① Solve the following recurrence relations

a) $x(n) = x(n-1) + 5 \quad n \geq +1 \quad x(1) = 0$

$x(n) = x(n-1) + 5 \rightarrow ①$

$x(n-1) = x(n-1-1) + 5$

$= x(n-2) + 5 \rightarrow ②$

$x(n-2) = x(n-3) + 5 — ③$

Sub 3 in ②

$x(n-1) = x(n-3) + 10 \rightarrow ④$

sub ④ in ①

$x(n) = x(n-3) + 15 \rightarrow ⑤$

for some k

$x(n) = x(n-k) + 5k \rightarrow ⑥$

$n-k = 1$

$n-1 = k$

Eq ⑥ $\Rightarrow x(n) = x(1) + 5(n-1)$

$x(n) = 0 + 5n - 5$

$O(n)$

b) $x(n) = 3x(n-1) \quad x(1) = 4$

$x(n) = 3x(n-1) \rightarrow ①$

$x(n-1) = 3x(n-2) — ②$

$x(n-2) = 3x(n-3) - ③$

Sub ③ in ②

$x(n-1) = 3(3x(n-3))$

$= 9x(n-3)$ — ④

sub 4 in ①

$x(n) = 3(9x(n-3))$

$= 27x(n-3)$ — 5

for some k

$x(n) = 3^k x(n-k)$ — ⑥

$n-k=1$

$n-1=k$

eq 6 in $x(n) = 3^{n-1} x(1)$

$= 3^{n-1} 4$

$= 3^n 3^{-1} 4$

$O(3^n) = (O^n)$

$x(n) = x(n/3)+1$ $x(1)=1$

$x(n) = x(n/3)+1$ — ①

$x(n/3) = x(n/3^2)+1$ — ②

$x(n/3^3) = x(n/3^3)+1$ — ③

sub 3 in ②

$x(n/3) = x(n/3^3)+2$ — 4

sub 4 in 1

$x(n) = x(n/3^3)+3$ — 5

some k

$x(n) = x(n/3^k)+k$

$= n/3^k = 1$ $n = 3^k$

$k = \log 3^k$

Eq ⑤

$$x(n) = x(1) + \log 3^n$$
$$= 1 + \log 3^n$$
$$O(\log 3^n)$$

→ Evaluate recurrence completely

1) $T(n) = T(n/2) + 1$

$T(n) = T(n/2) + 1 \quad = n = 2^k \quad n = 2^k$ for all $k \geq 0$

sub $n = 2^k$

$T(2^k) = t(2^k/2) + 1 = (2^{k-1}) + 1 \quad — ①$

$T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right) + 1 = T(2^{k-2}) + 1 \rightarrow ②$

$T(2^{k-2}) = T\left(\frac{2^{k-2}}{2}\right) + 1 = T(2^{k-3}) + 1 \rightarrow 3$

