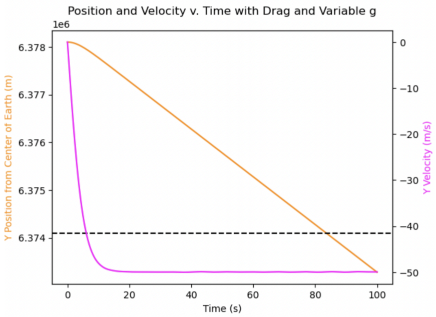
I. Introduction

This lab report is a detailed investigation into the physics of vertical motion through our planet Earth using one of the deepest vertical mine shafts. The calculations are for an idealized mineshaft which can be applied to many depths and environments. Initially, this report aims to calculate basics. However, as the research goes on, the analysis expands and simulates realistic and extreme geophysical conditions. With the help of Python's numerical integration and differential equation modeling capabilities, the investigation runs through six key stages. The aim of this report is to help our mining company build a comprehensive understanding of motion in mine shafts through considering multiple different realities from ideal physics to including many real life complexities.

II. Calculation of Fall Time (Including Drag and Variable g)

First was the calculation of fall time, or how long it takes for a 1kg test mass to reach the bottom of a 4000m mineshaft. Initially, choosing to ignore some forces, calculations for a highly idealized shaft were computed. This simplification allowed the use of kinematics equations, namely the equation x = 1/2at^2 + v0t + x0 which found a fall time of 28.6 seconds. However, in reality, there are other forces such as drag and variable gravity which impact fall time. While drag is common knowledge, variable gravity may need some more explanation. Earth's true gravitational field decreases linearly with depth due to variations in Earth's surface density. This means that the initial value for gravity, 9.81 m/s^2, is a simplification of real world conditions which now must be included in the report. The given second order differential equation is reduced to a first order by setting velocity equal to dy/dt. This produced an equation for the first derivative of velocity in terms of g, α, v, and Γ. α was assumed to be 0. Then using the function solve_ivp from the module scipy (scientific Python) to integrate differential equations numerically, the time was calculated. The solutions are extracted and plotted so the graph titled Figure 1 displays position and velocity versus time in an ideal system. Velocity is linear with a negative slope in accordance with gravity's constant negative acceleration. The position graph looks like half a parabola, aligning with theory, and demonstrating the descent part of typical projectile motion. At its maximum height, velocity is zero, and then it begins to accelerate downwards causing position to decrease and velocity to increase while acceleration remains constant and negative. We can compare two different types of fall time calculations, one using solve_ivp and one analytical. The difference between times is miniscule (10^-13 level) with both being approximately 28.6 seconds. The error may originate from approximations in the solver or the resolution of our t_eval grid which features 1000 points over 100 seconds and limits the precision of event detection.

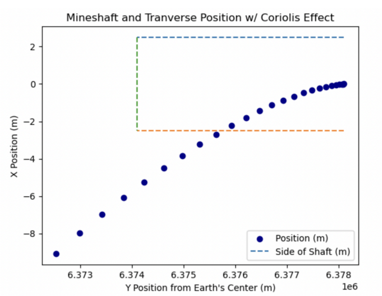
Now, variable gravity is added to the calculations by inputting the equation g(r) = g0 * (r/Re) and a new function is defined that accounts for variable gravity instead of constant. We use solve_ivp again and it makes another plot which includes variable gravity. However, the graph, Figure 2, appears to be nearly identical to the previous Figure 1. This makes sense because the mineshaft's depth of 4000m is small in comparison to Earth's radius of 6378.1km so variable gravity might not have a large effect. Still, it does



accelerate slower near the bottom which means that our velocity graph's curve becomes slightly less steep as time passes. When drag is included, much more changes. When the drag and gravity forces are equal, the object reaches something we call terminal velocity which is constant and can be defined as the maximum velocity attainable as an object moves through a medium: air, in this case. Alpha, the drag coefficient, is found to be 0.004 s/m^2. Calculations with solve_ivp are conducted again and there is a significant difference between this time and the previous ones with fall time being 83.5 seconds. With the distance the object must fall and the increasing velocity as it does, the force of drag becomes great and has a large impact on the fall time, slowing it down until it reaches and remains at terminal velocity. The position decreases exponentially for small t, but it becomes more linear until about 15 seconds when it becomes fully linear. The velocity behaves similarly, starting as decreasing and negative until it reaches around -50 m/s when it shifts to a constant value. This point is where the drag and gravity become equal, bringing terminal velocity into play.

