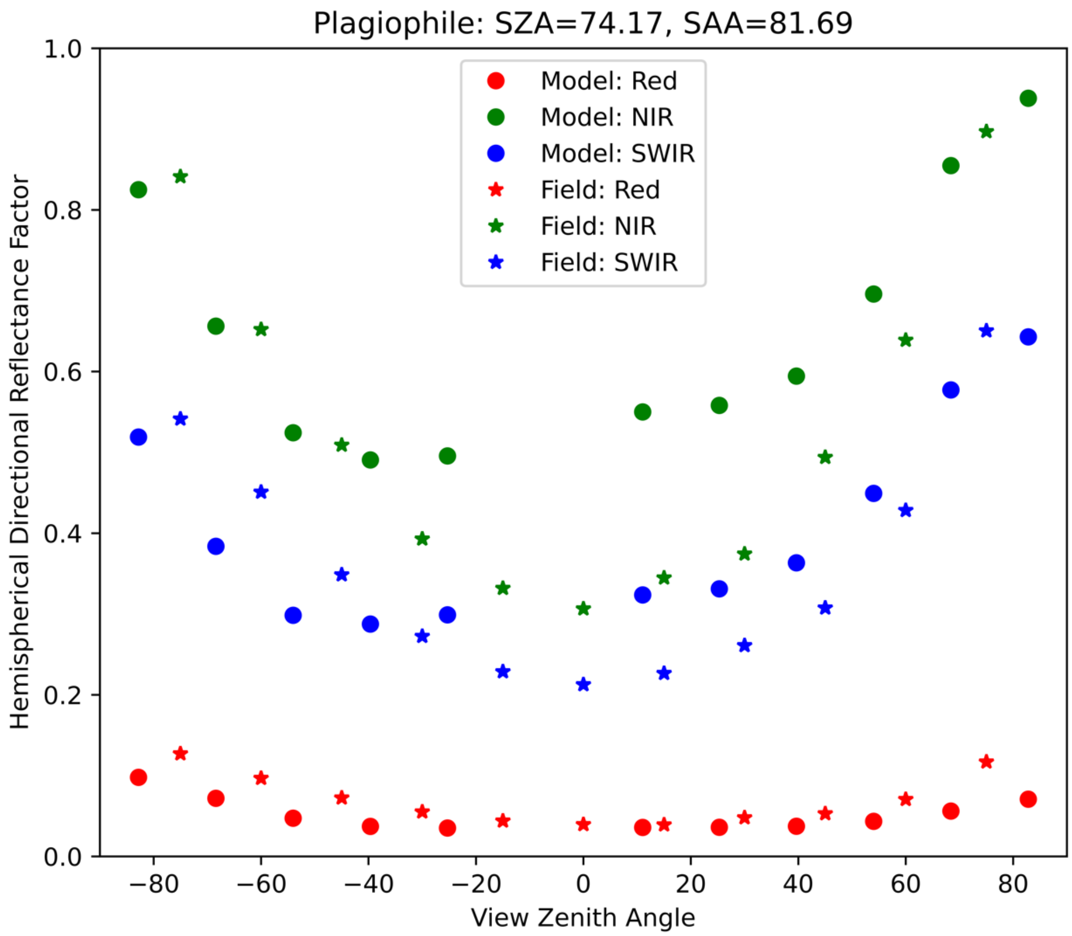

Figure 9. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 74.17º and Solar Azimuth Angle (SAA) is 81.69º. Modelled HDRF is shown for the plagiophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

