Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems

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Abstract

We consider a downlink multi-cell multiple-input multiple-output (MIMO) interference broadcast channel (IBC) using orthogonal frequency division multiplexing (OFDM) with multiple users contending for space-frequency resources in a given scheduling instant. The problem is to design precoders efficiently to minimize the number of backlogged packets queuing in the coordinating base stations (BSs). Conventionally, the queue weighted sum rate maximization (Q-WSRM) formulation with the number of backlogged packets as the corresponding weights is used to design the precoders. In contrast, we propose joint space-frequency resource allocation (JSFRA) formulation, in which the precoders are designed jointly across the space-frequency resources for all users by minimizing the total number of backlogged packets in each transmission instant, thereby performing user scheduling implicitly. Since the problem is nonconvex, we use the combination of successive convex approximation (SCA) and alternating optimization (AO) to handle nonconvex constraints in the JSFRA formulation. In the first method, we approximate the signal-to-interference-plus-noise ratio (SINR) by convex relaxations, while in the second approach, the equivalence between the SINR and the mean squared error (MSE) is exploited. We then discuss the distributed approaches for the centralized algorithms using primal decomposition and alternating directions method of multipliers. Finally, we propose a more practical iterative precoder design by solving the Karush-Kuhn-Tucker expressions for the MSE reformulation that requires minimal information exchange for each update. Numerical results are used to compare the proposed algorithms to the existing solutions.

Index Terms

Convex approximations, MIMO-IBC, MIMO-OFDM, precoder design, SCA, WSRM.

I. Introduction

In a network with multiple base stations (BSs) serving multiple users, the main driving factor for the transmission is the packets waiting at each BS corresponding to the different users existing in the network. We consider the problem of transmit precoder design and resource allocation over the space-frequency resources provided by the multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) framework in the downlink interference broadcast channel (IBC) to minimize the number of queued packets in the BSs. Thus, the problem of interest inherently includes resource allocation of users associated with different BSs.

In general, the resource allocation problems such as admission control can be formulated by assigning a binary variable for each user to indicate the presence or absence of a particular resource [1]. Alternatively, linear transmit precoders, which are complex vectors, can be implicitly modeled as decision variables, thereby avoiding the use of binary decision variables. After the design stage, the non-zero precoders are used to determine the transmission rates of users on a space-frequency resource, and the zero transmit precoder indicates the absence of the user on a given resource. In this way, the soft decisions are used in the optimization problem and the hard decisions are made after the algorithm convergence.

The queue minimizing precoder designs are closely related to the weighted sum rate maximization (WSRM) problem with additional rate constraints to limit the throughput beyond the number of backlogged packets associated with the users. The topics on MIMO IBC precoder design have been studied extensively with different performance criteria in the literature. Due to the nonconvex nature of the MIMO IBC precoder design problems, successive convex approximation (SCA) approach has become a powerful tool to solve these problems. For example, in [2], the nonconvex part of the objective is linearized around a fixed operating point to solve the WSRM problem in a iterative manner. A similar approach using arithmetic-geometric inequality was proposed in [3].

The relation between the achievable sum rate and the mean squared error (MSE) of the received symbol by using fixed minimum mean squared error (MMSE) receivers can also be used to solve the WSRM problem [4]. In [5], [6], the WSRM problem is reformulated via MSE, casting the problem as a convex one for fixed linearization coefficients. In this way, the original problem is expressed in terms of the MSE weight, precoders, and receivers. Then the problem is solved using an alternating optimization method, i.e., finding a subset of variables while the other variables are fixed. The MSE reformulation

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for the WSRM problem was also studied in [7] by using SCA to solve the problem in an iterative manner. Moreover, distributed precoder designs with quality of service (QoS) requirements as additional rate constraints are studied for the MSE reformulated WSRM problem in [8].

The problem of precoder design for the MIMO IBC system can be solved either by using a centralized controller or by using decentralized algorithms, where each BS handles the corresponding subproblem independently with the limited information exchange with other BSs via back-haul. The distributed approaches are usually based on the primal, or dual decompositions or the alternating directions method of multipliers (ADMM), as discussed in [9], [10]. In the primal decomposition, the so-called coupling interference variables are fixed for the subproblem at each BS to find the optimal precoders. The fixed interference is then updated using the subgradients as discussed in [11]. The dual and the ADMM approaches control the distributed subproblems by fixing the *'interference price'* for each BS as detailed in [12].

By adjusting the weights in the WSRM objective, we can find an arbitrary rate-tuple in the rate region that maximizes suitable objective measures. For example, if the weight of each user is set to be inversely proportional to its average data rate, the corresponding problem guarantees fairness on the average among the users. To reduce the number of backlogged packets, we can assign weights based on the current queue size of the users. Specifically, the queue states can be incorporated in the WSRM objective $\sum_k w_k R_k$ by replacing the weight w_k with the corresponding queue state Q_k or its function, which is the outcome of minimizing the Lyapunov drift between the current and the future queue states [13], where R_k denotes the achievable data rate of user k. In the *backpressure algorithm*, the differential queues between the source and the receiver nodes are used to scale the transmission rate [13].

Earlier studies on the queue minimization problem are summarized in the survey papers [14], [15]. In particular, the problem of power allocation to minimize the number of backlogged packets was considered in [16] using geometric programming. Since the problem addressed in [16] assumed single antenna transmitters and receivers, the queue minimizing problem reduces to the optimal power allocation problem. In the context of wireless networks, the *backpressure algorithm* mentioned above was extended in [17] by formulating the corresponding user queues as the weights in the WSRM problem. Recently, the precoder design for the video transmission over MIMO system was considered in [18]. In this design, the MU-MIMO precoders are designed by the MSE reformulation as in [5] with the higher layer performance objectives such as playback interruptions and buffer overflow probabilities.

Main Contributions: In this paper, we design precoders jointly over space-frequency resources to reduce the number of backlogged packets waiting at each BSs. The proposed formulation also limits the allocations beyond the number of backlogged packets without explicit rate constraints. Initially, we propose a centralized joint space-frequency resource allocation (JSFRA) formulation, which is solved by two iterative algorithms based on the combination of SCA and alternating optimization (AO) due to the nonconvex nature of the problem. The proposed algorithms solve a sequence of convex problems obtained by fixing a subset of optimization variables or by approximating the nonconvex constraints by the convex ones. The first approach is performed by directly relaxing the signal-to-interference-plus-noise ratio (SINR) expression, while in the second method, the equivalence between the MSE and the SINR is exploited. We then discuss the distributed implementation of the JSFRA methods using primal decomposition and the ADMM. Finally, we also propose a more practical iterative precoder design by directly solving the Karush-Kuhn-Tucker (KKT) system of equations for the MSE reformulation that is numerically shown to require minimal information exchange for each update. Note that the joint space-frequency channel matrix can be formed by stacking the channel of each sub-channel in a block-diagonal form for all users.

The rest of the paper is organized as follows. Section II introduces the system model and the problem formulation. The existing and the proposed centralized designs are presented in Section III. The distributed solutions for the proposed problem are provided in Section IV followed by the simulation results in Section V. Finally, conclusions are drawn in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a downlink MIMO IBC scenario in an OFDM framework with N sub-channels and N_B BSs each equipped with N_T transmit antennas, serving in total K users each with N_R receive antennas. The set of users associated with BS b is denoted by $\mathcal{U}_b$ and the set $\mathcal{U}$ represents all users in the system, i.e., $\mathcal{U} = \underset{b \in \mathcal{B}}{\cup} \mathcal{U}_b$, where $\mathcal{B}$ is the set of indices of all coordinating BSs. Data for user k is transmitted from only one BS which is denoted by $b_k \in \mathcal{B}$. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the set of all sub-channel indices available in the system.

We adopt linear transmit beamforming technique at BSs. Specifically, the data symbols $d_{l,k,n}$ for user k on the lth spatial stream over sub-channel n is multiplied with beamformer $\mathbf{m}_{l,k,n} \in \mathbb{C}^{N_T \times 1}$ before being transmitted. In order to detect multiple spatial streams at the user terminal, receive beamforming vector $\mathbf{w}_{l,k,n}$ is employed for each user. Consequently, the received data estimate corresponding to the lth spatial stream over sub-channel n at user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n} \tag{1}$$

where $\mathbf{H}_{b,k,n} \in \mathbb{C}^{N_R \times N_T}$ is the channel between BS b and user k on sub-channel n, and $\mathbf{n}_{l,k,n} \sim \mathcal{CN}(0, N_0)$ is the additive noise vector for user k on the nth sub-channel and lth spatial stream. In (1), $L = \text{rank}(\mathbf{H}_{b,k,n}) = \min(N_T, N_R)$ is the maximum number of spatial streams.[1] Assuming independent detection of data streams, we can write the signal-to-interference-plus-noise ratio (SINR) as

$$\gamma_{l,k,n} = \frac{\left| \mathbf{w}_{l,k,n}^{\mathrm{H}}\, \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n} \right|^2}{\grave{N}_0 + \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2} \tag{2}$$

where $\grave{N}_0 = N_0 \,\mathrm{tr}(\mathbf{w}_{l,k,n} \mathbf{w}_{l,k,n}^{\mathrm{H}})$ denotes the equivalent noise variance. In order to avoid the overhead involved in feeding back the user channels, we consider time division duplexing (TDD) system in which BSs can estimate the downlink channels from uplink pilots by using channel reciprocity property.

Let Q_k be the number of backlogged packets destined for user k at a given scheduling instant. The queue dynamics of user k are modeled using the Poisson arrival process with the average number of packet arrivals of $A_k = \mathbf{E}_i\{\lambda_k\}$ packets or bits, where $\lambda_k(i) \sim \text{Pois}(A_k)$ represents the instantaneous number of packets arriving for user k at the ith time instant.[2] The total number of queued packets at the $(i+1)$th instant for user k, denoted as $Q_k(i+1)$, is given by

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i) \Big]^+ + \lambda_k(i) \tag{3}$$

where $[x]^+ \equiv \max\{x, 0\}$ and t_k denotes the number of transmitted packets or bits for user k. At the ith instant, the transmission rate of user k is given by

$$t_k(i) = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}(i) \tag{4}$$

where $t_{l,k,n}$ denotes the number of transmitted packets or bits over the lth spatial stream on the nth sub-channel. The maximum rate achieved over the space-frequency resource (l,n) is given by $t_{l,k,n} \leq \log_2(1 + \gamma_{l,k,n})$ for the SINR $\gamma_{l,k,n}$.[3] Note that t_k and Q_k are represented by the same units, *i.e.*, in bits defined per channel use.

B. Problem Formulation

In order to minimize the total number of backlogged packets, we consider minimizing the weighted ℓ_q-norm of the queue deviation metric as

$$v_k = Q_k - t_k = Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \tag{5}$$

where $\gamma_{l,k,n}$ is given by (2) and the optimization variables are transmit precoders $\mathbf{m}_{l,k,n}$ and receive beamformers $\mathbf{w}_{l,k,n}$.

Explicitly, the objective of the problem considered is given as $\sum_{k \in \mathcal{U}} a_k |v_k|^q$. Thus, the formulation becomes

$$\underset{\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{6a}$$

$$\text{subject to} \quad \sum_{n=1}^{N} \sum_{k \in \mathcal{U}_b} \sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \forall b \tag{6b}$$

where $\tilde{v}_k \triangleq a_k^{1/q} v_k$ is the element of vector $\tilde{\mathbf{v}}$. Weighing factors a_k are used to alter the user priority based on the QoS constraints such as packet delay requirements and packet waiting time, since they are proportional to the corresponding number of backlogged packets. The BS specific power constraint for all sub-channels is considered in (6b).

For practical reasons, we impose a constraint on the maximum number of transmitted bits for the user k, since it is limited by the total number of backlogged packets available at the transmitter. As a result, the number of backlogged packets v_k for

[1] It can be easily extended for user specific streams L_k instead of using common L streams for all users. L streams are initialized but after solving the problem, only $L_{k,n} \leq L$ non-zero data streams are transmitted.

[2] The unit can either be packets or bits as long as the arrival and the transmission units are similar.

[3] The upper bound can be achieved by using Gaussian signaling.

user k remaining in the system is given by

$$v_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L}\log_2(1+\gamma_{l,k,n}) \geq 0. \tag{7}$$

The above positivity constraint needs to be satisfied by v_k to avoid the excessive allocation of the resources.

Before proceeding further, we show that the constraint in (7) is handled implicitly by the definition of ℓ_q norm in the objective of (6). Suppose that $t_k > Q_k$ for a certain k at the optimum, i.e., $-v_k = t_k - Q_k > 0$. Then there exists $\delta_k > 0$ such that $-v'_k = t'_k - Q_k < -v_k$ where $t'_k = t_k - \delta_k$. Since $\|\tilde{\mathbf{v}}\|_q = \||\tilde{\mathbf{v}}|\|_q = \||-\tilde{\mathbf{v}}|\|_q$, this means that the newly created vector $\mathbf{t}'$ achieves a strictly smaller objective which contradicts with the fact that the optimal solution has been obtained. The choice of ℓ_q norm used in the objective function [14], [16] alters the priorities for the queue deviation function as follows.

- ℓ_1 norm results in greedy allocation, *i.e*, emptying the queue of users with good channel states before considering the users with worse channel conditions. As a special case, it is easy to see that (6) reduces to the WSRM problem when the queue size is large enough for all users.
- ℓ_2 norm prioritizes users with a higher number of queued packets before considering the users with a smaller number of backlogged packets. For example, it would be more ideal for the delay limited scenario when the packet arrival rate of users are almost similar, since the number of queued packets waiting in the buffer is proportional to the transmission delay, by following Little's law [13].
- ℓ_∞ norm minimizes the maximum number of queued packets among users with the current transmission, thereby providing queue fairness by allocating resources proportional to the number of backlogged packets.

III. Proposed Queue Minimizing Precoder Designs

In general, the precoder design for the MIMO OFDM problem is difficult due to its nonconvex nature. In addition, the objective of minimizing the number of the queued packets over space-frequency dimensions adds further complexity. Since the scheduling of users in each sub-channel is achieved by allocating zero transmit power over certain sub-channels, our solutions perform joint precoder design and user scheduling. Before discussing the proposed solutions, we consider an existing algorithm to minimize the number of backlogged packets with additional constraints required by the problem.

A. Queue Weighted Sum Rate Maximization (Q-WSRM)

The queue minimizing algorithms have been studied extensively in the networking context to provide congestion-free routing between any two nodes in the network. One such is the *backpressure algorithm* [13]. It finds an optimal control policy in the form of rate or resource allocation by considering differential backlogged packets between any two entities.

