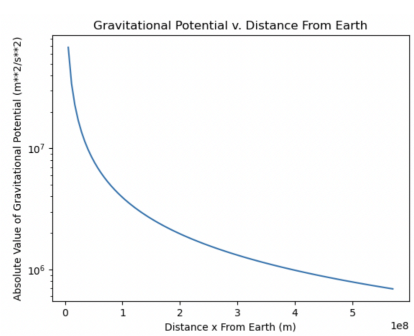
I. <u>Introduction</u>

As one of the engineers working on the Apollo Program to bring humankind to the moon, I have been investigating the forces of space relevant to our mission and the capabilities of the Saturn V Rocket that will get us there. This mission is ambitious but very feasible and if successful, may very well be humanity's greatest achievement yet. However, the Apollo Program's success relies on us having a thorough understanding of the gravitational forces governing the Earth and Moon system, which this report will provide. The precise calculation of these forces will allow us to develop and optimize spacecraft and navigational performance in order to deliver our astronauts safely to the Moon and back. Additionally, investigating Saturn V's capabilities will confirm its effectiveness and address any lingering questions. We will examine its burn rate, velocity, and expected altitude based on theoretical calculations. This report will provide evidence to Congress in favor of the Apollo Program's viability.

II. <u>Gravitational potential of the Earth and Moon system</u>

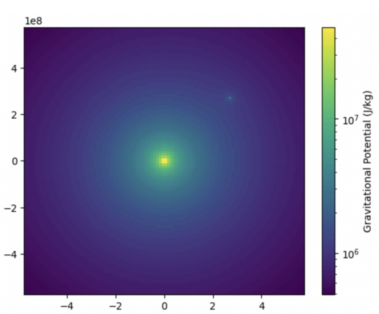
Gravitational potential can be defined as the energy per unit mass an object has due to Earth's gravity at a given distance. It influences the amount of energy that is needed for spacecraft to escape Earth's gravitational pull and reach the moon. In order to calculate and visualize this potential, Apollo's engineering team has coded a function for gravitational potential and drawn up a graph of distance from Earth versus gravitational potential.

First, we defined key constants including the universal gravitational constant, Earth's mass, the Moon's mass, and other values relevant to the mission. We created the function "grav_potential" which executes the formula $V = -GM/r$ where V is gravitational potential, G is the gravitational constant, M is the mass of the attracting body (Earth in our case), and r is the distance between the object and the Earth's center. To account for when the radius is zero, we replace undefined values with "NaN" (not a number).

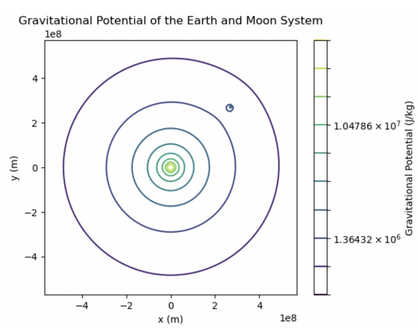


Next, the code on the left plots the relationship between gravitational potential and distance from Earth. We can see that as the distance from Earth increases, the gravitational potential decreases, meaning that Earth's pull on spacecraft lessens with distance. This data is vital to our engineers' work as they determine the exact amount of energy Saturn V must produce to escape Earth's gravitational pull during and after liftoff.

We also crafted a 2D color map of the gravitational potential of the Earth and Moon system. Here we can also view the gravitational effects of the moon which, of course, has a gravitational potential of its own. We use two functions, one for each celestial body. "Egp" calculates Earth's potential and "mgp" calculates the moon's at any point (x, y) in space. We use the same formula as seen in II for both calculations.



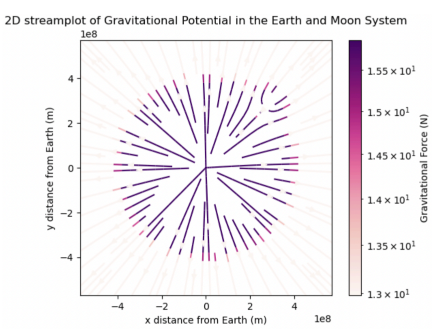
Next, the code creates a grid of points in 2D space to represent the different positions between the Earth and Moon. It calculates the gravitational potential at each of the points for the Earth and Moon. The final gravitational potential is a sum of both the Earth and Moon's effects. We represent the different gravitational potential values with different colors and apply logarithmic normalization to better visualize the large range of values by enhancing details and improving contrast so that we may still view the Moon's smaller effects. The engineering team also added a color bar to explain what each color means. As you can see, bright yellow represents the highest levels of potential and the dark blue-purple represents the lowest levels. We set the aspect ratio to be equal to ensure the x and y axes are scaled equally, preventing distortion.



