Precoder Design Approaches

I. Introduction

II. System Model

N_T	Number of transmit antenna
N_R	Number of receive antenna
N_B	Number of base stations (BSs)
N	Number of sub-channels
K	Number of users present in the system
Q_k	Number of backlogged packets for user k
A_k	Instantaneous arrival of user k
$\mathbf{H}_{b,k,n} \in \mathbb{C}^{N_R \times N_T}$	Channel between BS b and user k for sub-channel n
$\mathbf{m}_{l,k,n} \in \mathbb{C}^{N_T \times L}$	Transmit precoder for user k on the n^{th} sub-channel
$\mathbf{w}_{l,k,n} \in \mathbb{C}^{N_R \times L}$	Receive equalizer for user k on the n^{th} sub-channel
$\mathcal{B}$	Set of co-ordinating BSs
$\mathcal{U}_b$	Set of users belonging to BS b
N_{OTA}	Total number of over-the-air (OTA) signaling exchanges between BSs and users
N_{BH}	Total number of back-haul signaling performed between the coordinating BSs
$\kappa = N_F/N_{\text{OTA}}$	Denotes the efficiency of the OTA transmissions to the gain obtained
N_G	Number of transmission slots available to transmit the data before the next arrivals

Using above notations, received signal on the l^{th} spatial stream on the n^{th} sub-channel for user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{k,n}. \tag{1}$$

The received signal-to-interference-plus-noise ratio (SINR) of the corresponding spatial data is given as

$$\gamma_{l,k,n} = \frac{\left| \mathbf{w}_{l,k,n}^{\mathrm{H}}\, \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n} \right|^2}{\widetilde{N}_0 + \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2} \tag{2}$$

where $\widetilde{N}_0 = N_0 \,\mathrm{tr}(\mathbf{w}_{l,k,n} \mathbf{w}_{l,k,n}^{\mathrm{H}})$ denotes the equivalent noise variance. The achievable rate of user k on the l^{th} spatial stream and the n^{th} sub-channel is given by

$$t_{l,k,n} = \log_2(1 + \gamma_{l,k,n}). \tag{3}$$

III. Precoder Designs for Queue Minimization

The precoders can be designed by the combination of back-haul iterations among the coordinating BSs and by OTA exchanges between the BSs and the users present in the system. The total number of back-haul iterations and the OTA exchanges are represented by N_{BH} and N_{OTA}. In order to account for the resource utilization by the OTA exchanges, κ is used as the efficiency of resource usage for the actual data transmission. The objective of the problem is to minimize the total number of queued packets of each users in the system at each instant in a distributed manner, which is given by

$$\underset{t_{l,k,n}}{\text{minimize}} \quad \sum_{k \in \mathcal{U}} \left| Q_k - \kappa \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} t_{l,k,n} \right|^q \tag{4}$$

for a fixed κ. Since the problem is nonconvex, we use successive convex approximation (SCA) to solve it in an iterative manner for a fixed number of iterations. At each instant, the starting point of the iterative procedure is fixed with the operating point from the previous transmission instant. It provides improved speed of convergence in the iterative procedure.