The queue weighted sum rate maximization (Q-WSRM) formulation extends the *backpressure algorithm* to the downlink MIMO-OFDM framework, in which the BSs act as the source nodes and the users as the receivers. The control policy in the form of transmit precoders aims at minimizing the number of queued packets waiting in the BSs. To find an optimal strategy, we resort to the Lyapunov theory, which is predominantly used in control theory to achieve system stability. Since at each time slot, the system is described by the channel conditions and the number of backlogged packets for each user, the Lyapunov function is used to provide a scalar measure, which grows large when the system moves towards an undesirable state [13]. The scalar measure for queue stability is given by

$$\mathrm{L}\left[\mathbf{Q}(i)\right] = \frac{1}{2}\sum_{k\in\mathcal{U}} Q_k^2(i) \tag{8}$$

where $\mathbf{Q}(i) = [Q_1(i), Q_2(i), \ldots, Q_K(i)]^{\mathrm{T}}$. It provides a scalar measure of congestion in the system [13, Ch. 3].

To minimize the total number of backlogged packets for time instant i, the optimal transmission rate of all users is obtained by minimizing the Lyapunov drift expressed as

$$\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] = \frac{1}{2}\Big[\sum_{k\in\mathcal{U}}\Big([Q_k(i)-t_k(i)]^+ + \lambda_k(i)\Big)^2 - Q_k^2(i)\Big]. \tag{9}$$

To eliminate the nonlinear operator $[x]^+$, we bound (9) as

$$\leq \sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i)+t_k^2(i)}{2} + \sum_{k\in\mathcal{U}} Q_k(i)\left\{\lambda_k(i) - t_k(i)\right\} \tag{10}$$

by using the following inequality

$$\left[\max(Q-t,0)+\lambda\right]^2 \leq Q^2 + t^2 + \lambda^2 + 2Q(\lambda - t). \tag{11}$$

The total number of backlogged packets at any given instant i is reduced by minimizing the conditional expectation of the Lyapunov drift expression (10) given the current number of queued packets $\boldsymbol{Q}(i)$ waiting in the system. The expectation is taken over all possible arrival and transmission rates of the users to obtain the optimal rate allocation strategy.

Now, the conditional Lyapunov drift, denoted by $\Delta(\boldsymbol{Q}(i))$, is given by the infimum over the transmission rate as

$$\inf_{\mathbf{t}} \quad \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}} \left\{ \mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] | \mathbf{Q}(i) \right\} \tag{12a}$$

$$\leq \underbrace{\mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}} \Big\{ \sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} | \mathbf{Q}(i) \Big\}}_{\leq B} + \sum_{k\in\mathcal{U}} Q_k(i) A_k(i)$$
$$- \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}} \Big\{ \sum_{k\in\mathcal{U}} Q_k(i) t_k(i) | \mathbf{Q}(i) \Big\}, \tag{12b}$$

where $\mathbf{t}$ and $\boldsymbol{\lambda}$ are the vectors formed by stacking the transmission and the arrival rate of all users. Since they are bounded, the second order moments on the first term in (12b) can be bounded by a constant B without affecting the optimal solution [13]. The second term in (12b) follows the Poisson arrivals.

The expression in (12) looks similar to the WSRM formulation if the weights in the WSRM problem are replaced by the numbers of backlogged packets of the corresponding users. The above approach was extended to the wireless networks in [17], in which the queues were used as weights in the WSRM formulation to determine the transmit precoders. Since (12) is minimized by maximizing the function inside expectation, the Q-WSRM formulation is given by

$$\underset{\substack{\mathbf{m}_{l,k,n},\\ \mathbf{w}_{l,k,n}}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} Q_k \Big(\sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \Big) \tag{13a}$$

$$\text{subject to} \quad \sum_{n=1}^{N} \sum_{k\in\mathcal{U}_b} \sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \forall b. \tag{13b}$$

To avoid excessive allocation of the resources, we include an additional rate constraint $t_k \leq Q_k$ to address $[x]^+$ operation in (3). The rate constrained version of the Q-WSRM, denoted by Q-WSRM extended (Q-WSRME) problem for a cellular system, is given by with the additional constraints as

$$\sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \leq Q_k \; \forall k \in \mathcal{U} \tag{14}$$

where the precoders are associated with $\gamma_{l,k,n}$ defined in (2). By using the number of queued packets as the weights, the resources can be allocated to the user with more backlogged packets; this essentially results in greedy allocation.

B. JSFRA Scheme via SINR Relaxation

The problem defined in (13) ignores the second order term arising from the Lyapunov drift minimization objective by limiting it to a constant value. In fact, by using $\ell_{q=2}$ norm in (5), we obtain the objective function, similar to (13) as

$$\underset{t_k}{\text{minimize}} \sum_{k} v_k^2 = \underset{t_k}{\text{minimize}} \sum_{k} Q_k^2 - 2\, Q_k t_k + t_k^2 \tag{15}$$

The equivalence is achieved by either removing t_k^2 from (15) or when the number of queued packets is large enough.

By ignoring t_k^2 from (15), the Q-WSRM scheme requires an explicit rate constraint (14) to avoid over-allocation of the resources. In the proposed queue deviation approach, explicit rate constraints are not needed, since they are handled by the objective function (5) itself. In contrast to the WSRM formulation, the JSFRA and the Q-WSRME problems handle the sub-channels jointly to obtain an efficient allocation by identifying the optimal space-frequency resources for the users.

We present iterative algorithms to solve (6) using alternating optimization technique in conjunction with SCA approach presented [19]. The problem is to determine the transmit precoders $\mathbf{m}_{l,k,n}$ and the receive beamformers $\mathbf{w}_{l,k,n}$ to minimize the total number of backlogged packets in the system. Note that the SINR expression in (2) cannot be used to formulate the problem directly due to the equality constraint. However, by using additional variables, we can relax the SINR expression in (2) by inequality constraints to solve the problem (6) as

$$\underset{\substack{\gamma_{l,k,n}, \mathbf{m}_{l,k,n},\\ \beta_{l,k,n}, \mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{16a}$$

$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{l,k,n} \mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \tag{16b}$$

$$\beta_{l,k,n} \geq \grave{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2 \tag{16c}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right)\leq P_{\max}. \tag{16d}$$

The SINR expression in (2) is relaxed by the inequalities (16b) and (16c). Note that (16b) is an under-estimator for SINR $\gamma_{l,k,n}$, and (16c) provides an upper bound for the total interference seen by user $k\in\mathcal{U}_b$, denoted by variable $\beta_{l,k,n}$. Therefore, the problem formulation in (16) is an equivalent approximation for the problem presented in (6). Note that the JSFRA formulation in (16) can be reformulated as a WSRM problem, which is known to be NP-hard [20], and therefore it belongs to the class of NP-hard problems.

In order to find a tractable solution for (16), we note that (16d) is the only convex constraint with the involved variables. Thus, we need to deal with (16b) and (16c). To this end, we resort to the AO technique by fixing the linear receivers to solve for the transmit beamformers. For fixed receivers $\mathbf{w}_{l,k,n}$, *i.e.*, by fixing the receive beamformers of all users in the system, the problem now is to find optimal transmit precoders $\mathbf{m}_{l,k,n}$ which is still a challenging task. Now, by fixing $\mathbf{w}_{l,k,n}$, (16c) can be written as a second-order cone (SOC) constraint. Thus, the difficulty is due to the non-convexity of the constraint in (16b). Let

$$g(\mathbf{u}_{l,k,n})\triangleq\frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \tag{17}$$

be the r.h.s of (16b), where $\mathbf{u}_{l,k,n}\triangleq\{\mathbf{m}_{l,k,n},\mathbf{w}_{l,k,n},\beta_{l,k,n}\}$. Note that the function $g(\mathbf{u}_{l,k,n})$ is convex for a fixed $\mathbf{w}_{l,k,n}$, since it is in fact the ratio between a quadratic form of $\mathbf{m}_{l,k,n}$ over an affine function of $\beta_{l,k,n}$ as in [21]. The nonconvex set defined by (16b) can be decomposed as a series of convex subsets by linearizing the convex function $g(\mathbf{u}_{l,k,n})$ with the first order Taylor approximation around a fixed operating point $\tilde{\mathbf{u}}_{l,k,n}$ [22], [23], also referred to as SCA in [19]. By using the reduced convex subset for (16b), the problem in (16) is solved iteratively by updating the operating point in each iteration.

For this purpose, let the real and imaginary components of the complex number $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}$ be represented by

$$p_{l,k,n}\triangleq\Re\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{18a}$$

$$q_{l,k,n}\triangleq\Im\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{18b}$$

and hence $g(\mathbf{u}_{l,k,n})=(p_{l,k,n}^2+q_{l,k,n}^2)/\beta_{l,k,n}$.[4] Let $\tilde{\mathbf{u}}_{l,k,n}\triangleq\{\tilde{\mathbf{m}}_{l,k,n},\tilde{\mathbf{w}}_{l,k,n},\tilde{\beta}_{l,k,n}\}$ be a minimizer from the previous SCA iteration. Now, by using the first order Taylor approximation around the operating point $\tilde{\mathbf{u}}_{l,k,n}$, we can approximate (16b) as

$$2\frac{\tilde{p}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(p_{l,k,n}-\tilde{p}_{l,k,n}\right)+2\frac{\tilde{q}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(q_{l,k,n}-\tilde{q}_{l,k,n}\right)+\frac{\tilde{p}_{l,k,n}^2+\tilde{q}_{l,k,n}^2}{\tilde{\beta}_{l,k,n}}\left(1-\frac{\beta_{l,k,n}-\tilde{\beta}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\right)\geq\gamma_{l,k,n}. \tag{19}$$

In summary, for fixed receivers $\tilde{\mathbf{w}}_{l,k,n}$ and operating point $\tilde{\mathbf{u}}_{l,k,n}$ as in (19), obtained by using (18), the relaxed convex subproblem for finding transmit precoders is given by

$$\underset{\substack{\mathbf{m}_{l,k,n},\\ \gamma_{l,k,n},\beta_{l,k,n}}}{\text{minimize}}\quad \|\tilde{\mathbf{v}}\|_q \tag{20a}$$

$$\text{subject to}\quad \beta_{l,k,n}\geq\grave{N}_0+\sum_{(j,i)\neq(l,k)}|\tilde{\mathbf{w}}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{20b}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right)\leq P_{\max}\ \forall b \tag{20c}$$

$$\text{and (19).} \tag{20d}$$

Now, the optimal receivers for fixed transmit precoders $\tilde{\mathbf{m}}_{l,k,n}$ are obtained by minimizing (16) w.r.t. $\mathbf{w}_{l,k,n}$ as

$$\underset{\substack{\gamma_{l,k,n},\\ \mathbf{w}_{l,k,n},\beta_{l,k,n}}}{\text{minimize}}\quad \|\tilde{\mathbf{v}}\|_q \tag{21a}$$

$$\text{subject to}\quad \beta_{l,k,n}\geq\grave{N}_0+\sum_{(j,i)\neq(l,k)}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\tilde{\mathbf{m}}_{j,i,n}|^2 \tag{21b}$$

$$\text{and (19).} \tag{21c}$$

[4]Note that $p_{l,k,n}$ and $q_{l,k,n}$ are just symbolic notations. In CVX [24], for example, we declare $p_{l,k,n}$ and $q_{l,k,n}$ with the '*expression*' qualifier.

Solving (21) using the KKT conditions, we obtain the following iterative expression for an optimal receiver $\mathbf{w}^{o}_{l,k,n}$ as

$$\mathbf{A}_{l,k,n} = \sum_{(j,i)\neq(l,k)} \mathbf{H}_{b_i,k,n}\tilde{\mathbf{m}}_{j,i,n}\tilde{\mathbf{m}}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{22a}$$

$$\mathbf{w}^{(i)}_{l,k,n} = \left(\frac{\tilde{\beta}_{l,k,n}\tilde{\mathbf{m}}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}^{(i-1)}_{l,k,n}}{\|\mathbf{w}^{(i-1)}_{l,k,n}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\|^2}\right)\mathbf{A}^{-1}_{l,k,n}\mathbf{H}_{b_k,k,n}\tilde{\mathbf{m}}_{l,k,n} \tag{22b}$$

where $\mathbf{w}^{(i-1)}_{l,k,n}$ is the receive beamformer from the previous iteration, upon which the linear relaxation is performed for the nonconvex constraint in (16b), as used in the formulation (21). The optimal receiver $\mathbf{w}^{o}_{l,k,n}$ is obtained by either iterating (22b) until convergence or for a fixed number of iterations. Note that the receiver has no explicit relation with the choice of ℓ_q norm used in the objective. The dependency is implied by the precoders $\mathbf{m}_{l,k,n}$, which depends on the exponent q.

It can be seen that the optimal receiver in (22b) is in fact a scaled version of the MMSE receiver, which is given by

$$\mathbf{R}_{l,k,n} = \sum_{i\in\mathcal{U}}\sum_{j=1}^{L} \mathbf{H}_{b_i,k,n}\tilde{\mathbf{m}}_{j,i,n}\tilde{\mathbf{m}}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{23a}$$

$$\mathbf{w}_{l,k,n} = \mathbf{R}^{-1}_{l,k,n}\,\mathbf{H}_{b_k,k,n}\,\tilde{\mathbf{m}}_{l,k,n}. \tag{23b}$$

Note that the scaling present in the optimal receiver (22b) has no impact on the received SINRs, and therefore the MMSE receiver in (23b) can also be used. However, the convergence speed is different between the two receiver implementations.

The proposed solution involves two nested iterations, *i.e.*, one for the outer AO loop and the second for the inner SCA loop. Each AO iteration involves two steps, one for finding the transmit precoders by solving (20) iteratively until convergence for fixed receivers, and the other for updating the receive beamformers with the previously found fixed transmit precoders by either solving (22b) recursively or by using (23b).

Let us consider the jth SCA iteration in the ith AO step to find transmit precoders by solving (20). For brevity, let us consider $\mathbf{m}$ as the collection of all transmit precoders as $\mathbf{m} = \{\mathbf{m}_{l,k,n}\}_{\forall l,\forall k,\forall n}$. Similarly, we denote $\mathbf{w}$ and $\boldsymbol{\beta}$ as $\{\mathbf{w}_{l,k,n}\}_{\forall l,\forall k,\forall n}$ and $\{\beta_{l,k,n}\}_{\forall l,\forall k,\forall n}$ respectively. Let $\{\mathbf{m}^{(i)}_{j}, \boldsymbol{\beta}^{(i)}_{j}\}$ be a minimizer obtained in the $(j-1)$th SCA step. Since SCA is an iterative scheme, the next operating point $\tilde{\mathbf{u}}_{l,k,n}$ for the $(j+1)$th step is given as $\mathbf{z}^{(i)}_{j} \triangleq \{\mathbf{m}^{(i)}_{j}, \mathbf{w}^{(i-1)}_{*}, \boldsymbol{\beta}^{(i)}_{j}\}$. Note that $\mathbf{w}^{(i-1)}_{*}$ is obtained by solving either (22b) or by using (23b) in the $(i-1)$th AO step.