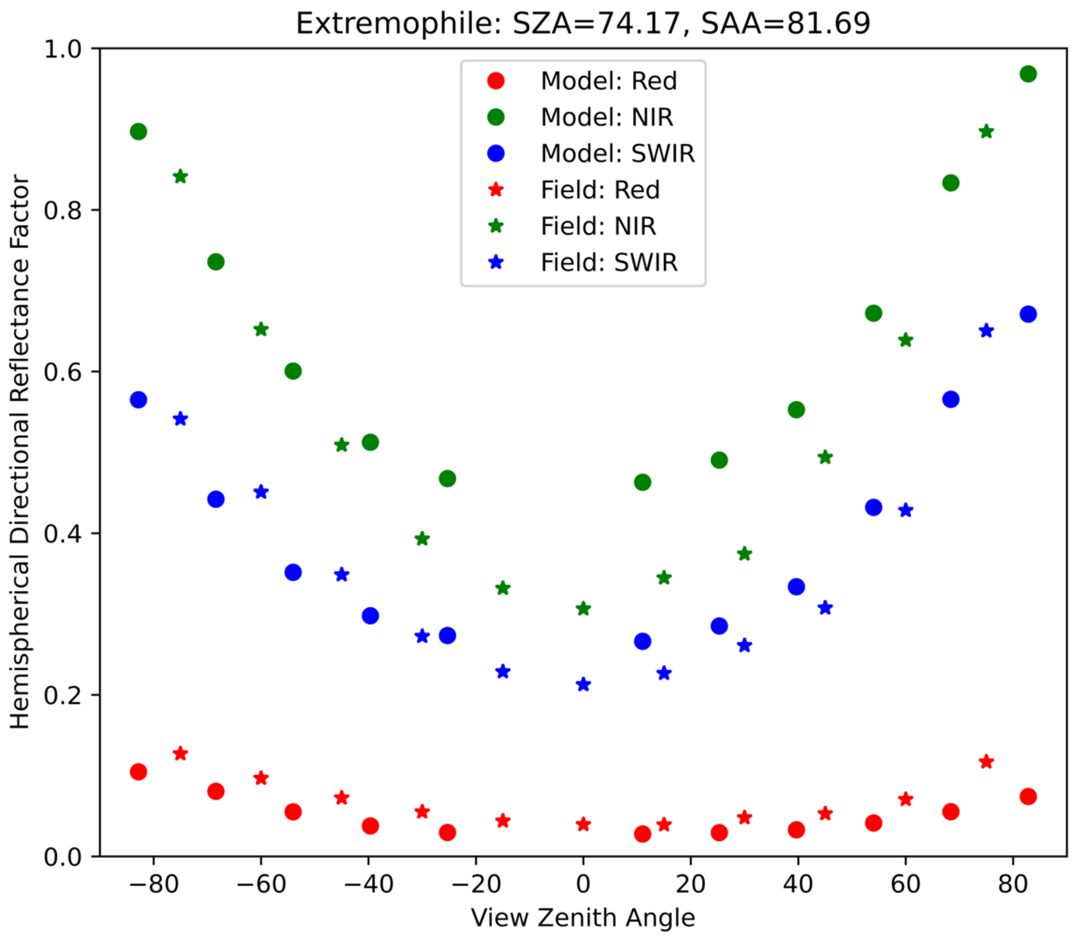

Figure 10. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 74.17º and Solar Azimuth Angle (SAA) is 81.69º. Modelled HDRF is shown for the extremophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

