Multi-scenario Extreme Weather Simulator Application to Heat Waves

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Abstract

Contents

2 Introduction

Quantifying heat wave intensity is an active area of research [11]

For many cases stochastic analysis only needs physics with reasonably good correlation between variables. For building energy modeling, the majority of energy consumption is explained by dry bulb temperature alone with humidity, and radiation following in order of importance making it justifiable to adopt a less physically rigorous approach through only altering temperature without associated changes to barometric pressure and humidity.

Extreme weather leads to additional concerns beyond though.

2.1 Literature Review

The accurate representation of extreme heat waves and winter cold is a new area of research

Much work is focussed on dynamic downscaling of regional climate models and earth systems modeling. The Anex 80 CORDEX project ... . Find and mention Ralph Mulheisen's project.

Studies concerning buildings and climate change have focussed on: 1) How robust energy retrofits are affected by climate change [4, 2, 6] 2) Clear trends in increased energy demand using current weather data over typical meteorological data based on the last couple decades [7, 6] 3) Heating and Cooling demand changes in entire regions [19].

Concerning resilience measures to extreme heat or cold, studies include loss of productivity due to power outages correlated to extreme heat conditions [10], thermal comfort and survivability [17, 16], changes in peak load and energy consumption [18]. A good review of resilence metrics to heat waves and power outages in the built environment is provided by [3].

These efforts provide datasets that often do not convey a straightforward procedure for estimating the effects of extreme events in an energy modeling resilience framework. Such

The work herein focusses on showing the dynamic that extreme weather variations for heat waves and extreme cold .

Data from the NOAA Global Historical Climatology Network daily (GHCNd) database [13, 12, 5] was used to quantify historical characteristics for heat waves for weather station XXXXXX located at YYYYYY. (https://www.ncei.noaa.gov/products/land-based-station/global-historical-climatology-network-daily). These daily records were combined with the NOAA United States Climate Normals for 1991-2020 data to determine consecutive exceedences of maximum or minimum 90% climate temperatures.

This information was then combined with climate normals average humidity

SPECULATION Each heat wave was then

The procedure uses maximum and minimum daily temperature to quantify heat wave severity (peak temperature) and duration using techniques similar to Li et. al. [8].

A minimum of fifty years of data was required for maximum and minimum daily temperatures using a threshold temperature equal to the

Of even greater concern is the possibility of the need for increased probability of compound heat waves where events are more likely just after a previous event. Baldwin et. al. [**?**]

3 Methods

A minimal complexity process is sought that characterizes the historical statistics of heat waves and then extrapolates increasing frequency and severity as defined by IPCC.

3.1 Extreme Temperature Events Functional Form

In this discussion, both hot and cold extreme temperature events are addressed. Hot temperature extreme events are called "heat waves" and cold temperature extreme events are called "cold snaps." Though humidity is important, the present discussion only focusses on temperature with the understanding that temperature can be defined as a heat index should historic humidity data become more prevalent.

The model used here assumes the form of a heat wave that allows a mixture of increases in daily maximum temperature and increases of temperture throughout the entire heat wave. This function must result in zero change in temperature at the beginning and end of the heat wave. A function that satisfies this, is shown below.

$$\Delta T(t, D, \Delta t_{min}) = \begin{cases} A\sin\left(\frac{\pi t}{D_{odd}}\right) + B\left(1-\cos\left(\frac{2\pi t}{\Delta t_{min}}\right)\right) + C & t \leq D_{odd} \\ B\left(1-\cos\left(\frac{2\pi t}{\Delta t_{min}}\right)\right) + C & t > D_{odd} \end{cases} \tag{1}$$

Here ΔT is the change from the original temperature signal due to the heat wave and Δt_{min} is the shortest permissible length of a heat wave which is one earth day here. The parameter D_{odd} is the closest odd multiple of Δt_{min} that is less than the heat wave duration D simulated by the Markov process. The parameters A and B are determined once the maximum temperature and energy log-normal distributions have been sampled for a given heat wave.

$$D_{odd} = \Delta t_{min}\left[\lfloor\frac{D}{\Delta t_{min}}\rfloor - \delta\left(\lfloor\frac{D}{\Delta t_{min}}\rfloor \mod 2\right)\right] \tag{2}$$

Here $\mod$ is the modulus operator, $\lfloor\rfloor$ indicates the floor function or closest integer less than the input, and δ is the Dirac delta function. Using D_{odd} instead of D in equation 1 avoids erratic variations in the maximum temperature condition with respect to the heat wave duration which greatly simplifies mapping ΔT to sampled maximum temperature change, ΔT_{max}, and heat wave energy per duration E. Integration of equation 1 from 0 to D produces the following total energy in degrees temperature times time.

