SSA 7

Improved Numerical Optimization for Truss Structure Design

Csaba Benedek

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Goal

- Create a multi-parameter numerical optimization algorithm that accounts for a diverse range of multiple indicators.

Conclusion

- Moving points along the upper chord of the crane towards the rear (left side of the canvas) may help better distribute compressive forces and reduce the overall internal loadings in the structure.
- *The Croissant* design may show promise in efficient, low-material, and high-stability truss design.

Problems

- The main failure point in some of our designs was buckling.
- The second thing that piqued my interest was the diagonal extension part.

Follow up Steps

Work Division

- Multi-parameter approach is constructed (parameters selected and objective functions created).
- Rewriting code for multi-parameter objective function.
- Creating a graphical user interface for the optimizer algorithm.
- Writing the background information in the SSA.
- Optimizing the latest crane design (tuning weights and simulation attempts).

Time Division

• Selecting new parameters and expressing them mathematically	**2 hours**
• Programming the multi-parameter objective function.	**3 hours**
• Creating a graphical user interface for the optimizer.	**2 hour**
• Writing the background information in the SSA	**2 hour**
• Application to the latest crane design (Crane Design 3)	**2 hour**
• Finishing writing and revision	**1 hour**

Total of 12 hours

Goal 1 - Multi-parameter approach

Synopsis of previous work

In SSA 4, a multi-point numerical optimization algorithm was developed in Python using a Finite Element Method solver to obtain values for the internal forces in a truss structure, along with an iterative optimization process. Within this optimization process, an 'objective' function was created, which the optimizer attempted to minimize by varying the position of nodes in the truss. This objective function optimizes for the buckling distribution factor Γ, which is the following:

$$\Gamma = \gamma + 2s_\mu$$

where γ is the weighted average of the utilization ratio μ_i and s_μ is the weighted standard deviation of the utilization ratios.

However, by focusing solely on reducing this single metric, the optimizer can inadvertently increase the total compressive loadings within the whole truss structure. This behavior arises from the nature of the buckling distribution factor itself, which is a function of the γ, s_μ, and μ_i. The standard deviation, s_μ, quantifies the spread of these ratios among the compressive members. Therefore, an algorithm seeking to reduce the buckling distribution factor Γ may find a simple mathematical solution by minimizing the spread of utilization ratios. This can be achieved by distributing the compressive forces more uniformly, which may increase the loads on some members to bring them closer to other higher-loaded members. While this uniform loading effectively reduces s_μ and, consequently, the buckling distribution factor Γ, it can lead to a design with higher overall compressive forces, and thus a less stable and less efficient structure overall. Therefore, this SSA aims to update the objective function to account for a diverse range of indices, thereby alleviating the issues from the previous single-indicator approach.

Multi-parameter optimization framework

To address the limitations of the previous single-parameter approach, a multi-parameter optimization framework shall be developed. Therefore, a new objective function will be created which simultaneously evaluates and balances multiple, sometimes competing, design goals: safety, efficiency, and load distribution. The multi-parameter approach is built upon a weighted objective function in the general form:

$$\Omega = \sum \omega_i O_i$$

where Ω is the total design score, ω_i is a user-adjustable weighting factor, and O_i is the normalized value for a single-parameter objective function for any component parameter i. This approach not only optimizes the truss design for various factors but offers flexibility during the optimization process via adjustable weighting factors that the user may adjust according to the importance of each parameter.

To create a multi-parameter objective function, a set of single-parameter core objective functions is first selected. The parameters are selected such that they encapsulate various key design considerations. The following sections will discuss each parameter, why it was selected, and how it is calculated and processed quantitatively.

Objective 1: Buckling distribution factor

The buckling distribution factor remains incorporated into the multi-parameter approach as it serves to optimize the distribution of compressive loadings within the truss structure. Rather than having single elements under significant compressive loadings, the optimization of this parameter shall shift the position of nodes to relieve loadings from the elements under the greatest compression and spread it among elements under smaller loadings. The function for calculating this remains unchanged and will be written as follows:

$$O_d = \gamma + 2s_\mu \tag{1}$$

The optimizer shall thus aim to minimize the value of the buckling distribution O_d in order to have a more efficient spread of loadings within the truss structure at a global level of analysis.