Algorithm 1: Algorithm of JSFRA scheme

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b\in\mathcal{B}$, $\forall k\in\mathcal{U}, \forall n\in\mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}\forall l\in\{1,2,\ldots,L\}$
Initialize: $i=0, j=0$ and $\tilde{\mathbf{m}}_{l,k,n}$ using single user beamformer satisfying power constraint (20c)
update $\tilde{\mathbf{w}}_{l,k,n}, \tilde{\beta}_{l,k,n}$ using (23b) and (20b) with $\tilde{\mathbf{m}}_{l,k,n}$
repeat
    repeat
        solve for the transmit precoders $\mathbf{m}_{l,k,n}$ using (20)
        update the constraint set (19) with $\mathbf{u}_{l,k,n}$ and $\mathbf{m}_{l,k,n}$ using (18) and increment $j=j+1$
    until *SCA convergence or* $j\geq K_{\max}$
    update the receive beamformers $\mathbf{w}_{l,k,n}$ using (21) or (23b) with the updated precoders $\mathbf{m}_{l,k,n}$
    $i=i+1$, $j=0$
until *Queue convergence or* $i\geq I_{\max}$

Upon convergence of the objective sequence or for some fixed $K_{\max}$, receivers are then updated using fixed $\mathbf{m}^{(i)}_{*}$ found in previous SCA step. Once the receivers are updated, transmit precoders are again evaluated using the newly found receivers by solving (20) or (23b). The above procedure is repeated until $i\to\infty$ or for some fixed $I_{\max}$ as outlined in Algorithm 1.

A feasible initial point $\tilde{\mathbf{u}}_{l,k,n}$ is obtained by fixing $\tilde{\mathbf{m}}_{l,k,n}$ with the respective single-user precoders satisfying total power constraint in (20c) and $\tilde{\mathbf{w}}_{l,k,n}$'s are updated by using the MMSE receiver in (23b). Now, for fixed transmit and receive beamformers, $\tilde{\beta}_{l,k,n}$'s are evaluated using (20b).

C. JSFRA Scheme via MSE Reformulation

In the second method, we solve the JSFRA problem by exploiting the relation between the MSE and the achievable SINR when the MMSE receivers are used at the user terminals [4], [5]. The MSE $\epsilon_{l,k,n}$, for a data symbol $d_{l,k,n}$ is given by

$$\mathbb{E}\big[(d_{l,k,n}-\hat{d}_{l,k,n})^2\big] = \big|1-\mathbf{w}^{\mathrm{H}}_{l,k,n}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\big|^2$$

$$+\sum_{(j,i)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2+\grave{N}_0=\epsilon_{l,k,n} \quad (24)$$

where $\hat{d}_{l,k,n}$ is the estimate of the transmitted symbol. Plugging the MMSE receivers in (23b) into the MSE expression in (24) and into the SINR expression in (2), we arrive at

$$\epsilon_{l,k,n}=(1+\gamma_{l,k,n})^{-1}. \quad (25)$$

The above equivalence is valid only if the receivers are based on the MMSE criterion. Using the equivalence in (25), the WSRM objective can be reformulated as the weighted minimum mean squared error (WMMSE) to obtain the precoders for the MU-MIMO scenario as discussed in [5]–[7]. Note that the receive beamformers based on the MMSE criterion are independent of the choice of the ℓ_q norm used in the objective function to obtain the optimal transmit precoders $\mathbf{m}_{l,k,n}$.

Before proceeding futher, let $v_k'=Q_k-\sum_{n=1}^{N}\sum_{l=1}^{L}t_{l,k,n}$ denotes the queue deviation corresponding to user k and $\tilde{v}_k'\triangleq a_k^{1/q}v_k'$ be the corresponding weighted objective.By using the relaxed MSE expression in (24), we can reformulate (6) as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \quad (26a)$$

$$\text{subject to } t_{l,k,n}\leq-\log_2(\epsilon_{l,k,n}) \quad (26b)$$

$$\sum_{(j,i)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2+\grave{N}_0 + \left|1-\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2\leq\epsilon_{l,k,n} \quad (26c)$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right)\leq P_{\max}\ \forall b. \quad (26d)$$

The alternative MSE formulation given by (26) is nonconvex even for a fixed $\mathbf{w}_{l,k,n}$ due to the constraint (26b). Again we resort to the SCA approach [19] by relaxing the constraint by a sequence of convex subsets using the first order Taylor approximation around a fixed MSE point $\tilde{\epsilon}_{l,k,n}$ as

$$-\log_2(\tilde{\epsilon}_{l,k,n})-\frac{(\epsilon_{l,k,n}-\tilde{\epsilon}_{l,k,n})}{\log(2)\,\tilde{\epsilon}_{l,k,n}}\geq t_{l,k,n} \quad (27)$$

Using the above approximation for the rate constraint, the problem defined in (26) is solved for optimal transmit precoders $\mathbf{m}_{l,k,n}$, MSEs $\epsilon_{l,k,n}$, and the user rates over each sub-channel $t_{l,k,n}$ for fixed receivers. The optimization subproblem to find transmit precoders for fixed $\mathbf{w}_{l,k,n}$ is given by

$$\underset{t_{l,k,n},\mathbf{m}_{l,k,n},\epsilon_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \quad (28a)$$

$$\text{subject to} \quad (26c), (26d), \text{ and } (27). \quad (28b)$$

For fixed receivers, transmit precoders are obtained by solving (28) iteratively by updating $\tilde{\epsilon}_{l,k,n}$ with $\epsilon_{l,k,n}$ found in previous step until termination as in Section III-B and Algorithm 1.

In all our numerical simulations, the objective sequence generated by Algorithm 1 converges. However, to provide a formal convergence analysis, we consider using a regularized objective in (46b) instead of using (16a) and (26a). The proximal term in (46b) ensures strong convexity, thereby enforcing the uniqueness of the minimizer in each iteration. Note that the feasible set of problems (6) and (26) is bounded. However, for problem (16), the feasible set need not be bounded due to newly introduced variables in (16b) and (16c) to relax the SINR expression in (2). This follows from the fact that $\beta_{l,k,n}$ can take any value by satisfying (16c) without violating the constraints of problem (16) when $\gamma_{l,k,n}=0$. Nevertheless, we can limit $\beta_{l,k,n}$'s by using a maximum interference threshold depending on $P_{\max}$ and the channel gains, and thus the feasible set can be bounded without affecting the optimality of the considered problem. With the above assumptions, convergence analysis of Algorithm 1 is discussed in Appendix A.

D. Reduced Complexity Spatial Resource Allocation (SRA)

The JSFRA algorithm complexity scales with the number of sub-channels, since the complexity of interior point method, used to solve the problem, increases with the problem size. Thus, we can use the decomposition methods in [9], [10] for designing precoders of each sub-channel with less complexity by using additional coupling variables among the subproblems.

As an alternative sub-optimal solution, we present queue minimizing spatial resource allocation (SRA), which solves for precoders using JSFRA formulation for a specific sub-channel i with fixed transmit power $P_{\max,i}$. For each sub-channel, power sharing can either be equal or based on some predetermined limits as in partial frequency reuse satisfying

$$\sum_{i=1}^{N} P_{\max,i} = P_{\max}. \tag{29}$$

Even though N sub-channels are present at any given scheduling instant, precoders are computed for each sub-channel sequentially with $P_{\max,i}$ and the residual number of backlogged packets. Let $Q_{k,i}$ be the number of backlogged packets associated with user k while designing the precoders for the ith sub-channel. Since the precoder design is sequential, *i.e*, the precoders are designed for sub-channels $[0, i-1]$ before the ith sub-channel, the number of queued packets for the first chosen sub-channel is given by $Q_{k,1} = Q_k$. The queues associated with the consecutive sub-channels are updated as

$$Q_{k,i+1} = \max\left(Q_k - \sum_{j=1}^{i}\sum_{l=1}^{L} t_{l,k,j}, 0\right) \forall\, k \in \mathcal{U} \tag{30}$$

where $t_{l,k,j}$ is the kth user rate on sub-channel j.

For simplicity we use random sub-channel ordering in our paper, *i.e.*, after finding the precoders for a current sub-channel, we can choose any previously unselected sub-channels as the next candidate sub-channel for which the precoders are identified using the updated backlogged packets. Alternatively, we can also use greedy ordering to select the sub-channels by sorting the norm of the channel seen between the users and the respective serving BSs. However, it comes at the cost of increased complexity. Nevertheless, the SRA scheme will be insensitive to the sub-channel ordering as the number of users in the system increases, due to the available multi-user diversity in the system.

IV. Distributed Solutions

In this section, we discuss the distributed designs for the formulations proposed in Sections III-B and III-C for problem (16). Note that the convex subproblems in (20) or (28) requires a centralized controller to design the precoders for all users in the system. However, the amount of channel state information (CSI) that needs to be exchanged between the BSs and the controller grows significantly as the network size increases (e.g., the number of BSs and users, and the number of associated antennas). The overhead involved in the CSI exchanges can be avoided, if we design the respective precoders independently at each BS with the minimal information exchange, which is determined by the distributed schemes discussed herein.

Let us consider the convex subproblem with fixed receivers $\mathbf{w}_{l,k,n}$ as presented in (20) based on the SINR relaxation for the nonconvex constraint (16b). The following discussions are equally valid for the MSE based solution outlined in (28) as well. Since the objective of (20) can be decoupled across each BS, the centralized problem can be equivalently written as

$$\underset{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\beta_{l,k,n}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}} \|\tilde{\mathbf{v}}_b\|_q \tag{31a}$$

$$\text{subject to} \quad (20\text{b}) - (20\text{d}) \tag{31b}$$

where $\tilde{\mathbf{v}}_b$ denotes the stacked vector of weighted queue deviations corresponding to all users in BS b as $\forall k \in \mathcal{U}_b$.

To begin with, let $\bar{\mathcal{B}}_b$ be the set $\mathcal{B}\backslash\{b\}$ and $\bar{\mathcal{U}}_b$ represent the set $\mathcal{U}\backslash\mathcal{U}_b$. Following an approach similar to the one presented in [11], [12], the coupling constraint (20b) or (26c) can be expressed by stacking the interference from all BSs in $\bar{\mathcal{B}}_{b_k}$ as

$$\grave{N}_0 + \sum_{j=1,j\neq l}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,k,n}|^2 + \sum_{b\in\bar{\mathcal{B}}_{b_k}} \zeta_{l,k,n,b} + \sum_{i\in\mathcal{U}_{b_k}\backslash\{k\}}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n} \tag{32}$$

where $\zeta_{l,k,n,b}$ is the total interference caused by the transmission of BS b to user $k \in \mathcal{U}_{b_k}$ in spatial stream l and sub-channel n. In order to ensure (20b), we impose

$$\zeta_{l,k,n,b} \geq \sum_{i\in\mathcal{U}_b}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{j,i,n}|^2 \;\forall b \in \bar{\mathcal{B}}_{b_k}. \tag{33}$$

The decentralization is achieved by decomposing the original convex problem in (31) to a parallel subproblems coordinated by either primal or dual decomposition update. The coupling variables are updated in each iteration by exchanging limited information among the BSs. Before proceeding further, let $\bar{\boldsymbol{\zeta}}_b$ be the vector formed by stacking interference terms (33) from the neighboring BSs to the users of BS b and $\hat{\boldsymbol{\zeta}}_b$ be the stacked interference terms caused by BS b to all users in the neighboring BSs $\bar{\mathcal{B}}_b$, represented for each $b \in \mathcal{B}$ as

$$\bar{\boldsymbol{\zeta}}_b = \left[\zeta_{1,\mathcal{U}_b(1),1,\bar{\mathcal{B}}_b(1)}, \ldots, \zeta_{L,\mathcal{U}_b(|\mathcal{U}_b|),N,\bar{\mathcal{B}}_b(|\bar{\mathcal{B}}_b|)}\right]^{\mathrm{T}} \tag{34a}$$

$$\hat{\boldsymbol{\zeta}}_b = \left[\zeta_{1,\bar{\mathcal{U}}_b(1),1,b}, \ldots, \zeta_{L,\bar{\mathcal{U}}_b(|\bar{\mathcal{U}}_b|),N,b}\right]^{\mathrm{T}}. \tag{34b}$$

Let us define the vector $\boldsymbol{\zeta}_b$, formed by stacking the interference terms corresponding to the BS b as

$$\boldsymbol{\zeta}_b = \left[\hat{\boldsymbol{\zeta}}_b^{\mathrm{T}}, \bar{\boldsymbol{\zeta}}_b^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{35}$$

Since the decentralization solution is an iterative procedure, we represent the jth iteration index as $x^{(j)}$. Let $\boldsymbol{\zeta}_b(b_k)$ denote the interference terms corresponding to BS b_k in BS b as

$$\boldsymbol{\zeta}_b(b_k) = [\zeta_{1,\mathcal{U}_b(1),1,b_k}, \ldots, \zeta_{L,\mathcal{U}_b(|\mathcal{U}_b|),N,b_k}, \\ \zeta_{1,\mathcal{U}_{b_k}(1),1,b}, \ldots, \zeta_{L,\mathcal{U}_{b_k}(|\mathcal{U}_{b_k}|),N,b}]. \tag{36}$$

To decentralize (31), the BS specific vector $\boldsymbol{\zeta}_b$ in (35), which are relevant for the BS b, can either be fixed or treated as a variable in accordance to the decomposition method. To decouple the precoder design across BSs, the equivalent downlink channels $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}, \forall k \in \mathcal{U}$ are to be known at each BS b through the precoded uplink pilots from all the users in the system, where the linear precoders are evaluated at the user using MMSE formulation in (28). Similarly, to update the MMSE receivers at each user k, the equivalent channels $\mathbf{H}_{b,k,n}\mathbf{m}_{l,k',n}, \forall k' \in \mathcal{U}_b, \forall b \in \mathcal{B}$ need to be known. It is obtained through the user specific downlink pilots precoded with the newly obtained transmit beamformers $\mathbf{m}_{l,k,n}, \forall k \in \mathcal{B}$ evaluated at the BS b using equivalent downlink channels [25].