This plot provides an accurate 2D visualization of the gravitational effects of both the Earth and Moon. We can view where the spacecraft will shift from being primarily affected by the gravity of Earth to the gravity of the Moon. Furthermore, this will help us plan the best trajectory for our mission by taking these features into account.

Finally, we also have provided you a visualization of the gravitational potential of the Earth and Moon system by using contour lines to make the potential levels clearer. Each contour represents a region where the gravitational potential has the same value. The spacing between the lines increases for higher potential values thanks to our decision to use logarithmic scaling which is once again useful due to the range of gravitational potential values. You can view the colorbar to the right of the plot to aid with interpreting the contour values.

III. <u>Gravitational force of the Earth and Moon system</u>



The engineering team also calculated the gravitational force between the Moon and the Earth. Gravitational force can be described as the pull an object feels from gravity whereas gravitational potential can be equated to the effort needed to escape that pull. Our code created a function for the gravitational force where F = (G*M1*m2)r^2 in both the x and y directions. We found the distance between two points with the Pythagorean Theorem and prevented zero division errors again. The total gravitational force was split into x and y components and then returned by our function so we could visualize the force in 2D.

We generated a streamplot as a visual representation of the gravitational forces between the Moon and Earth with both showing direction and intensity. The colorbar indicates where the force is strongest and weakest. The streamplot was created by making a grid in 2D space for some distance from Earth and then computing the total gravitational force at each point. Once again, we summed the forces from both the Earth and Moon. Then, we plotted the streamplot. This code also provides essential information about the forces of space and is crucial to Apollo.

IV. <u>Projected performance of the Saturn V Rocket during Stage 1</u>

The Saturn V rocket is the launch vehicle to be used for the Apollo missions and we have given it three stages. Stage 1 (S-IC) includes the initial liftoff and the journey through Earth's lower atmosphere. The rocket will be powered by five F-1 engines. Our code models the performance of the engines' burn time or how long they will fire, the velocity change they cause, and then the altitude after burnout (end of S-IC).

To calculate burn time, we created a function which divides the total amount of mass lost (or fuel burned) by the rate of fuel consumption. The rocket starts with a mass that includes its body and fuel, but as the engines burn, the mass of the fuel is lost at a constant rate which we call the mass flow rate. This function determines the amount of time the F-1 engines will be able to fire before the fuel runs out. Our engineers will use this to time the stage separation and ignition of Stage 2.

Following that, we made a function to find the velocity change or speed our rocket will gain as a result of the S-IC's thrust. The Rocket Equation states that a rocket's velocity increases when it burns fuel and expels exhaust gases. These gases propel the rocket through space and conserve momentum of the system. The faster the gases' velocity, the more efficiently our rocket gains speed. However, the gravity of nearby celestial bodies works against the rocket's acceleration which is why it is important that we calculated the gravitational effects of the Earth and Moon previously. This helps find out if our rocket will reach the required velocity for the next stage.

Lastly, we calculated the rocket's final altitude as Stage 1 separates. We calculate it by finding the velocity over time and using an integral to sum the small altitude changes at each moment. An integral is necessary because velocity is not constant as the rocket accelerates through the air. We use the burn time previously calculated to obtain the bounds for our integral.

V. Discussion and future work

Comparing our answers to NASA's test results, SI-C was recorded to burn for 160 seconds and reach an altitude of 70km. Our functions found a burn time of 158 seconds and our final altitude was 5e+06 meters. Our burn time was about accurate, but our altitude was an overestimate. Some of this error can be explained by the fact that we made several approximations to simplify our calculations. For one, we ignored air resistance which in reality will reduce Saturn V's velocity change and therefore the final altitude. Also, we assumed an entirely constant gravitational field when it actually decreases slightly with altitude. In order to improve accuracy in the future, we should use a different equation which incorporates drag and accounts for the rocket's shape and air density as well. We could also refine our gravitational model by using a different equation to calculate the gravitational force which considers the exact altitude of the rocket from Earth's center. However, our calculations for final altitude are still too far off to confirm that S-IC performs as expected. While our model does not yet match NASA's testing, with a few more days of work, I feel confident that it will. Unfortunately, we did not have time to correct the error in this code before submitting it to you, Dr. Kranz. I am certain that for someone as incredible at coding as you the error will be easy to spot. Returning back to this report's point, the Apollo Program will expand human knowledge and secure the United States' position as the leading power in space exploration. We have put much effort into our mission plan and will not rest until every error is eliminated. Landing on the Moon would inspire more scientific advancements and benefit the country and future generations for years to come.