$$\Delta E = \frac{2AD_{odd}}{\pi} + BD - \frac{B\Delta t_{min}}{2\pi}\sin\left(\frac{2\pi D}{\Delta t_{min}}\right) \tag{3}$$

Though D_{odd} makes the energy relationship more complex, It makes the heat wave maximum temperature much more simple.

$$T_{max} = A + 2B + C \tag{4}$$

To close the mathematical system for any heat wave, E and T_{max} can be sampled as random variables that are normalized by the duration of the heat wave (the subsequent derivation will explain this). The relationships in equations 3 and 4 can be solved to provide values for A and B:

$$A = \frac{T_{max} - \frac{\pi}{2D_{odd}}E}{2 - \frac{\pi D}{2D_{odd}} + \frac{\Delta t_{min}}{4D_{odd}}\sin\left(\frac{2\pi D}{\Delta t_{min}}\right)} \tag{5}$$

$$B = \frac{T_{max} - A}{2} \tag{6}$$

Both the values of A and B must be greater than zero for heat wves (or less than zero for cold snaps) for the solution to be physically meaningful. If this is not the case, then ΔE and T_{max} can be resampled until a physically admissible solution is found.

3.2 Extreme temperature event classification

The historical NOAA daily summary data must be classified into heat wave days and non-heat-wave days. Consecutive days then have to be found so that the probability of initiating a heat wave and the probability of sustaining a heat wave can be characterized for the Markov chain model.

Let D_d be the entire set of days for which daily maximum T_{max} and minium T_{min} temperatures are available with no gaps in the data for at least 50 years. Let H be the set of hourly climate normals from NOAA that has 10% CI on minimum hourly temperature, average climate temperature and 90% CI on maximum hourly temperature ($T_{min10}, T_{50}, T_{max90}$). If data has daily CI's, that can work as well. Here, heat wave days are defined as any two or more consecutive day for which one or more of the following are true: 1) The daily maximum temperature is above the NOAA climate normals 90% CI for the hourly maximum of daily maximum temperature or 2) The daily minimum temperature is above the climate normals 90% CI for the hourly maximum of daily minimum temperature. The set of heat waves HW is therefore the set of sets of days for which the duration D for which these conditions are true is greater than or equal to 2:

$$HW = \left\{ \left\{ D_{d_i} : \bigwedge_{i=S_{HW_m}}^{S_{HW_m}+D_{HW_m}-1} \left[T_{max_i} > \max\left(\left\{ T_{max90_j} \right\}_{j=k\Delta t_{min}}^{(k+1)\Delta t_{min}} \right) | T_{min_i} > \max\left(\left\{ T_{min90_j} \right\}_{j=k\Delta t_{min}}^{(k+1)\Delta t_{min}} \right) \right] \right\}_{m=1}^{N_{hw}} \right\} \tag{7}$$

Here S_{HW} is the set of heat wave start days, N_{hw} is the number of heat waves found in D_d, and k is the modulus of the current value of i by the number of Δt_{min} in the current year which varies between normal and leap years. The set of cold snaps is defined similarly but using the 10% CI's:

$$CS = \left\{ \left\{ D_{d_i} : \bigwedge_{i=S_{CS_m}}^{S_{CS_m}+D_{CS_m}-1} \left[T_{min_i} < \min\left(\left\{ T_{min10_j} \right\}_{j=k\Delta t_{min}}^{(k+1)\Delta t_{min}} \right) | T_{max_i} < \min\left(\left\{ T_{max10_j} \right\}_{j=k\Delta t_{min}}^{(k+1)\Delta t_{min}} \right) \right] \right\}_{m=1}^{N_{cs}} \right\} \tag{8}$$

3.3 Frequency of extreme events

Once HW and CS have been formed, the Markov chain transition matrix can be calculated. The transition matrix parameters are calculated for each month of the year. This is necessary because the statistics of extreme temperture events are seasonal. This is especially true for heat waves where the summer contains many more heat waves than winter does. Smoothing out this affect is unacceptable because winter heat waves do occur but are benign or even desirable whereas summer heat waves have potential to cause harm. The probability of a heat wave or cold snap beginning is the ratio of the total number of extreme events found in the current month divided by the total number of time steps within the number of years of data for the current month represented in D_d:

$$P_{hw_m} = \Delta t_{min} \frac{|S_{HW_m}|}{\Delta t_m} \tag{9}$$

$$P_{cs_m} = \Delta t_{min} \frac{|S_{CS_m}|}{\Delta t_m} \tag{10}$$

Here m is the month number, S_{HW_m} is all members of S_{HW} that fall in month m, and S_{CS_m} is all members of S_{CS} that fall in month m. If an extreme event starts in one month and ends in the next month, then the month that contains the majority of days for the extreme event is asigned the extreme event. If an extreme event is split equally between two months, then the extreme event is assigned to the first month that contains the extreme event where months are in calendar order and December precedes January. Once a heat wave has begun, the conditional probability that a extreme event continues is needed. This requires finding the histogram of total hours for all extreme events and each duration.

