Traffic Aware Precoder Design for Space Frequency Resource Allocation

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Outline

Section 1

Introduction

Introduction and Motivation

- Complex algorithms are adopted to maximize the throughput to satisfy the data requirements of the higher layers
- Available wireless resources are to be utilized efficiently to minimize the backlogged packets
- Spatial and Frequency resources are exploited to empty the packets waiting at the BSs
- In this work, we discuss the precoder design for the multiple users MIMO-OFDM setup to minimize the number of queued packets

Section 2

System Model & Problem Formulation

Symbols used

- OFDM system with N sub-channels and N_B base stations (BSs), each equipped with N_T transmit antennas
- Let K be the total number of users with N_R antennas
- Let $\mathcal{B}$ and $\mathcal{U}$ denote the set of coordinating BSs and users in the system
- The set of users belonging to BS b is denoted by $\mathcal{U}_b \in \mathcal{U}$
- Let $b_k \in \mathcal{B}$ denotes the BS serving the user k
- Let L be the total available spatial streams for a user k, given by $\min(N_T, N_R)$

System Model

- The lth spatial signal received on sub-channel n of user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n} \quad (1)$$

- where $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are transmit and receive beamformers corresponding to the lth spatial stream on the nth sub-channel of user k

System Model

- $\mathbf{H}_{b_k,k,n} \in \mathbb{C}^{N_R \times N_T}$ denotes the channel between BS b_k and user k
- $d_{l,k,n}$ and $n_{l,k,n}$ correspond to data symbol and equivalent noise on lth spatial stream of user k
- Using the above notations, the SINR seen by the lth spatial stream on the nth sub-channel for user k is given by

$$\gamma_{l,k,n} = \frac{\left| \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2}{\grave{N}_0 + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}\right|^2} \tag{2}$$

- where $\grave{N}_0 = \|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n}\|^2$

Queueing Model

- Each user is associated with backlogged packets of size Q_k packets.
- Queued packets Q_k of each user follows dynamic equation at the ith instant as

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i)\Big]^+ + \lambda_k(i) \tag{3}$$

- where $t_k = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$ denotes the total number of transmitted packets corresponding to user k in the previous ith instant
- λ_k represents the fresh arrivals of user k at BS b_k

Problem Formulation

- Objective is to transmit the queued packets waiting at BSs to corresponding users in the system.
- The available spatial and frequency resources are to be efficiently utilized to minimize the queued packets
- In order to achieve this, precoders are to be designed with certain objective that involves the backlogged packets as well
- Precoders can perform scheduling of users by providing zero powers to exclude from the resource elements

Section 3

Centralized Solutions

Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Q-WSRM formulation is the result of minimizing the conditional Lyapunov drift [1]
- Q-WSRM formulation is also called as back pressure algorithm, since it acts greedily in minimizing the backlogged packets at each instant

$$\underset{t_{l,k,n}}{\text{minimize}} \quad \sum_{k \in \mathcal{U}} \left\{ Q_k(i)^2 - Q_k(i-1)^2 \right\},$$

- where Q_k follows the dynamic Queue expression in (3) and $t_k = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$

Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Upon solving the Lyapunov drift expression, we obtain the Q-WSRM formulation as

$$\underset{t_{l,k,n}}{\text{maximize}} \quad \sum_{k \in \mathcal{U}} Q_k \left(\sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right) \tag{4a}$$

$$\sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \leq Q_k \ / \ Q_{k,n} \tag{4b}$$

- Performs better in cell-edge scenarios
- Complexity can be reduced if precoders are designed for each sub-channel independently

Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Complexity can be reduced if precoders are designed for each sub-channel independently
- Coupling across sub-channels is obtained by the queues, which are updated after evaluating the rate from previously chosen sub-channels as

$$Q_{k,n} = \max\left\{Q_k - \sum_{j=1}^{n-1}\sum_{l=1}^{L} t_{l,k,j}, 0\right\}, \ \forall\ k \in \mathcal{U}$$

JSFRA Formulation (SINR Relaxation)

- Precoders are designed by a centralized controller, which are then used by all BSs in $\mathcal{B}$
- The objective used to design transmit precoders is

$$v_k = \left| Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right|^q$$

- To generalize the objective, we use $\tilde{v}_k \triangleq a_k\, v_k$, where a_k is arbitrary weights used control the priorities
- Exponent q plays different role based on the value it assumes
  - $\ell_{q=1}$ results in greedy allocation
  - $\ell_{q=2}$ ideal for the delay or buffer size limited scenarios
  - $\ell_{q=\infty}$ provides fair resource allocation in each transmission instant

