Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems

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***Abstract*—We consider a downlink multi-cell multiple-input multiple-output (MIMO) interference broadcast channel (IBC) scenario using orthogonal frequency division multiplexing (OFDM) with multiple-users contending for space-frequency resources in a given scheduling instant. The problem is to determine the transmit precoders in a coordinated approach to minimize the total number of backlogged packets in the base stations (BSs), which are destined for the users in the system. Traditionally, it is solved using weighted sum rate maximization (WSRM) objective with the number of backlogged packets being the corresponding weights, *i.e*, longer the queue size, higher the priority. In contrast, we design the precoders jointly across the space-frequency resources by directly minimizing the total user queue deviations. The problem is inherently nonconvex and therefore we employ successive convex approximation (SCA) technique to solve the problem by a sequence of convex subproblems. At first, we propose a centralized joint space-frequency resource allocation (JSFRA) solution using two different approaches by employing SCA technique, namely the direct formulation and the mean squared error (MSE) reformulation. We then introduce distributed precoder designs using primal and alternating directions method of multipliers for the JSFRA solutions. Finally, we propose a distributed iterative precoder design based on MSE reformulation by solving the Karush-Kuhn-Tucker conditions with closed form expressions. Numerical results are provided to compare the proposed algorithms with the existing solutions.**

***Index Terms*—convex approximations, MIMO-IBC, MIMO-OFDM, precoder design, SCA, WSRM.**

I. Introduction

In a network with multiple base stations (BSs) serving multiple-userss (MUs), the main driving factor for downlink transmission is the number of packets waiting at each BS corresponding to the different users present in the network. These available packets are transmitted over the shared wireless resources subject to certain system limitations and constraints. We consider the problem of transmit precoder design over the space-frequency resources provided by the multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) framework in the downlink interference broadcast channel (IBC) to minimize the number of queued packets. Since the space-frequency resources are shared by multiple users in different BSs, it can be viewed as a resource allocation problem.

In general, the resource allocation problems are formulated by assigning a binary variable for each user to indicate whether the user is allocated to a particular resource [1]. In this paper, we follow a different approach in which linear transmit precoders are used as decision variables. The purpose is two-fold. Firstly, it determines the transmission rate on a particular resource. Secondly, in certain resource, if the transmit precoder of a user turns out to be a zero vector after solving the design problem, the corresponding user will not be scheduled.

The queue minimizing precoder designs are closely related to the weighted sum rate maximization (WSRM) problem with additional rate constraints determined by the number of backlogged packets for each user in the system. The topics on MIMO IBC precoder design have been studied extensively with different performance criteria in the literature. Due to the nonconvex nature of the MIMO IBC precoder design problems, the successive convex approximation (SCA) method has become a powerful tool to deal with these problems [2]. For example, in [3], the nonconvex part of the objective has been linearized around an operating point in order to solve the WSRM problem in an iterative manner. Similar approach of solving the WSRM problem by using arithmetic-geometric inequality has been proposed in [4].

The connection between the achievable capacity and the mean squared error (MSE) for the received symbol by using the fixed minimum mean squared error (MMSE) receivers as shown in [5], [6] can also be used to solve the WSRM problem. In [7], [8], the WSRM problem is reformulated via MSE, casting the problem as a convex one for fixed linearization coefficients. In this way, the original problem is expressed in terms of the MSE weights, precoders, and decoders. Then the problem is solved using an alternating optimization method, i.e., finding a subset of variables while the remaining others are fixed. The MSE reformulation for the WSRM problem has also been studied in [9] by using the SCA to solve the problem in an iterative manner. Additional rate constraints based on the quality of service (QoS) requirements were included in the WSRM problem and solved via MSE reformulation in [10], [11].

The problem of precoder design for the MIMO IBC system is solved either by using a centralized controller or by using decentralized algorithms where each BS handles the corresponding subproblem independently with the limited information exchange with the other BSs via back-haul. The distributed approaches are based on primal, dual or alternating

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directions method of multipliers (ADMM) decomposition, which has been discussed in [12], [13]. In the primal decomposition, the so-called coupling interference variables are fixed for the subproblem at each BS to find the optimal precoders. The fixed interference are then updated by using the subgradient method as discussed in [14]. The dual and ADMM approaches control the distributed subproblems by fixing the *'interference price'* for each BS as detailed in [15].

By adjusting the weights in the WSRM problem properly, we can find a rate-tuple in the rate region that maximizes a performance measure of interest. For example, if the weights of each user is set to be inversely proportional to user's average data rate, the problem guarantees fairness on average achieved rate among the users. As an approximation to our considered problem, we may assign weights based on the current queue size of the users. More specifically, the queue states can be incorporated to the traditional WSRM objective $\sum_k w_k R_k$ by replacing the weight w_k with the corresponding queue state Q_k or a related function, which is the outcome of minimizing the Lyapunov drift between the current and the future queue states [16], [17]. In backpressure algorithm, the differential queues between the source and the destination nodes are used as the weights scaling the transmission rate [18].

Earlier studies on the queue minimization problem were summarized in the survey paper [19], [20]. In particular, the problem of power allocation to minimize the number of backlogged packets was considered in [21] using geometric programming. Since the problem addressed in [21] assumed single antenna transmitters and receivers, the queue minimizing problem reduces to the optimal power allocation problem. In the context of wireless networks, the backpressure algorithm mentioned above was extended in [22] by formulating the corresponding user queues as the weights in the WSRM problem. Recently, the precoder design for the video transmission over MIMO system is considered in [23]. In this design, the MU-MIMO precoders are designed by the MSE reformulation as in [7] with the higher layer performance objective such as playback interruptions and buffer overflow probabilities.

Main Contributions: In this paper, we design the precoders jointly across space-frequency resources to minimize the total number of backlogged packets waiting at the BSs. Since the problem is nonconvex, we adopt the SCA technique to solve by a sequence of convex subproblems using first order approximations. First, we propose two centralized joint space-frequency resource allocation (JSFRA) algorithms, which employs the SCA technique for the nonconvex constraints. The first method is based on applying the SCA technique to the nonconvex constraints of the problem formulation directly, while the second one applies SCA technique to the equivalent formulation obtained by the MSE equivalence. In the second part, we propose a distributed precoder designs based on the primal and the ADMM. Finally, we propose an iterative algorithm to decouple the precoder design across the coordinating BSs with limited information exchange by solving the Karush-Kuhn-Tucker (KKT) conditions for the MSE reformulation solution.

The paper is organized as follows. In Section II, we introduce the system model and the problem formulation for the queue minimizing precoder design. The existing and the proposed centralized precoder designs are presented in Section III. The distributed solutions are provided in Section IV followed by the simulation results in Section V. Conclusions are drawn in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a downlink MIMO IBC scenario in an OFDM framework with N sub-channels and N_B BSs each equipped with N_T transmit antennas, serving in total K users each with N_R receive antennas. The set of users associated with BS b is denoted by $\mathcal{U}_b$ and the set $\mathcal{U}$ represents all users in the system, i.e., $\mathcal{U} = \bigcup_{b \in \mathcal{B}} \mathcal{U}_b$, where $\mathcal{B}$ is the set of indices of all coordinating BSs. Data for user k is transmitted from only one BS which is denoted by $b_k \in \mathcal{B}$. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the set of all sub-channel indices present in the system.

We adopt linear transmit beamforming technique at BSs. Specifically, the data symbols $d_{l,k,n}$ for user k on the l^{th} spatial stream over the sub-channel n is multiplied with beamformer $\mathbf{m}_{l,k,n} \in \mathbb{C}^{N_T \times 1}$ before being transmitted. In order to detect multiple spatial streams at the user terminal, receive beamformer $\mathbf{w}_{l,k,n}$ is employed for each user. Consequently, the received data symbol estimate corresponding to the l^{th} spatial stream over sub-channel n at user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \, \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n}, \quad (1)$$

where $\mathbf{H}_{b,k,n} \in \mathbb{C}^{N_R \times N_T}$ is the channel between BS b and user k on sub-channel n, and $\mathbf{n}_{k,n} \sim \mathcal{CN}(0, N_0)$ is the additive noise vector for the user k on the n^{th} sub-channel and l^{th} spatial stream. In (1), $L = rank(\mathbf{H}_{b,k,n}) = min(N_T, N_R)$ is the maximum number of spatial streams[1]. Assuming independent detection of data streams, we can write the signal-to-interference-plus-noise ratio (SINR) as

$$\gamma_{l,k,n} = \frac{\left| \mathbf{w}_{l,k,n}^{\mathrm{H}} \, \mathbf{H}_{b_k,k,n} \, \mathbf{m}_{l,k,n} \right|^2}{N_0 \|\mathbf{w}_{l,k,n}\|^2 + \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2}. \quad (2)$$

Let Q_k be the number of backlogged packets destined for user k at a given scheduling instant. The queue dynamics of user k are modeled using the Poisson arrival process with the average number of packet arrivals of $A_k = \mathbf{E}_i\{\lambda_k\}$ packets/bits, where $\lambda_k(i) \sim \text{Pois}(A_k)$ represents the instantaneous number of packets arriving for user k at the i^{th} time instant[2]. The total number of queued packets at the $(i+1)^{\text{th}}$ instant for user k, denoted as $Q_k(i+1)$, is given by

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i) \Big]^+ + \lambda_k(i), \quad (3)$$

where $[x]^+ \equiv \max\{x, 0\}$ and t_k denotes the number of transmitted packets or bits for user k. At the i^{th} instant,

[1] L streams are initialized but after solving the problem, only $L_{k,n} \leq L$ non-zero data streams are transmitted

[2] The unit can be either packets or bits as long as the arrival and the transmission units are similar

transmission rate of user k is given by

$$t_k(i) = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}(i), \tag{4}$$

where $t_{l,k,n}$ denotes the number of transmitted packets or bits over l^{th} spatial stream on the n^{th} sub-channel. The maximum rate achieved over the (l, n) space-frequency resource is given by $t_{l,k,n} \leq \log_2(1+\gamma_{l,k,n})$ for the signal-to-interference-plus-noise ratio (SINR) of $\gamma_{l,k,n}$[3]. Note that the units of t_k and Q_k are in bits per channel use.