$$D_{HW_{Pm}} = \left\{\sum D_{HW_m} \delta\left(D_{HW_m} - i\right)\right\}_{i=2}^{\max\left(D_{HW}\right)} \tag{11}$$

$$D_{CS_{Pm}} = \left\{\sum D_{CS_m} \delta\left(D_{CS_m} - i\right)\right\}_{i=2}^{\max\left(D_{CS}\right)} \tag{12}$$

The probability of an extreme event derived from the data is then estimated as the ratio of the number of hours within all events of a given duration divided by the total time steps of all extreme events of a given type:

$$P_{HWD_m} = \frac{D_{HW_{Pm}}}{\Delta t_{min} \sum\left(D_{HW_{Pm}}\right)} \tag{13}$$

$$P_{CSD_m} = \frac{D_{CS_{Pm}}}{\Delta t_{min} \sum\left(D_{CS_{Pm}}\right)} \tag{14}$$

The Markov transition matrix probability of sustaining a heat wave in the next time step is then estimated as the least squares minimal fit of itself to the power of the number of steps taken.

$$P(D_{HW_{Pm}} \Delta t_{min}) = (P_{hws_m})^{D_{HW_{Pm}} \Delta t_{min}} \tag{15}$$

$$P(D_{CS_{Pm}} \Delta t_{min}) = (P_{css_m})^{D_{CS_{Pm}} \Delta t_{min}} \tag{16}$$

Taking the logarithm of equations 15 and 16 enables using a linear least squares estimation for P_{hws_m} and P_{css_m} using paired sets $(D_{HW_{Pm}}, P_{HWD_m})$ and $(D_{CS_{Pm}}, P_{CSD_m})$ as the observations. The Markov transition matrix for each month, m, is then defined for normal conditions, heat waves, and cold snaps as:

$$M_m = \begin{bmatrix} 1 - P_{hw_m} - P_{cs_m} & P_{cs_m} & P_{hw_m} \\ 1 - P_{css_m} & P_{css_m} & 0 \\ 1 - P_{hws_m} & 0 & P_{hws_m} \end{bmatrix} \tag{17}$$

3.4 Severity of extreme events

The severity of extreme events requires estimation of both the peak delta in temperature above (heat wave) or below (cold snap) the average daily temperature T_{50} and total energy in $°C \cdot days$ above T_{50} as well. These values need to be normalized with respect to how long an extreme temperature wave lasts in order to assure that distributions compare between The following four sets for extreme events need to be calculated for each month m: 1) Maximum heat wave temperature:

$$\Delta T_{hw_m} = \left\{ \max_{i=S_{HW_m}}^{S_{HW_m}+D_{HW_m}-1} \left(T_{max90_i} - T_{50_k}\right) \right\} \tag{18}$$

2) Minimum cold snap temperture:

$$\Delta T_{cs_m} = \left\{ \min_{i=S_{CS_m}}^{S_{CS_m}+D_{CS_m}-1} \left(T_{min10_i} - T_{50_k}\right) \right\} \tag{19}$$

3) Heat wave energy:

$$\Delta E_{hw_m} = \left\{ \Delta t \sum_{i=S_{HW_m}}^{S_{HW_m}+D_{HW_m}-1} \left(T_{max90_i} - T_{50_k}\right) \right\} \tag{20}$$

4) Cold snap energy reduction:

$$\Delta E_{cs_m} = \left\{ \Delta t \sum_{i=S_{CS_m}}^{S_{CS_m}+D_{CS_m}-1} \left(T_{min10_i} - T_{50_k}\right) \right\} \tag{21}$$

Here the sets S_{HW_m} and D_{HW_m} have the same number of elements and the sets are formed by applying the functions seen with each element in the start days and durations sets. Also recall that k is the modulus of i with respect to the day of the year.

Performing statistical fits of the sets derived by equations 18 to 21 would lead to poor results because peak temperatures and energy both grow as heat waves increase in duration. It is therefore necessary to normalize both temperature and energy by a measure of the average temperature and energy of a heat wave given its duration. Once this normalization is complete, then the statistical scatter about the normalized results represents the actual variation per duration time of heat waves. For this analysis, it is assumed that energy added and peak temperature change both grow as linear functions of duration. These linear functions are determined by regressing normalized heat wave energy Δe and normalized temperature τ.

$$\Delta e_{w,m}(D) = \alpha_{E_{w,m}} \frac{D}{\max\left(D_{w,m}\right)} \tag{22}$$

$$\Delta \tau_{w_m} = \alpha_{T_w} \frac{D}{\max\left(D_{w,m}\right)} + \beta_{T_w} \tag{23}$$