JSFRA Formulation (SINR Relaxation)

- Now, the precoder design problem is given as

$$\underset{t_{l,k,n},\mathbf{M}_{k,n},\mathbf{W}_{k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{5a}$$

$$\text{subject to} \quad t_{l,k,n} \leq \log_2(1+\gamma_{l,k,n}) \tag{5b}$$

$$\gamma_{l,k,n} \leq \frac{\left|\mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2}{\beta_{l,k,n}} \triangleq f(\tilde{\mathbf{u}}_{l,k,n}), \tag{5c}$$

$$\beta_{l,k,n} \geq \grave{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^H \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2, \tag{5d}$$

$$\sum_{n=1}^{N} \sum_{k\in\mathcal{U}_b} \operatorname{tr}\left(\mathbf{M}_{k,n}\mathbf{M}_{k,n}^H\right) \leq P_{\max}, \forall b \tag{5e}$$

- where $\tilde{\mathbf{u}}_{l,k,n} \triangleq \{\mathbf{w}_{l,k,n}^H, \mathbf{H}_{b_k,k,n}, \mathbf{m}_{l,k,n}, \beta_{l,k,n}\}$

JSFRA Formulation (SINR Relaxation)

- To maximize the received SINR, $\mathbf{w}_{l,k,n}$ are modeled with the MMSE receivers
- The JSFRA formulation in (5) is nonconvex due to the constraint defined by (5c) as $\gamma_{l,k,n} \leq \frac{\left|\mathbf{w}_{l,k,n}^{H}\,\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}\right|^2}{\beta_{l,k,n}}$
- In order to solve the problem in (5), we use successive convex approximation (SCA) approach for the constraint defined by (5c)
- Let, $f(\tilde{\mathbf{u}}_{l,k,n}) = (p_{l,k,n}^2 + q_{l,k,n}^2)/\beta_{l,k,n}$, where

$$p_{l,k,n} \triangleq \Re\left\{\mathbf{w}_{l,k,n}^{H}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\}, \quad (6a)$$

$$q_{l,k,n} \triangleq \Im\left\{\mathbf{w}_{l,k,n}^{H}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \quad (6b)$$

JSFRA Formulation (SINR Relaxation)

- Now, the first order Taylor approximation is used for the function $f(\tilde{\mathbf{u}}_{l,k,n})$ around an arbitrary point $\tilde{\mathbf{u}}_{l,k,n}^{(i-1)}$
- With this approximation, the problem in (5) can be solved using the well known solvers for the optimal precoders and $\tilde{\mathbf{u}}_{l,k,n}$
- Once the precoders $\mathbf{m}_{l,k,n}$ are evaluated, $\mathbf{w}_{l,k,n}$ are updated using the MMSE receivers
- The local point $\tilde{\mathbf{u}}_{l,k,n}^{(i-1)}$ is updated with the current optimal point $\tilde{\mathbf{u}}_{l,k,n}^{(i)}$
- With the updated precoders and $\tilde{\mathbf{u}}_{l,k,n}^{(i)}$, optimization is carried out in an iterative manner until convergence

JSFRA Formulation (MSE Reformulation)

- In this approach, we utilize the relation between the MSE and the SINR as $\epsilon_{l,k,n} = (1 + \gamma_{l,k,n})^{-1}$
- Equivalence is valid only when the receivers are designed with the mean squared error (MSE) objective, *i.e.*, using MMSE receivers

$$\mathbb{E}\left[(d_{l,k,n} - \hat{d}_{l,k,n})^2\right] = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2 + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}\right|^2 + \grave{N}_0 = \epsilon_{l,k,n} \quad (7)$$

JSFRA Formulation (MSE Reformulation)

- Using the above reformulation, we can formulate the JSFRA problem as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \tag{8a}$$

$$\text{subject to} \quad t_{l,k,n} \le -\log_2(\epsilon_{l,k,n}) \tag{8b}$$

$$\sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2 + \grave{N}_0 + \left|1-\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2 \le \epsilon_{l,k,n} \tag{8c}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \le P_{\max}\ \forall b. \tag{8d}$$

JSFRA Formulation (MSE Reformulation)

- The nonconvex constraint in (8b) is approximated by a sequence of convex constraints
- It is achieved by using SCA technique as earlier
- The iterative procedure is performed until convergence or for suitable number of iterations
- The above reformulation works only with the MMSE receiver