III. Feasibility of Depth Measurement Approach (Including Coriolis Forces)

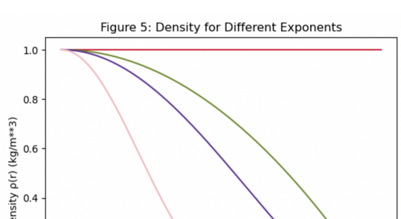
The effect of the Coriolis Force creates an apparent deflection of objects' paths in a rotating system. In this case, Earth is rotating and this force has an influence on the transverse (x-direction) motion of the test mass. To account for it, our calculations must become multi-dimensional (x and y). The z-direction is excluded because it is unaffected. The differential equations are updated by adding a Coriolis acceleration term. Now, when the x and y positions of the falling object are plotted, it can be observed that as the y position from the center of the Earth increases and the object falls farther, the x position strays farther from the initial position with the data points getting farther apart (see the graph on the left). A test calculation with a 5 m wide mineshaft and test mass dropped from the center finds that the mass hits the wall of the shaft at __m and __ seconds which is before it reaches the ground. When drag is included, the object hits the mineshaft wall much earlier than without drag, but both still hit the wall before the ground. Given this, proceeding with the depth measurement technique seems like a poor idea. The test mass consistently makes contact with the wall of the shaft before reaching the bottom because of the Coriolis force. This problem could lead to inaccurate measurements if this technique is continued to be used so it is recommended that the company switch to another method.



IV. Calculation of Crossing Times for Homogenous and Non-Homogenous Earth

Next, the team decided to investigate several hypothetical mineshafts to gain a deeper understanding of the physics behind motion inside them. First, an infinitely deep mine was considered. Variable gravity was still included in the calculations and it was determined that the position of an object would follow simple harmonic motion, oscillating from one side of the Earth to the other forever. Using solve_ivp, it was found that it would take the object 2,532.6 seconds to reach the other side of Earth while falling in an infinite mineshaft. Calculations were repeated for the crossing time of the object and the velocity at that time which were found to be 1,266.5 seconds and 7,910.8 m/s downwards respectively. This is the point where the object passes through the center of the Earth. It was also necessary to calculate orbital speed which we did using the basic orbital velocity equation and it came out to be 7,905.3 m/s which is about 5 m/s off from the crossing time we calculated previously. This difference is miniscule compared to their respective values which are in the thousands. In addition, another relationship was established as the orbital period is about four times that length of the crossing time. Four times the crossing time is 5,065.9 seconds and the orbital period is 5,069.4 seconds, meaning the two are less than four seconds apart. However, these calculations are for a homogenous Earth when in truth Earth is non-homogenous so additional calculations must be carried out.

Four density values were compared based on four n values from zero to nine to provide a broad profile of the ranging densities on Earth. These values were used to calculate the forces produced by each and were found to have a large range of over 30N. It can be concluded from



this work that increasing the density also increases the force. For the two most extreme cases considered, the force produced by an n of nine was over 40N while that produced by an n of zero was less than 10N. It is evident that the shape of the curves change drastically as n increases with the curvature increasing rapidly from the initial linear graph produced by an n of zero. Regarding density concentration, if mass is more concentrated towards the center, gravity inside will increase quicker as the radius decreases. In other words, a denser core will decrease the crossing time. For example, the moon has an average density of about 3.3 g/cm^3 while Earth's is 5.5g/cm^3 so the moon is less dense than Earth. In that way, it would take an object 1,625.0 seconds to reach the center of the moon whereas it would take that same object 1,266.5 seconds to reach the center of the Earth.

V. Discussion and Future Work

Though this report has given the company much insight on mineshaft physics, it has also used many approximations that could influence the results. Physically, while we accounted for some changes in Earth's density, Earth was always treated as a perfect sphere. The equatorial bulge and flattening near the poles was ignored which affects the gravity distribution on the planet. Even in our models of the density distribution, everything was simplified. There are much more detailed Earth density models like PREM that could produce much more accurate and realistic results. Thirdly, there was no consideration of Earth's internal pressure. It was assumed that the test mass could survive these conditions without being deformed or damaged by the heat in some way. Changes to the mass' shape as a result of these high pressures would influence the length of its fall time and the effect of drag on it. The drag force that was used also neglected to utilize altitude-dependent air density, instead opting for constant air density to make the model simpler. While this was beneficial for this report's purposes, if the company were to conduct a more thorough and accurate investigation, we would need to consider these factors. Additionally, there were computational approximations made such as the rounding error introduced by using numerical integration methods and the fact that our code implemented discrete time steps which have limited our results' precision. Though our work was done in multiple dimensions, it would have been best to also include the z-direction in many of the calculations to increase realisticness. Given this, in the future this research should be improved by using realistic Earth density models, modeling Earth as an ellipsoid rather than sphere, and accounting for effects from pressure and heat on the motion of the falling test mass. Going even further, it would be helpful to take material properties of the planet into consideration such as the matter states of Earth's layers, different rock types, and the lithosphere. Computationally, it would be best to implement more accurate integration methods and to improve our step size sensitivity for better resolution.

Signed,
Madeline Springer