Here, $w = \{hw, cs\}$ is an index over the extreme temperature wave type. The 6 coefficients α_{E_w}, α_{T_w}, and β_{T_w} are determined by linear regression of $D/\max\left(D_{w,m}\right)$ as the indepedent variable. For $\Delta e_{w,m}$, $\Delta E_{w_m}/\max\left(\Delta E_{w_m}\right)$ is the dependent variable, and for $\Delta\tau_{w,m}$, $\Delta T_{w_m}/\max\left(\Delta T_{w_m}\right)$ is the dependent variable. The energy per duration ε and peak temperature change per duration $\mathscr{T}$ normalizations. These linear regressions can be used to form two new sets that are suitable for

$$\varepsilon_{w,m}(D) = \frac{\Delta E_{w,m}}{\max\left(\Delta E_{w,m}\right) \Delta e_{w,m}(D)} \tag{24}$$

$$\mathscr{T}_{w,m}(D) = \frac{\Delta T_{w,m}}{\max\left(\Delta T_{w,m}\right) \Delta \tau_{w,m}(D)} \tag{25}$$

Here, the sets of duration normalized energies and temperatures for historic heat waves is represented by $\varepsilon_{w,m}$ and $\mathscr{T}_{w,m}$ whereas $\varepsilon_{w,m}(D)$ and $\mathscr{T}_{w,m}(D)$ refer to the functions used to evaluate future sampled heat waves. These sets are mapped to the interval $-1 ... 1$ using a linear transform and the sample mean μ and standard deviation σ taken.

$$I(X) = 2\frac{(X - \min(X))}{\max(X) - \min(X)} - 1 \tag{26}$$

Here X is any of the sets in equations 24 and 25 and I is the corresponding $-1 \ldots 1$ transformed data. The inverse transform to return to X is:

$$X(I) = \frac{(I+1)(\max(X) - \min(X))}{2} + \min(X) \tag{27}$$

The maximum and minimum values of each set (e.g. $\min(\varepsilon_{w,m})$, $\max(\varepsilon_{w,m})$ have to therefore be retained. So that reverse transformation can be accomplished. From now on, the symbols σ and μ will be used with a subscript with the entire variable symbol will be used to indicate what variable is being referred to. For example, $\sigma_{\varepsilon_{w,m}}$ is the standard deviation of $I(\varepsilon_{w,m})$. Finally the maximum and minimum values of $\varepsilon_{w,m}$ and $\mathscr{T}_{w,m}$ are taken as the boundaries a and b of a truncated Gaussian distribution.

Extreme event severity ΔE and ΔT are determined by sampling from these truncated Gaussian distributions for the appropriate month, transforming via X, and denormalizing. A truncated Gaussian distribution enables modeling of any skew that exists in the data such that very extreme values in the data are less probable but still possible. This is also consistent with the assumption that increases in severity are fully attributable to the climate change factors from IPCC in section 3.5 rather than from random variations.

3.5 Increases in frequency and severity

The IPCC Physical Basis report for policy makers [1] provides some of the first global estimates of how extreme events will shift and stretch probability distributions with climate change. Though the information does not comprehensively fit to the MEWS implementation, a best fit procedure is formulated here to produce changes in the truncated normal distributions fits that originate from $\Delta T^{max}_{hw_m}$, $\Delta T^{min}_{cs_m}$, ΔE_{hw_m}, and ΔE_{cs_m}. It is necessary to input a climate change temperture history, $\Delta T_G(y)$ for the scenario being modeled. Here y is the number of years since the start year. ΔT_G only needs resolution for each year into the future simulated. Histories that increase temperature beyond ΔT_G of $4°C$ are rejected by MEWS due to the limits on the information available. Figure SPM.6 of [1] contains numerical information that is summarized in Table 1. The confidence bounds represent the spread of results accross different climate change models averaged over the entire land surface of the earth. These bounds therefore do not necessarily reflect a specific location's spread but are a reasonable estimation globally. For now, this work demonstrates feasible variations and the average values are used here. This is defensible based on Table TS.5 of the IPCC technical summary [9] where all of North America is expected to increase in heat extremes with high confidence making it likely that the average or higher values in Table 1 are representative of North America. Also, the data available from NOAA daily summaries is seldom within the 1850-1900 baseline required for Table 1. The change factors are therefore taken as the current level minus the 1.0°C global warming levels for temperature. For the frequency multiplication factors the interpolated value is divided by the factor reflected with ΔT_G = 1.0°C.