Section 4

Distributed Solutions

Distributed Methods

- When the system size is small, centralized approach is viable if the channel remains constant for multiple transmission slots
- However, the overhead involved in the centralized design scales up significantly as the network size grows
- Distributed schemes based on primal decomposition or ADMM can be used to reduce the signaling requirements
- The overhead involved in the design of precoders are only scalar interference variables
- Only the convex approximated subproblem in each SCA step is performed via distributed approaches

Primal Decomposition Method

- Precoder design is performed by a master-slave approach
- Interference created to the neighboring BS users are bounded by a scalar variable
- The interference thresholds are determined by the master problem, which is performed at each BS with the signaling exchange

$$\zeta_{l,k,n,b} \geq \sum_{i \in \mathcal{U}_b} \sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b,k,n} \mathbf{m}_{j,i,n}|^2 \ \forall b \in \bar{\mathcal{B}}_{b_k}. \qquad (9)$$

- Each BS subproblem includes (9) as an additional interference constraint in the respective optimization problem

ADMM based Decomposition Method

- The alternating directions method of multipliers (ADMM) is superior to other distributed schemes in terms of the convergence speed
- The ADMM includes an additional quadratic term in the objective as $\|\zeta_b - \zeta_b^{(j)}\|^2$, where $\zeta_b^{(j)}$ is global consensus variable
- Unlike primal decomposition method, $\zeta_{l,k,n,b}$ in (9) is treated as an optimization variable in ADMM
- The consensus variables are updated as

$$\zeta_{b_k}(b)^{(j+1)} = \zeta_b(b_k)^{(j+1)} = \frac{\zeta_b(b_k) + \zeta_{b_k}(b)}{2}. \qquad (10)$$

- where $\zeta_{b_k}(b)$ denotes the entries corresponding to BS b in BS b_k

KKT based Distributed Solution

- The decentralization methods considered so far involve considerable signaling exchanges via backhaul
- However, when the users are equipped with multiple receive antennas, the overhead requirement is significantly large
- Since the signaling requirements are large, the iterative algorithm should design efficient precoders in few number of iterations to reduce the backlogged packets
- In order to achieve that, we design an iterative procedure based on solving the Karush-Kuhn-Tucker (KKT) equations for the JSFRA problem via MSE reformulation
- Unlike the earlier schemes, we perform the group update of all the involved optimization variables to speed up the convergence of precoder design

KKT based Distributed Solution

$$\mathbf{m}_{l,k,n}^{(i)} = \left(\sum_{x\in\mathcal{U}} \sum_{y=1}^{L} \alpha_{y,x,n}^{(i-1)} \mathbf{H}_{b_k,x,n}^{\mathrm{H}} \mathbf{w}_{y,x,n}^{(i-1)} \mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)} \mathbf{H}_{b_k,x,n} + \delta_b \mathbf{I}_{N_T} \right)^{-1} \alpha_{l,k,n}^{(i-1)} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n}^{(i-1)}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \left(\sum_{x\in\mathcal{U}} \sum_{y=1}^{L} \mathbf{H}_{b_x,k,n} \mathbf{m}_{y,x,n}^{(i)} \mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)} \mathbf{H}_{b_x,k,n}^{\mathrm{H}} + N_0 \mathbf{I}_{N_R} \right)^{-1} \mathbf{H}_{b_k,k,n}\ \mathbf{m}_{l,k,n}^{(i)} \qquad (11a)$$

$$\epsilon_{l,k,n}^{(i)} = \left| 1 - \mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)} \right|^2 + \sum_{(x,y)\neq(l,k)} \left| \mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n}^{(i)} \right|^2 + \grave{N}_0 \qquad (11b)$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{\left(\epsilon_{l,k,n}^{(i)} - \epsilon_{l,k,n}^{(i-1)}\right)}{\log(2)\ \epsilon_{l,k,n}^{(i-1)}} \qquad (11c)$$

$$\sigma_{l,k,n}^{(i)} = \left[\frac{a_k\, q}{\log(2)} \left(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}^{(i)} \right)^{(q-1)} \right]^{+} \qquad (11d)$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)} + \rho^{(i)} \left(\frac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}} - \alpha_{l,k,n}^{(i-1)} \right) \qquad (11e)$$

Section 5

Simulation Results

SISO Example

- Let us consider a simple example with $N_T = N_R = 1$, $N = 3$ sub-channels, and $K = 3$ users