B. Problem Formulation

To minimize the total number of backlogged packets, we consider minimizing the weighted ℓ_q-norm of the queue deviation given by

$$v_k = Q_k - t_k = Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}). \tag{5}$$

Explicitly, the objective of the problem considered is given by $\sum_{k\in\mathcal{U}} a_k |v_k|^q$. With this objective function, the weighted queued packet minimization formulation is given by

$$\underset{\mathbf{M}_{k,n},\mathbf{W}_{k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{6a}$$

$$\text{subject to} \quad \sum_{n=1}^{N} \sum_{k\in\mathcal{U}_b} \text{tr}\left(\mathbf{M}_{k,n}\mathbf{M}_{k,n}^{\text{H}}\right) \leq P_{\max},\ \forall b, \tag{6b}$$

where $\tilde{v}_k \triangleq a_k^{1/q} v_k$ is the element of vector $\tilde{\mathbf{v}}$, and a_k is the weighting factor which is incorporated to control user priority based on their respective QoS, $\mathbf{M}_{k,n} \triangleq [\,\mathbf{m}_{1,k,n}\,\mathbf{m}_{2,k,n} \ldots \mathbf{m}_{L,k,n}\,]$ comprises the beamformers associated with user k for n^{th} sub-channel, and $\mathbf{W}_{k,n} \triangleq [\,\mathbf{w}_{1,k,n}\,\mathbf{w}_{2,k,n} \ldots \mathbf{w}_{L,k,n}\,]$ stacks the receive beamformers respectively[4]. In (6b), we consider a BS specific sum power constraint for each BS across all sub-channels.

For practical reasons, we may impose a constraint that the maximum number of transmitted bits for the user k is limited by the total number of backlogged packets available at the transmitter. As a result, the number of backlogged packets v_k for user k remaining in the system is given by

$$v_k = Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \geq 0. \tag{7}$$

The above positivity constraint need to be satisfied by v_k to avoid the excessive allocation of the resources.

Before proceeding further, we show that the constraint in (7) is handled implicitly by the definition of ℓ_q norm in the objective of (6). Suppose that $t_k > Q_k$ for certain k at the optimum, i.e., $-v_k = t_k - Q_k > 0$. Then there exists $\delta_k > 0$ such that $-v'_k = t'_k - Q_k < -v_k$ where $t'_k = t_k - \delta_k$. Since $\|\tilde{\mathbf{v}}\|_q = \||\tilde{\mathbf{v}}|\|_q = \|| - \tilde{\mathbf{v}}|\|_q$, this means that the newly created vector $\mathbf{t}'$ achieves a smaller objective which contradicts with the fact that the optimal solution has been obtained. The choice of the ℓ_q norm used in the objective function [19], [21] has different effects on the priorities for the queue deviation function as follows

- the ℓ_1 norm results in greedy allocation *i.e*, emptying the queue of users with good channel conditions before considering the users with worse channel conditions. As a special case, it can be seen that (6) reduces to the WSRM problem when the queue size is large enough for all users.
- the ℓ_2 norm prioritizes users with higher number of queued packets before considering the users with a smaller number of backlogged packets. For example, it could be more ideal for the delay limited scenario when the packet arrival rates of the users are similar, since the number of backlogged packets is proportional to the delay in the transmission following the Little's law [17].
- the ℓ_∞ norm minimize the maximum queue deviation in each transmission instant by allocating the resources proportional to the number of backlogged packets. The queue deviation of all users will be the same after each allocation instant, irrespective of the user channel gains.

III. Proposed Queue Minimizing Precoder Designs

In general, the precoder design for the MIMO OFDM problem is difficult due to the combinatorial and the nonconvex nature of the problem. In addition, the objective of minimizing the number of the queued packets over space-frequency dimensions adds further complexity to the existing problem. Since the scheduling of users in each space-frequency resource can be controlled by the transmit precoders, our solutions perform precoder design and user scheduling jointly. Before discussing the proposed solutions, we consider the existing algorithm to minimize the number of backlogged packets with additional constraints required by the problem.

A. Queue Weighted Sum Rate Maximization (Q-WSRM) Formulation

The queue minimizing algorithms are discussed extensively in the networking literature to provide congestion-free routing between any two nodes in the network. One such algorithm is the *backpressure algorithm* [16]–[18]. It determines an optimal control policy in the form of rate or resource allocation for the nodes in the network by considering the differential backlogged packets between the source and the destination nodes. Even though the algorithm is primarily designed for wired infrastructures, it can be extended to wireless networks by designing the user rate variable t_k in accordance to wireless networks.

The queue weighted sum rate maximization (Q-WSRM) formulation extends the *backpressure algorithm* to the downlink MIMO-OFDM framework, in which the multiple BSs act as the source nodes and the user terminals as the receiver nodes. The control policy in the form of transmit precoders aims at minimizing the number of queued packets waiting in the BSs. In order to find the optimal strategy, we resort to the Lyapunov theory, which is predominantly used in the control theory to achieve system stability. Since at each time slot, the system is described by the channel conditions and the number

[3]Upper bound is achieved by using Gaussian signaling

[4]It can be easily extended for user specific streams $L_{k,n}$ instead of using the common L streams for all users

of backlogged packets of each user, the Lyapunov function is used to provide a scalar measure, which grows large when the system moves toward the undesirable state. Following similar approach as in [17], the scalar measure for the queue stability is given by

$$\mathrm{L}\left[\mathbf{Q}(i)\right] = \frac{1}{2}\sum_{k\in\mathcal{U}} Q_k^2(i), \tag{8}$$

where $\mathbf{Q}(i) = [Q_1(i), Q_2(i), \ldots, Q_K(i)]^{\mathrm{T}}$ and $\frac{1}{2}$ is used for the convenience. It provides a scalar measure of congestion present in the system [17, Ch. 3].

To minimize the total number of backlogged packets for an instant i, the optimal transmission rate of all users are obtained by minimizing the Lyapunov function drift expressed as

$$\begin{aligned}&\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] = \\ &\quad \frac{1}{2}\Big[\sum_{k\in\mathcal{U}}\Big([Q_k(i) - t_k(i)]^+ + \lambda_k(i)\Big)^2 - Q_k^2(i)\Big].\end{aligned} \tag{9}$$

In order to eliminate the nonlinear operator $[x]^+$, we bound the expression in (9) as

$$\le \sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} + \sum_{k\in\mathcal{U}} Q_k(i)\left\{\lambda_k(i) - t_k(i)\right\}, \tag{10}$$

by using the following inequality

$$\left[\max(Q - t, 0) + \lambda\right]^2 \le Q^2 + t^2 + \lambda^2 + 2Q(\lambda - t). \tag{11}$$

The total number of backlogged packets at any given instant i is reduced by minimizing the conditional expectation of the Lyapunov drift expression (10) given the current number of queued packets $\boldsymbol{Q}(i)$ waiting in the system. The expectation is taken over all possible arrival and transmission rates of the users to obtain the optimal rate allocation strategy.

Now, the conditional Lyapunov drift, denoted by $\Delta(\boldsymbol{Q}(i))$, is given by the infimum over the transmission rate as

$$\inf_{\mathbf{t}} \quad \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\left\{\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] | \mathbf{Q}(i)\right\} \tag{12a}$$

$$\begin{aligned}\le\ & \underbrace{\mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{\sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} |\mathbf{Q}(i)\Big\}}_{\le B} + \sum_{k\in\mathcal{U}} Q_k(i) A_k(i) \\ & - \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{\sum_{k\in\mathcal{U}} Q_k(i) t_k(i) |\mathbf{Q}(i)\Big\},\end{aligned} \tag{12b}$$

where the subscripts $\mathbf{t}$ and $\boldsymbol{\lambda}$ represents the vector formed by stacking the transmission and the arrival rate of all users in the system. Since the transmission and the arrival rates are bounded, the second order moments in the first term of (12b) can be bounded by a constant B without affecting the optimal solution of the problem [17]. The second term in (12b) follows from the Poisson arrival process.

The expression in (12) looks similar to the WSRM formulation if the weights in the WSRM problem are replaced by the number of backlogged packets corresponding to the users. The above discussed approach was extended for the wireless networks in [22], where the queue weighted sum rate maximization is considered as the objective function to determine the transmit precoders. Since the expectation can be minimized by minimizing the function inside, the Q-WSRM formulation is given by

$$\underset{\mathbf{M}_{k,n}, \mathbf{W}_{k,n}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} Q_k \left(\sum_{n=1}^{N}\sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n})\right) \tag{13a}$$

$$\text{subject to.} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b} \operatorname{tr}\left(\mathbf{M}_{k,n}\mathbf{M}_{k,n}^{\mathrm{H}}\right) \le P_{\max},\ \forall\, b. \tag{13b}$$

In order to avoid the excessive allocation of the resources, we include an additional rate constraint $t_k \le Q_k$ to address $[x]^+$ operation in (3). The rate constrained version of the Q-WSRM, denoted by Q-WSRM extended (Q-WSRME) problem for a cellular system, is given by with the additional constraint

$$\sum_{n=1}^{N}\sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \le Q_k,\ \forall\, k \in \mathcal{U}, \tag{14}$$

where the precoders are associated with $\gamma_{l,k,n}$ defined in (2). By using the number of queued packets as the weights, the resources can be allocated to users with more backlogged packets, which essentially a greedy resource allocation scheme.