The markov process cannot perfectly shift 10 year and 50 year events due to its single parameter nature of the probability of any extreme temperature event starting. Let N_{10} and N_{50} be the number of time steps in 10 and 50 years respectively. Recalling the P_{hw_m} and P_{cs_m} probabilities from equations 9 and 10. It

Table 1: IPCC heat wave intensity and frequency multiplying factors

Event	Global Warming Levels $\Delta T_G(y)$ (°C)	5% CI Increase in Intensity $\Delta T^{max05}_{ipcc_Y}$ (°C)	Avg Increase in Intensity $\Delta T^{max50}_{ipcc_Y}$ (°C)	95% Increase In Intensity $\Delta T^{max95}_{ipcc_Y}$ (°C)	5% CI Increase in Frequency $f^{ipcc}_{05_Y}$	Average Increase in Frequency $f^{ipcc}_{50_Y}$	95% CI Increase in Frequency $f^{ipcc}_{95_Y}$
Heat wave 10 year event	1.0	0.7	1.2	1.5	1.8	2.8	3.2
Heat wave 10 year event	1.5	1.3	1.9	2.3	2.8	4.1	4.7
Heat wave 10 year event	2.0	1.8	2.6	3.1	3.8	5.6	6
Heat wave 10 year event	4.0	4.3	5.1	5.8	8.3	9.4	9.6
Heat wave 50 year event	1.0	0.7	1.2	1.6	2.3	4.8	6.4
Heat wave 50 year event	1.5	1.3	2	2.4	4.3	8.6	10.7
Heat wave 50 year event	2.0	1.8	2.7	3.2	6.9	13.9	16.6
Heat wave 50 year event	4.0	4.4	5.3	6.0	27	39.2	41.4

Y = 10 or 50

is postulated that the ideal value for an adjusted value for P_{hw_m} is the weighted average of $f^{ipcc}_{50_{10}} P_{hw_m}$ and $f^{ipcc}_{50_{50}} P_{hw_m}$ where the weightings are the probability that a heat wave is the 10 year heat wave $P^{-1}_{hw_m} N^{-1}_{10}$ and the probability that a heat wave is the 50 year heat wave $P^{-1}_{hw_m} N^{-1}_{50}$. Simplification of the resulting equation produces:

$$P^{'}_{hw_m} = P_{hw_m} \frac{f^{ipcc}_{50_{50}} N_{10} + f^{ipcc}_{50_{10}} N_{50}}{N_{10} + N_{50}} \tag{28}$$

Using the weighted average gives more emphasis on increasing rates for the 10 year event while still providing some weight to the 50 year event. Though similar values can be calculated for P_{cs_m}, cold snaps are actually expected to decrease in severity [REFERENCE??] and the change in cold snaps is estimated as zero in this work.

Changes to peak temperature due to Table 1 can be exactly incorporated because the truncated Gaussian distributions can be shifted and stretched such that the new events take place accordingly. From Table 1 we know the shift of 10 and 50 year events. We assume that these events occur at probabilities equal to $1 - P^{-1}_{hw_m} N^{-1}_{10}$ and $1 - P^{-1}_{hw_m} N^{-1}_{50}$ for the truncated Gaussian cumulative distribution function F. We approximate the 10 year and 50 year $\mathscr{T}_{hw,m}$ by solving the 2 independent nonlinear equations for duration normalized temperature $\mathscr{T}^{Y}_{hw,m}$ where $Y = \{10, 50\}$:

$$F\left(I\left(\mathscr{T}^{Y}_{hw,m}\right), \mu_{\mathscr{T}_{hw,m}}, \sigma_{\mathscr{T}_{hw,m}}, -1, 1\right) = 1 - \frac{1}{N_Y P_{hw_m}} \tag{29}$$

These equations are suitably solved by bisection. An estimate of the expected value for duration of the 10 and 50 year events is needed so that the duration normalization can be carried out on the IPCC table values. Also, the IPCC table does not provide information concerning the probability of heat waves increasing in duration. Here, it is assumed that increases in temperture are proportionately matched by increases in duration as seen in Figure ..[YOU NEED TO MAKE THIS FIGURE]. The expected value for duration of 10 and 50 year heat waves can be estimated by setting the probability of a heat wave being the 10 or 50 year events equal to the probability of a heat wave being sustained P_{hws_m} to the power of the duration for the same amount of time and solving for the exponent:

$$D_Y = \frac{\ln\left(\frac{1}{P_{hw_m} N_Y}\right)}{\ln\left(P_{hws_m}\right)} \tag{30}$$