A. Primal Decomposition

In the primal decomposition, the convex problem in (31) is solved for the transmit precoders in an iterative manner by fixing the BS specific interference terms $\boldsymbol{\zeta}_{b_k}$ using master-slave model [11]. The slave subproblem is solved in each BS for the optimal transmit precoders only for the associated users by assuming fixed interference terms $\boldsymbol{\zeta}_{b_k}^{(i)}$ in each ith iteration. Upon finding the associated transmit precoders by each slave subproblems, the master problem is used to update the BS specific interference terms $\boldsymbol{\zeta}_{b_k}^{(i+1)}$ for the next iteration by using dual variables corresponding to the interference constraint (32) as discussed in [11]. In this manner, the interference variables are updated until the global consensus is reached. The master problem treats $\boldsymbol{\zeta}_b$ as a variable and the slave subproblems assumes it to be a constant in each iteration. Convergence discussions are presented in Appendix B.

B. Alternating Directions Method of Multipliers (ADMM)

The ADMM approach can also be used to decouple the precoder design across multiple BSs to solve the convex subproblem in (31). Generally, the ADMM is preferred over the dual decomposition in [12] for its robustness and improved convergence behavior [10]. In contrast to the primal decomposition, the ADMM relaxes the interference constraints by including in the objective function of each subproblem with a penalty pricing [9], [10]. Similar approach for the precoder design in the minimum power context was considered in [26].

Using the formulation presented in [10], [26], we can write the BS b specific ADMM subproblem for the jth iteration as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n} \\ \beta_{l,k,n},\boldsymbol{\zeta}_b}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}_b\|_q + \boldsymbol{\nu}_b^{(j)\,\mathrm{T}}(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j)}) + \frac{\rho}{2}\|\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j)}\|^2 \tag{37a}$$

$$\text{subject to} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \tag{37b}$$

$$\sum_{\bar{b}\in\bar{\mathcal{B}}_b}\zeta_{l,k,n,\bar{b}} + \sum_{\{\bar{l},\bar{k}\}\neq\{l,k\}}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{\bar{l},\bar{k},n}|^2 + \grave{N}_0 \leq \beta_{l,k,n} \tag{37c}$$

$$\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}|\mathbf{w}_{\bar{l},\bar{k},n}^{\mathrm{H}}\mathbf{H}_{b,\bar{k},n}\mathbf{m}_{l,k,n}|^2 \leq \zeta_{\bar{l},\bar{k},n,b}\ \forall \bar{k}\in\bar{\mathcal{U}}_b\ \forall n \tag{37d}$$

$$\text{and (19)} \tag{37e}$$

where $\boldsymbol{\zeta}_b^{(j)}$ denotes the interference vector updated from the earlier iteration and $\boldsymbol{\nu}_b^{(j)}$ represents the dual vector corresponding to the equality constraint at the jth iteration as

$$\boldsymbol{\zeta}_b = \boldsymbol{\zeta}_b^{(j)}. \tag{38}$$

Upon solving (37) for $\boldsymbol{\zeta}_b \forall b$ in the jth iteration, the next iterate is updated by exchanging the corresponding interference terms between two BSs b and b_k as

$$\boldsymbol{\zeta}_{b_k}(b)^{(j+1)} = \boldsymbol{\zeta}_b(b_k)^{(j+1)} = \frac{\boldsymbol{\zeta}_b(b_k) + \boldsymbol{\zeta}_{b_k}(b)}{2}. \tag{39}$$

The dual vector for the next iteration is updated by using the subgradient search to maximize the dual objective as

$$\boldsymbol{\nu}_b^{(j+1)} = \boldsymbol{\nu}_b^{(j)} + \rho \left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(j+1)} \right) \tag{40}$$

where step size parameter ρ is chosen in accordance with [10] to depend on the system model. The convergence rate is susceptible to the choice of step size parameter ρ. In numerical simulations, we consider step size $\rho = 2$. The above iterative procedure is performed until convergence or terminated when exceeding a predetermined number of steps, say, $J_{\max}$. Algorithm 2 outlines a practical way of implementing the ADMM based precoder design with minimal signaling overhead.

Algorithm 2: Distributed JSFRA scheme using ADMM

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}$, $\forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l, k, n$
Initialize: $i = 0, j = 0$ and $\mathbf{m}_{l,k,n}$ satisfying (37b)
update $\mathbf{w}_{l,k,n}$ with (23b) and $\tilde{\mathbf{u}}_{l,k,n}$ using (16c) and (18)
initialize the vectors $\boldsymbol{\nu}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \boldsymbol{\zeta}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \forall b \in \mathcal{B}$
foreach *BS* $b \in \mathcal{B}$ **do**
  repeat
    repeat
      solve for $\mathbf{m}_{l,k,n}$ and $\boldsymbol{\zeta}_b$ with (37) using $\boldsymbol{\zeta}_b^{(j)}$
      exchange $\boldsymbol{\zeta}_b$ among BSs in $\mathcal{B}$
      update $\boldsymbol{\zeta}_b^{(j+1)}$ and $\boldsymbol{\nu}_b^{(j+1)}$ using (39) and (40)
      update $j = j + 1$
    until *convergence or* $j \geq J_{\max}$, $\forall b \in \mathcal{B}$
    evaluate $\mathbf{w}_{l,k,n}$ using (23b) at each user
    update $\tilde{\mathbf{u}}_{l,k,n}$ using (16c) and (18) for SCA point or $\tilde{\epsilon}_{l,k,n}$ using (26c) for MSE operating point
    $i = i + 1$, $j = 0$
  until *convergence or* $i \geq I_{\max}$
end

Assuming that the following conditions are satisfied by the distributed schemes in each SCA step, *i.e.*, (i) by ensuring the uniqueness of the minimizer, and (ii) the distributed methods are carried out until convergence or for $j \to \infty$, then the convergence analysis follows the discussions presented in Appendices A and B. Moreover, it is still valid even if the receivers are updated together with the transmit precoders as in Algorithm 2, since the strict monotonic decrease in the objective sequence can still be ensured. The reason is that the MMSE receiver is optimal for the fixed transmit precoders obtained after each SCA update. However, in practice, to reduce the amount of information exchange between the coupling BSs, the distributed schemes are often iterated for a limited number of iterations, say, for $J_{\max}$ only. In such a case, convergence of the distributed algorithms cannot be ensured, since it is impossible to show that the objective value is monotonically decreasing in each SCA update step. We recall that in each primal or ADMM iteration, the global objective cannot be guaranteed to decrease monotonically.

C. Decomposition via KKT Conditions for MSE Formulation

In this section, we discuss an alternative way to decentralize the precoder design across the coordinating BSs in $\mathcal{B}$ based on the MSE reformulation method discussed in Section III-C. In contrast to Sections IV-A and IV-B, the problem is solved using the KKT conditions in which the transmit precoders, receive beamformers and the subgradient updates are performed at the same time to minimize the queue deviation with few number of iterations. The alternative way is motivated by the fact that distributed approaches presented in the preceding sections may not be efficient for large systems in terms of signaling overhead involved in exchanging the coupling variables and the receivers.

In this section, we provide an algorithm that can be of practical importance owing to lower signaling requirements. We consider an idealized TDD system due to the knowledge of local CSI at the transmitter that includes channel between BS b and all interfering users in the system. Similar work has been considered for the WSRM problem with minimum guaranteed rate constraints in [8]. However, in the proposed scheme, the maximum rate constraints are implicitly handled by the objective function without any need for explicit constraints. However, due to the non-differentiability of objective, the KKT conditions are not computationally useful to find the optimization variables. In order to make the objective function differentiable, we consider the following two conditions in which the absolute operator can be ignored without affecting the problem, namely,

- when the exponent q is even, or
- when the number of backlogged packets waiting to be transmitted for each user is large enough, *i.e*, $Q_k \gg \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$ to ignore the absolute operator in the objective and queues in the first place as well.

Assuming that either one of the above conditions is satisfied, the problem in (28) can be written as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}}\sum_{k\in\mathcal{U}_b} a_k\left(Q_k-\sum_{n=1}^{N}\sum_{l=1}^{L}t_{l,k,n}\right)^q \tag{41a}$$

subject to

$$\alpha_{l,k,n}: \left|1-\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2+\grave{N}_0 + \sum_{(x,y)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}\right|^2 \le \epsilon_{l,k,n} \tag{41b}$$

$$\sigma_{l,k,n}: \log_2(\tilde{\epsilon}_{l,k,n})+\frac{(\epsilon_{l,k,n}-\tilde{\epsilon}_{l,k,n})}{\log(2)\tilde{\epsilon}_{l,k,n}} \le -t_{l,k,n} \tag{41c}$$

$$\delta_b: \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right)\le P_{\max}\ \forall b \tag{41d}$$

where $\alpha_{l,k,n}, \sigma_{l,k,n}$ and δ_b are the dual variables corresponding to the constraints defined in (41b), (41c) and (41d).

The problem in (41) is solved using the KKT conditions which include stationarity, complementary slackness, and primal and dual feasibility requirement as shown in Appendix C. In particular, we propose an iterative algorithm to compute a solution to the system of equations (61) in Appendix C as

$$\mathbf{m}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L}\alpha_{y,x,n}^{(i-1)}\mathbf{H}_{b_k,x,n}^{\mathrm{H}}\mathbf{w}_{y,x,n}^{(i-1)}\mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)}\mathbf{H}_{b_k,x,n} + \delta_b\mathbf{I}_{N_T}\Bigg)^{-1}\alpha_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)} \tag{42a}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L}\mathbf{H}_{b_x,k,n}\mathbf{m}_{y,x,n}^{(i)}\mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_x,k,n}^{\mathrm{H}} + N_0\mathbf{I}_{N_R}\Bigg)^{-1}\mathbf{H}_{b_k,k,n}\ \mathbf{m}_{l,k,n}^{(i)} \tag{42b}$$

$$\epsilon_{l,k,n}^{(i)} = \left|1-\mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}^{(i)}\right|^2+\grave{N}_0 + \sum_{(x,y)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}^{(i)}\right|^2 \tag{42c}$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{\left(\epsilon_{l,k,n}^{(i)}-\epsilon_{l,k,n}^{(i-1)}\right)}{\log(2)\,\epsilon_{l,k,n}^{(i-1)}} \tag{42d}$$

$$\sigma_{l,k,n}^{(i)} = \left[\tfrac{a_k\,q}{\log(2)}\left(Q_k-\sum_{n=1}^{N}\sum_{l=1}^{L}t_{l,k,n}^{(i)}\right)^{(q-1)}\right]^{+} \tag{42e}$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)}+\rho^{(i)}\left(\frac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}}-\alpha_{l,k,n}^{(i-1)}\right) \tag{42f}$$

Since the dual variables $\alpha^{(i)}$ and $\sigma^{(i)}$ are interdependent in (42), one has to be fixed to optimize for the other. So, $\alpha^{(i)}$ is fixed to evaluate $\sigma^{(i)}$ using (42e). At iteration i, the dual variable $\alpha^{(i)}$ is a point in the line segment between $\alpha^{(i-1)}$ and $\frac{\sigma^{(i)}}{\epsilon^{(i)}}$ determined by using a diminishing or a fixed step size $\rho^{(i)}\in(0,1)$. The choice of $\rho^{(i)}$, which depends on the system, affects the convergence behavior and also controls the oscillations in the users' rate when $\sigma^{(i)}$ is negative (before projection) due to over-allocation. However, in all numerical simulations, the step size $\rho^{(i)}$ is fixed to 0.1 irrespectively.

For a physical interpretation, when the allocated rate $t_k^{(i-1)}$ is greater than Q_k for a user k, the corresponding dual variable $\sigma^{(i)}$ will be negative and due to the projection operator $[x]^+$ in (42e), it will be zero, thereby forcing $\alpha_k^{(i)} < \alpha_k^{(i-1)}$ as in (42f). Once $\alpha_k^{(i)}$ is reduced, the precoder weight in (42a) is lowered to make the rate $t_k^{(i)} < t_k^{(i-1)}$ eventually.

The KKT expressions in (42) are solved in an iterative manner by initializing the transmit and the receive beamformers $\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}$ with the single user beamforming and the MMSE vectors. The dual variable α's are initialized with ones to have equal priorities to all the users in the system. Then the transmit and the receive beamformers are evaluated using the expressions in (42). The transmit precoder in (42a) depends on the BS specific dual variable δ_b, which can be found by bisection search satisfying the total power constraint (41d). Note that the fixed SCA operating point is given by $\tilde{\epsilon}_{l,k,n}=\epsilon_{l,k,n}^{(i-1)}$, which is considered in the expression (42).

To obtain a practical distributed precoder design, we assume that each BS b knows the corresponding equivalent channels $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}, \forall k\in\mathcal{U}$, which embeds the receivers $\mathbf{w}_{l,k,n}$, through precoded uplink pilot signaling. We extend the decentralization methods discussed in [25], for the current problem as follows. Upon receiving the updated transmit precoders from all BSs in $\mathcal{B}$, each user will evaluate the MMSE receiver (42b) and notify to all BSs by using precoded uplink pilots. On receiving the pilots, each BS updates the MSE in (24) as

$$\epsilon_{l,k,n}^{(i)} = 1 - \mathbf{w}_{l,k,n}^{(i)\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)}. \tag{43}$$

Using $\epsilon_{l,k,n}^{(i)}$, the variables $t_{l,k,n}^{(i)}, \sigma_{l,k,n}^{(i)}$, and $\alpha_{l,k,n}^{(i)}$ are updated using (42d), (42e) and (42f) respectively, and the obtained dual variables $\alpha_{l,k,n}$ are exchanged between the BSs to evaluate the transmit precoders $\mathbf{m}_{l,k,n}^{(i+1)}$ for the next iteration. The SCA operating point is also updated with the current MSE value.

