Выполнил Ширшов Александр Юрьевич. Группа БФЗ171.Потратитл времени очень много, изучил курсы по LaTeX, так как некоторые задачи были реально трудоемкими и сложными для осознания, то желаемая оценка -15.

1. Задача 1

a)

$s_2 = 0$
$e_2 = 1110$
$Mantissa_2 = (1.f)_2 = 1.1010\ 0000\ 0000\ 0000\ 0000\ 000$

b)

$e_{10} = 14$
$p_{10} = e_{10} - 127 = -113$

c)

$Mantissa_{10} = 1 + 2^{-1} \cdot 1 + 2^{-2} \cdot 0 + 2^{-3} \cdot 1 + 0 = 1.625$

d)

$A_{10} = (-1)^0 \cdot 1.f \cdot 2^p \approx 1.56 \cdot 10^{-34}$

2. Задача 2

a)

```
N=1100
under=1.
over=1.
for i in range (N):
       under=under / 2.
       over=over*2.
       print(i,'under=' , under, 'over=' , over)
```

Граница *underflow* и *overflow* для чисел с плавающей запятой с двойной точностью:
$under \approx 5 \cdot 10^{-324}$
$over \approx 9 \cdot 10^{307}$

В Python 3.0 нет чисел с плавающей запятой одинарной точности

b)

```
import math
under = -1
over = 1
N = 1000000000
for i in range (N):
       under = abs(under)*1000000
       over = over*1000000
       print ("Loop: ", i, " ", -math.log10(abs(under)), " ", math.log10
          (
       over))
```

Для целых чисел:
under стремится к бесконечности
over стремится к бесконечности

3. Задача 3

a)

```
eps = 1.0
one_Plus_eps = 2.0
while (one_Plus_eps != 1):
       eps = eps /2
       one_Plus_eps = 1.0 + eps
       print ( " eps = " , eps , " , one + eps = " , one_Plus_eps )
```

Для типа *float* $\varepsilon \approx 1 \cdot 10^{-16}$

a)

```
eps = complex(1.0, 0.0)
one_Plus_eps = (2.0, 0.0)
while (one_Plus_eps != complex(1.0, 0.0)):
       eps = eps /2
       one_Plus_eps = 1.0 + eps
       print ( " eps = " , eps , " , one + eps = " , one_Plus_eps )
```

Для типа *complex* $\varepsilon \approx 1 \cdot 10^{-16}$

4. Задача 4

1)

```
import math
def my_sin(x):
	term = x
	sum = x
	eps = 10**(-8)
	n = 1
	while (abs(term) > abs(sum * eps)):
		n += 1
		term = -term * x**2 /( (2*n-1) * (2*n-2) )
		sum = sum + term
	return sum
while (True):
	try:
		x = float(input())
	except:
		print ("Exit")
		break
	if (math.sin(x) != 0):
		print (abs ((my_sin(x) - math.sin(x))/(math.sin(x))))
	else:
		print ("sin(x) = 0, my_sin(x) = ", my_sin(x))
```

2)

$\mathbf{x}$	$\varepsilon_{\mathbf{sin}(x)}$
0 (0)	0.0/0.0
3.14159 (π)	$4 \cdot 10^{-11}$
3.141592654 (π)	$8 \cdot 10^{-7}$
6.28318 (2π)	$2 \cdot 10^{-10}$
6.283185307 (2π)	$6 \cdot 10^{-6}$
1.57 ($\pi/2$)	$6 \cdot 10^{-12}$
4.71 ($3\pi/2$)	$6 \cdot 10^{-12}$
31.4159 (10π)	$7 \cdot 10^{0}$
31.415926 (10π)	$1 \cdot 10^{3}$
314.159 (100π)	$6 \cdot 10^{121}$
314.1592654 ($3\pi/2$)	$2 \cdot 10^{126}$
32.99 ($10\pi + \pi/2$)	$4 \cdot 10^{-4}$
32.98672286 ($10\pi + \pi/2$)	$8 \cdot 10^{-4}$
315.73 ($100\pi + \pi/2$)	$5 \cdot 10^{118}$
315.7300617 ($100\pi + \pi/2$)	$2 \cdot 10^{119}$

4)

Убедились, что при малых x алгоритм сходится к правильному ответу.