The distribution is then shifted by $\Delta\mu_{\mathscr{T}_{hw,m}}$ and $\Delta\sigma_{\mathscr{T}_{hw,m}}$ that satisfy the following dependent equations for $Y = 10, 50$:

$$F\left(I\left(\mathscr{T}_{hw,m}^{Y}+\frac{\Delta T_{ipcc_Y}^{max50}}{\max\left(\Delta T_{hw,m}\right)\Delta\tau(D_Y)}\right),\mu_{\mathscr{T}_{hw,m}}+\Delta\mu_{\mathscr{T}_{hw,m}},\sigma_{\mathscr{T}_{hw,m}}+\Delta\sigma_{\mathscr{T}_{hw,m}},-1+S_{-1},1+S_1\right)=1-\frac{1}{N_Y P_{hw_m}} \tag{31}$$

$$S_{-1}=\Delta\mu_{\mathscr{T}_{hw,m}}-\frac{1+\mu_{\mathscr{T}_{hw,m}}}{\sigma_{\mathscr{T}_{hw,m}}}\Delta\sigma_{\mathscr{T}_{hw,m}} \tag{32}$$

$$S_{1}=\Delta\mu_{\mathscr{T}_{hw,m}}+\frac{1-\mu_{\mathscr{T}_{hw,m}}}{\sigma_{\mathscr{T}_{hw,m}}}\Delta\sigma_{\mathscr{T}_{hw,m}} \tag{33}$$

The expected value for duration, D_Y is increased by the same percentage that the increase in temperature changes:

$$S_{D_Y}=\frac{\Delta T_{ipcc_Y}^{max50}+\max\left(\Delta T_{hw,m}\right)\Delta\tau(D_Y)\mathscr{T}_{hw,m}^{Y}}{\max\left(\Delta T_{hw,m}\right)\Delta\tau(D_Y)\mathscr{T}_{hw,m}^{Y}} \tag{34}$$

$$D_Y^{'}=S_{D_Y}D_Y \tag{35}$$

The probability of sustaining a heat wave is then shifted to a higher value but requires combining two constraints for a single variable the same as $P_{hw_m}^{'}$ in equation 28. The same weighted average with weights equal to the probability of a heat wave being a 10 or 50 year event are therefore used.

$$P_{hws_m}^{'}=\frac{N_{10}P_{hwsm}^{S_{D_{50}}}+N_{50}P_{hwsm}^{S_{D_{10}}}}{N_{10}+N_{50}} \tag{36}$$

This new sustained probability implies a scaling S_{D_m} of the original duration data $D_{HW_{Pm}}$ (equation 11) such that the probability of the longer heat waves remains equal to the constant probabilities P_{HWD_m} (equation 13). This scaling factor is can be solved algebraically as:

$$S_{D_m}=\frac{\ln\left(P_{hws_m}\right)}{\ln\left(P_{hws_m}^{'}\right)} \tag{37}$$

Finally the, the heat wave is assumed to scale in proportion to temperature times duration (S_{D_m}). Once again assuming a weighted average for the 10 and 50 year temperature changes, the resulting energy therefore scales by the following expression:

$$S_{E_m}=S_{D_m}\sum_{Y=10,50}\frac{\Delta T_{ipcc_Y}^{max50}+\max\left(\Delta T_{hw,m}\right)\Delta\tau(D_Y)\mathscr{T}_{hw,m}^{Y}}{2\max\left(\Delta T_{hw,m}\right)\Delta\tau(D_Y)\mathscr{T}_{hw,m}^{Y}} \tag{38}$$

We then assume that this scaling occurs on the expected value of the energy for heat waves but that standard deviation in energy spreads out proportionately to change in standard deviation for maximum temperature:

$$\Delta\mu_{\varepsilon_{hw,m}}=I\left(S_{E_m}X\left(\mu_{\varepsilon_{hw,m}}\right)\right)-\mu_{\varepsilon_{hw,m}} \tag{39}$$

$$\Delta\sigma_{\varepsilon_{hw,m}}=I\left(\frac{X\left(\sigma_{\mathscr{T}_{hw,m}}+\Delta\sigma_{\mathscr{T}_{hw,m}}\right)}{X\left(\sigma_{\mathscr{T}_{hw,m}}\right)}X\left(\sigma_{\varepsilon_{hw,m}}\right)\right)-\sigma_{\varepsilon_{hw,m}} \tag{40}$$

Here transformation of the terms must return to physical energy via the reverse X transformation in equation 27 so that multiplicative scaling is valid. The amplified term is then transformed back to the