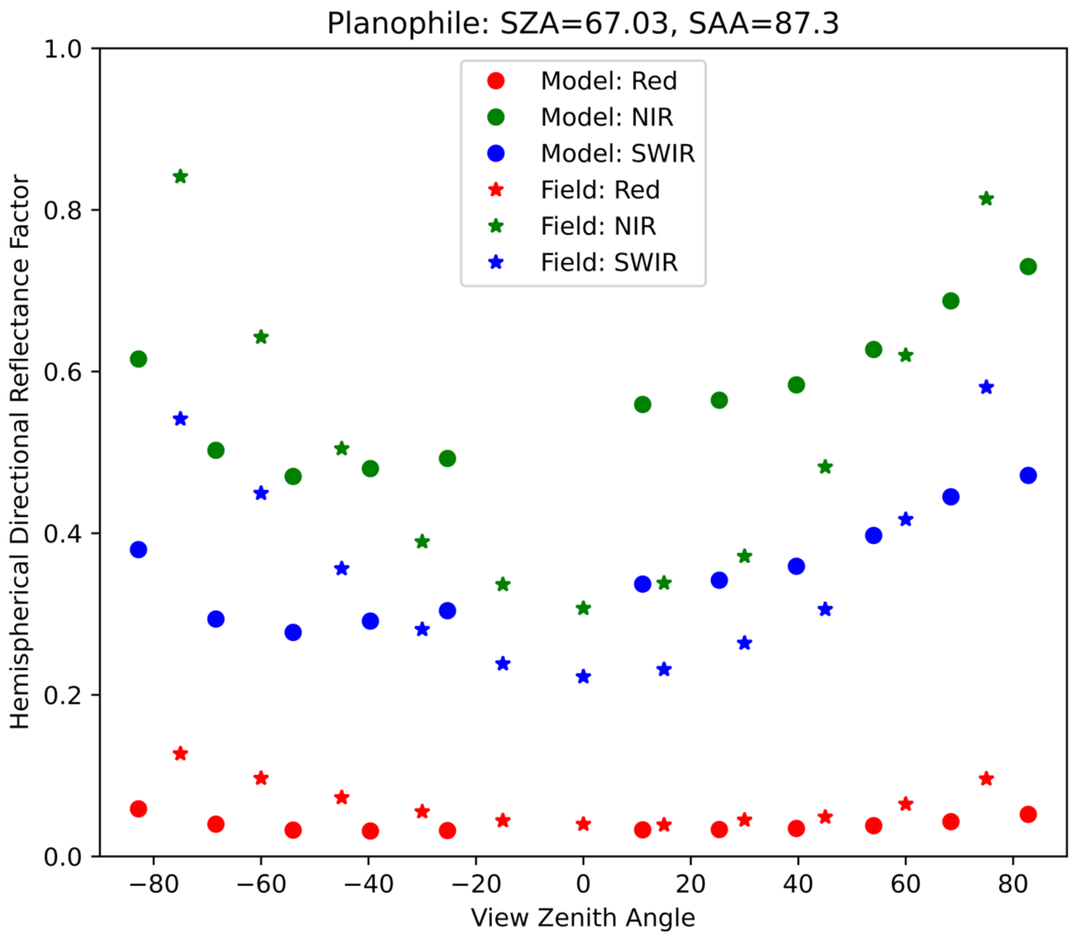

Figure 11. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 67.03º and Solar Azimuth Angle (SAA) is 87.3º. Modelled HDRF is shown for the planophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

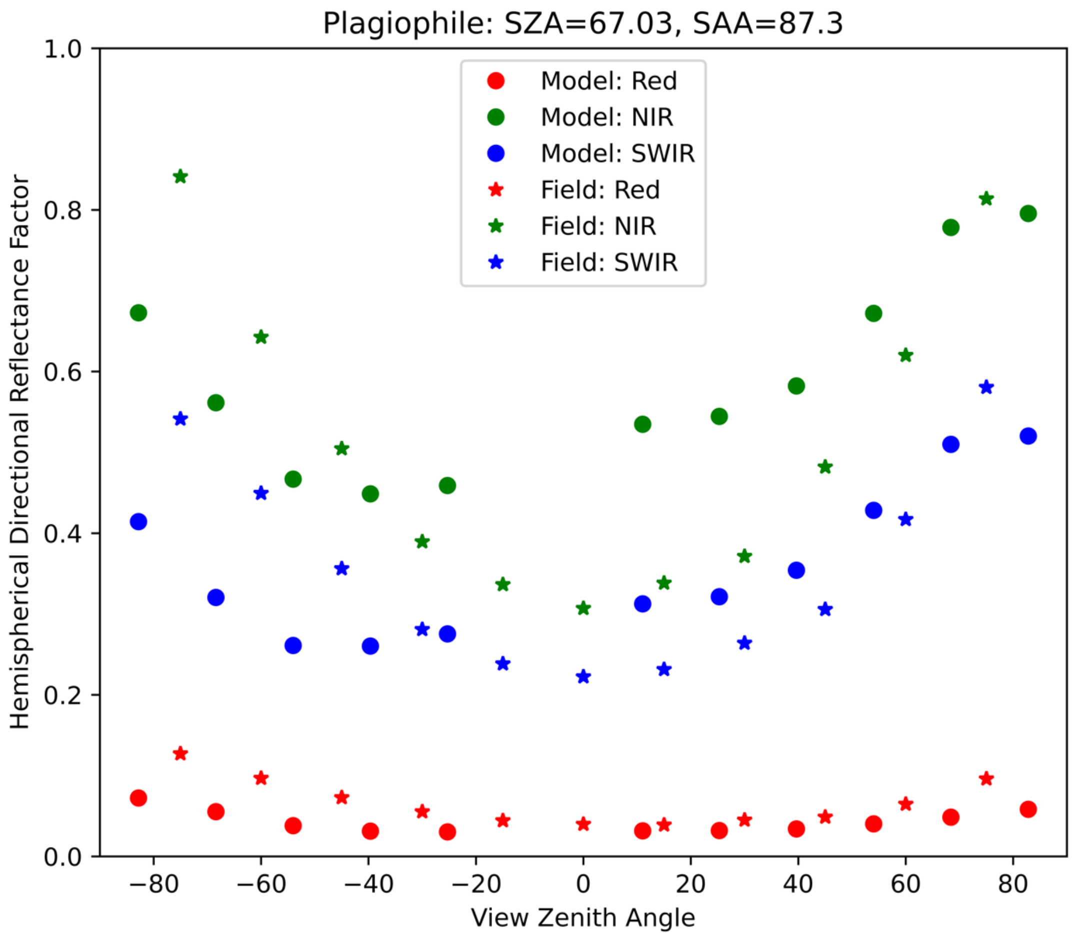

Figure 12. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 67.03º and Solar Azimuth Angle (SAA) is 87.3º. Modelled HDRF is shown for the plagiophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