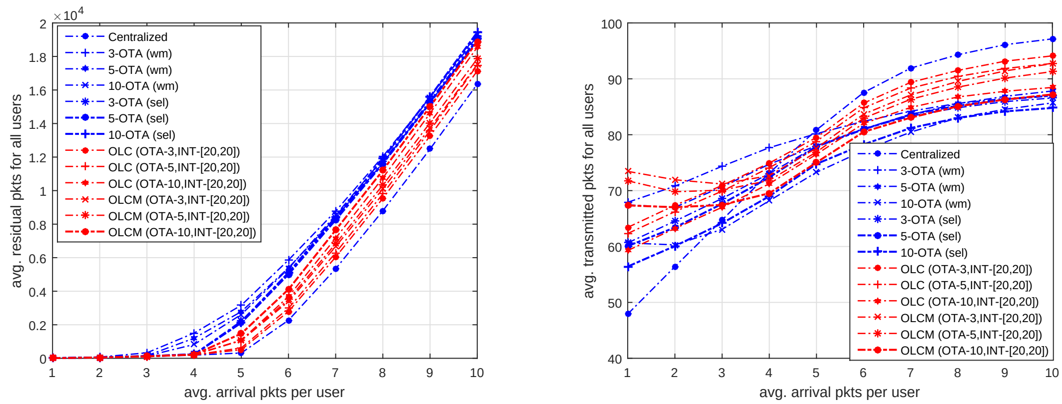
The objective is to design the precoders in a distributed approach with minimal number of information exchanges between the coordinating BSs and the users. Since the frequency of arrival packets is larger than the physical layer transmissions, we assume N_G transmission slots are available to empty to queues of all users before the next arrival packets from the higher layers.

A. KKT based Decomposition

In this approach, the precoders are designed iteratively by performing OTA exchanges between the users and the coordinating BSs in the system. It is performed by the following approach. Each BS will design the transmit precoders and broadcast to all users in the system by the precoded downlink pilots as where the initial transmit beamformers $\mathbf{m}_{l,k,n}^{(0)}$ and the receive equalizers $\mathbf{w}_{l,k,n}^{(0)}$ are initialized with the operating point of the previous transmission.

$$\mathbf{m}_{l,k,n}^{(j)} = \Big(\sum_{x \in \mathcal{U}} \sum_{y=1}^{L} \alpha_{y,x,n}^{(j-1)} \mathbf{H}_{b_k,x,n}^{\mathrm{H}} \mathbf{w}_{y,x,n}^{(j-1)} \mathbf{w}_{y,x,n}^{\mathrm{H}\,(j-1)} \mathbf{H}_{b_k,x,n} + \delta_b \mathbf{I}_{N_T} \Big)^{-1} \alpha_{l,k,n}^{(j-1)} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n}^{(j-1)} \tag{5}$$

Upon receiving the downlink pilots, each user evaluate equivalent downlink channel $\mathbf{H}_{b,k} \mathbf{m}_{l,\bar{k},n}$ $\forall \bar{k} \in \mathcal{U}$.



(a) Average Transmitted Packets after each of 500 transmission instants

(b) Average Residual packets after each of 500 transmission instants

Fig. 1: Queue dynamics for a system $\{N, N_B, K, N_T, N_R\} = \{4, 2, 16, 4, 2\}$

It is then used to calculate the effective receive equalizers using

$$\mathbf{w}_{l,k,n}^{(j)} = \left(\sum_{x \in \mathcal{U}} \sum_{y=1}^{L} \mathbf{H}_{b_x,k,n} \mathbf{m}_{y,x,n}^{(j)} \mathbf{m}_{y,x,n}^{\mathrm{H}\,(j)} \mathbf{H}_{b_x,k,n}^{\mathrm{H}} + \mathbf{I}_{N_R} \right)^{-1} \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n}^{(j)} \tag{6}$$

$$\epsilon_{l,k,n}^{(j)} = \left| 1 - \mathbf{w}_{l,k,n}^{\mathrm{H}\,(j)} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(j)} \right|^2 + N_0 \, \|\mathbf{w}_{l,k,n}^{(j)}\|^2 + \sum_{(x,y) \neq (l,k)} \left| \mathbf{w}_{l,k,n}^{\mathrm{H}\,(j)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n}^{(j)} \right|^2 \tag{7a}$$