Objective 2: Material consumption

For an efficient truss structure, the material consumption is also important to consider. A lighter and less complex truss structure is generally more cost-effective but also more efficient in its use of members. Therefore, minimizing this parameter can further help improve any designs to make better use of the resources available. To calculate this, the following expression may be used:

$$O_m = \sum_{i=1}^{n} A_i L_i \tag{2}$$

where the objective O_m is the sum of the product of the cross-sectional area A_i and element length L_i for any given truss member i in a structure of n total members. This parameter also introduces a dilemma of trade-offs, as lighter cross-sections which utilize less material (such as B and C), will have significantly lower buckling resistance than the heavier and larger elements like the L-shaped profile (A).

Objective 3: Buckling Failure

This parameter objective function is not a continuous function that must be minimized, but rather a hard limit that acts as a penalty to prevent any member from experiencing compressive loadings that would cause buckling–and therefore catastrophic failure. As the buckling utilization ratio μ_i is defined as $\mu_i = \frac{T_i}{P_{c,i}}$ which is the compressive force T_i divided by the critical buckling load $P_{c,i}$ for any element i. Thus, when $\mu_i \geq 1$, an element will undergo buckling. In order to prevent this risk, a slight margin is included in a piecewise function that either returns 0 for any $\mu_i < 0.95$ and returns an exponentially increasing function for any $\mu_i \geq 0.95$ until $\mu_i = 1$. This may be written as:

$$O_b = 0 \text{ for } \mu_i < 0.95 \tag{3}$$

$$O_b = c \cdot e^{\mu_i - 0.95} \text{ for } \mu \geq 0.95 \tag{4}$$

where O_b is the buckling failure objective and c is a large positive constant based on the weight of this parameter. Therefore, regardless of the value for any other objective function, the value of O_b will increase exponentially when elements are at risk of buckling, which should effectively 'steer' the optimizer away from positioning nodes that create individual members susceptible to buckling.

Objective 4: Compression uniformity

This objective parameter has been designed to directly counter the negative behavior from the previous single-parameter approach. A low-weighted standard deviation s_μ may mislead the optimizer to believe that the overall structure has become more stable. However, the coefficient of variation ν_μ normalizes the spread of utilization ratios by their weighted average γ. As such, the following expression may be used:

$$O_u = \nu_\mu = \gamma s_\mu \tag{5}$$

where O_u expresses the objective for compression uniformity. A truss design with a low average utilization ratio but a few highly stressed members will have a high value for s_μ, thereby still increasing this value. However, when the average utilization ratio increases, the value of ν_μ will also increase with it, thereby eliminating the previous issue where only the spread of compressive loadings is taken into account. Therefore, a low coefficient of variation indicates both a low spread of compressive loading but also a low average utilization ratio, thereby minimizing the compressive loadings in the structure alongside minimizing the spread.

Objective 5: Average magnitude of internal forces

Additionally, the objective parameter for the average magnitude of internal forces is included as the optimizer should aim not only to minimize compressive loadings within the truss structure but to also minimize the overall internal loading. Therefore, the magnitude of all internal forces, whether tensile or compressive, may be used to obtain an average magnitude that expresses how heavily loaded the truss structure is. This may be expressed as:

$$O_a = \frac{1}{n} \sum_{i=1}^{k} |T_i| \tag{6}$$

where O_a expresses the average magnitude of forces T_i for the sum of members i in a set of n total members. This expression therefore characterizes the overall loading of the truss from the perspective of tensile forces as well. Thus, this objective function provides a more diverse approach for analysis and optimization.

Multi-parameter summary

Objective	Description	Mathematical Formulation	Dependencies
O_d	Buckling distribution	$O_d = \gamma + 2s_\mu$	T_i, $L_{0,i}$, λ, cross-section
O_m	Material consumption	$O_m = \sum_{i=1}^{n} A_i L_i$	$L_{0,i}$, cross-section type
O_b	Buckling failure penalty	$O_b = 0$ for $\mu_i < 0.95$ $O_b = c \cdot e^{\mu_i - 0.95}$ for $\mu \geq 0.95$	T_i, $L_{0,i}$, λ, cross-section
O_u	Compressive load uniformity	$O_u = \nu_\mu = \gamma s_\mu$	T_i, $L_{0,i}$
O_a	Average loading magnitude	$O_a = \frac{1}{n} \sum_{i=1}^{k} \lvert T_i \rvert$	T_i

Table 1: Table of new multi-parameter objectives.

Table 1 may be used as a quick reference for how the various new objectives are obtained and what variables they depend on. These parameters may then be used to effectively optimize a given crane design.