13)

```
import math
def my_sin(x):
	while (abs(x) > math.pi):
		x -= 2*math.pi
	term = x
	sum = x
	eps = 10**(-8)
	n = 1
	while (abs(term) > abs(sum * eps)):
		n += 1
		term = -term * x**2 /( (2*n-1) * (2*n-2) )
		sum = sum + term
	return sum
while (True):
	try:
		x = float(input())
	except:
		print ("Exit")
		break
	if (math.sin(x) != 0):
		print (abs ((my_sin(x) - math.sin(x))/(math.sin(x))),
```

```
                my_sin(x +
                    2*math.pi) - my_sin(x))
        else:
               print ("sin(x) = 0, my_sin(x) = ", my_sin(x))
```

Точность увеличилась на диапазоне $|x| \in (\pi, +\infty)$ и стала такой же, как на диапазоне $|x| \in [0, \pi]$

13)

```
import math
def my_sin(x):
        term = x
        sum = x
        eps = 10**(-8)
        n = 1
        while (abs(term) > abs(sum * eps)):
               n += 1
               term = -term * x**2 /( (2*n-1) * (2*n-2) )
               sum = sum + term
        return sum
x = 0
while (True):
        x+= 0.25
        if (math.sin(x) != 0):
               print ("x = ", x, " ", abs ((my_sin(x) - math.sin(x))/(
                  math.sin(x))), my_sin(x + 2*math.pi) - my_sin(x))
        else:
               print ("sin(x) = 0, my_sin(x) = ", my_sin(x))
```

Алгоритм резко теряет точность где-то при $x \in [30 - 34]$. Из-за потери точности соотвественно алгоритм перестает сходится.

15)

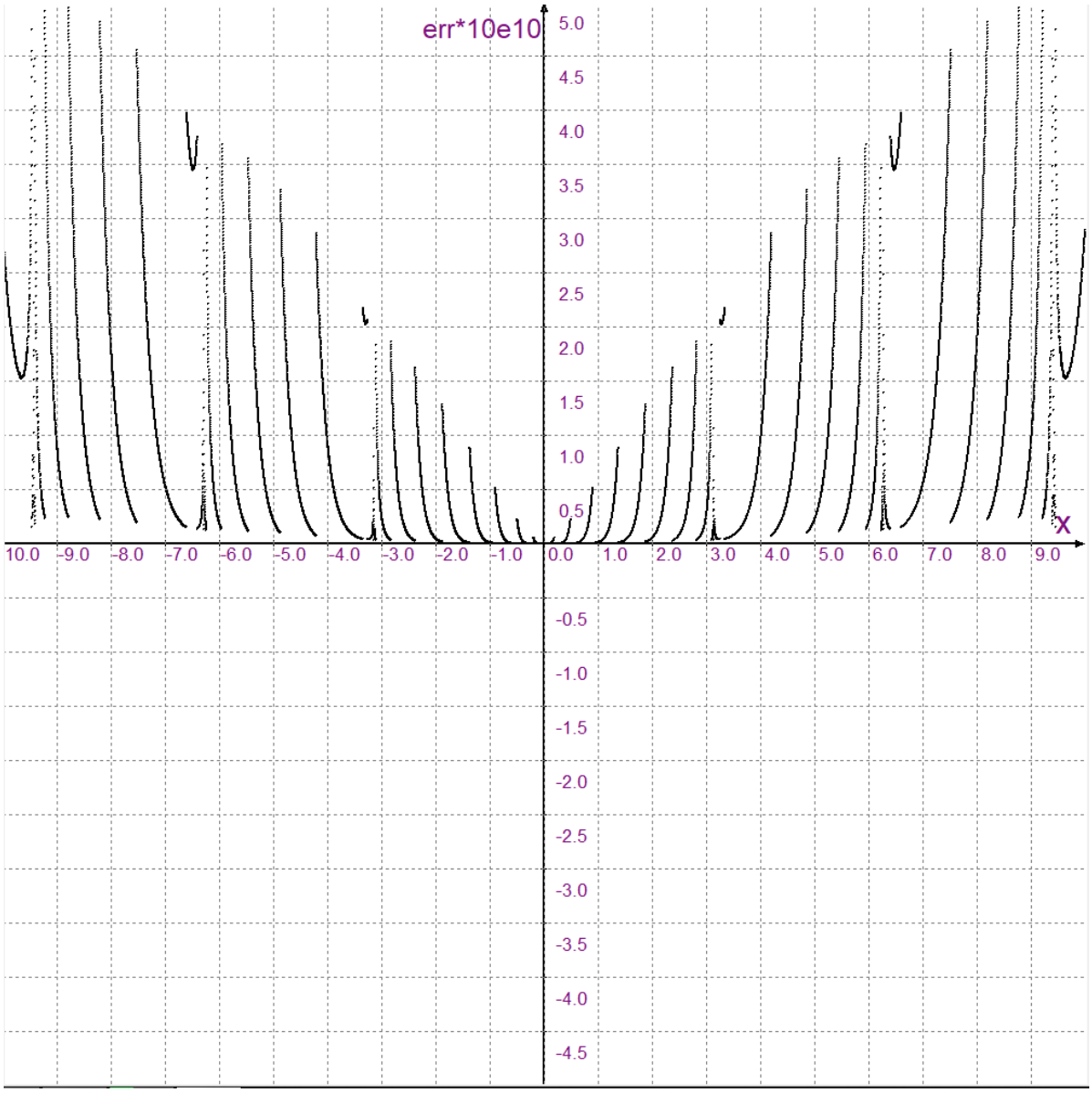
Построим график зависмости $\varepsilon_{\sin(x)}(x)$.