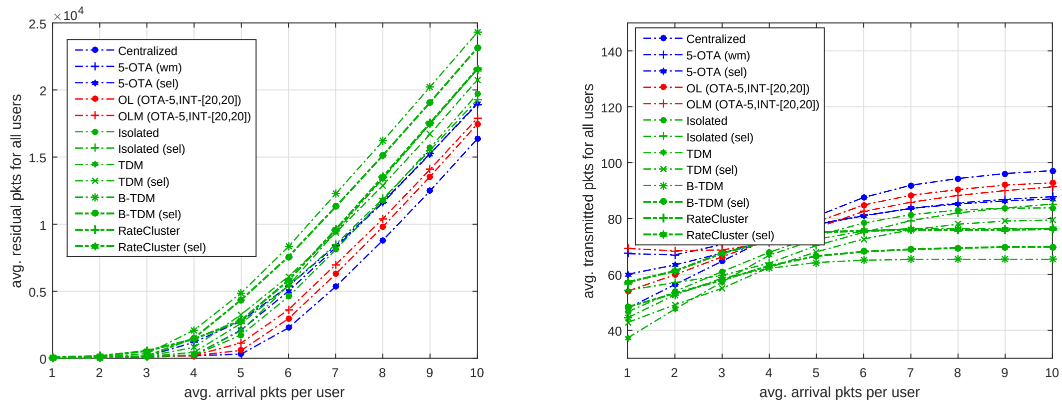
$$t_{l,k,n}^{(j)} = -\log_2(\epsilon_{l,k,n}^{(j-1)}) - \frac{\left(\epsilon_{l,k,n}^{(j)} - \epsilon_{l,k,n}^{(j-1)}\right)}{\log(2)\, \epsilon_{l,k,n}^{(j-1)}} \tag{7b}$$

$$\sigma_{l,k,n}^{(j)} = \left[\frac{a_k\, q}{\log(2)} \left(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}^{(j)} \right)^{(q-1)} \right]^{+} \tag{7c}$$

$$\alpha_{l,k,n}^{(j)} = \alpha_{l,k,n}^{(j-1)} + \rho \left(\frac{\sigma_{l,k,n}^{(j)}}{\epsilon_{l,k,n}^{(j)}} - \alpha_{l,k,n}^{(j-1)} \right) \tag{7d}$$

With the updated receive equalizer, the uplink precoded pilots are transmitted from all users to inform the effective channel $\sqrt{\alpha_{l,k,n}^{(j)}}\mathbf{H}_{b,k,n}^{\mathrm{T}} \mathbf{w}_{l,k,n}^{*}$ to all BSs in the system to obtain the next set of transmit beamformers.

To reduce the number of OTA exchanges in the system, we limit the iteration count to a fixed value N_{OTA}. The SCA operating points are initialized with the operating point obtained from the previous transmission after N_{OTA} iterations.



(a) Average Transmitted Packets after each of 500 transmission instants (b) Average Residual packets after each of 500 transmission instants

Fig. 2: Queue dynamics for a system $\{N, N_B, K, N_T, N_R\} = \{4, 2, 16, 4, 2\}$

B. ADMM based Decomposition

The alternating directions method of multipliers (ADMM) based approach uses traditional decomposition scheme without OTA signaling requirement. The information is exchanged between the coordinating BSs in $\mathcal{B}$ to update the dual and the coupling variables. The BS specific subproblem is given by

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n}\\ \beta_{l,k,n},\boldsymbol{\zeta}_b}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}_b\|_q + \boldsymbol{\nu}_b^{(j)\,\mathrm{T}}\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j)}\right) + \frac{\rho}{2}\left\|\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j)}\right\|^2 \tag{8a}$$

$$\text{subject to} \quad \sum_{n=1}^{N-1}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L-1}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \tag{8b}$$

$$\sum_{\bar{b}\in\bar{\mathcal{B}}_b}\zeta_{l,k,n,\bar{b}} + \sum_{\{\bar{l},\bar{k}\}\neq\{l,k\}}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{\bar{l},\bar{k},n}|^2 + \widetilde{N}_0 \leq \beta_{l,k,n} \tag{8c}$$