Algorithm 3: KKT approach for the JSFRA scheme

Input: $a_k,\, Q_k,\, \mathbf{H}_{b,k,n},\ \forall b \in \mathcal{B},\, \forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l,k,n$
Initialize: $i = 1$, $\mathbf{w}_{l,k,n}^{(0)}, \tilde{\epsilon}_{l,k,n}$ randomly, dual variables $\alpha_{l,k,n}^{(0)} = 1$, and $I_{\max}$ for certain value
foreach *BS* $b \in \mathcal{B}$ **do**
  repeat
    update $\mathbf{m}_{l,k,n}^{(i)}$ using (42a), and perform precoded downlink pilot transmission
    find $\mathbf{w}_{l,k,n}^{(i)}$ using (42b) at each user
    evaluate $\epsilon_{l,k,n}^{(i)}$, $t_{l,k,n}^{(i)}$, $\sigma_{l,k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ using (42c) and (42d), (42e) and (42f) at each user with the updated $\mathbf{w}_{l,k,n}^{(i)}$
    using precoded uplink pilots, $\mathbf{w}_{l,k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ are notified to all BSs in $\mathcal{B}$
    $i = i + 1$
  until *until convergence or* $i \geq I_{\max}$
end

To avoid the backhaul exchanges between the BSs, as an alternative approach, users can perform all the required processing and BSs will update the precoders based on the feedback from all the users. Upon receiving the transmit precoders from the BSs, each user will update the receive beamformer $\mathbf{w}_{l,k,n}$, the MSE $\epsilon_{l,k,n}$, and the dual variables $\sigma_{l,k,n}$ and $\alpha_{l,k,n}$. Then, the updated $\alpha_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are notified to the BSs using two separate precoded uplink pilots with $\tilde{\mathbf{w}}_{l,k,n}^{(i)} = (\alpha_{l,k,n}^{(i)})^{1/2}\, \mathbf{w}_{l,k,n}^{*(i)}$ and $\bar{\mathbf{w}}_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i)} \mathbf{w}_{l,k,n}^{*(i)}$ as the precoders, where $\mathbf{x}^*$ is the complex conjugate of $\mathbf{x}$. Upon receiving the precoded uplink pilots, each BS evaluates the equivalent channels $\mathbf{H}_{b,k,n}^{\mathrm{T}} \tilde{\mathbf{w}}_{l,k,n}^{(i)}$ and $\mathbf{H}_{b,k,n}^{\mathrm{T}} \bar{\mathbf{w}}_{l,k,n}^{(i)}$ to update the transmit precoders using (42a). The algorithmic description of the above scheme is presented in Algorithm 3.

We conclude this section by providing some remarks on selecting an algorithm within centralized and distributed approaches for certain scenarios. Among the centralized algorithms proposed in Section III, both the SINR relaxation and the MSE reformulation schemes are equally attractive when $N_R > 1$. However, when $N_R = 1$ the SINR relaxation is preferable, since the receiver update is not needed, unlike the MSE reformulation which requires the scalar MMSE receiver update. Similarly, for the distributed approaches with $N_R = 1$, the SINR relaxation is preferred to the MSE reformulation schemes, since the variables to be exchanged among BSs are only the scalar interference values. However, when $N_R > 1$ the KKT scheme in Section IV-C is favorable, since the overhead required to achieve a certain throughput improvement is less, as compared to the primal decomposition or ADMM.

V. Simulation Results

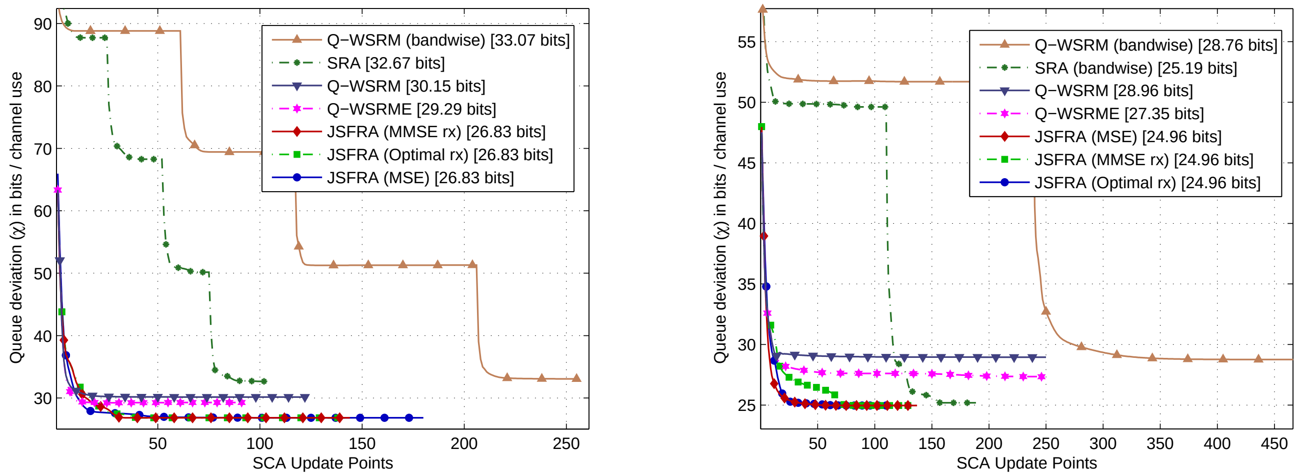
The simulations carried out in this work consider the path loss (PL) varying uniformly across all users in the system with the small-scale fading drawn from the *i.i.d.* samples. The queues are generated based on the Poisson process with the average values specified for each numerical experiments.

A. Centralized Solutions

We discuss the performance of the centralized algorithms in Section III for some system configurations. To begin with, we consider a single cell single-input single-output (SISO) model operating at 10 dB signal-to-noise ratio (SNR) with $K = 3$

TABLE I
Sub-channel-wise listing of channel gains and rate allocations by different algorithms for a scheduling instant

Users	Queued Packets	Channel Gains			Q-WSRME approach (modified *backpressure*)			JSFRA Scheme			Q-WSRM band Alloc Scheme		
		SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3
1	4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
2	8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
3	4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining backlogged packets (χ)					3.92 bits			2.51 bits			5.89 bits		



(a). System model $\{N, N_B, K, N_T, N_R\} = \{4, 3, 9, 4, 1\}$, number of backlogged bits for each user $Q_k = [14, 15, 14, 8, 12, 9, 12, 11, 11]$ bits

(b). System model $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$, number of backlogged bits for each user $Q_k = [9, 12, 8, 12, 5, 4, 10, 8, 5]$ bits

Fig. 1. Total number of backlogged packets χ present in the system after each SCA updates using $\ell_1(q = 1)$ norm for JSFRA schemes

TABLE II

NUMBER OF BACKLOGGED BITS ASSOCIATED WITH EACH USER FOR A SYSTEM $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$.

q	user indices								χ
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
∞	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
Q_k	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	

users sharing $N = 3$ sub-channel resources. The total number of backlogged packets waiting at the transmitter for each user is given by $Q_1 = 4$, $Q_2 = 8$ and $Q_3 = 4$ bits, respectively.

Table I tabulates the channels of the users over each sub-channel followed by the rates assigned by three different algorithms, Q-WSRME allocation, JSFRA approach and the sub-channel wise Q-WSRM scheme using the MSE reformulation for all cases [5]. The metric used for the comparison is the total number of backlogged bits left over after each transmission, which is denoted as $\chi = \sum_{k=1}^{K} [Q_k - t_k]^+$. Even though $\mathcal{U}(1)$ and $\mathcal{U}(3)$ have equal number of backlogged packets of $Q_1 = Q_3 = 4$ bits, user $\mathcal{U}(3)$ is scheduled in the first sub-channel due to the better channel condition. In contrast, the JSFRA approach assigns the first user on the first sub-channel, which reduces the total number of backlogged packets. The rate allocated for $\mathcal{U}(2)$ on the second sub-channel is higher in JSFRA scheme compared to the others. It is due to the efficient allocation of the total power shared across the sub-channels.

For a MIMO setup, we consider a system with $N = 4$ and $N = 2$ sub-channels with $N_B = 3$ BSs, each equipped with $N_T = 4$ transmit antennas operating at 10dB SNR, serving $|\mathcal{U}_b| = 3$ users each. The PL difference between the BSs and the users are uniformly generated from $[0, -3]$ dB and the BS-user associations are made by selecting the BS with the lowest PL component. Fig. 1(a) and Fig. 1(b) compares the performance of the centralized schemes for $N_R = 1$ and $N_R = 2$ receive antenna cases respectively.

The comparison in Fig. 1 is made in terms of the total number of residual bits remaining in the system after each SCA update. The Q-WSRM scheme is not optimal due to the problem of over-allocation when the number of queued packets is small. In contrast, the Q-WSRME algorithm provides better allocation with the explicit rate constraint to avoid the over-allocation. For both scenarios in Fig. 1, the Q-WSRME performs marginally inferior to the JSFRA algorithms due to the weights used in the algorithm, since the Q-WSRME scheme favors the users with the large number of backlogged packets as compared to the users with better channel conditions. Note that the receivers are updated along with the SCA update instants *i.e*, $K_{\max} = 1$ for both Fig. 1(a) and Fig. 1(b). However, the performance loss incurred by the combined update of the SCA operating point and the receiver is marginal.

The behavior of the JSFRA algorithm for different exponents q is outlined in the Table II for the users located at the cell-edge of the system employing $N_T = 4$ transmit antennas. It is evident that the JSFRA algorithm minimizes the total number of queued bits for the ℓ_1 norm compared to the ℓ_2 norm, which is shown in the column displaying the total number of left over packets χ in bits. The ℓ_∞ norm provides fair allocation of the resources by making the leftover packets to be equal for all users to $\chi_k = 3.58$ bits.

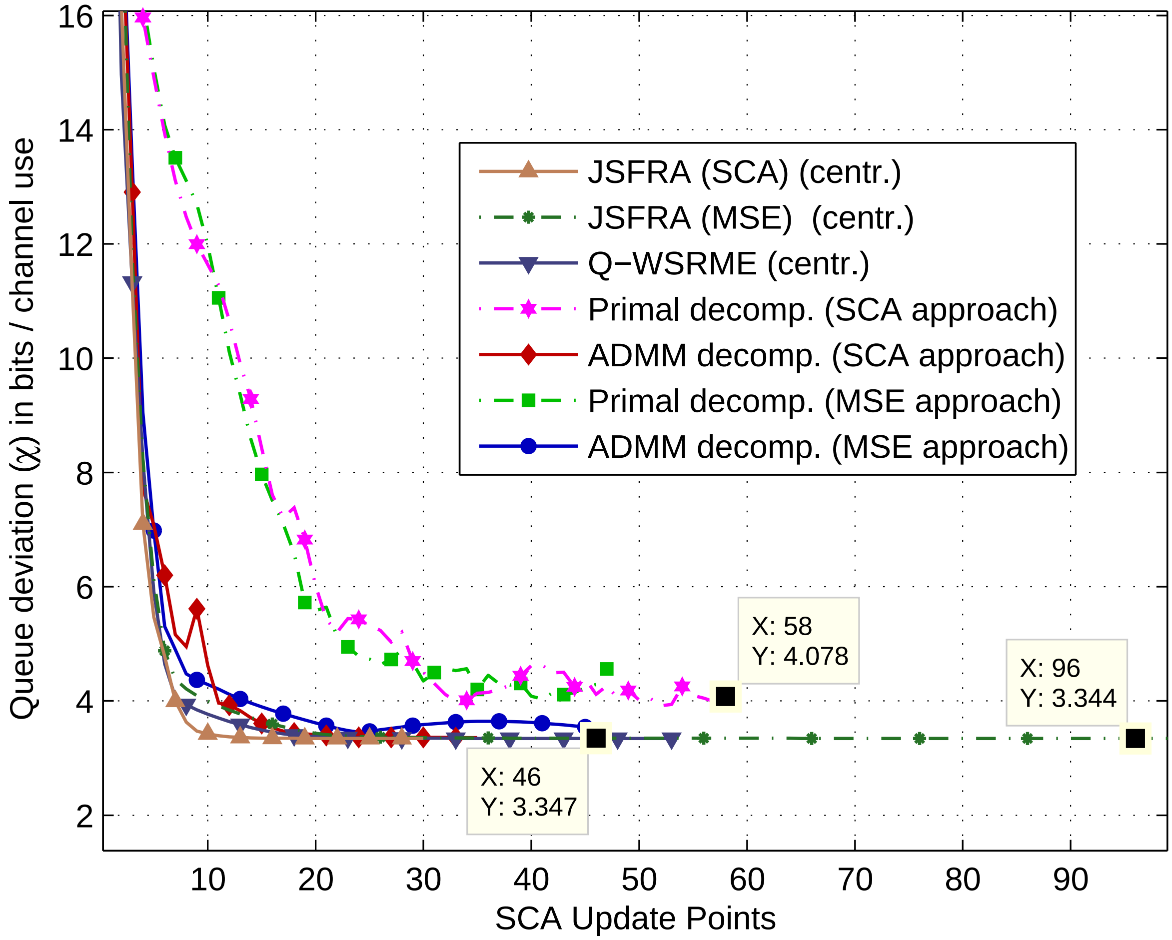


Fig. 2. Convergence of the centralized and the distributed algorithms for $\{N, N_B, K, N_T, N_R\} = \{3, 2, 8, 4, 1\}$ using ℓ_1 norm for JSFRA schemes with $Q_k = [5, 7, 9, 11, 8, 12, 5, 4]$ bits

B. Distributed Solutions

The distributed algorithms are compared using the total number of backlogged packets after each SCA update. Fig. 2 compares the performances of the algorithms with the PL varies uniformly between $[0, -6]$ dB and each BS serves $|\mathcal{U}_b| = 4$ users. As discussed in Section IV, the performance and the convergence speed of the distributed algorithms are susceptible to the step size $\rho^{(i)}$. Due to the fixed interference levels in the primal approach, it may lead to infeasible solutions if the initial or any intermediate update is not feasible.

Fig. 2 compares the performance of the JSFRA schemes discussed in Sections III-B and III-C using primal and ADMM approaches. For each SCA update, the primal or the ADMM scheme is performed for $J_{\max} = 20$ iterations to exchange the respective coupling variables. The number of backlogged packets only at the SCA points are marked in the figure. The performance of the distributed approaches is similar to that of the centralized schemes if the distributed algorithms are allowed converge. However, in our simulations, we observe that $J_{\max} = 20$ is sufficient for the ADMM to converge.

Fig. 3 compares the performance of the centralized and the KKT algorithm in Section IV-C for different exponents with PL chosen uniformly between $[0, -3]$ dB. The ℓ_1 norm JSFRA scheme performs better over other ℓ_q norms due to the greedy nature of the objective. The KKT approach for ℓ_1 norm is not defined due to the non-differentiability of the objective as discussed in Section IV-C. If used for ℓ_1 norm, the over-allocation will not affect the dual variables $\sigma_{l,k,n}$ and $\alpha_{l,k,n}$ since the queue deviation is raised to the power zero in (42e). A heuristic method is proposed in Fig. 3 by assigning zero for $\sigma_{l,k,n}$ when $Q_k - t_k < 0$ to addresses the over-allocation. The heuristic approach oscillates near the converging point with the deviation determined by the factor ρ used in (42f). The objective values are mentioned in the legend for all the schemes and the ℓ_1 norm is used for comparison.