As such, the final expression for the total design score Ω can be expressed as:

$$\Omega = \omega_d O_d + \omega_m O_m + \omega_b O_b + \omega_u O_u + \omega_a O_a \tag{7}$$

where each single-parameter objective component O is multiplied by a user-alterable weight ω when running the algorithm.

Implementation, weighting heuristics, and analysis

Implementation

The proposed multi-parameter framework may be seamlessly integrated into the current multipoint numerical algorithm. The user shall be able to define the adjustable weighting factors–ω_d, ω_m, ω_u, and ω_a–as input parameters via a graphical user interface (GUI) or else. The buckling failure penalty is a fixed, high-value component derived from the total material consumption, designed to override all other objectives if such a failure condition is approached. The optimizer will then seek to minimize the total design score Ω by adjusting the position of nodes within the design. It is important to note that it does not vary the cross-sections, the trusses (members) between nodes, or the attachment methods between members. Therefore, it gives insights into the optimal positioning of nodes in an existing design but cannot completely alter the fundamental layout of truss elements.

Weighting heuristics

It is important to note that the effectiveness of the multi-parameter optimization algorithm depends heavily on the selection of weighting factors by the user. The effect of each design factor represented by each parameter is complicated and non-linear; thus, iterative tuning of weighting parameters with user oversight and verification shall be used to guide the algorithm toward ideal solutions. Additionally, it is up to the user to select the attachment methods and cross-sections used for various components, as well as to propose any alterations to the fundamental truss connections between nodes in the design. The goal of the optimizer is to assist the user in the design process and find optimal designs that fit the user's selected optimization parameters and trade-offs as given by the selected weighting factors (i.e., it is a design tool, not an all-in-one solution).

The following table describes the behavior of certain weighting factors, which may serve as a guide for using the optimizer:

Scenario Name	ω_d	ω_m	ω_u	ω_a	**Rationale**	**Expected Outcome**
Safety-First	High	Low	High	High	Prioritizes structural integrity and robust load distribution.	A highly robust but potentially heavy and material-intensive design.
Material-Optimized	Low	High	Low	Low	Focuses on minimizing cost and material usage.	A lightweight, cost-effective design that operates closer to safety margins.
Balanced	Med	Med	Med	Med	A compromise between safety, efficiency, and robustness.	A well-rounded design that balances all key performance indicators.
Uniformity-Focused	Med	Low	High	Med	Aims to achieve even compressive load distribution across all members.	A stable design with evenly shared forces, though possibly less material-efficient.
Lightweight-Safety Hybrid	High	Med	Low	Med	Attempts to combine reduced material consumption with sufficient safety.	A lighter design than "Safety-First," with better safety margins than "Material-Optimized."
Performance-Driven	Low	Med	Med	High	Prioritizes reducing overall loading magnitudes, accepting closer operation to safety limits.	An efficient design with low internal forces, but potentially greater compressive spread.

Table 2: Scenario weightings for different optimization strategies.

As such, Table 2 may be used to tune the weighting parameters in the optimizer algorithm to achieve the best results.

Goal 2 - Application to crane design

The numerical optimization algorithm was run using Crane Design 3 as depicted below:

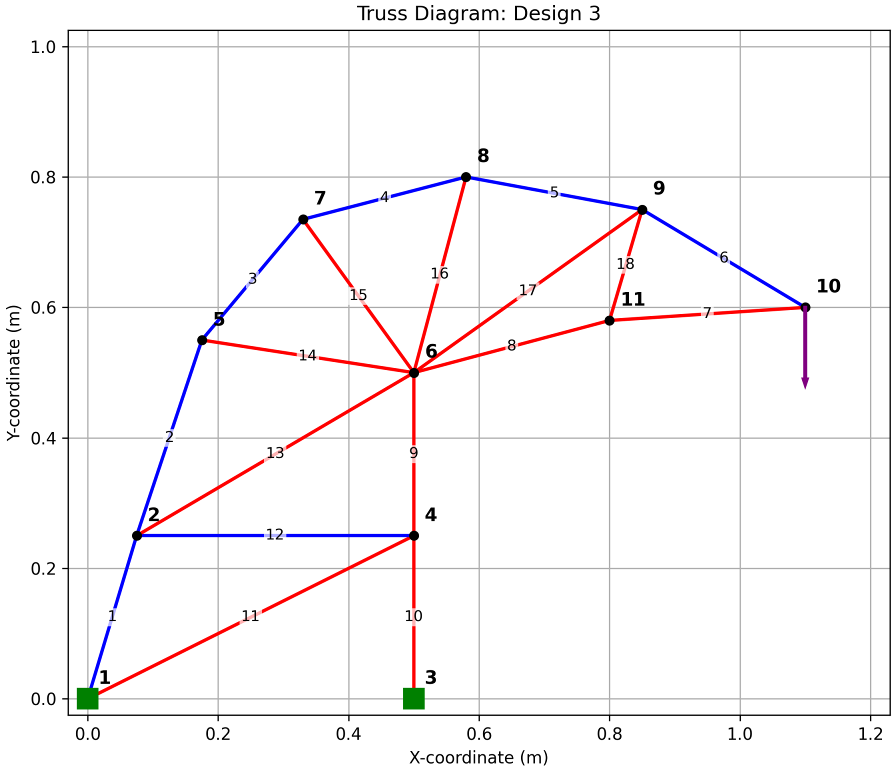

Figure 1: Original Crane Design 3

The numerical optimization algorithm was run with the following parameter weighting:

- Buckling Distribution Weight: 15.00
- Material Consumption Weight: 23.34
- Compressive Load Uniformity Weight: 27.87
- Average Loading Magnitude Weight: 42.56

These weights were briefly tuned, and the algorithm was run several times to determine whether they result in chaotic (nonsensical) results or legitimate and plausible optimizations. As this requires several attempts and some 'random' guessing, more time would be required to find the most optimal weights, and additional tuning to the initial input design may help coerce the algorithm to produce better optimization results.

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The points selected for optimization were Nodes 5, 6, and 7. However, the set of points targeted may be expanded later, and the weights may be further tuned. After running the numerical optimization algorithm, the following was obtained: As can be seen in Figure 2, this optimized crane

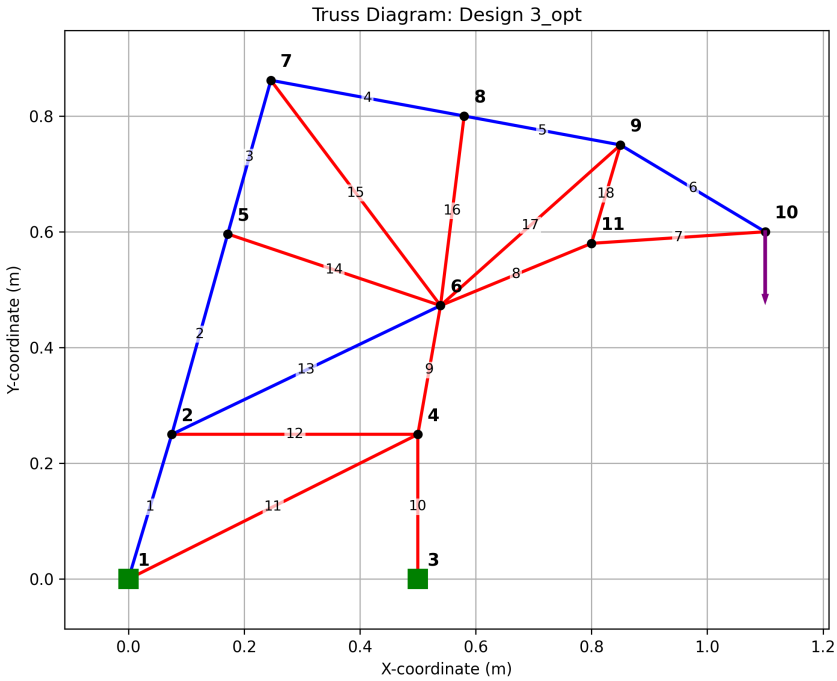

Figure 2: Optimized Crane Design 3

design has points shifting toward the left of the canvas. Therefore, the proposed *The Croissant* design during the SSA meetings may prove to show promise for better distributing compressive forces while also minimizing the overall magnitude of forces throughout the structure.

As a follow-up, *The Croissant* crane design may be passed into the numerical optimizer to determine whether it is already in the most optimal state or whether the algorithm reinforces the characteristics that it encompasses.

Additionally, the current numerical optimizer algorithm has been created for 2D, but further research shall be conducted to determine whether it may be easily expanded into 3D. This would enable multivariate optimization for the 3D designs, however, the complexity of the algorithm and the runtime for each optimization would increase from a factor of n^2 as it currently stands to n^3 due to an increase in the input matrices from nodes in $\mathbb{R}^2$ to $\mathbb{R}^3$.