As a special case of the problem defined in (13), we can formulate the sum rate maximization problem by setting the weights in (13a) to be unity, leading to the problem as in (13) with $Q_k = 1, \forall k \in \mathcal{U}$. This approach provides a greedy queue minimizing allocation as compared to Q-WSRME, since the resource allocation is driven by the channel conditions in comparison to the number of queued packets as in Q-WSRME. Note that in both formulations, the resources allocated to the users are limited by the number of backlogged packets with an explicit maximum rate constraint given in (14).

B. JSFRA Scheme via SCA approach

The problem defined in (13) ignores the second order term arising from the Lyapunov drift minimization objective by limiting it to a constant value. In fact, (5) provides similar expression of $\ell_{q=2}$ norm as

$$\underset{t_k}{\text{minimize}} \sum_{k} v_k^2 = \underset{t_k}{\text{minimize}} \sum_{k} Q_k^2 - 2\, Q_k t_k + t_k^2. \tag{15}$$

It is evident that (15) is equivalent to (12) if the second order terms are ignored. Limiting t_k^2 by a constant value, the Q-WSRM formulation requires the explicit rate constraint (14) to avoid the resource wastage in the form of over-allocation. In the proposed queue deviation formulation, the explicit rate constraint is not needed, since it is handled by the objective function itself. This makes the problem simpler and allows us to employ efficient algorithms to distribute the precoder design problem across each BSs independently by exchanging minimal information exchange [13]. In contrast to the WSRM formulation, the JSFRA and the Q-WSRM problems include the sub-channels jointly to achieve an efficient allocation by identifying the optimal space-frequency resource for each user in the system. The queue deviation objective provides an alternative approach to perform the resource allocation without the additional rate constraints as in the Q-WSRME formulation.

In this approach, we present an algorithm to solve (6) to obtain the transmit precoders in a centralized manner by using the idea of alternating optimization and successive convex approximation. Using (2), we can reformulate the problem defined in (6) as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\\ \beta_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{16a}$$
$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \triangleq f(\tilde{\mathbf{u}}_{l,k,n}) \tag{16b}$$
$$\beta_{l,k,n} \geq \tilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{16c}$$
$$\text{and (6b)}, \tag{16d}$$

where $\tilde{\mathbf{u}}_{l,k,n} \triangleq \{\mathbf{w}_{l,k,n}^{\mathrm{H}}, \mathbf{H}_{b_k,k,n}, \mathbf{m}_{l,k,n}, \beta_{l,k,n}\}$ is the vector which needs to be identified for the optimal allocation and $\tilde{N}_0 = N_0\|\mathbf{w}_{l,k,n}\|^2$ be the effective noise variance. In this formulation, we relaxed the equality constraint in (2) by the inequalities in (16b) and (16c). However, this step leads to the same solution without loss of optimality, since the inequalities in (16b) and (16c) are active for an optimal solution, following the same arguments as those in [4]. Intuitively, (16b) denotes the SINR constraint for $\gamma_{l,k,n}$, and (16c) gives an upper bound for the total interference seen by user $k \in \mathcal{U}_b$, denoted by the optimization variable $\beta_{l,k,n}$.

Similar to the WSRM problem in [4], the proposed formulation in (16) can be shown to be NP-hard even for the single antenna case. The reformulation in (16) allows a tractable solution as presented below. First, we note that the constraints (6b) are convex with involved variables. Thus, we only need to deal with (16b) and (16c). Towards this end, we resort to the traditional coordinate descent technique by fixing the linear receivers, and finding the optimal transmit beamformers. Recall that the coordinate descent method assumes that the optimization variables belong to disjoint sets and the problem must be convex for a given set of variables while keeping other variables fixed as discussed in [24].

By fixing the receivers, the problem now is to find the optimal transmit beamformers for a given set of linear receivers which is still a challenging task. We note that for fixed $\mathbf{w}_{l,k,n}$, (16c) can be written as a second-order cone (SOC) constraint. Thus, the difficulty is due to the non-convexity in (16b). To arrive at a tractable formulation, we adopt the SCA method to handle (16b) by replacing the original non-convex constraint by a series of convex constraints. Note that function $f(\tilde{\mathbf{u}}_{l,k,n})$ in (16b) is convex for fixed $\mathbf{w}_{l,k,n}$, since it is in fact the ratio between a quadratic form of $\mathbf{m}_{l,k,n}$ over an affine function of $\beta_{l,k,n}$ [25]. According to the SCA method, we relax (16b) to a convex constraint in each iteration of the iterative procedure. Since $f(\tilde{\mathbf{u}}_{l,k,n})$ is convex, a concave approximation of (16b) can be easily found by considering the first order approximation of $f(\tilde{\mathbf{u}}_{l,k,n})$ around the current operation point. For this purpose, let the real and imaginary component of the complex number $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}$ be represented by

$$p_{l,k,n} \triangleq \Re\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{17a}$$
$$q_{l,k,n} \triangleq \Im\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{17b}$$

and hence $f(\tilde{\mathbf{u}}_{l,k,n}) = (p_{l,k,n}^2 + q_{l,k,n}^2)/\beta_{l,k,n}$[5]. Suppose that the current value of $p_{l,k,n}$ and $q_{l,k,n}$ at a specific iteration are $\tilde{p}_{l,k,n}$ and $\tilde{q}_{l,k,n}$, respectively. Using the first order Taylor approximation around the local point $[\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}, \tilde{\beta}_{l,k,n}]^T$, we can approximate (16b) by the following linear inequality constraint as

$$2\frac{\tilde{p}_{l,k,n}}{\tilde{\beta}_{l,k,n}}(p_{l,k,n} - \tilde{p}_{l,k,n}) + 2\frac{\tilde{q}_{l,k,n}}{\tilde{\beta}_{l,k,n}}(q_{l,k,n} - \tilde{q}_{l,k,n}) + \frac{\tilde{p}_{l,k,n}^2 + \tilde{q}_{l,k,n}^2}{\tilde{\beta}_{l,k,n}}\left(1 - \frac{\beta_{l,k,n} - \tilde{\beta}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\right) \geq \gamma_{l,k,n}. \tag{18}$$

In summary, for the fixed linear receivers, the JSFRA problem to find transmit beamformers is shown by

$$\underset{\substack{\mathbf{m}_{l,k,n},\\ \gamma_{l,k,n},,\beta_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{19a}$$
$$\text{subject to} \quad \beta_{l,k,n} \geq \tilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{19b}$$
$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b} \mathrm{tr}\left(\mathbf{M}_{k,n}\mathbf{M}_{k,n}^{\mathrm{H}}\right) \leq P_{\max},\ \forall\, b \tag{19c}$$
$$\text{and (18)}. \tag{19d}$$

Now, the optimal linear receivers for the fixed transmit precoders $\mathbf{m}_{j,i,n}\,\forall i \in \mathcal{U}$, $\forall n \in \mathcal{C}$ are obtained by minimizing (6) with respect to $\mathbf{w}_{l,k,n}$ as

$$\underset{\gamma_{l,k,n},\mathbf{w}_{l,k,n},\beta_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{20a}$$
$$\text{subject to} \quad \text{(19b), (19c), (19d), and (18)}. \tag{20b}$$

Solving (20) using the KKT conditions, we obtain the following iterative expression for the receiver $\mathbf{w}_{l,k,n}$ as

$$\widetilde{\mathbf{R}}_{l,k,n} = \sum_{(j,i)\neq(l,k)} \mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{21a}$$
$$\mathbf{w}_{l,k,n}^{(i)} = \left(\frac{\tilde{\beta}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)}}{\|\mathbf{w}_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\|^2}\right)\widetilde{\mathbf{R}}_{l,k,n}^{-1}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}, \tag{21b}$$

where $\mathbf{w}_{l,k,n}^{(i-1)}$ is the receive beamformer from the earlier iteration, upon which the linear relaxation is performed for the nonconvex constraint in (20). Note that (21b) is obtained by iterating over the fixed $\mathbf{w}_{l,k,n}^{(i-1)}$ at each SCA iteration until convergence or for fixed number of iterations. It can be seen that the optimal receiver expression in (21b) is in fact a scaled version of the MMSE receiver, which is given by

$$\mathbf{R}_{l,k,n} = \sum_{i\in\mathcal{U}}\sum_{j=1}^{L} \mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{22a}$$
$$\mathbf{w}_{l,k,n} = \mathbf{R}_{l,k,n}^{-1}\,\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}. \tag{22b}$$

The proposed algorithm is referred to as queue minimizing JSFRA scheme with a per BS power constraint, and it is outlined in Algorithm 1. The iterative procedure repeats until the improvement on the objective is less than a predetermined tolerance parameter or the maximum number of iterations is

[5]Note that $p_{l,k,n}$ and $q_{l,k,n}$ are just symbolic notation and not the newly introduced optimization variables. In CVX [26], for example, we declare $p_{l,k,n}$ and $q_{l,k,n}$ with the '*expression*' qualifier

reached. Instead of initializing $\tilde{\mathbf{u}}_{l,k,n}$ arbitrarily to a feasible point, transmit precoders can also be initialized with any feasible point $\tilde{\mathbf{m}}_{l,k,n}$, which is then used to find $\tilde{\mathbf{u}}_{l,k,n}$ in an efficient manner as briefed in Algorithm 1. For a fixed receive beamformer $\mathbf{w}_{l,k,n}$, the SCA iteration is carried out until convergence or for the predefined iterations, say, $J_{\max}$ for the optimal transmit precoders $\mathbf{m}_{l,k,n}$. Next, the receive beamformers are updated based on either (21b) or (22b) using the fixed transmit precoders $\mathbf{m}_{l,k,n}$. This procedure is carried out until convergence of the queue deviation or for fixed number of iterations by $I_{\max}$ as outlined in Algorithm 1.