C. Queuing Analysis over Multiple Transmission Slots

In this section, we numerically study the performance of the centralized algorithms with different ℓ_q values over multiple transmission slots. The system model examined for the illustrations is provided in Fig. 4. For all users in the system, the

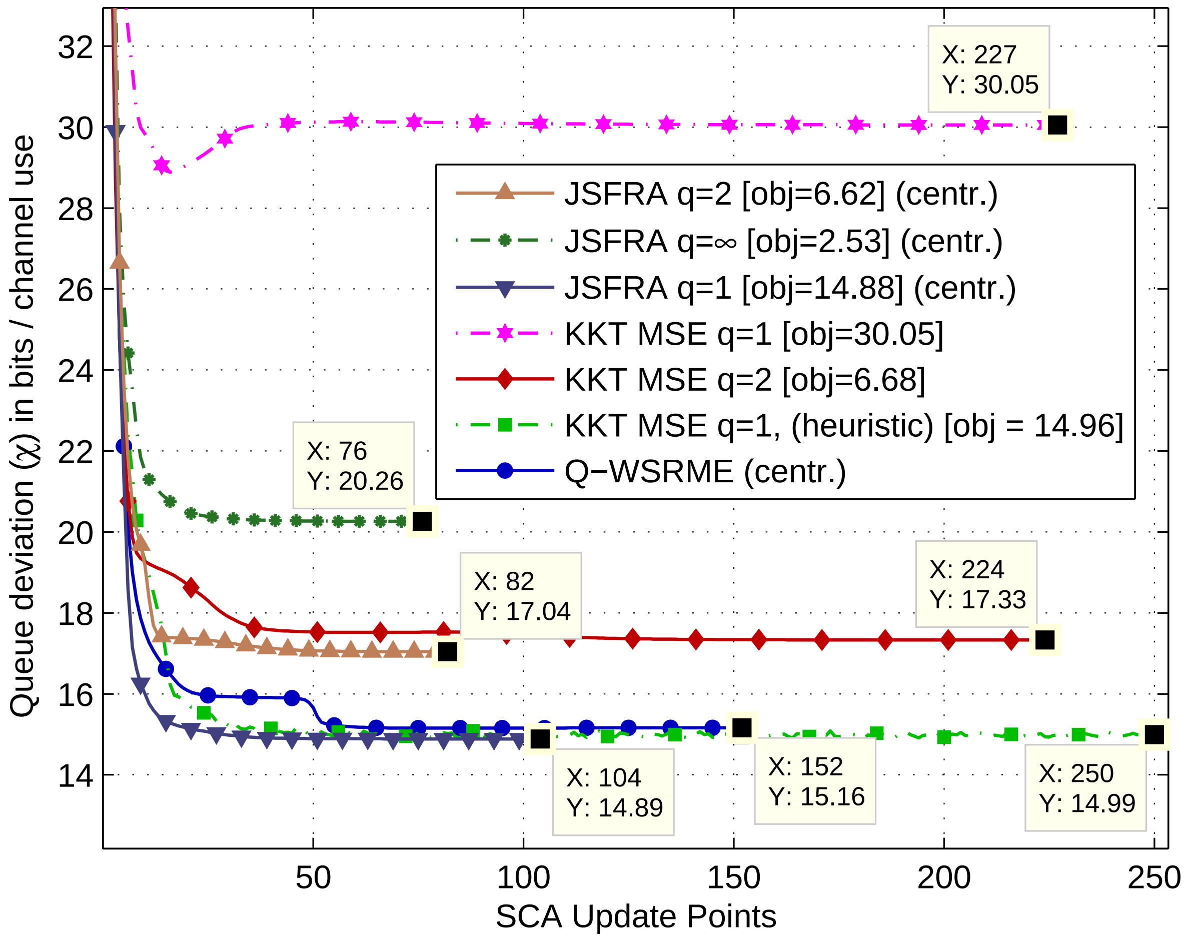
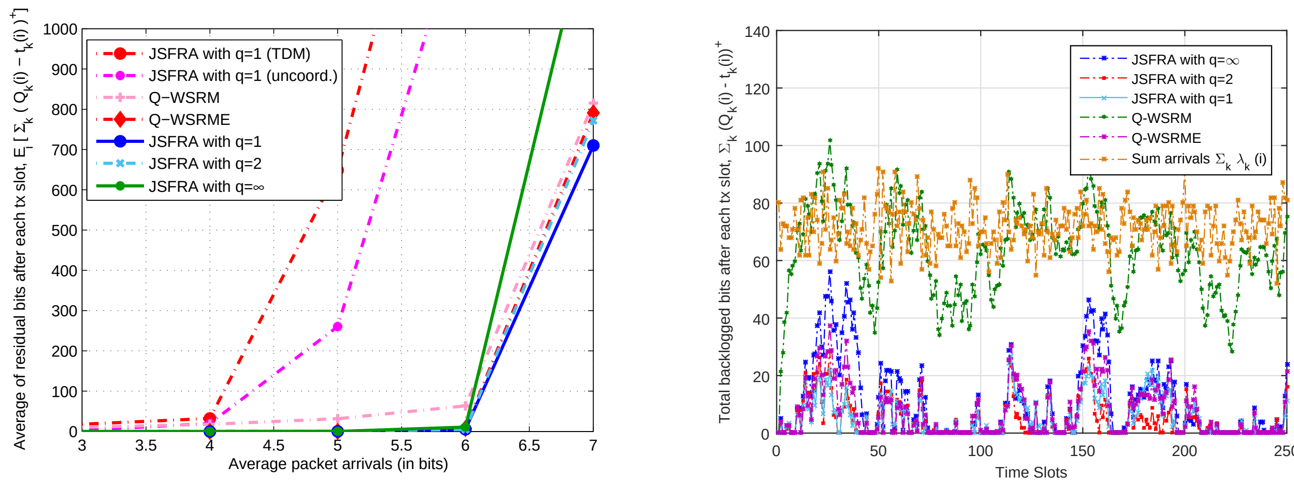


Fig. 3. Impact of varying q on the total number of backlogged packets after each SCA update for a system $\{N, N_B, K, N_T, N_R\} = \{5, 2, 8, 4, 1\}$ and $Q_k = [9, 16, 14, 16, 9, 13, 11, 12]$ bits

average arrivals A_k's are fixed and varied equally for the model considered in Fig. 4(a), and for Fig. 4(b), the average arrival is fixed to be $A_k = 6$ bits. Note that the instantaneous arrivals $\lambda_k(i)$ are all different and it follows the Poisson process. The PL is modeled as a uniform random variable $[0, -6]$ dB.

Fig. 4(a) plots the average of the total number of backlogged packets left out in the system after each transmission instant, *i.e*, $\mathbb{E}_i\left[\sum_k \left[Q_k(i) - t_k(i)\right]^+\right]$. Unlike the Q-WSRM scheme, the average backlogged packets of the ℓ_2 JSFRA scheme is comparable to the Q-WSRME approach for all average arrival rates due to the explicit rate constraints (14). However, when $A_k \geq 7$ bits in Fig. 4(a), both Q-WSRM and Q-WSRME schemes perform the same since the problem of over-allocation is negligible. The performance of the ℓ_1 JSFRA scheme outperforms all other schemes in terms of the average number of residual packets due to the greedy allocation at each instant.

Fig. 4(a) also includes the uncoordinated ℓ_1 JSFRA scheme and the time division multiplexing (TDM) mode, which ignores the inter-cell interference terms in the SINR expressions while designing the precoders. The performance of the TDM scheme with the total power constraint is inferior to the uncoordinated transmission due to the diverse user PL variations in the system model. Fig. 4(b) compares the number of backlogged packets left in the system after each transmission slot by different centralized algorithms. The total number of residual packets for the Q-WSRM scheme is noticeably large in comparison with the other schemes in Fig. 4(b). This performance loss is due to the inability in controlling the over-allocations at each instant. The instantaneous fairness constraint imposed by the ℓ_∞ JSFRA scheme is effective in reducing the number of backlogged packets for the average arrival rate considered in Fig. 4(b). However, in Fig. 4(a), the performance of the ℓ_∞ JSFRA is inferior to the Q-WSRM scheme, since the fairness is not effective when the system is unstable, *i.e*, when $A_k \geq 7$ in the current model.



(a). Average backlogged packets in the system after 250 transmission instants (b). Total backlogged packets at each transmission slot for $A_k = 6$ bits

Fig. 4. Time analysis of the Queue dynamics for a system $\{N, N_B, K, N_T, N_R\} = \{4, 2, 12, 4, 1\}$

VI. Conclusions

In this paper, we have addressed the problem of allocating downlink space-frequency resources to the users in a multi-cell MIMO IBC system using OFDM. The resource allocation is considered as a joint space-frequency precoder design problem since the allocation of a resource to a user is obtained by a non-zero precoding vector. We have proposed the JSFRA scheme by relaxing the nonconvex constraint by a sequence of convex subsets using the SCA for designing the precoders to minimize the total number of user queued packets. Additionally, an alternative MSE reformulation approach has been proposed by using the SCA to address the nonconvex constraints for a fixed MMSE receivers. We also proposed various methods to decentralize the precoder designs for the JSFRA problem using primal and ADMM methods. Finally, we proposed a practical iterative algorithm to obtain the precoders in a decentralized manner by solving the KKT expressions of the MSE reformulated problem. The proposed iterative algorithm requires few iterations and limited signaling exchange between the coordinating BSs to obtain the efficient precoders for a given number of iterations. Numerical results are used to compare the performance of the proposed schemes.

Appendix A
Convergence Proof for Centralized Algorithm

The following conditions are required to show the convergence of Algorithm 1, which is used to solve problems (16) and (26) in an iterative manner.

(a) Uniqueness of the minimizer in each step
(b) Objective function should be bounded below
(c) Feasible set should be compact
(d) Sequence of objective values should be strictly decreasing

The uniqueness condition in (a) is required to ensure strict monotonicity in the objective sequence. Therefore, upon satisfying the conditions (a)-(d), we can show that the objective sequence generated by Algorithm 1 converges, by using [27, Th. 3.14]. Then, by following the discussions in [19], [22], [23], we can show that every limit point of sequence of iterates is a stationary point of nonconvex problems in (16) and (26).

Before discussing further on the analysis, let us consider a generalized formulation for the problems in (16) and (26) as

$$\underset{\mathbf{m},\mathbf{w},\boldsymbol{\gamma}}{\text{minimize}} \quad \hat{f}(\mathbf{m},\mathbf{w},\boldsymbol{\gamma}) \tag{44a}$$
$$\text{subject to} \quad h(\boldsymbol{\gamma}) - g_0(\mathbf{m},\mathbf{w}) \leq 0 \tag{44b}$$
$$g_1(\mathbf{m},\mathbf{w}) \leq 0 \tag{44c}$$
$$g_2(\mathbf{m}) \leq 0 \tag{44d}$$

where g_2 and $\hat{f}$ are convex, h is linear, and g_0 and g_1 are convex w.r.t either $\mathbf{m}$ or $\mathbf{w}$, but not jointly convex on both variables. The constraint (44b) corresponds to either (16b) or (26b) and the constraint (44c) represents to either (16c) or (26c) correspondingly. Other convex constraints are handled by (44d) and the feasible set of (44) is given by

$$\mathcal{F} = \{\, \mathbf{m},\mathbf{w},\boldsymbol{\gamma} \mid \; h(\boldsymbol{\gamma}) - g_0(\mathbf{m},\mathbf{w}) \leq 0, \; g_1(\mathbf{m},\mathbf{w}) \leq 0, g_2(\mathbf{m}) \leq 0 \}. \tag{45}$$

We use the following notations to discuss the convergence of Algorithm 1. Since it involves two nested loops, *i.e.*, one for SCA and another for AO, we denote the AO step by a superscript (i) and the SCA iteration by a subscript k. Let $\mathbf{m}$, $\mathbf{w}$, and $\boldsymbol{\gamma}$ be the vector stacking all transmit precoders, receive beamformers, and other optimization variables used in (16) and (26), respectively. Now, let us consider AO step i with fixed $\mathbf{w}$. The solution obtained in the kth SCA step is given by $\mathbf{m} = \mathbf{m}_k^{(i)}$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}_k^{(i)}$. As $k \to \infty$, objective sequence converges, and the respective solution is denoted by $\mathbf{m}_*^{(i)}$ and $\boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}$. Similarly, while alternating the optimization variables for fixed $\mathbf{m}$, as $k \to \infty$, the solution obtained is represented as $\mathbf{w} = \mathbf{w}_*^{(i)}$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}_{*|\mathbf{m}}^{(i)}$. In the following discussion we will show that the conditions listed in (a)-(d) are satisfied by Algorithm 1 using the above notations.

A. Uniqueness of the Iterates and Strong Convexity

The uniqueness of the iterates $\{\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k|\mathbf{w}}^{(i)}\}$ can be ensured for the MSE reformulated problem in (28) when all the constraints are active. However, if $\hat{f}(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k|\mathbf{w}}^{(i)}) = 0$ after some iteration, say k and i, then the uniqueness is not ensured as there can be multiple solutions in the feasible set.

To ensure the uniqueness of transmit and receive beamformers in each iteration, the objective function of subproblems (20) and (28) is regularized by a strongly convex function in each SCA iteration k and AO update step i as

$$\hat{f}(\mathbf{z}) = \|\tilde{\mathbf{v}}\|_q \tag{46a}$$

$$f(\mathbf{z}) = \hat{f}(\mathbf{z}) + \tau_k^{(i)} \|\mathbf{z} - \mathbf{z}_k^{(i)}\|_2^2 \tag{46b}$$

where $\mathbf{z}$ is a vector formed by stacking all the optimization variables, *i.e.*, $[\mathbf{m}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}]$ or $[\mathbf{m}_*^{(i)}, \mathbf{w}, \boldsymbol{\gamma}]$ depending on the variables being optimized, and $\mathbf{z}_k^{(i)} \triangleq [\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)}]$ is a vector formed by stacking the solution obtained from SCA step $k-1$ and AO iteration i, respectively. The positive constant $\tau_k^{(i)} > 0$ ensures strong convexity of $f(\mathbf{z})$ in each step [28, Sec. 3.4.3]. Thus, due to strong convexity in $f(\mathbf{z})$, uniqueness of the minimizer is ensured in each iteration [29], [30].