$n = k - 2$

$n = 2^k \Rightarrow k = \log 2^n$

$t(2^k) = T(2^{k-1}) + 1 = t(2^{k-2}) + 1 + 1 \ldots$

since

$2^0 = 1 \quad T(2^0) = T(1)$

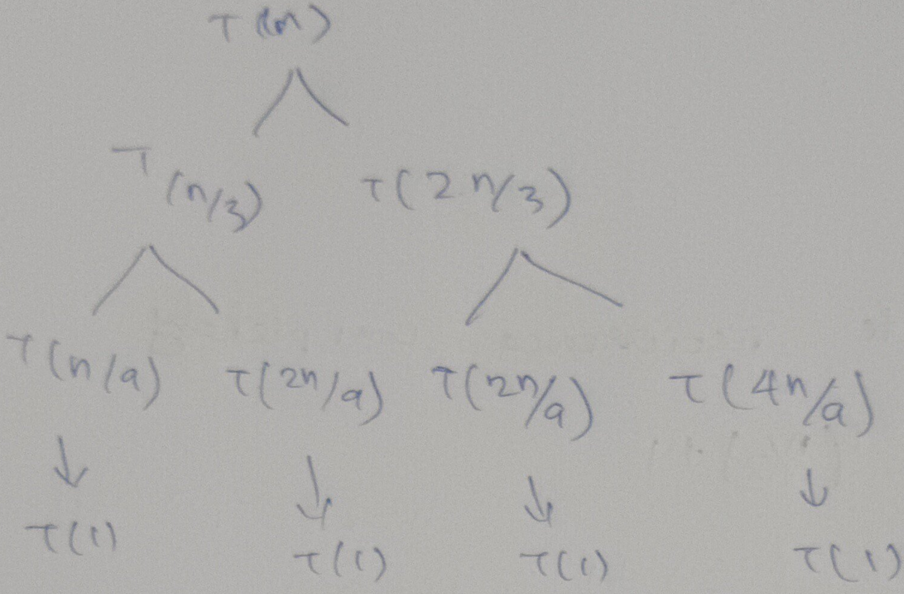
$T(2^k) = 1 + k$

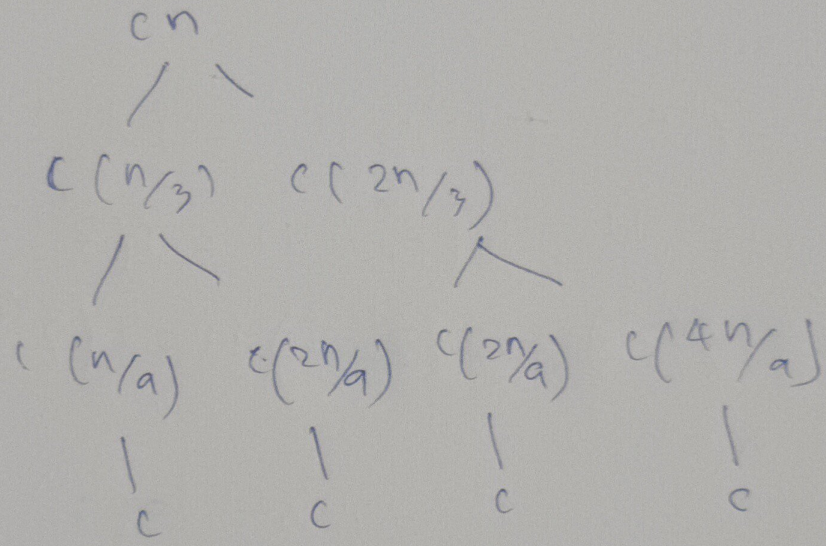
$t(n) = 1 + \log_3 n$

Time complexity $O(\log^n)$

2) $T(n) = T(n/3) + T(2n/3) + cn \ldots$

$T(n) = T(n/3) + T(2n/3)$





consider following algorithm Min1(A[0]...(n-1])

if n = 1 return A[0].

Else temp = Min1(A[0.... n-2])

If temp <= A[n-1] return temp

Else

return [A[n-1]

What does this Algorithm compute

This Algorithm compute minimum element in array A of size n

if $i < n$, $A[i]$ is smaller than all element then, $A[i]$ is smaller than $A[i]$, $j = i+i$ to $n-1$, then it returns $A[i]$. It also returns the left most minimal element

comparison occurs during recursion and solve it ?

$$T(n) = T(n-1) + 1$$

$$T(1) = 0$$

$$T(n) = T(1) + (n-1) + 1$$

$$= 0 + (n-1)$$

$$= n-1$$

time complexity $O(n)$

Analyses order of growth.

$F(n) = 2n^2 + 5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation.

$F(n) = 2n^2+5$

$F(n) \geq c\, g(n)$

$c \cdot g(n) = 7n$

$n=1$

$F(1) = 2(1)^2+5$

$g(1) = 7$

$n=2$

$F(2) = 2(2^2)+5$

$= 8+5 = 13$

$g(2) = 7\times 2 = 14$

$n=1,\ 7=7$

$n=2,\ 13 = 14$

$n=3,\ 23 = 21$

$n \geq 3,\ F(n) \geq g(n) \cdot c$

$n=3$

$F(3) = 2(3^2)+5$

$= 2(9)+5$

$= 18+5 = 23$

$g(3) = 7\times 3 = 21$

$F(n) = \Omega(g(n))$