Table 2: MEWS input variables

Variable	Description	Derivation
NOAA and IPCC data:		
$\Delta T_G(y)$	Expected trajectory for global warming	From IPCC information [1, 9] choose a region and scenario (SSP51-1.9 to SSP5-8.5) and estimate average temperature anomaly for the location of interest for each year into the future. The length of $\Delta T_G(y)$ determines how many years to project into the future
T_{max}, T_{min}	NOAA daily summary data [13] maximum and minimum temperatures	Find a weather station ID close to the location of interest. MEWS downloads the data automatically
T_{10}, T_{50}, and T_{90}	NOAA climate normals data for 1991-2020 [14]	Find the climate normals for the same station ID close to the location of interest. MEWS downloads the data automatically
Historical or Typical Weather Data	Energy Plus or DOE2 weather file	[15]
IPCC extreme event data	Information portraying the change in frequency and temperature anomaly due to 10 year and 50 year heat wave events. Cold snap data is not given and the warming trend indicates cold snaps will decrease in severity	For now, from IPCC policy summary [1] Figure SPM.6 recorded in Table 1
Other inputs:		
Random seed	Optional input only needed if MEWS output needs to be replicable	
Start year	First year to simulate	
Number of realizations	Number of independent sets of weather files to generate such that a stochastic ensemble of files is generated	
Time zone name	A valid time zone identifier per python library pytz	
M_m and ΔM_m Markov transition matrix inputs calculated for each month m and year y:		
P_{hw_m}	Markov matrix probability in month, m, for any time step where a heat wave or cold snap are not happening, that a heat wave will occur	Use start of heat wave event data from 50 or more years of NOAA daily-summaries and 1 year of NOAA climate normals built into HW of equation 7. Final value calculated using equation 9
P_{hws_m}	Markov matrix probability in month, m, for any time step in a heat wave to continue as a heat wave	Use duration of heat wave event data from same data as P_{hw_m} and determine P_{hws_m} using the least squares problem posed in equation 15
P_{cs_m}	Same as P_{hw_m} but for cold snaps	Same as P_{hw_m} but using the cold snap event set CS of equation 8 instead of the HW. Final value calculated using equation 10
P_{css_m}	Same as P_{hws_m} but for cold snaps	Same as P_{hws_m} but via equation 16
$P^{'}_{hw_m}$	Adjusted Markov matrix probability P_{hw_m} taking into account IPCC changes in frequency for 10 year and 50 year heat wave events	Interpolate current year's global warming anomaly $\Delta T_G(y)$ using data in Table 1 to determine $f^{ipcc}_{50_{50}}$ and $f^{ipcc}_{50_{10}}$, then use equation 28
$P^{'}_{hws_m}$	Adjusted Markov matrix probability P_{hws_m} taking into account IPCC changes in frequency for 10 year and 50 year heat wave events	Estimate 10 and 50 year event durations based on equation 30 adjust this duration using equation 35 extrapolating changes in temperture from Table 1 and then calculate the final value via equation 36
$P^{'}_{cs_m}$	Same as $P^{'}_{hw_m}$ but for cold snaps	Global warming makes this term best to estimate as equal to zero until more information is available
$P^{'}_{css_m}$	Same as $P^{'}_{hws_m}$ but for cold snaps	Global warming makes this term best to estimate as equal to zero until more information is available. Future information is likely to make this value less than P_{css_m}
Distributions for T_{max} and E of extreme events:		
$\mu_{\mathscr{T}_{hw,m}}$, $\sigma_{\mathscr{T}_{hw,m}}$, $\min\left(\mathscr{T}_{hw,m}\right)$, $\max\left(\mathscr{T}_{hw,m}\right)$, $\max\left(\Delta T_{hw,m}\right)$, $\alpha_{T_{hw,m}}$, $\beta_{T_{hw,m}}$	Heat wave mean, standard deviation, and bounds of $\mathscr{T}_{hw,m}$ truncated Gaussian distribution	From heat wave event set HW derive Gaussian parameters per equation 18 set bounds as the maximum and minimum of the set $\left(\mathscr{T}_{hw,m}\right)$
$\mu_{\mathscr{T}_{cs,m}}$, $\sigma_{\mathscr{T}_{cs,m}}$, $\min\left(\mathscr{T}_{cs,m}\right)$, $\max\left(\mathscr{T}_{cs,m}\right)$, $\max\left(\Delta T_{cs,m}\right)$, $\alpha_{T_{cs,m}}$, $\beta_{T_{cs,m}}$	Cold snap mean, standard deviation, and bounds of $\mathscr{T}_{cs,m}$ truncated Gaussian distribution	From cold snap event set CS derive Gaussian parameters per equation 19 set bounds as the maximum and minimum of the set $\left(\mathscr{T}_{cs,m}\right)$
$\mu_{\varepsilon_{hw,m}}$, $\sigma_{\varepsilon_{hw,m}}$, $\min\left(\varepsilon_{hw,m}\right)$, $\max\left(\varepsilon_{hw,m}\right)$, $\max\left(\Delta E_{hw,m}\right)$, $\alpha_{E_{hw,m}}$	Heat wave mean, standard deviation, and bounds of energy $\varepsilon_{hw,m}$ truncated Gaussian distribution	From heat wave event set HW derive Gaussian parameters per equation 20 set bounds as the maximum and minimum of the set $\left(\varepsilon_{hw,m}\right)$
$\mu_{\varepsilon_{cs,m}}$, $\sigma_{epsilon_{cs,m}}$, $\min\left(\varepsilon_{cs,m}\right)$, $\max\left(\varepsilon_{cs,m}\right)$, $\max\left(\Delta E_{cs,m}\right)$, $\alpha_{E_{cs,m}}$	Cold snap mean, standard deviation, and bounds of energy $\varepsilon_{cs,m}$ truncated Gaussian distribution	From cold snap event set CS derive Gaussian parameters per equation 21 set bounds as the maximum and minimum of the set $\left(\varepsilon_{cs,m}\right)$
$\Delta\mu_{\mathscr{T}_{hw,m}}$, $\Delta\sigma_{\mathscr{T}_{hw,m}}$, $\Delta\min\left(\mathscr{T}_{hw,m}\right)$, $\Delta\max\left(\mathscr{T}_{hw,m}\right)$	Shift and stretching due to IPCC data of heat wave mean, standard deviation, and bounds of $\mathscr{T}_{hw,m}$ truncated Gaussian distribution	Solve 2 sets of 2 nonlinear equations 29 and 31
$\Delta\mu_{\mathscr{T}_{cs,m}}$, $\Delta\sigma_{\mathscr{T}_{cs,m}}$, $\Delta\min\left(\mathscr{T}_{cs,m}\right)$, $\Delta\max\left(\mathscr{T}_{cs,m}\right)$	Shift and stretching due to IPCC data of cold snap mean, standard deviation, and bounds of $\mathscr{T}_{cs,m}$ truncated Gaussian distribution	For the present, IPCC data available values of 0 are assigned to all of these.
$\Delta\mu_{\varepsilon_{hw,m}}$, $\Delta\sigma_{\varepsilon_{hw,m}}$, $\Delta\min\left(\varepsilon_{hw,m}\right)$, $\Delta\max\left(\varepsilon_{hw,m}\right)$	Shift and stretching due to IPCC data of heat wave mean, standard deviation, and bounds of energy $\varepsilon_{hw,m}$ truncated Gaussian distribution	Assume shifting and stretching of temperature times expected value of change in duration is proportionate to energy shift and stretch. Calculate with equations 38 to 40
$\Delta\mu_{\varepsilon_{cs,m}}$, $\Delta\sigma_{(\varepsilon_{cs,m}}$, $\Delta\min\left(\varepsilon_{cs,m}\right)$, $\Delta\max\left(\varepsilon_{cs,m}\right)$	Shift and stretching due to IPCC data of cold snap mean, standard deviation, and bounds of energy $\varepsilon_{cs,m}$ truncated Gaussian distribution	For the present, IPCC data available values of 0 are assigned to all of these.