$$\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}|\mathbf{w}_{\bar{l},\bar{k},n}^{\mathrm{H}}\mathbf{H}_{b,\bar{k},n}\mathbf{m}_{l,k,n}|^2 \leq \zeta_{\bar{l},\bar{k},n,b}\ \forall \bar{k}\in\bar{\mathcal{U}}_b\ \forall n \tag{8d}$$

$$\text{linearize SINR} \tag{9}$$

where $\boldsymbol{\zeta}_b^{(j)}$ denotes the interference vector updated from the earlier iteration and $\boldsymbol{\nu}_b^{(j)}$ represents the dual vector corresponding to the equality constraint at the i^{th} iteration as

$$\boldsymbol{\zeta}_b = \boldsymbol{\zeta}_b^{(j)}. \tag{10}$$

Upon solving (8) for $\boldsymbol{\zeta}_b \forall b$ in the i^{th} iteration, the next iterate is updated by exchanging the corresponding interference terms between two BSs b and b_k as

$$\boldsymbol{\zeta}_{b_k}(b)^{(j+1)} = \boldsymbol{\zeta}_b(b_k)^{(j+1)} = \frac{\boldsymbol{\zeta}_b(b_k) + \boldsymbol{\zeta}_{b_k}(b)}{2}. \tag{11}$$

The dual vector for the next iteration is updated by using the subgradient search to maximize the dual objective as

$$\boldsymbol{\nu}_b^{(j+1)} = \boldsymbol{\nu}_b^{(j)} + \rho\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j+1)}\right) \tag{12}$$

where step size parameter ρ is chosen to depend on the system model under consideration. The convergence rate of the distributed algorithm is susceptible to the choice of step size parameter ρ. For our simulation models, we consider step size parameter $\rho = 2$. The above iteration is performed until convergence or for certain accuracy in the variation of the objective value between two consecutive updates.

C. Decomposition over Clusters

D. Hybrid Design with Initial Bias

In this approach, initialization of the resources are performed for multiple time slots, in particular N_G, which is then used as the initial operating for the forthcoming transmissions with the actual channel. It reduces the overhead involved in the iterative procedure and therefore requires less number of iterations for the precoder convergence. The problem can be given as

$$\underset{\substack{\gamma_{l,k,n}, \mathbf{m}_{l,k,n}^{(i)} \\ \beta_{l,k,n}, b_k^{(i)}}}{\text{minimize}} \quad \sum_{k \in \mathcal{U}} \left| b_k^{(N_G-1)} \right|^q \tag{13}$$

$$\text{subject to} \quad \sum_{n=1}^{N-1} \sum_{k \in \mathcal{U}_b} \sum_{l=1}^{L-1} \operatorname{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \tag{14a}$$

$$b_k^{(i-1)} - \mu_k^{(i)} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} t_{l,k,n}^{(i)} \leq b_k^{(i)} \tag{15}$$

$$\sum_{\substack{\{\bar{l},\bar{k}\} \neq \{l,k\} \\ \forall \bar{k} \in \mathcal{U}_b}} |\mathbf{w}_{l,k,n}^{\mathrm{H}(i)} \mathbf{H}_{b,k,n}^{(0)} \mathbf{m}_{\bar{l},\bar{k},n}^{(i)}|^2 + \widetilde{N}_0 \leq \beta_{l,k,n} \tag{16a}$$

$$\text{linearize SINR} \tag{17}$$

where $b_k^{(-1)} = Q_k$ and $\mu_k \leq 1$ is a measure of the Doppler associated with user k. In the above formulation, $\mu_k^{(i)}$ denotes the degradation in the achievable rate corresponding to the change in the channel gain. Once the precoders are designed, the users are preallocated over the time as well with the channel knowledge at the 0^{th} instant, where the user arrivals are happening.

Using the predetermined precoders from the above centralized problem as the initialization points for the SCA operating point, the current beamformers are updated through limited OTA signaling.

IV. Precoder Designs with Delay Constraints