Algorithm 1: Algorithm of JSFRA scheme

Input: $a_k,\ Q_k,\ \mathbf{H}_{b,k,n},\ \forall\, b \in \mathcal{B},\ \forall\, k \in \mathcal{U},\ \forall\, n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}\, \forall\, l \in \{1, 2, \ldots, L\}$
Initialize: $i = 0$ and transmit precoders $\tilde{\mathbf{M}}_{k,n}$ randomly satisfying the total power constraint (6b)
update $\mathbf{W}_{k,n}$ and $\tilde{\mathbf{u}}_{l,k,n}$ using (22b) and (18) using $\tilde{\mathbf{M}}_{k,n}$
repeat
  initialize $j = 0$
  repeat
    solve for the transmit precoders $\mathbf{m}_{l,k,n}$ using (19)
    update the constraint set (18) with $\tilde{\mathbf{u}}_{l,k,n}$ and $\mathbf{m}_{l,k,n}$ using (17)
    $j = j + 1$
  until *SCA convergence or* $j \geq J_{\max}$
  update the receive beamformers $\mathbf{w}_{l,k,n}$ using (20) or (22b) with the updated precoders $\mathbf{m}_{l,k,n}$
  $i = i + 1$
until *Queue convergence or* $i \geq I_{\max}$

Convergence: In order to prove the convergence of the proposed iterative algorithm, following conditions are to be satisfied [27]

- convergence of the SCA subproblem
- uniqueness of the transmit and the receive beamformers
- monotonic convergence of the objective function

In the proposed solution, we replaced (16b) by a convex constraint using the first order approximation, which is majorized by the quadratic-over-linear function in (16b) from below around a fixed point $\tilde{\mathbf{u}}^{(i)}_{l,k,n}$. Since the SCA method is adopted in the proposed algorithm, the constraint approximation satisfies the following conditions as in [28]

$$f(\tilde{\mathbf{u}}_{l,k,n}) \leq \bar{f}(\tilde{\mathbf{u}}_{l,k,n}, \tilde{\mathbf{u}}^{(i)}_{l,k,n}) \tag{23a}$$
$$f(\tilde{\mathbf{u}}^{(i)}_{l,k,n}) = \bar{f}(\tilde{\mathbf{u}}^{(i)}_{l,k,n}, \tilde{\mathbf{u}}^{(i)}_{l,k,n}) \tag{23b}$$
$$\nabla f(\tilde{\mathbf{u}}^{(i)}_{l,k,n}) = \nabla \bar{f}(\tilde{\mathbf{u}}^{(i)}_{l,k,n}, \tilde{\mathbf{u}}^{(i)}_{l,k,n}), \tag{23c}$$

where $\bar{f}(\mathbf{x}, \mathbf{x}^{(i)})$ is the approximate function of $f(\mathbf{x})$ around the point $\mathbf{x}^{*(i)}$. The stationary point of the relaxed convex problem satisfies the KKT conditions of the original nonconvex problem, which can be obtained by using conditions in (23). It can be seen that the SCA relaxed formulation converges to a local stationary point at each iteration.

The uniqueness of the transmit and the receive beamformers can be justified by forcing one antenna to be real valued to exclude the phase ambiguity arising from the complex precoders. The monotonic convergence of the objective function can be justified by the following arguments. At each SCA iteration, the relaxed subproblem is solved for the locally optimal transmit precoders to minimize the objective function. Since the SCA subproblem is relaxed around the $i-1^{\text{th}}$ optimal point, *i.e*, $\mathbf{x}^{*(i-1)}$ for the i^{th} iteration, the domain of the problem in the i^{th} step includes optimal point from the $i-1^{\text{th}}$ iteration as well. Therefore, at each SCA step, the objective function can either be equal to or smaller than the previous value, thereby leading to the monotonic convergence of the objective function.

Once the problem is converged to a stationary transmit precoders, the receive beamformers are updated based on the receivers in (21b) or (22b). The monotonic nature of the objective function is preserved by the receive beamformer update, since the receiver minimizes the objective value for the fixed transmit precoders, and hence the proposed JSFRA scheme is guaranteed to converge to a stationary point of the original nonconvex problem.

C. JSFRA Scheme via MSE Reformulation

In this section, we solve the JSFRA problem by exploiting the equivalence between the MSE and the achievable sum rate for MMSE receivers [5], [6]. The MSE $\epsilon_{l,k,n}$, for the data symbol is given by

$$\mathbb{E}\big[(d_{l,k,n} - \hat{d}_{l,k,n})^2\big] = \big|1 - \mathbf{w}^{\mathrm{H}}_{l,k,n}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\big|^2 + \sum_{(j,i)\neq(l,k)} \big|\mathbf{w}^{\mathrm{H}}_{l,k,n}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\big|^2 + N_0\,\|\mathbf{w}_{l,k,n}\|^2 = \epsilon_{l,k,n}, \tag{24}$$

where $\hat{d}_{l,k,n}$ is the estimate of the transmitted symbol. Now, replacing the receive beamformer in (24) with the MMSE receiver in (22b), we obtain the following relation between the MSE and the SINR as

$$\epsilon_{l,k,n} = \frac{1}{1 + \gamma_{l,k,n}}, \tag{25}$$

where $\gamma_{l,k,n}$ is the received SINR as in (2). Using the equivalence in (25), the WSRM objective can be reformulated as the weighted minimum mean squared error (WMMSE) equivalent to obtain the precoders for the MU-MIMO scenario as discussed in [7]–[9].

Let $v'_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}$ denote the queue deviation corresponding to user k and $\tilde{v}'_k \triangleq a_k^{1/q} v'_k$ represents the weighted equivalent. By using the relaxed MSE expression in (24), the problem in (6) can be expressed as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \tag{26a}$$
$$\text{subject to} \quad t_{l,k,n} \leq -\log_2(\epsilon_{l,k,n}) \tag{26b}$$
$$\big|1 - \mathbf{w}^{\mathrm{H}}_{l,k,n}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\big|^2 + N_0\,\|\mathbf{w}_{l,k,n}\|^2 + \sum_{(j,i)\neq(l,k)} \big|\mathbf{w}^{\mathrm{H}}_{l,k,n}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\big|^2 \leq \epsilon_{l,k,n} \tag{26c}$$
$$\text{and (6b).} \tag{26d}$$

An alternative MSE formulation given by (26) is non-convex even for the fixed $\mathbf{w}_{l,k,n}$ due to the constraint (26b). We adopt the SCA method as in Section II-B to relax the constraint by a sequence of convex subsets using first order Taylor series approximation around a fixed MSE point $\tilde{\epsilon}_{l,k,n}$ as

$$-\log_2(\tilde{\epsilon}_{l,k,n}) - \frac{(\epsilon_{l,k,n} - \tilde{\epsilon}_{l,k,n})}{\log(2)\,\tilde{\epsilon}_{l,k,n}} \geq t_{l,k,n}, \tag{27}$$

Using the above approximation for the rate constraint, the problem defined in (26) is solved for optimal transmit precoders $\mathbf{m}_{l,k,n}$, MSEs $\epsilon_{l,k,n}$, and the user rates over each sub-channel $t_{l,n,k}$ given the fixed receive beamformers. Once the optimal precoders are obtained, the local MSE variable $\tilde{\epsilon}_{l,k,n}$ is updated with the current update $\epsilon_{l,k,n}$. The optimization problem for a fixed receive beamformers $\mathbf{w}_{l,k,n}$ is given as

$$\underset{t_{l,k,n},\mathbf{m}_{l,k,n},\epsilon_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \tag{28a}$$
$$\text{subject to} \quad \text{(6b), (26c), and (27).} \tag{28b}$$

Convergence: Following similar approach as in Section III-B, at each iteration, the SCA subproblems converge to a stationary point of the original nonconvex problem. The uniqueness of the precoders are justified if there is no phase ambiguity in the stationary solution. By reorganizing (26c) as

$$\begin{aligned}\epsilon_{l,k,n} \geq 1 - 2\Re\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\} \\ + \sum_{\forall(j,i)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2 + N_0\,\|\mathbf{w}_{l,k,n}\|^2,\end{aligned} \tag{29}$$

we can see that the ambiguity in the phase rotations for the transmit and the receive beamformers are eliminated by the presence of real component in the MSE expression.

At each SCA update, the transmit precoders are obtained uniquely by minimizing (26) due to the convex nature of the relaxed problem. For a fixed transmit precoders, the MMSE receiver improves the objective value [7], [8], leading to the monotonic convergence of the objective function. At each SCA step, the optimal value of the previous iteration is also included in the domain of the problem, and the objective value can either decrease or stays the same after each iteration. Note that the objective function improves at each iteration, whereas the sum rate need not follow the same behavior.

D. Reduced Complexity Resource Allocation

The complexity involved in the JSFRA scheme scales significantly with the increase in the number of sub-channels considered in the formulation. In addition to the increased complexity, the rate of convergence to the optimal precoders also degrades due to its dependency on the problem size. In order to address these issues, we provide an alternative sub-optimal solution, in which the precoders are designed over each sub-channel independently in a sequential manner by taking the remaining number of queued bits in the formulation. The optimal approach is to decompose the problem over each sub-channel with a fixed transmit power constraint for each sub-channel. The power allocated for each sub-channel is controlled by a master problem based on different algorithms as discussed in [12], [13].