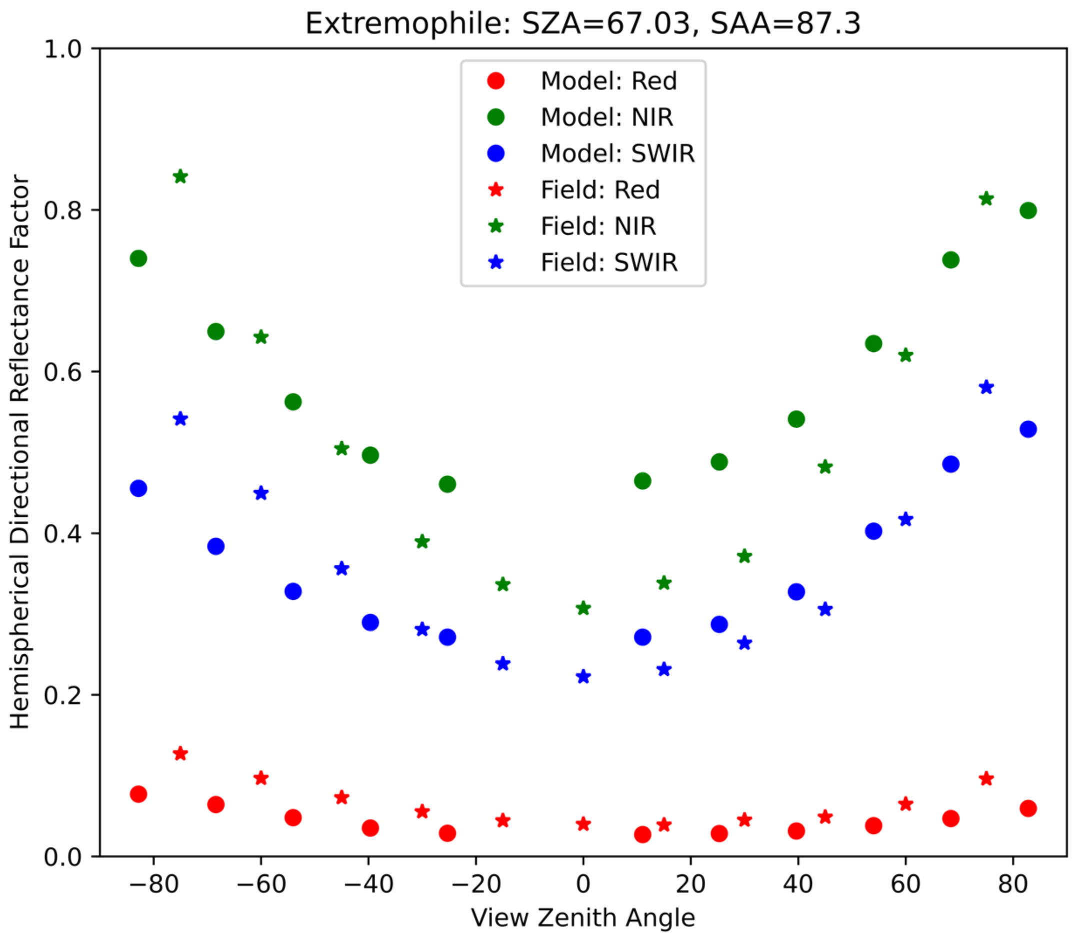

Figure 13. View Zenith Angle against HDRF in the principal plane. Solar Zenith Angle (SZA) is 67.03º and Solar Azimuth Angle (SAA) is 87.3º. Modelled HDRF is shown for the extremophile leaf normal distribution in the Red (0.63-0.69 µm), NIR (0.76-0.90 µm), and SWIR (1.55-1.75 µm) bands.

Part 4. Inversion

(1) Inversion using two bands and one direction in principal plane

Gauss Direction: VZA=11.02º (i = 12); VAA=285.72º (j=9)
Measured HDRF: Red=0.0228; NIR=0.3315

LAI_Red = **2.5**; LAI_NIR = **1.375**

Mean Estimated LAI = **1.94**

(2) Inversion using two bands and four directions in principal plane

Gauss Direction 1: VZA=11.02º (i = 12); VAA=285.72º (j = 9)
Measured HDRF: Red=0.0228; NIR=0.3215

Gauss Direction 2: VZA=11.02º (i=12); VAA= 113.79º (j=5)
Measured HDRF: Red=0.0253; NIR=0.3390

Gauss Direction 3: VZA=25.29 (i=11); VAA=285.72º (j=9)
Measured HDRF: Red=0.0391; NIR=0.3824

Gauss Direction 4: VZA=25.29º (i=11); VAA=113.79º (j=5)
Measured HDRF: Red=0.0299; NIR=0.3545

LAI_Red = **2.875**; LAI_NIR = **1.625**

Mean Estimated LAI = **2.25**

Estimating LAI from the NIR band was more slightly more accurate when using more directions in the principal plane. However, LAI was under-estimated for both inversions in the NIR band. LAI was over-estimated for both inversions in the red band. Averaging estimated LAI for both methods produced the most accurate result, with the one direction inversion being slightly closer to the "measured" data (LAI = 2.0).