B. Bounded Objective Function and Compact Feasible Set

The feasible set of (16) and (26) is bounded and closed due to the total transmit power constraint given by (16d) and (26d), respectively. Therefore, the feasible region is compact.

Note that the norm function present in the objective satisfies $\|\tilde{\mathbf{v}}\|_q \geq 0$ for all exponents q used in ℓ_q norm, therefore, it is bounded from below. Moreover, due to the compactness of the feasible set, it is bounded from above as well.

C. Strict Monotonicity of the Objective Sequence

In the following discussions, we consider the modified objective $f(\mathbf{z})$ in (46b) instead of $\hat{f}(\mathbf{z})$ to exploit the uniqueness of the minimizer at each iteration. However, note that upon convergence $f(\mathbf{z})$ is equal to $\hat{f}(\mathbf{z})$. Therefore, the discussions are valid for the JSFRA problems in (16) and (26) by regularizing the objective functions (16a) and (26a) as (46b).

To begin with, let us consider the variable $\mathbf{w}$ begin fixed for the ith AO with the optimal value found in previous iteration $i-1$ as $\mathbf{w}_*^{(i-1)}$. In order to solve for $\mathbf{m}$ in SCA iteration k, we linearize the nonconvex function g_0 using previous SCA iterate $\mathbf{m}_k^{(i)}$ of $\mathbf{m}$ as

$$\begin{aligned}\hat{g}_o(\mathbf{m}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \triangleq\ & g_0(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}) \\ & + \nabla g_0(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)})^{\mathrm{T}}(\mathbf{m} - \mathbf{m}_k^{(i)}).\end{aligned} \tag{47}$$

Let $\mathcal{X}_k^{(i)}$ be the feasible set for the ith AO iteration and the kth SCA point for a fixed $\mathbf{w}_*^{(i-1)}$ and $\mathbf{m}_k^{(i)}$. Similarly, $\mathcal{Y}_k^{(i)}$ denotes the feasible set for a fixed $\mathbf{m}_*^{(i)}$ and $\mathbf{w}_k^{(i)}$. Using (47), the convex subproblem for the ith AO iteration and the kth SCA point for variables $\mathbf{m}$ and $\boldsymbol{\gamma}$ is given by

$$\underset{\mathbf{m}, \boldsymbol{\gamma}}{\text{minimize}} \quad f(\mathbf{m}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}) \tag{48a}$$

$$\text{subject to} \quad h(\boldsymbol{\gamma}) - \hat{g}_0(\mathbf{m}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \leq 0 \tag{48b}$$

$$g_1(\mathbf{m}, \mathbf{w}_*^{(i-1)}) \leq 0, \quad g_2(\mathbf{m}) \leq 0 \tag{48c}$$

The feasible set defined by the problem (48) is denoted by $\mathcal{X}_k^{(i)} \subset \mathcal{F}$. To prove the convergence of the SCA updates in the ith AO iteration, let us consider that (48) yields $\mathbf{m}_{k+1}^{(i)}$ and $\boldsymbol{\gamma}_{k+1}^{(i)}$ as the solution in the kth iteration. The point $\mathbf{m}_{k+1}^{(i)}$ and $\boldsymbol{\gamma}_{k+1}^{(i)}$, which minimize the objective function satisfy

$$h(\boldsymbol{\gamma}_{k+1}^{(i)}) - g_0(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}) \leq$$

$$h(\boldsymbol{\gamma}_{k+1}^{(i)}) - \hat{g}_0(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \leq 0. \quad (49)$$

Using (49), we can show that $\{\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k+1}^{(i)}\}$ is feasible, since the initial SCA operating point $\mathbf{m}_*^{(i-1)}$ was chosen to be feasible from the $(i-1)$th AO iteration. In each SCA step, the feasible set includes the solution from the previous iteration as $\{\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k+1}^{(i)}\} \in \mathcal{X}_{k+1}^{(i)} \subset \mathcal{F}$, therefore, it decreases the objective as [22], [23], [31]

$$\begin{aligned} f(\mathbf{m}_0^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_0^{(i)}) &\geq f(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)}) \\ &\geq f(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k+1}^{(i)}) \geq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}). \quad (50)\end{aligned}$$

Thus, the sequence $\{f(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)})\}$ is nonincreasing. Using strong convexity of the objective, we can show that the inequalities in (50) are strict, *i.e.*, the sequence of objectives returned by Algorithm 1 is strictly decreasing. To this end, let us consider $\mathbf{z}_k^{(i)} \triangleq [\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)}]$ as the minimizer for (48) in the $(k-1)$th SCA step and let $\mathbf{z}_{k+1}^{(i)}$ be the minimizer in the kth step. At the kth SCA iteration, $\forall \mathbf{z} \in \mathcal{X}_k^{(i)}$, it follows

$$\nabla f(\mathbf{z}_{k+1}^{(i)})^{\mathrm{T}}(\mathbf{z} - \mathbf{z}_{k+1}^{(i)}) \geq 0 \quad (51a)$$

$$f(\mathbf{z}) - f(\mathbf{z}_{k+1}^{(i)}) \geq c\,\|\mathbf{z} - \mathbf{z}_{k+1}^{(i)}\|^2 \quad (51b)$$

where $c > 0$ be the constant of strong convexity, defined as $c \triangleq \min_k\{\tau_k^{(i)}\}$, and $\mathbf{z}_{k+1}^{(i)}$ is the solution. Therefore, by using (51) and $\mathbf{z}_k^{(i)} \in \mathcal{X}_k^{(i)}$, we can show that $f(\mathbf{z}_k^{(i)}) > f(\mathbf{z}_{k+1}^{(i)})$ holds strictly in each SCA step unless $\mathbf{z}_k^{(i)} \to \mathbf{z}_*^{(i)}$ as $k \to \infty$. Now, by utilizing the facts that (i) $\{f(\mathbf{z}_k^{(i)})\}$ is monotonic, and (ii) the uniqueness of the minimizer (see (51)), strict monotonicity of $\{f(\mathbf{z}_k^{(i)})\}$ is ensured in each SCA step [32]. Furthermore, by using the fact that $\{f(\mathbf{z}_k^{(i)})\}$ is also bounded, we can show that the objective sequence converges as $k \to \infty$ in AO step i, and let $\mathbf{z}_*^{(i)} \triangleq [\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}]$ be a stacked vector of the corresponding minimizer of (48) when $k \to \infty$ in AO step i.

Once $\mathbf{m}_*^{(i)}$ is obtained for fixed $\mathbf{w}$, then (44) is solved for $\mathbf{w}$ with fixed $\mathbf{m}$. However, after fixing $\mathbf{m}$ as $\mathbf{m}_*^{(i)}$ in (44), the problem is still nonconvex due to (44b). Following similar approach as above, the minimizer $\{\mathbf{m}_*^{(i)}, \mathbf{w}_{k+1}^{(i)}, \boldsymbol{\gamma}_{k+1}^{(i)}\}$ can be found in each SCA step k by solving (48) iteratively. Note that $\boldsymbol{\gamma}_{k+1}^{(i)}$ is reused since the variable $\mathbf{m}$ is fixed in the ith AO iteration. The convergence and the nonincreasing behavior of the objective follow similar arguments as above.[5] The solution obtained by solving (48) iteratively until convergence of the objective value is given as $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i)}, \boldsymbol{\gamma}_{*|\mathbf{m}}^{(i)}\} \in \mathcal{Y}_*^{(i)} \subset \mathcal{F}$.

Finally, to prove the global convergence of the objective sequence, we need to show that the AO updates also produce a nonincreasing sequence of objectives, *i.e.*,

$$f(\mathbf{m}_*^{(i)}, \mathbf{w}_0^{(i)}, \boldsymbol{\gamma}_0^{(i)}) \leq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}). \quad (52)$$

Let $\mathbf{m}_*^{(i)}$ and $\boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}$ be the solution obtained by solving (48) with respect to $\mathbf{m}$ and $\boldsymbol{\gamma}$ by performing SCA update as $k \to \infty$ in the ith AO step with fixed $\mathbf{w} = \mathbf{w}_*^{(i-1)}$. In order to find $\mathbf{w}_0^{(i)}$, we fix $\mathbf{m}$ as $\mathbf{m}_*^{(i)}$ and solve problem (48) for $\mathbf{w}$ and $\boldsymbol{\gamma}$. Since the constraint (48b) is linearized around $\mathbf{z}_*^{(i)}$, the fixed operating point is also included in the feasible set by following (49) as $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}\} \in \mathcal{Y}_0^{(i)}$, therefore, we have

$$f(\mathbf{m}_*^{(i)}, \mathbf{w}_0^{(i)}, \boldsymbol{\gamma}_0^{(i)}) \leq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}). \quad (53)$$

Now, by using (48b), we can ensure that $\mathbf{z}_*^{(i)} \in \{\mathcal{X}_*^{(i)} \cap \mathcal{Y}_0^{(i)}\}$ in each AO update i while alternating the optimization variables from $\mathbf{m}, \boldsymbol{\gamma}$ to $\mathbf{w}, \boldsymbol{\gamma}$. Moreover, due to strong convexity of the objective function (46b), by following (51), we can show that (53) holds with strict inequality in each AO update unless the overall objective sequence converges. Now, by combining (50), (51) and (53), we can ensure strictly nonincreasing nature of the objective sequence $\{f(\mathbf{z}_k^{(i)})\}$ in each SCA and AO step.

D. Stationarity of Limit Points

We now discuss the convergence properties of sequence of iterates. For convenience, let us consider a unified superscript t to refer to the index of both SCA and AO procedures (*i.e.*, index t denotes SCA step k and AO step i). Then, we denote iterate $\mathbf{x}^t$ as a vector formed by stacking the minimizer of (48) at iteration t as $\mathbf{x}^t = \mathbf{z}_k^{(i)}$, and let $\{\mathbf{x}^t\}$ be the sequence formed by collecting the minimizers of (48) from each iteration.

Note that the objective sequence $\{f(\mathbf{x}^t)\}$ is bounded and nonincreasing (see Appendices A-B and A-C). Therefore, by using [33, Prop. A.3], we can ensure the convergence of $\{f(\mathbf{x}^t)\}$. Unfortunately, such a claim cannot be made on the convergence of sequence of iterates $\{\mathbf{x}^t\}$. However, due to compactness of the feasible set, the sequence of iterates $\{\mathbf{x}^t\} \subset \mathcal{F}$ is bounded.

[5] Note that the receive beamformers can also be designed by the MMSE receiver using closed-form (23b) instead designing recursively by updating the SCA operating point as in optimal receiver (22b).

Therefore, the sequence $\{\mathbf{x}^t\}$ has at least one limit such that there exists a subsequence of $\{\mathbf{x}^t\}$ that converges to it, by following [33, Prop. A.5].

We now show that every limit point of the sequence $\{\mathbf{x}^t\}$ is a stationary point, *i.e.,* the limit point of every convergent subsequence is a stationary point. To do this, let us consider a subsequence $\{\mathbf{x}^{t_j}|j=0,1,\dots\}$ of $\{\mathbf{x}^t\}$ that converges to $\bar{\mathbf{x}}$, where $\bar{\mathbf{x}} = \lim_{j\to\infty}\mathbf{x}^{t_j}$ is a limit point of $\{\mathbf{x}^t\}$. Since $\{\mathbf{x}^{t_j}\}\subset\{\mathbf{x}^t\}$ and $\{f(\mathbf{x}^{t_j})\}\subset\{f(\mathbf{x}^t)\}$, we have from Appendix A-C that $f(\mathbf{x}^{t_{j+1}})\le f(\mathbf{x}^{t_j})$. Now, by taking the limit as $j\to\infty$ and by using the continuity of objective function f, we obtain

$$\lim_{t\to\infty} f(\mathbf{x}^t) = \lim_{j\to\infty} f(\mathbf{x}^{t_j}) = f(\lim_{j\to\infty}\mathbf{x}^{t_j}) = f(\bar{\mathbf{x}}). \tag{54}$$

Let us prove the stationarity of $\bar{\mathbf{x}}$ by contradiction. Assume that $\bar{\mathbf{x}}$ is not a stationary point. Then there exists some other point $\mathbf{x}'\in\{\mathbf{x}^t\}$, such that $\nabla f(\bar{\mathbf{x}})^{\mathrm{T}}(\mathbf{x}'-\bar{\mathbf{x}})<0$ and $f(\bar{\mathbf{x}})>f(\mathbf{x}')$. Since $\mathbf{x}'$ is a point in $\{\mathbf{x}^t\}$, there exists a subsequence that converges to $\mathbf{x}'$, for which $\mathbf{x}'$ is a limit point. However, by using (54), we have $f(\bar{\mathbf{x}})=f(\mathbf{x}')$ for all limit points of $\{\mathbf{x}^t\}$, which is a contradiction to our initial assumption.

Now, by using the strict monotonicity of $\{f(\mathbf{x}^{t_j})\}$, it follows that as $j\to\infty$, $\mathbf{x}^{t_j}\to\bar{\mathbf{x}}$ and $f(\mathbf{x}^{t_j})-f(\mathbf{x}^{t_{j+1}})\to 0$. Moreover, since $\bar{\mathbf{x}}$ is the solution of (48), it satisfies (51) over convex subset $\mathcal{Q}$, which can either be $\mathcal{X}_k^{(i)}$ or $\mathcal{Y}_k^{(i)}$ as $k\to\infty, i\to\infty$. Using the above statements, we can show that $\bar{\mathbf{x}}$ satisfies the optimality condition [33, Prop. 2.1.2] as

$$\nabla f(\bar{\mathbf{x}})^{\mathrm{T}}(\mathbf{x}-\bar{\mathbf{x}})\ge 0,\ \forall\mathbf{x}\in\mathcal{Q}\subset\mathcal{F}. \tag{55}$$

Thus, $\bar{\mathbf{x}}$ is a stationary point, and by using (54), we can show that every limit point of $\{\mathbf{x}^t\}$ is a stationary point of (48).

Before proceeding further, we also ensure the convergence of $\{\hat{f}(\mathbf{x}^t)\}$ by showing that the original objective function in (46a) also decreases at each iteration. To do so, let us consider the minimizers $\mathbf{x}^t$ and $\mathbf{x}^{t+1}$ of problem (48) in iterations $t-1$ and t, respectively. Since $\|\mathbf{x}-\mathbf{x}^t\|^2$ is the proximal term in (46b) at iteration t, the relation between $\mathbf{x}^t$ and $\mathbf{x}^{t+1}$ satisfies

$$\hat{f}(\mathbf{x}^{t+1})+\tau^t\|\mathbf{x}^{t+1}-\mathbf{x}^t\|^2\le\hat{f}(\mathbf{x}^t)+\tau^t\|\mathbf{x}^t-\mathbf{x}^t\|^2 \tag{56}$$

where τ^t denotes $\tau_k^{(i)}$ in (46b) and $\|\mathbf{x}^{t+1}-\mathbf{x}^t\|^2\ge 0$. Thus, by extending (56) for each update and by using Appendix A-B, convergence of $\{\hat{f}(\mathbf{x}^t)\}$ can be shown using [33, Prop. A.3].