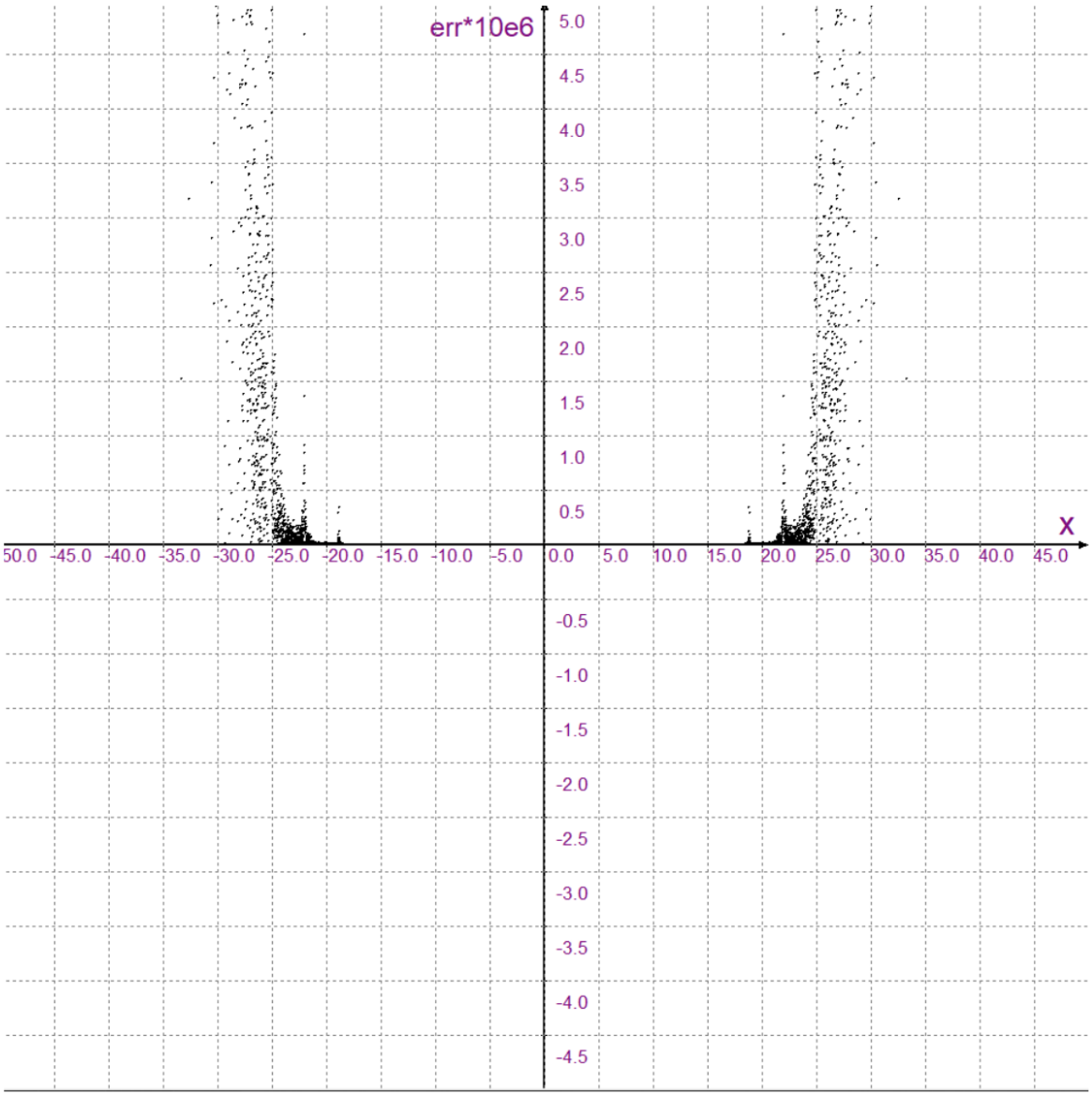
```
from math import *
from tkinter import *
def my_sin(x):
        term = x
        sum = x
        eps = 10**(-8)
```

```
        n = 1
        while (abs(term) > abs(sum * eps)):
                n += 1
                term = -term * x**2 /( (2*n-1) * (2*n-2) )
                sum = sum + term
        return sum


def err(x):
        if (sin(x) != 0):
                return abs((my_sin(x) - sin(x))/sin(x))
        else:
                return 0


root = Tk()
x_0 = 100
y_0 = 10**(12)
x_sc = 10**(10)
canv = Canvas(root, width = 1000, height = 1000, bg = "white")
canv.create_line(500, 1000, 500, 0, width = 2, arrow = LAST)
canv.create_line(0, 500, 1000, 500, width = 2, arrow = LAST)
canv.create_text(980, -20 +500, font = ("Purisa", 18), text = "x", fill
= "purple")
canv.create_text(-57 + 500, 25, font = ("Purisa", 15), text = "err*" +
str(x_sc), fill = "purple")
First_x = -500
for i in range(16000):
        if (i % 800 == 0):
                k = First_x + (1 / 16) * i
                canv.create_line(k + 500, -3 + 500, k + 500, 3 + 500,
                   width=0.5,ill='black')
                canv.create_line(k + 500, 0, k + 500, 1000, width=0.1,
                   fill='grey', dash=(1, 1))
                canv.create_text(k + 515, 10 + 500, font = ("Purisa", 10),
                    text=
                       str(k/x_0), fill="purple")
                if (k != 0):
                       canv.create_line(-3 + 500, k + 500, 3 + 500, k +
                          500, width
                              =0.5, fill='black')
                       canv.create_line(0, k + 500, 1000, k + 500, width
                          =0.1, fill='grey', dash=(1, 1))
                       canv.create_text(25 + 500, k + 500 + 20, font = ("
```

```
                        Purisa", 10), text=str(-k/y_0*x_sc), fill="
                        purple")
        try:
                x = First_x + (1 / 16) * i
                y = -err(x/x_0)*y_0 + 499
                x += 499
                canv.create_oval(x, y, x + 1, y + 1, fill='black')
        except:
                First_x = -500


canv.pack()
root.mainloop()
```