The proposed queue minimizing spatial resource allocation (SRA) formulation enables us to solve for the transmit precoders of all the users associated with the coordinating BS in the set $\mathcal{B}$ over each sub-channel independently by fixing the transmit power on each sub-channel to a constant value $P_{\max,n}$ as compared to the global power constraint defined by (6b). In contrast to the decomposition based approach for the sub-channel-wise resource allocation, where the primal/dual variables are exchanged, this method requires the update on the number of queued bits before each sub-channel-wise optimization. The total number of queued bits for each user is updated by the difference between the total number of queued bits present during the current slot to the total number of bits that are guaranteed by the earlier sub-channel allocations for the same slot as

$$Q_{k,n} = \max\Big\{Q_k - \sum_{r=1}^{n-1}\sum_{l=1}^{L} t_{l,k,r}, 0\Big\},\ \forall\, k \in \mathcal{U}, \tag{30}$$

where $Q_{k,n}$ is the total number of queued bits used in the optimization problem carried out for the sub-channel n. In the expression in (30), Q_k denotes the total number of queued bits waiting to be transmitted for the user k during the current slot and $t_{l,k,r}$ is the rate or guaranteed bits allocated over the sub-channel r. However, the proposed scheme is sensitive to the order in which the sub-channels are selected for the optimization problem.

IV. Distributed Solutions

This section addresses the distributed precoder designs for the proposed JSFRA scheme. The formulation in (19) or (28) requires a centralized controller to perform the precoder design for all users belonging to the BSs in $\mathcal{B}$. In order to design the precoders independently at each BS with the minimal information exchange via back-haul, iterative decentralization methods are considered. In particular, the primal decomposition and the ADMM which is basically based on dual decomposition approaches are followed in this paper.

To begin with, let $\bar{\mathcal{B}}_b$ denote the set $\mathcal{B}\backslash\{b\}$ and $\bar{\mathcal{U}}_b$ represents the set $\mathcal{U}\backslash\mathcal{U}_b$. To facilitate decomposition based solutions, we consider the solution proposed in (19), which is based on the Taylor series approximation for the nonconvex constraint. The following discussions are equally valid for the MSE based solution outlined in (28) as well. Since the objective of (19) can be decoupled across each BS, the centralized problem can be equivalently written as

$$\underset{\gamma_{l,k,n},\mathbf{M}_{k,n},\mathbf{W}_{k,n},\beta_{l,k,n}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}} \|\tilde{\mathbf{v}}_b\|_q \tag{31a}$$
$$\text{subject to} \quad \text{(19b)} - \text{(19d)}, \tag{31b}$$

where $\tilde{\mathbf{v}}_b$ denotes the vector of of weighted queue deviation corresponding to users $k \in \mathcal{U}_b$.

Following similar approach as in [14], [15], the coupling constraint (19b) or (26c) can be expressed by grouping the interference contribution from each BS in $\mathcal{B}$ as

$$N_0\|\mathbf{w}_{l,k,n}\|^2 + \sum_{j=1,j\neq l}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,k,n}|^2 + \sum_{b\in\bar{\mathcal{B}}_{b_k}} \zeta_{l,k,n,b}$$

$$+\sum_{i\in\mathcal{U}_{b_k}\setminus\{k\}}\sum_{j=1}^{L}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n}, \quad (32)$$

where $\zeta_{l,k,n,b}$ is the total interference caused by BS b to the l^{th} stream of user $k \in \mathcal{U}_{b_k}$ on the n^{th} sub-channel, is upper bounded by

$$\zeta_{l,k,n,b} \geq \sum_{i\in\mathcal{U}_b}\sum_{j=1}^{L}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2, \forall b \in \bar{\mathcal{B}}_{b_k}. \quad (33)$$

The coupling variable $\beta_{l,k,n}$ can be decoupled using the variable $\zeta_{l,k,n,b}$, which limits the total interference caused by the transmission from BS b to the l^{th} spatial stream of user k over the sub-channel n. In order the solve for the global optimal precoders, we need to find the coupling variables $\zeta_{l,k,n,b}$ by either the primal or dual decomposition method. In both approaches, the coupling constraint (19b) for the SCA and (26c) for the MSE relaxation schemes are decoupled to perform the distributed precoder design problem.

A. Decomposition based Approaches

1) Primal Decomposition Approach: The primal decomposition approach decomposes the problem by fixing the interference variables $\zeta_{l,k,n,b}\forall k,b$ in order to perform the precoder design independently across each BS. Once the optimal precoders are designed at each BS with the fixed interference constraints (32), the dual variables corresponding to the interference constraints are exchanged between the cooperating BSs in $\mathcal{B}$ to update the interference variables $\zeta_{l,k,n,b}$ for the next iteration until convergence. The primal approach is discussed extensively for the min-power problem in [14] and much of the current work follows similar approach.

Convergence: The convergence of the primal decomposition is similar to that of the centralized problem if the interference variables $\zeta_{l,k,n,b}$ are allowed to converge to a stationary point. In practice, we can limit the number of exchanges to $J_{\max}$ after which the SCA update is performed until convergence or for $I_{\max}$ times. The update of $\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}$ and $\tilde{\beta}_{l,k,n}$ can be made in conjunction with the receiver update $\mathbf{W}_{k,n}$. The receiver update can be made by using the precoded pilot transmission from each user as in [29].

2) ADMM approach: The ADMM decomposition method is based on the dual decomposition, however it shows better convergence properties. In contrast to the primal decomposition problem, the ADMM method relaxes the interference constraints by including it in the objective function of each subproblem with a penalty pricing [12], [13]. In order to decouple the problem (31), the coupling variables $\zeta_{l,k,n,b}$ in (32) are replaced by the respective local copies $\boldsymbol{\zeta}^{\{b\}}, \forall b \in \mathcal{B}$, which are then solved for an optimal solution. Now the sub problems are coupled by the global consensus vector $\boldsymbol{\zeta}$ maintaining the complete stacked interference profile of all users in the system as

$$\begin{aligned}\boldsymbol{\zeta} &= \Big[\zeta_{1,\bar{\mathcal{U}}_1(1),1,1},\ldots,\zeta_{L,\bar{\mathcal{U}}_1(1),1,1},\ldots,\zeta_{L,\bar{\mathcal{U}}_1(|\bar{\mathcal{U}}_1|),1,1},\\ &\quad \ldots,\zeta_{L,\bar{\mathcal{U}}_{N_B}(|\bar{\mathcal{U}}_{N_B}|),1,N_B},\ldots,\zeta_{L,\bar{\mathcal{U}}_{N_B}(|\bar{\mathcal{U}}_{N_B}|),N,N_B}\Big] \quad (34a)\\ n_{b_k} &= |\boldsymbol{\zeta}^{\{b_k\}}| = NL\sum_{b\in\mathcal{B}}|\bar{\mathcal{U}}_b|. \quad (34b)\end{aligned}$$

Let $\boldsymbol{\zeta}(b_k)$ denote the consensus entries corresponding to BS b_k. Let $\boldsymbol{\nu}^{\{b_k\}}$ represent the stacked dual variables corresponding to the equality condition $\boldsymbol{\zeta}^{\{b_k\}} = \boldsymbol{\zeta}(b_k)$ used in the subproblems. In order to limit the local interference assumptions $\zeta_{l,k,n,b}^{\{b_k\}}$ in BS b_k, the ADMM method augments a scaled quadratic penalty of the interference deviation between the local and consensus value for the interference from the BS b as $\zeta_{l,k,n,b}$ in the objective function. At optimality, the locally assumed and the consensus interference values will be equal, providing no contribution to the objective function. The optimal step size used to update the dual variables is the scaling factor ρ used to scale the penalty term in the objective function [2], [13]. The equality constraint for the local and the consensus interference vector $\boldsymbol{\zeta}^{\{b_k\}} = \boldsymbol{\zeta}(b_k)$ present in each subproblem is relaxed by the taking the partial Lagrangian. Now, the subproblem at BS b for the i^{th} iteration is given by

$$\begin{aligned}\underset{\substack{\gamma_{l,k,n},\mathbf{W}_{k,n},\mathbf{M}_{k,n}\\ \beta_{l,k,n},\boldsymbol{\zeta}^{\{b\}(i)}}}{\text{minimize}} \quad & \|\tilde{\mathbf{v}}_b\|_q + \boldsymbol{\nu}^{\{b\}(i-1)T}\left(\boldsymbol{\zeta}^{\{b\}(i)} - \boldsymbol{\zeta}^{(i-1)}(b)\right)\\ & + \frac{\rho}{2}\Big\| \underbrace{\boldsymbol{\zeta}^{\{b\}(i)}}_{\text{local}} - \underbrace{\boldsymbol{\zeta}^{(i-1)}(b)}_{\text{consensus}} \Big\|_2^2 \quad (35a)\\ \text{subject to} \quad & \\ \beta_{l,k,n} \geq & \sum_{j=1,j\neq l}^{L}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{j,k,n}|^2 + \sum_{\hat{b}\in\bar{\mathcal{B}}_b}\zeta_{l,k,n,\hat{b}}^{\{b\}(i-1)}\\ + & \sum_{i\in\mathcal{U}_b\setminus\{k\}}\sum_{j=1}^{L}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{j,i,n}|^2 + N_0\|\mathbf{w}_{l,k,n}\|^2 \quad (35b)\\ \zeta_{l',k',n,b}^{\{b\}(i)} \geq & \sum_{k\in\mathcal{U}_{\hat{b}}}\sum_{l=1}^{L}|\mathbf{w}_{l',k',n}^{\mathrm{H}}\mathbf{H}_{b,k',n}\mathbf{m}_{l,k,n}|^2, \forall k' \in \bar{\mathcal{U}}_b \quad (35c)\\ & \text{(18) and (6b)}, \quad (35d)\end{aligned}$$

where the superscript i represents the current iteration or the information exchange index and $\boldsymbol{\zeta}^{(i-1)}$ denotes the updated global interference level from the $(i-1)^{\mathrm{th}}$ information exchange of the local interference vector $\boldsymbol{\zeta}^{\{b\}(i-1)}, \forall b \in \mathcal{B}$.