original -1 ..1 interval via I. These terms are then applied to change the bounds of the $-1...1$ interval distributions similar to equations 31 to 33. All of the terms needed to reconstruct the change in the markov transition matrix :

$$\Delta M_m = \begin{bmatrix} P_{hw_m} + P_{cs_m} - P^{'}_{hw_m} - P^{'}_{cs_m} & P^{'}_{cs_m} - P_{cs_m} & P^{'}_{hw_m} - P_{hw_m} \\ P_{css_m} - P^{'}_{css_m} & P^{'}_{css_m} - P_{css_m} & 0 \\ P_{hws_m} - P^{'}_{hws_m} & 0 & P^{'}_{hws_m} - P_{hws_m} \end{bmatrix} \tag{41}$$

As required, the rows of this matrix add to zero. Also recall that, for the present analysis that no change to cold snaps is assumed (i.e $P^{'}_{cs_m} = P_{cs_m}$ and $P^{'}_{css_m} = P_{css_m}$). For each year of data, this matrix is reformulated based on the global warming trajectory $\Delta T_G(y)$. The Markov process has a transition matrix for each year and month m of $M_m + \Delta M_m$. Table 2 shows the 40 different inputs to MEWS that have been derived in this discussion. These, combined with the input data from IPCC, NOAA, and either DOE-2 or Energy Plus, provide the basis for creating a stochastic ensemble of future heat wave scenarios with gradual climate change trend within MEWS.

4 Results

5 Conclusion

Continuation of this research requires demonstration that the statistical approaches used do not introduce significantly different statistical distributions than dynamic downscaling approaches using regional climate models. Specifically, resilience metric distributions for this study need to be vetted against a dynamic downscaling approach to assess whether there are systematic differences.

GAPS:

1) This research needs to extend the methods to allow a Monte-carlo type sampling of the relationship between Duration, Temperature, and Energy instead of using just expected values.

2) The heat waves need to be added either on weather that falls within the climate normals. Otherwise, it is possible that heat waves will be added to heat waves amplifying the effects seen.

3) A non-sinusoidal function eventually needs to be adapted that is developed by machine learning that accurately captures the heat up characteristics of the day reflecting accurate peak temperature during day and night. This function probably needs to include humidity affects.

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