Ниже представлено два масштаба:


err*10e10
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
X
10.0
-9.0
-8.0
-7.0
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
-4.5



Наблюдаем резкое уменьшение точности

5. Задача 5

a)

$ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} = \frac{b(-1 \pm sign(b) \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)n!^2 r^n} x^n)}{2a} = \frac{2c}{b(-1 \mp sign(b) \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)n!^2 r^n} x^n)}$$

Третий и четвёртый способ вычисления даёт выигрыш в точности при вычисле-

нии x_1, а первый и второй - при вычислении x_2 при $a = b = 1$, $c = 10^{-n}, n > 0$
$a = 1$
$b = 1$
$c = 0.0001$

x_1	x_2
-0.00010001000200049459	-0.9998999899979994 (1)
-0.00010001000200049459	-0.9998999899979994 (2)
-0.00010001000200050015	-0.9998999899980551 (3)
-0.00010001000200050015	-0.9998999899980551 (4)

```
from math import *
def solve(a, b, c):
       if (b**2 >= 4*a*c) and (a != 0) and (c != 0):
              print ((-b + sqrt(b**2-4*a*c))/(2*a), (-b - sqrt(b**2-4*a*
                 c))/(2*a))
              print ((2*c)/(-b - sqrt(b**2-4*a*c)), (2*c)/((-b + sqrt(b
                 **2-4*a*c))))
              sum = 0
              x = -4*a*c/b**2
              f = 1
              term = 1
              sum = 1
              eps = 10 ** (-40)
              n = 0
              while (abs(term) > abs(sum * eps)):
                     term = term * x * (1-2*n)/(2*(n+1))
                     sum = sum + term
                     n += 1
              print (b*(-1+b/abs(b)*sum)/(2*a), b*(-1-b/abs(b)*sum)/(2*a
                 ))
              print ((2*c)/(b*(-1-b/abs(b)*sum)), (2*c)/(b*(-1+b/abs(b)*
                 sum)))
       else:
              if (a == 0) and ( b != 0):
                     print ((-c/b), (-c/b))
              else:
                     if (a == 0) and ( b == 0):
                            if (c == 0):
                                   print ("x in (-inf, +inf)")
                            else:
                                   if ( c != 0):
                                          print("NaN, NaN")
                                   else:
```

```
                                                print("Error: D < 0")
while (True):
        try:
                a = float(input("a = "))
                b = float(input("b = "))
                c = float(input("c = "))
        except:
                print ("Exit")
                break
        solve(a, b, c)
```

6. Задача 5

a)

```
from math import *
def s_1(N):
        n = 1
        sum = -0.5
        term = -0.5
        while (n < 2*N):
                term = term * (-1)*((n+1)**2)/(n*(n+2))
                sum += term
                n += 1
        return sum
def s_2(N):
        n = 1
        sum = 0
        while (n < N):
                sum += -(2*n-1)/(2*n) + (2*n)/(2*n+1)
                n += 1
        return sum
def s_3(N):
        n = 1
        sum = 1/6
        term = 1/6
        while (n < N):
                term = term * (2*n*(2*n+1))/((2*n+2)*(2*n+3))
                sum += term
                n += 1
        return sum
```

b)