Now, the local problem (35) at each BS b is solved either by the SCA approach discussed in Section III-B or by using the MSE reformulation approach outlined in Section III-C. Once the local problems are solved at each BS, the new update for the global interference vector $\boldsymbol{\zeta}^{(i)}$ and the dual variables $\boldsymbol{\nu}^{\{b\}(i)}$ are performed at each BS independently by exchanging the corresponding local copies of the interference vector $\boldsymbol{\zeta}^{\{b\}(i)}, \forall b \in \mathcal{B}$. Since the entries in $\boldsymbol{\zeta}^{(i)}$ relate exactly to two BSs only, each entry in $\boldsymbol{\zeta}^{(i)}$ can be updated by exchanging the local copies from the corresponding two BSs. For instance, the entry $\zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{(i)}$ depends on the local interference value $\zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{\{b_k\}(i)}$ assumed by the BS b_k and the actual interference caused by BS b as in $\zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{\{b\}(i)}$ as

$$\zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{(i)} = \frac{1}{2}\left(\zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{\{b\}(i)} + \zeta_{l,\mathcal{U}_{b_k}(1),n,b}^{\{b_k\}(i)}\right). \quad (36)$$

The dual variable vector $\boldsymbol{\nu}^{\{b_k\}}$, which includes the stacked dual variables of the interference equality constraint at BS b_k, are updated using the subgradient as

$$\nu_{l,k,n,b}^{\{b_k\}(i)} = \nu_{l,k,n,b}^{\{b_k\}(i-1)} + \rho \left(\zeta_{l,k,n,b}^{\{b_k\}(i)} - \zeta_{l,k,n,b}^{(i)} \right). \tag{37}$$

The distributed precoder design using ADMM approach is shown in Algorithm 2.

Algorithm 2: Distributed JSFRA scheme using ADMM

Input: $a_k,\ Q_k,\ \mathbf{H}_{b,k,n},\ \forall\, b \in \mathcal{B},\ \forall\, k \in \mathcal{U}, \forall\, n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall\, l \in \{1, 2, \ldots, L\}$
Initialize: $i = 0$ and the transmit precoders $\tilde{\mathbf{m}}_{l,k,n}$ randomly satisfying total power constraint (6b)
update $\mathbf{w}_{l,k,n}$ with (22b) and $\tilde{\mathbf{u}}_{l,k,n}$ with (18)
initialize the global interference vectors $\boldsymbol{\zeta}^{(0)} = \mathbf{0}^{\mathrm{T}}$
initialize the interference threshold $\boldsymbol{\nu}^{\{b\}(0)} \forall b \in \mathcal{B} = 0$
foreach *BS* $b \in \mathcal{B}$ **do**
  repeat
    initialize $j = 0$
    repeat
      solve for $\mathbf{M}_{k,n}$ and the local interference $\boldsymbol{\zeta}^{\{b\}}$ using (35)
      exchange $\boldsymbol{\zeta}^{\{b\}(j)}$ among BSs in $\mathcal{B}$
      update dual variables in $\boldsymbol{\nu}^{\{b\}(j+1)}$ using (37)
      update the consensus vector $\boldsymbol{\zeta}^{(j+1)}$ using (36)
      $j = j + 1$
    until *convergence or* $j \geq J_{\max}$
    downlink precoded pilot transmission with $\mathbf{M}_{k,n}$
    update $\mathbf{W}_{k,n}$ and notify to all BSs in $\mathcal{B}$ using uplink precoded pilots as in [29]
    update $\tilde{\mathbf{u}}_{l,k,n}$ using (16c) and (17) for SCA or $\tilde{\epsilon}_{l,k,n}$ using (26c) for MSE approach
    $i = i + 1$
  until *convergence or* $i \geq I_{\max}$
end

Convergence: The convergence of the ADMM method follows the same argument as the centralized algorithm if each distributed algorithm is allowed to converge to a stationary value for the fixed SCA point. Since the subproblem solved at each BS is convex, the ADMM method converges to a stationary point [13] for the fixed SCA value. The receive beamformers are updated along with the SCA update of $\tilde{\mathbf{u}}_{l,k,n}$. Combining the receiver update with the SCA update improves the convergence speed due to the fact that the MMSE receivers are optimal for the fixed transmit beamformers, providing monotonic increase in the objective function.

B. Decomposition via KKT Conditions for MSE Formulation

In this section, we discuss different ways to decentralize the precoder design across the coordinating BSs in $\mathcal{B}$ based on the MSE reformulation method discussed in Section III-C. In contrast to Section IV-A, the problem is solved using the KKT conditions in which the transmit precoders, receive beamformers and the subgradient updates are performed at the same instant to minimize the global queue deviation objective with few number of iterations. The proposed methods in this section provide algorithms that can be of practical importance due to the limited signaling requirements. Similar work has been considered for the WSRM problem with minimum rate constraints in [10], [11]. Since the formulation in [10], [11] are similar to the Q-WSRME scheme with an additional maximum rate constraint (14), it requires explicit dual variables to handle the maximum rate constraint, thereby making the problem difficult to solve in an iterative manner.

In the proposed JSFRA formulation, the maximum rate constraints are implicitly handled by the objective function without the need of explicit constraints. However, the KKT conditions cannot be formulated due to the non-differential objective function. The non-differentiability is due to the absolute value operator present in the norm function. In order to make the objective function differentiable, we consider the following two cases for which the absolute operator can be ignored without affecting the optimal solution, namely,

- when the exponent q is even, or
- when the number of backlogged packets of each user is large enough, *i.e*, $Q_k \gg \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$ to ignore the absolute operator, which means also ignoring the queues in the first place as well.

With the assumption of either one of the above conditions to be true, the problem in (28) can be written as

$$\underset{\substack{t_{l,k,n},\mathbf{M}_{k,n},\\ \epsilon_{l,k,n},\mathbf{W}_{k,n}}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}} \sum_{k\in\mathcal{U}_b} a_k \left(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right)^q \tag{38a}$$

subject to

$$\alpha_{l,k,n} : \left| 1 - \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2 + N_0 \|\mathbf{w}_{l,k,n}\|^2 + \sum_{(x,y)\neq(l,k)} \left| \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n} \right|^2 \leq \epsilon_{l,k,n} \tag{38b}$$

$$\sigma_{l,k,n} : \log_2(\tilde{\epsilon}_{l,k,n}) + \frac{(\epsilon_{l,k,n} - \tilde{\epsilon}_{l,k,n})}{\log(2)\tilde{\epsilon}_{l,k,n}} \leq -t_{l,k,n} \tag{38c}$$

$$\delta_b : \sum_{n=1}^{N} \sum_{k\in\mathcal{U}_b} \sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}} \right) \leq P_{\max},\ \forall\, b, \tag{38d}$$

where $\alpha_{l,k,n}, \sigma_{l,k,n}$ and δ_b are the dual variables corresponding to the constraints defined in (38b), (38c) and (38d).

The problem in (38) is solved using the KKT expressions, which are obtained by the derivative of the Lagrangian function w.r.t the primal variables, complementary slackness conditions, and the primal, dual feasibility requirements as shown in Appendix A. Upon solving, we obtain the iterative solution as

$$\mathbf{m}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}} \sum_{y=1}^{L} \alpha_{y,x,n}^{(i-1)} \mathbf{H}_{b_k,x,n}^{\mathrm{H}} \mathbf{w}_{y,x,n}^{(i-1)} \mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)} \mathbf{H}_{b_k,x,n} + \delta_b \mathbf{I}_{N_T} \Bigg)^{-1} \alpha_{l,k,n}^{(i-1)} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n}^{(i-1)} \tag{39a}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}} \sum_{y=1}^{L} \mathbf{H}_{b_x,k,n} \mathbf{m}_{y,x,n}^{(i)} \mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)} \mathbf{H}_{b_x,k,n}^{\mathrm{H}} + \mathbf{I}_{N_R} \Bigg)^{-1} \mathbf{H}_{b_k,k,n}\ \mathbf{m}_{l,k,n}^{(i)}. \tag{39b}$$

$$\epsilon_{l,k,n}^{(i)} = \left| 1 - \mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)} \right|^2 + N_0 \|\mathbf{w}_{l,k,n}^{(i)}\|^2$$

$$+\sum_{(x,y)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}^{(i)}\right|^2 \tag{39c}$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{\left(\epsilon_{l,k,n}^{(i)}-\epsilon_{l,k,n}^{(i-1)}\right)}{\log(2)\,\epsilon_{l,k,n}^{(i-1)}} \tag{39d}$$

$$\sigma_{l,k,n}^{(i)} = \left[\tfrac{a_k\, q}{\log(2)}\left(Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}^{(i)}\right)^{(q-1)}\right]^+ \tag{39e}$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)} + \rho\left(\tfrac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}} - \alpha_{l,k,n}^{(i-1)}\right) \tag{39f}$$

Since the dual variables $\alpha^{(i)}$ and $\sigma^{(i)}$ are interdependent in (39), one has to be fixed to optimize for the other. So, the variable $\alpha^{(i)}$ is fixed as in (39f) to obtain the other variables in (39). At each iteration, the dual variables $\alpha^{(i)}$ are updated linearly from the earlier $\alpha^{(i-1)}$ by a step size of $\rho \in [0,1]$. When the allocated rate $t_k^{(i-1)}$ is greater than the number of queued packets Q_k for a user k, the corresponding dual variable $\sigma^{(i)}$ will be negative and due to the projection operator $[x]^+$ in (39e), it will be zero, thereby forcing $\alpha_k^{(i)} < \alpha_k^{(i-1)}$ as in (39f). Once the $\alpha_k^{(i)}$ is reduced, the precoder weight in (39a) is lowered to make the rate $t_k^{(i)} < t_k^{(i-1)}$.