Now to prove that $\bar{\mathbf{x}}$ is also a stationary point of the original problem (44), we show the equivalence between the gradients of (48) and (44) at $\bar{\mathbf{x}}$. In order to ensure that $\bar{\mathbf{x}}$ is a KKT point of (44), it must satisfy the gradient condition of (44) as

$$\nabla\hat{f}(\bar{\mathbf{x}})+\mu_0\big[\nabla h(\bar{\mathbf{x}})-\nabla g_0(\bar{\mathbf{x}})\big]+\textstyle\sum_{i=1}^{2}\mu_i\nabla g_i(\bar{\mathbf{x}})=0 \tag{57}$$

for some Lagrange multipliers $\mu_i\ge 0$ and feasible as $\bar{\mathbf{x}}\in\mathcal{F}$.

Using (49), we have $\mathcal{X}_k^{(i)}\subset\mathcal{F}$ and $\mathcal{Y}_k^{(i)}\subset\mathcal{F}$, therefore, $\bar{\mathbf{x}}$ is a feasible point for (44). Moreover, the interiors of $\mathcal{X}_k^{(i)}$ and $\mathcal{Y}_k^{(i)}$ are non-empty. Thus, the Slater's constraint qualification holds for (48). Since $\bar{\mathbf{x}}$ is the solution for (48) as $k\to\infty, i\to\infty$, there exist Lagrange multipliers $\mu_i\ge 0$ which satisfy

$$\nabla f(\bar{\mathbf{x}})+\mu_0\big[\nabla h(\bar{\mathbf{x}})-\nabla\hat{g}_0(\bar{\mathbf{x}};\bar{\mathbf{x}})\big]+\textstyle\sum_{i=1}^{2}\mu_i\nabla g_i(\bar{\mathbf{x}})=0. \tag{58}$$

The relation between (57) and (58) is evident by the following facts: (i) The quadratic term in (46b) $\|\mathbf{x}^{t_{j+1}}-\mathbf{x}^{t_j}\|^2\to 0$ as $j\to\infty, \mathbf{x}^{t_j}\to\bar{\mathbf{x}}$, since we assume that $\bar{\mathbf{x}}$ is the limit point of a convergent subsequence $\{\mathbf{x}^{t_j}\}$. Therefore, the gradient evaluated at $\bar{\mathbf{x}}$ satisfies $\nabla f(\bar{\mathbf{x}})=\nabla\hat{f}(\bar{\mathbf{x}})$. (ii) Additionally, by using the continuity of function g_o and (47), equivalence between the gradient of (44b) and (48b) is obtained as $\mathbf{x}^{t_j}\to\bar{\mathbf{x}}$

$$\nabla h(\bar{\mathbf{x}})-\nabla\hat{g}_o(\bar{\mathbf{x}};\bar{\mathbf{x}})=\nabla h(\bar{\mathbf{x}})-\nabla g_o(\bar{\mathbf{x}}). \tag{59}$$

Now, by applying the relation (59) and $\nabla f(\bar{\mathbf{x}})=\nabla\hat{f}(\bar{\mathbf{x}})$ in (58), we can show that limit point $\bar{\mathbf{x}}$ satisfies (57). Finally, by following [22, Thm. 2] and [31, Prop. 3.2], we can show that every limit point of sequence of iterates $\{\mathbf{x}^t\}$ generated by the iterative method is a stationary point of problem (44).

Appendix B
Convergence Proof for Distributed Algorithms

Convergence of the primal decomposition and the ADMM follow the same discussions as in Appendix A, if in each SCA step, the distributed schemes converge to the centralized solution. To show that, let us consider subproblem (31) in the kth SCA step, solved by distributed method using a regularized objective as in (46b). At first, the convergence of the primal decomposition method is discussed by using the following properties. (i) The feasible set is non-empty and bounded. Therefore, each subproblem satisfies the Slater's constraint qualifications. (ii) Due to the strong convexity of the objective (46b) in subproblem, the minimizer is unique. Therefore, by using the above facts and as $j\to\infty$, the convergence of the primal decomposition method follows [34, Prop. 8.2.6] by using a diminishing step size in each subgradient update.

Now, to show the convergence of ADMM, we use the discussion in [28, Prop. 4.2] and [10] by writing (37) as

$$\underset{\mathbf{x}\in\mathcal{C}_1,\mathbf{z}\in\mathcal{C}_2}{\text{minimize}}\quad G(\mathbf{x})+H(\mathbf{z}) \tag{60a}$$

$$\text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{z} \tag{60b}$$

where the constraint in (60b) is identical to that of (38) used in the ADMM subproblem (37). To show the convergence, we rely on the following conditions. (i) G, H should be convex. (ii) $\mathcal{C}_1, \mathcal{C}_2$ should be a convex set and bounded. (iii) $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ should be invertible. It is evident from the equality constraints (60b) and (38) that $\mathbf{A} = \mathbf{I}$, and therefore it is invertible. Note that the objective functions G, H include ℓ_q norm and an additional quadratic term as in (46b), thereby exhibiting strong convexity. Moreover, the feasible set defined by the constraints of (31) is convex and has a nonempty interior. Now, by using [28, Prop. 4.2], we can show that the ADMM algorithm converges to the centralized solution as $j \to \infty$ by using diminishing step size in each ADMM update as discussed in [10]. Therefore, if both the primal and the ADMM methods are allowed to converge to the centralized solution in each SCA step, then every limit point of the whole sequence of iterates generated by the distributed algorithms is a stationary point of (44), by following the discussions in Appendix A.

Unlike the primal or the ADMM approaches, decomposition via KKT conditions, presented in Section IV-C, updates all the optimization variables at once, *i.e*, the SCA update of $\epsilon^{(i-1)}$, the AO update of $\mathbf{w}_{l,k,n}$ and the dual variable update of α using subgradient method. Therefore, it is difficult to theoretically prove the convergence of the algorithm to a stationary point of the nonconvex problem in (16).

The algorithm in (42) is identical to (28), if the receivers $\mathbf{w}_{l,k,n}$ and the MSE operating point $\epsilon_{l,k,n}^{(i-1)}$ are fixed to find the optimal transmit precoders $\mathbf{m}_{l,k,n}$ and the dual variable $\alpha_{l,k,n}$. Note that it requires four nested loops to obtain the centralized solution, namely, the receive beamformer loop, MSE operating point loop, the dual variable update loop and the bisection method to find the transmit precoders.

However, to avoid the nested iterations, the proposed method performs block update of all variables at once to obtain transmit and receive beamformers for limited number of iterations, thus, achieving improved speed of convergence. Since the optimization variables are updated together, it is theoretically difficult to prove the monotonicity of the objective in each block update. Moreover, the objective function used to obtain the iterative algorithm is not strongly convex, and therefore the uniqueness of the iterates is also not guaranteed in each update. Thus, the convergence of iterative scheme outlined in Algorithm 3 cannot be guaranteed.

Appendix C
KKT conditions for MSE approach

To design transmit precoders iteratively, we solve the KKT conditions of problem (41) by assuming the constraints (41b) and (41c) are tight. Hence, we obtain the following equations.

$$\nabla_{\epsilon_{l,k,n}} : -\alpha_{l,k,n} + \frac{\sigma_{l,k,n}}{\tilde{\epsilon}_{l,k,n}} = 0 \tag{61a}$$

$$\nabla_{t_{l,k,n}} : -q\, a_k \Big(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \Big)^{(q-1)} + \frac{\sigma_{l,k,n}}{\log_2(e)} = 0 \tag{61b}$$

$$\begin{aligned} \nabla_{\mathbf{m}_{l,k,n}} :& \sum_{y \in \mathcal{U}} \sum_{x=1}^{L} \alpha_{x,y,n} \mathbf{H}_{b_k,y,n}^{\mathrm{H}} \mathbf{w}_{x,y,n} \mathbf{w}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_k,y,n} \mathbf{m}_{l,k,n} \\ &+ \delta_b \mathbf{m}_{l,k,n} = \alpha_{l,k,n} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n} \end{aligned} \tag{61c}$$

$$\begin{aligned} \nabla_{\mathbf{w}_{l,k,n}} :& \sum_{(x,y) \neq (l,k)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n} \mathbf{m}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_y,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n} \\ &+ N_0 \mathbf{I}_{N_R} \mathbf{w}_{l,k,n} = \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n}. \end{aligned} \tag{61d}$$

In addition to the primal constraints (41b), (41c) and (41d), the complementary slackness criterion must also be satisfied for the optimality of solution. Upon solving the above expressions in (61) with the complementary slackness conditions, we formulate an algorithm to determine transmit and receive beamformers iteratively as shown in formulation (42).

References

[1] E. Matskani, N. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, "Convex Approximation Techniques for Joint Multiuser Downlink Beamforming and Admission Control," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2682–2693, July 2008.

[2] C. Ng and H. Huang, "Linear Precoding in Cooperative MIMO Cellular Networks with Limited Coordination Clusters," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1446–1454, December 2010.

[3] L.-N. Tran, M. Hanif, A. Tölli, and M. Juntti, "Fast Converging Algorithm for Weighted Sum Rate Maximization in Multicell MISO Downlink," *IEEE Signal Process. Lett.*, vol. 19, no. 12, pp. 872–875, 2012.

[4] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and Sum-MSE Minimization," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5436–5446, Nov 2007.

[5] S. S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, 2008.

[6] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, sept. 2011.

[7] J. Kaleva, A. Tölli, and M. Juntti, "Weighted Sum Rate Maximization for Interfering Broadcast Channel via Successive Convex Approximation," in *Global Communications Conference*. IEEE, 2012, pp. 3838–3843.
[8] ——, "Decentralized Beamforming for Weighted Sum Rate Maximization with Rate Constraints," in *24th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC Workshops)*. IEEE, 2013, pp. 220–224.
[9] D. P. Palomar and M. Chiang, "A Tutorial on Decomposition Methods for Network Utility Maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1439–1451, 2006.
[10] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.
[11] H. Pennanen, A. Tölli, and M. Latva-Aho, "Decentralized Coordinated Downlink Beamforming via Primal Decomposition," *IEEE Signal Process. Lett.*, vol. 18, no. 11, pp. 647–650, 2011.
[12] A. Tölli, H. Pennanen, and P. Komulainen, "Decentralized Minimum Power Multi-Cell Beamforming with Limited Backhaul Signaling," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 570–580, 2011.
[13] M. Neely, *Stochastic Network Optimization with Application to Communication and Queueing Systems*, ser. Synthesis Lectures on Communication Networks. Morgan & Claypool Publishers, 2010, vol. 3, no. 1.
[14] R. A. Berry and E. M. Yeh, "Cross-Layer Wireless Resource Allocation," *IEEE Signal Process. Mag.*, vol. 21, no. 5, pp. 59–68, 2004.
[15] M. Chiang, S. Low, A. Calderbank, and J. Doyle, "Layering as Optimization Decomposition: A Mathematical Theory of Network Architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–312, Jan 2007.
[16] K. Seong, R. Narasimhan, and J. Cioffi, "Queue Proportional Scheduling via Geometric Programming in Fading Broadcast Channels," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1593–1602, 2006.
[17] P. C. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, "Resource Allocation for Cross-Layer Utility Maximization in Wireless Networks," *IEEE Trans. Veh. Technol.*, vol. 60, pp. 2790–2809, 2011.
[18] F. Zhang and V. Lau, "Cross-Layer MIMO Transceiver Optimization for Multimedia Streaming in Interference Networks," *IEEE Trans. Signal Process.*, vol. 62, no. 5, pp. 1235–1244, March 2014.
[19] B. R. Marks and G. P. Wright, "A General Inner Approximation Algorithm for Nonconvex Mathematical Programs," *Operations Research*, vol. 26, no. 4, pp. 681–683, 1978.
[20] Z.-Q. Luo and S. Zhang, "Dynamic Spectrum Management: Complexity and Duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 57–73, Feb 2008.
[21] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.
[22] G. Scutari, F. Facchinei, L. Lampariello, and P. Song, "Distributed Methods for Constrained Nonconvex Multi-Agent Optimization – Part I: Theory." [Online]. Available: http://arxiv.org/abs/1410.4754v1
[23] G. R. Lanckriet and B. K. Sriperumbudur, "On the Convergence of the Concave-Convex Procedure," in *Advances in Neural Information Processing Systems*, 2009, pp. 1759–1767.
[24] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.0 beta," http://cvxr.com/cvx, Sep. 2013.
[25] P. Komulainen, A. Tölli, and M. Juntti, "Effective CSI Signaling and Decentralized Beam Coordination in TDD Multi-Cell MIMO Systems," *IEEE Trans. Signal Process.*, vol. 61, no. 9, pp. 2204–2218, 2013.
[26] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Distributed Robust Multicell Coordinated Beamforming With Imperfect CSI: An ADMM Approach," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2988–3003, June 2012.
[27] W. Rudin, *Principles of Mathematical Analysis*. McGraw-Hill New York, 1964, vol. 3.
[28] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Prentice Hall Englewood Cliffs, NJ, 1989.
[29] Y. Yang, G. Scutari, P. Song, and D. Palomar, "Robust MIMO Cognitive Radio Systems Under Interference Temperature Constraints," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2465–2482, November 2013.
[30] G. Scutari, F. Facchinei, P. Song, D. Palomar, and J.-S. Pang, "Decomposition by Partial Linearization: Parallel Optimization of Multi-Agent Systems," *IEEE Trans. Signal Process.*, vol. 62, no. 3, pp. 641–656, Feb 2014. [Online]. Available: http://arxiv.org/abs/1302.0756v2
[31] A. Beck, A. Ben-Tal, and L. Tetruashvili, "A sequential parametric convex approximation method with applications to nonconvex truss topology design problems," *Journal of Global Optimization*, vol. 47, no. 1, pp. 29–51, 2010.
[32] G. Scutari, D. P. Palomar, F. Facchinei, and J.-S. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 35–49, 2010.
[33] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Athena Scientific, sep 1999.
[34] D. P. Bertsekas, A. Nedic, A. E. Ozdaglar *et al.*, *Convex Analysis and Optimization*. Athena Scientific, 2003.