Я внес все точки в файл и построил график в GnuPlot

```
from math import *
from tkinter import *

def s_1(N):
	n = 1
	sum = -0.5
	term = -0.5
	while (n < 2*N):
		term = term * (-1)*((n+1)**2)/(n*(n+2))
		sum += term
		n += 1
	return sum

def s_2(N):
	n = 1
	sum = 0
	while (n < N):
		sum += -(2*n-1)/(2*n) + (2*n)/(2*n+1)
		n += 1
	return sum

def s_3(N):
	n = 1
	sum = 1/6
	term = 1/6
	while (n < N):
		term = term * (2*n*(2*n+1))/((2*n+2)*(2*n+3))
		sum += term
		n += 1
	return sum

def err(x):
	try:
		return abs((s_1(x)-s_3(x))/s_3(x))
	except:
		return 1
root = Tk()
x_0 = 10**(0)
y_0 = 10**(2)
```

```
x_sc = 10**(0)
canv = Canvas(root, width=1000, height=1000, bg="white")
canv.create_line(500, 1000, 500, 0, width=2, arrow=LAST)
canv.create_line(0, 500, 1000, 500, width=2, arrow=LAST)
canv.create_text(980, -20 + 500, font=("Purisa", 18), text="x", fill="
   purple")
canv.create_text(-57 + 500, 25, font = ("Purisa", 15), text="err*"+ str(
x_sc), fill="purple")
First_x = -500;
my_file = open("lg.dat", "w")
for i in range(18000):
       if (i % 1800 == 0):
              k = First_x + (1 / 18) * i
              canv.create_line(k + 500, -3 + 500, k + 500, 3 + 500,
                 width=0.5,fill='black')
              canv.create_line(k + 500, 0, k + 500, 1000, width=0.1,
                 fill='grey', dash=(1, 1))
              canv.create_text(k + 515, 10 + 500, font = ("Purisa", 10),
                  text=str(k/x_0), fill="purple")
              if (k != 0):
                     canv.create_line(-3 + 500, k + 500, 3 + 500, k +
                        500, width=0.5, fill='black')
                     canv.create_line(0, k + 500, 1000, k + 500, width
                        =0.1, fill='grey', dash=(1, 1))
                     canv.create_text(25 + 500, k + 500 + 20, font = ("
                        Purisa",10), text=str(-k/y_0*x_sc), fill="
                        purple")
       try:
              x = First_x + (1 / 18) * i
              y = -err(x/x_0)*y_0 + 499
              x += 499
              canv.create_oval(x, y, x + 1, y + 1, fill='black')
              my_file.write(str(x - 499) + " " + str(499-y) + "\n")
       except:
              First_x = -500
my_file.close()
canv.pack()
root.mainloop()
```