The KKT expressions in (39) are solved in an iterative manner by initializing the transmit and the receive beamformers $\mathbf{M}_{k,n}, \mathbf{W}_{k,n}$ with the single user beamforming and the MMSE vectors. The dual variable α's are initialized with ones to have equal priorities to all the users in the system. Then the transmit and the receive beamformers are evaluated using the expressions in (39). The transmit precoder in (39a) depends on the BS specific dual variable δ_b, which can be found by bisection search satisfying the total power constraint (38d). Note that the fixed SCA operating point is given by $\tilde{\epsilon}_{l,k,n} = \epsilon_{l,k,n}^{(i-1)}$, which is considered in the expression (39).

To devise an algorithm for a more practical implementation, we extend the decentralization methods discussed in [29], for our problem of minimizing the total number of backlogged packets as follows. After receiving the updated transmit precoders from all BSs in $\mathcal{B}$, each user evaluates the MMSE receiver in (39b) and notify them to the BSs via uplink precoded pilots. On receiving pilot signals, BSs update the MSE in (24) as

$$\epsilon_{l,k,n}^{(i)} = 1 - \mathbf{w}_{l,k,n}^{(i)\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}^{(i)}. \tag{40}$$

Using the current MSE value, $t_{l,k,n}^{(i)}, \sigma_{l,k,n}^{(i)}$, and $\alpha_{l,k,n}^{(i)}$ are evaluated using (39d), (39e) and (39f), and the updated dual variables $\alpha_{l,k,n}$ are exchanged between the BSs to evaluate the transmit precoders $\mathbf{m}_{l,k,n}^{(i+1)}$ for the next iteration. The SCA operating point is also updated with the current MSE value.

To avoid the back-haul exchanges between BSs, as an alternative approach, users may perform all processing required and BSs will update the precoders based on the feedback information from the users. Upon receiving the transmit precoders from BSs, each user will update the receive beamformer $\mathbf{w}_{l,k,n}$, the MSE $\epsilon_{l,k,n}$, and the dual variables $\lambda_{l,k,n}$ and $\alpha_{l,k,n}$. The updated $\alpha_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are notified to the BSs using two separate precoded uplink pilot symbols with $\tilde{\mathbf{w}}_{l,k,n}^{(i)} = \sqrt{\alpha_{l,k,n}^{(i)}}\mathbf{w}_{l,k,n}^{*(i)}$ and $\bar{\mathbf{w}}_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i)}\mathbf{w}_{l,k,n}^{*(i)}$ as the precoders. On receiving the precoded uplink pilots, each BS use the effective channel $\mathbf{H}_{b,k,n}^{\mathrm{T}}\tilde{\mathbf{w}}_{l,k,n}^{(i)}$ and $\mathbf{H}_{b,k,n}^{\mathrm{T}}\bar{\mathbf{w}}_{l,k,n}^{(i)}$ in (39a) to update the transmit precoders, where $\mathbf{x}^*$ is the complex conjugate of $\mathbf{x}$. Finally, Algorithm 3 outlines the distributed precoder design using the KKT based MSE reformulated JSFRA problem.

Algorithm 3: KKT approach for the JSFRA scheme

Input: $a_k,\ Q_k,\ \mathbf{H}_{b,k,n},\ \forall\, b \in \mathcal{B},\ \forall\, k \in \mathcal{U}, \forall\, n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}\, \forall\, l \in \{1,2,\ldots,L\}$
Initialize: $i=1$, $\mathbf{w}_{l,k,n}^{(0)}, \tilde{\epsilon}_{l,k,n}$ randomly, dual variables $\alpha_{l,k,n}^{(0)} = 1$, and $I_{\max}$ for certain value
foreach *BS* $b \in \mathcal{B}$ **do**
- initialize $i = 0$
- **repeat**
  - update $\mathbf{M}_{k,n}^{(i)}$ using (39a), and perform downlink transmission
  - find $\mathbf{W}_{k,n}^{(i)}$ using (39b) at each user
  - evaluate $\epsilon_{l,k,n}^{(i)},\ t_{l,k,n}^{(i)},\ \sigma_{l,k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ using (39c) and (39d), (39e) and (39f) at each user with the updated $\mathbf{W}_{k,n}^{(i)}$
  - using precoded uplink pilots, $\mathbf{W}_{k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ are notified to all BSs in $\mathcal{B}$
  - $i = i + 1$
- **until** *until convergence or* $i \geq I_{\max}$

end

V. Simulation Results

The simulations carried out in this work consider the path loss varying uniformly across all users in the system with the channels drawn from the *i.i.d.* samples. The queues are generated based on the Poisson process with the average values specified in each section presented.

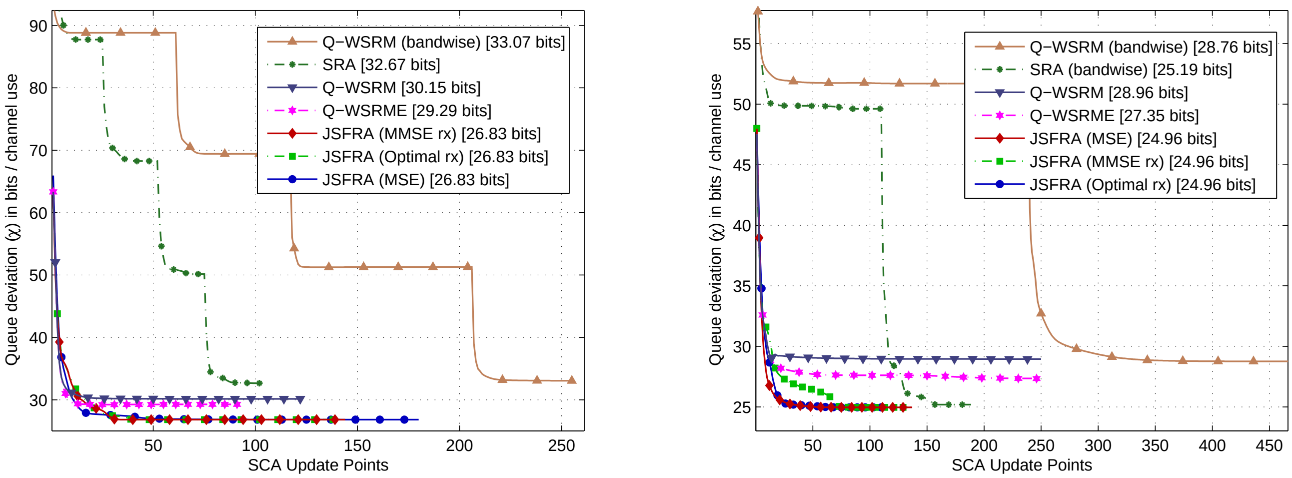
A. Centralized Solutions

We discuss the performance of the centralized algorithms in Section III for some system configurations. To begin with, we consider a single cell single-input single-output (SISO) model operating at 10 dB signal-to-noise ratio (SNR) with $K = 3$ users sharing $N = 3$ sub-channel resources. The number of packets waiting at the transmitter for each user is given by $Q_k = 4, 8$ and 4 bits, respectively.

Table I tabulates the channel seen by the users over each sub-channel followed by the rates assigned by three different algorithms, Q-WSRME allocation, JSFRA approach and the band-wise Q-WSRM scheme using the WMMSE design [8]. The performance metric used for the comparison is the total number of backlogged bits left over at each slot after the allocation, which is denoted as $\chi = \sum_{k=1}^{K}\ [Q_k - t_k]^+$. Even though $\mathcal{U}(1)$ and $\mathcal{U}(3)$ has equal number of backlogged packets of $Q_1 = Q_3 = 4$ bits, user $\mathcal{U}(3)$ is scheduled in the first sub-channel due to the better channel condition. In contrast, the JSFRA approach assigns the first user on the first sub-channel, which reduces the total number of backlogged packets waiting at the transmitter. The rate allocated for

TABLE I
SUB-CHANNEL-WISE LISTING OF CHANNEL GAINS AND RATE ALLOCATIONS BY DIFFERENT ALGORITHMS FOR A SCHEDULING INSTANT

Users	Queued Packets	Channel Gains			Q-WSRME approach (modified *backpreassure*)			JSFRA Scheme			Q-WSRM band Alloc Scheme		
		SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3
1	4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
2	8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
3	4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining backlogged packets (χ)					3.92 bits			2.51 bits			5.89 bits		



(a). System Model $\{N, N_B, K, N_T, N_R\} = \{4, 3, 9, 4, 1\}$ (b). System Model $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

Fig. 1. Total number of backlogged packets χ present in the system after each SCA updates

TABLE II
NUMBER OF BACKLOGGED BITS ASSOCIATED WITH EACH USER FOR A SYSTEM $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$.

q	user indices								χ
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
∞	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
Q_k	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	

$\mathcal{U}(2)$ on the second sub-channel is higher in JSFRA scheme compared to the other schemes. It is due to the efficient allocation of the total power shared across the sub-channels.

For a MIMO framework, we consider a system with $N = 3$ sub-channels and $N_B = 3$ BSs, each equipped with $N_T = 4$ transmit antennas operating at 10dB SNR, serving $|\mathcal{U}_b| = 3$ users each. The path loss between the BSs and the users are uniformly generated from $[0, -3]$ dB and the association is made by selecting the BS with the lowest path loss component. Fig. 1(a) shows the performances of the centralized schemes for a single receive antenna system. The total number of queued packets for Fig. 1(a) is given by $Q_k = [14, 15, 14, 8, 12, 9, 12, 11, 11]$ bits and for Fig. 1(b) is $Q_k = [9, 12, 8, 12, 5, 4, 10, 8, 5]$ bits respectively.