Table: Sub-channel-wise listing of channel gains and rate allocations

Q_k	$\mathbf{H}_{b_k,k}$			(A)			(B)			(C)		
	1	2	3	1	2	3	1	2	3	1	2	3
4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining packets (χ)				3.92 bits			2.51 bits			5.89 bits		

- $\zeta = \sum_{k=1}^{K} [\, Q_k - t_k \,]^+$
- $\zeta = 5.89$ bits for Q-WSRM (C), $\zeta = 3.92$ bits for Q-WSRME scheme (A) and, $\zeta = 2.51$ bits for JSFRA scheme (B)

Performance Comparison with Number of Residual Packets

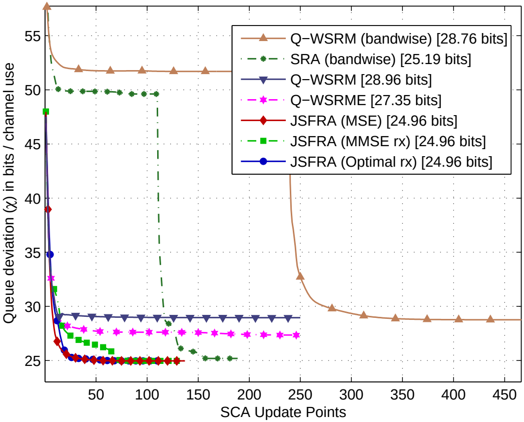

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

Performance Comparison with Number of Residual Packets

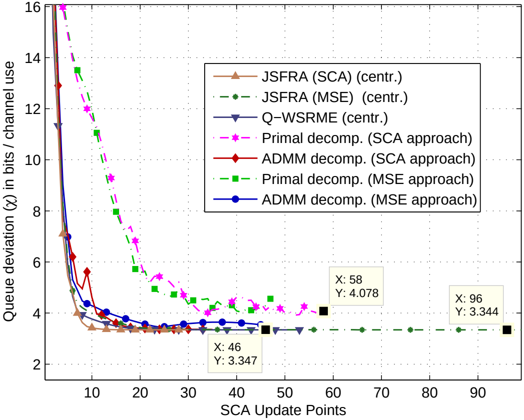

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{3, 2, 8, 4, 1\}$

Effect on Residual Packets with different ℓ_q norms

Table: Number of backlogged bits associated with each user for a system $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$.

q	user indices								χ
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
∞	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
Q_k	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	

Performance Comparison with Number of Residual Packets

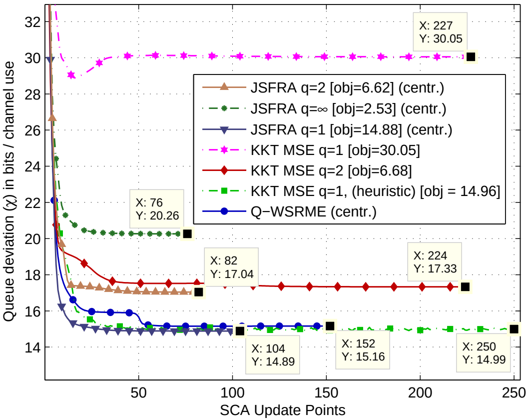

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{5, 2, 8, 4, 1\}$

Average Residual Packets after each Transmission Slot

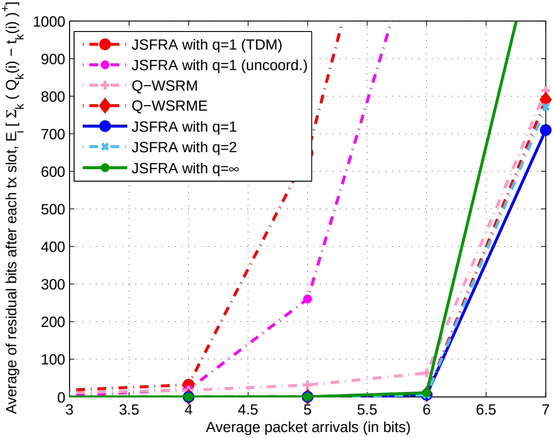

Figure: System Model - $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ after 250 transmissions

Section 6

Conclusions

Conclusions

- We discussed the problem of wireless resource allocation to minimize backlogged packets in an efficient way
- The proposed approach uses SCA method by using linear approximation for the nonconvex constraint
- We also addressed different distributed methods for the precoder design across each BSs with minimal information exchange
- An iterative algorithm for the JSFRA scheme using MSE reformulation is also studied

Questions !