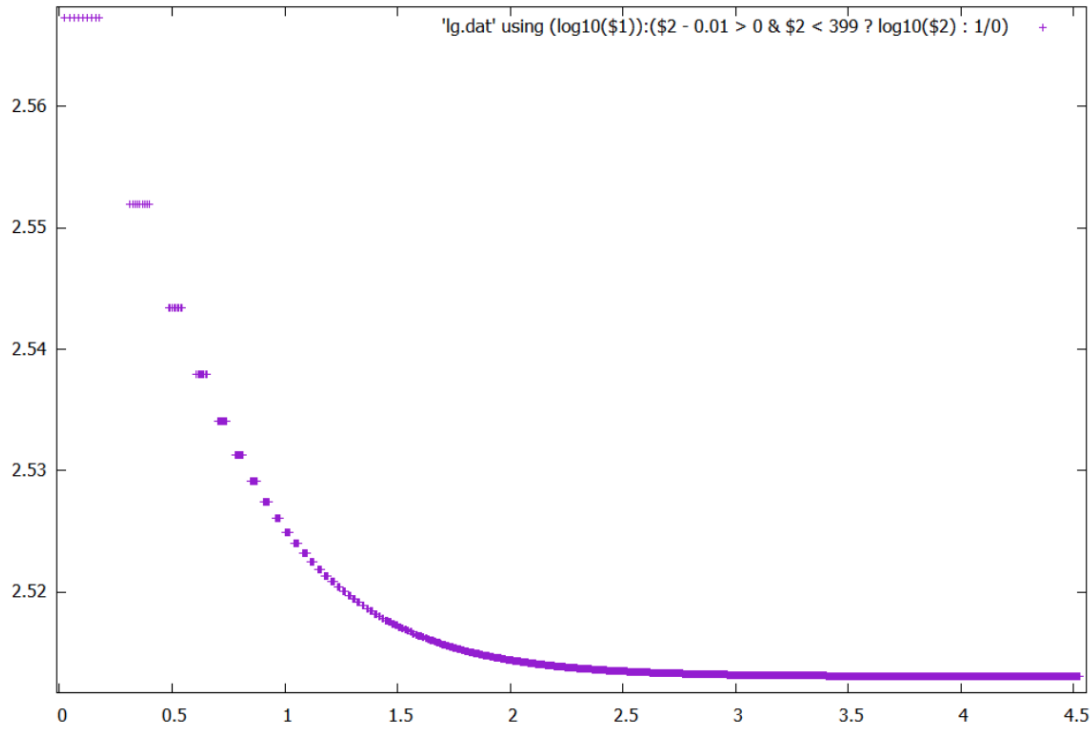
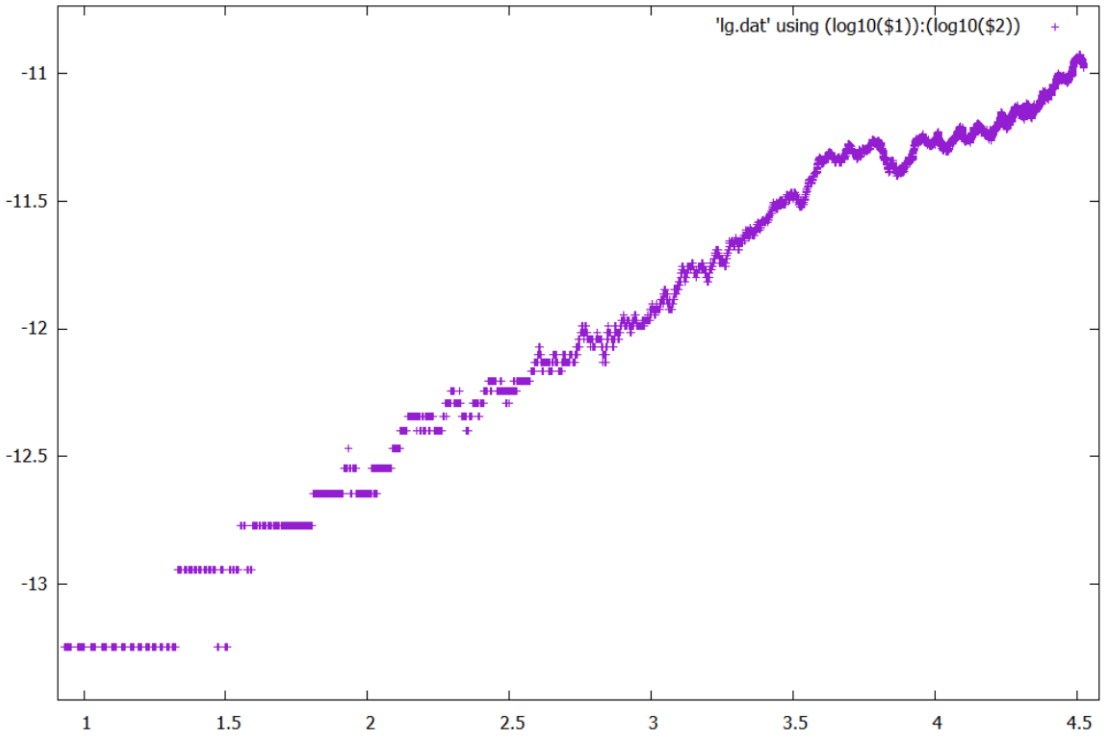


График показывает зависимость $\lg|(S_N^{(1)} - S_N^{(3)})/S_N^{(3)}|$ от $\lg(N)$. Программа работала очень долго, было получено огромное множество точек. При увеличении N можно увидеть, что приближенно зависимость становится линейной.

Но на этом же графике, в области отрицательных чисел есть ещё точки:


'lg.dat' using (log10($1)):(log10($2))
-11
-11.5
-12
-12.5
-13
1
1.5
2
2.5
3
3.5
4
4.5