The performances of the centralized algorithms are compared in terms of the total number of residual bits remaining in the system after each SCA update in Fig. 1. The Q-WSRM algorithm is not optimal due to the problem of over-allocation when the number of queued packets are few in number. In contrast, the Q-WSRME algorithm provides more favorable allocation by including the explicit rate constraint to avoid the over-allocation. It can be seen that the JSFRA algorithms converge to the optimal point for all formulations proposed in Section III-B. All the algorithms are Pareto-optimal and provide different performances based on the weights used to find a point in the rate region.

For both scenarios in Fig. 1, the Q-WSRME performs marginally inferior to the JSFRA algorithms due to the weights used in the algorithm. The performance loss is attributed to the fact that the Q-WSRME algorithm favors the users with the large number of backlogged packets as compared to the users with better channel conditions. Fig. 1(b) compares the algorithms for $N_R = 2$ receive antenna case. In all figures, the receivers are updated along with the SCA update instants *i.e*, $J_{\max} = 1$ in Algorithm 1. It is also noted that the performance degradation by performing the group update is very minimal. Since the receiver minimizes the objective for the fixed transmit precoders, the convergence is monotonic as can be seen from the figures.

The behavior of the JSFRA algorithm for different exponents q is outlined in the Table II for the users located at the cell-edge of the system employing $N_T = 4$ transmit antennas. It is evident that the JSFRA algorithm minimizes the total number of queued bits for the ℓ_1 norm compared to the ℓ_2 norm, which is shown in the column displaying the total number of left over packets χ in bits. The ℓ_∞ norm provides fair allocation of the resources by making the left over packets to be equal for all users to $\chi_k = 3.58$ bits.

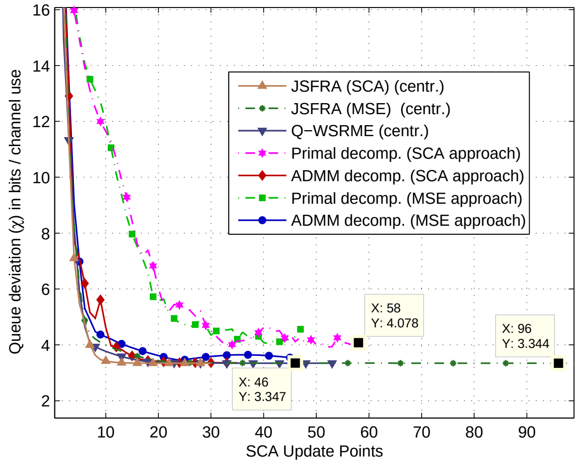


Fig. 2. Convergence behaviour of the centralized and the distributed algorithms for a system $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$

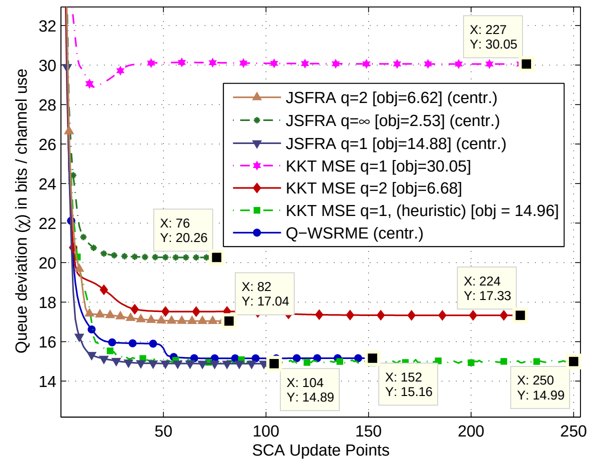


Fig. 3. Impact of varying q in the total number of backlogged packets after each SCA update for a system $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$

B. Distributed Solutions

The performances of the distributed algorithms are compared using the total number of backlogged packets after each SCA update points. Fig. 2 compares the performances of the algorithms for the system configuration $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ with $N_T = 4$ transmit antennas at the BSs. Each BS serves $|\mathcal{U}_b| = 4$ users in a coordinated manner to reduce the total number of backlogged packets at each BS. The total number of queued packets assumed for both figures is $Q_k = [5, 7, 9, 11, 8, 12, 5, 4]$ bits. As pointed out in Section IV, the performance and the convergence speed of the distributed algorithms are susceptible to the step size used in the subgradient update. Due to the fixed interference levels in the primal approach, it may lead to infeasible solutions if the initial or any intermediate update is not feasible.

Fig. 2 plots the performance of the primal and the ADMM solutions for the JSFRA scheme using the SCA and by MSE relaxation at each SCA point. In between the SCA updates, the primal or the ADMM scheme is performed for $J_{\max} = 20$ iterations to exchange the respective coupling variables. In Fig. 2, the total number of backlogged packets at each SCA points are plotted without the inner loop iterations of $J_{\max}$ times for the primal or the dual variables convergence. It can be seen from Fig. 2 that the distributed algorithms approach the centralized performance by exchanging minimal information between the coordinating BSs.

Fig. 3 compares the performance of the centralized and the KKT algorithm in Section IV-B for different exponents by plotting the total number of backlogged packets at each SCA update point. The ℓ_1 norm JSFRA scheme provides better performance over other schemes due to the greedy objective. The KKT approach for ℓ_1 norm is not defined due to the non-differentiability of the objective as discussed in the Section IV-B. If used for ℓ_1 norm, the problem of over-allocation will not affect the dual variables $\sigma_{l,k,n}$ and $\alpha_{l,k,n}$ since the queue deviation is raised to the power zero in (39e), which will always be equal to one. A heuristic method based on subdifferential calculus in [2] is proposed in Fig. 3 by assigning zero for $\sigma_{l,k,n}$ when the queue deviation is negative, *i.e*, $Q_k - t_k < 0$. It is required to address the problem of over-allocation in the ℓ_1 norm for dropping the absolute value operator from the objective function. It can be seen that the heuristic method oscillates near the optimal point with the deviation determined by the factor ρ used in (39f).

The objective values are mentioned in the legend for all the schemes and the objective of the ℓ_2 norm is not the same as that of the ℓ_1 norm used for plotting. For simulations, we update all variables in (39) at once at each iteration, *i.e*, $J_{\max} = 1$, which is well justified for the practical implementations due to the signaling overheads. The ℓ_2 norm for the JSFRA and the KKT approach achieves nearly the same value of 6.62 with different χ, due to the limited number of iterations for the dual variable convergence between each SCA update. Fig. 3 also shows the effect of dropping the squared rate variable from the objective in the Q-WSRME scheme compared to the ℓ_2 norm which includes it. By dropping it, the Q-WSRME scheme minimizes the number of queued packets in a prioritized manner based on the respective queues. On contrary, the ℓ_2 norm allocate rates to the users with the higher number of queued packets before addressing the users with the smaller number of queued packets.

VI. Conclusions

In this paper, we addressed the problem of allocating downlink space-frequency resources to the users in a multi-cell MIMO IBC system using OFDM transmission. The resource allocation is considered as a joint space-frequency precoder design problem since the allocation of a resource to a user is obtained by a non-zero precoding vector. We proposed the JSFRA scheme by employing the SCA technique to relax the nonconvex constraint by a sequence of convex subsets for designing the precoders to minimize the total number of user queued packets. Additionally, an alternative MSE relaxation approach is also proposed by using SCA technique to address the nonconvex constraints for a fixed MMSE receivers. We then introduced distributed precoder designs for the JSFRA problem using primal and ADMM methods. Finally, we proposed a practical iterative algorithm to obtain the precoders in a decentralized manner by solving the KKT conditions of the MSE reformulated JSFRA method. The proposed iterative algorithm requires few iterations and limited signaling exchange between the coordinating BSs to obtain the efficient precoders for a given number of iterations. Numerical results are provided to compare the performance of the proposed algorithms with the existing solutions.

Appendix A
KKT conditions for MSE approach

In order to solve for an iterative precoder design algorithm, the KKT expressions for the problem in (38) are obtained by differentiating the Lagrangian by assuming the equality constraint for (38b) and (38c). At the stationary points, following conditions are satisfied.

$$\nabla_{\epsilon_{l,k,n}} : -\alpha_{l,k,n} + \frac{\sigma_{l,k,n}}{\tilde{\epsilon}_{l,k,n}} = 0 \tag{41a}$$

$$\nabla_{t_{l,k,n}} : -q\, a_k \Big(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \Big)^{(q-1)} + \frac{\sigma_{l,k,n}}{\log_2(e)} = 0 \tag{41b}$$

$$\begin{aligned}\nabla_{\mathbf{m}_{l,k,n}} : &\sum_{y \in \mathcal{U}} \sum_{x=1}^{L} \alpha_{x,y,n} \mathbf{H}_{b_k,y,n}^{\mathrm{H}} \mathbf{w}_{x,y,n} \mathbf{w}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_k,y,n} \mathbf{m}_{l,k,n} \\ &+ \delta_b \mathbf{m}_{l,k,n} = \alpha_{l,k,n} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n},\end{aligned} \tag{41c}$$

$$\begin{aligned}\nabla_{\mathbf{w}_{l,k,n}} : &\sum_{(x,y) \neq (l,k)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n} \mathbf{m}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_y,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n} \\ &+ \mathbf{I}_{N_R} \mathbf{w}_{l,k,n} = \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n}.\end{aligned} \tag{41d}$$

In addition to the primal constraints given in (38b), (38c) and (38d), the complementary slackness criterion must also be satisfied at the stationary point. Upon solving the above expressions in (41) with the complementary slackness conditions, we obtain the iterative algorithm to determine the transmit and the receive beamformers as shown in (39).

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