Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems

Ganesh Venkatraman† *Student Member, IEEE*, Antti Tölli† *Senior Member, IEEE*, Markku Juntti† *Senior Member, IEEE*, and Le-Nam Tran‡ *Member, IEEE*

***Abstract*—We consider a downlink multi-cell multiple-input multiple-output (MIMO) interference broadcast channel (IBC) scenario using orthogonal frequency division multiplexing (OFDM) with multiple-user contending for space-frequency resources in a given scheduling instant. The problem is to determine the transmit precoders by the base stations (BSs) in a coordinated approach to minimize the total number of backlogged packets in the BSs, which are destined for the users in the system. Traditionally, it is solved using weighted sum rate maximization (WSRM) objective with the number of backlogged packets as the corresponding weights, *i.e*, longer the queue size, higher the priority. In contrast, we design the precoders jointly across the space-frequency resources by minimizing the total user queue deviations. The problem is nonconvex and therefore we employ successive convex approximation (SCA) technique to solve the problem by a sequence of convex subproblems using first order Taylor approximations. At first, we propose a centralized joint space-frequency resource allocation (JSFRA) solution using two different formulations by employing SCA technique, namely the sum rate formulation and the mean squared error (MSE) reformulation. We then introduce distributed precoder designs using primal and alternating directions method of multipliers method for the JSFRA solutions. Finally, we propose a practical distributed iterative precoder design based on MSE reformulation approach by solving the Karush-Kuhn-Tucker conditions with closed form expressions. Numerical results are used to compare the proposed algorithms with the existing solutions.**

***Index Terms*—Convex approximations, MIMO-IBC, MIMO-OFDM, Precoder design, SCA, WSRM.**

I. INTRODUCTION

In a network with multiple base stations (BSs) serving multiple-users (MUs), the main driving factor for the transmission are the packets waiting at each BS corresponding to the different users present in the network. These available packets are transmitted over the shared wireless resources subject to certain system limitations and constraints. We consider the problem of transmit precoder design over the space-frequency resources provided by the multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) framework in the downlink interference broadcast channel (IBC) to minimize the number of queued packets. Since the space-frequency resources are shared by multiple users associated with different BSs, it can be viewed as a resource allocation problem.

In general, the resource allocation problems are formulated by assigning a binary variable for each user to indicate the presence or the absence in a particular resource [1]. In contrast, the linear transmit precoders, which are complex vectors, are implicitly used as decision variables, thereby avoiding the explicit modeling of binary decision variables. It is used to determine the transmission rate of a user on a space-frequency resource if the precoder vector is non-zero. A zero transmit precoder indicates the absence of the user on a given resource. In this way, the soft decisions are used in the optimization problem and the hard decisions are made after the algorithm convergence.

The queue minimizing precoder designs are closely related to the weighted sum rate maximization (WSRM) problem with additional rate constraints determined by the number of backlogged packets for each user in the system. The topics on MIMO IBC precoder design have been studied extensively with different performance criteria in the literature. Due to the nonconvex nature of the MIMO IBC precoder design problems, the successive convex approximation (SCA) approach has become a powerful tool to deal with these problems. For example, in [2], the nonconvex part of the objective has been linearized around an operating point in order to solve the WSRM problem in an iterative manner. Similar approach of solving the WSRM problem by using arithmetic-geometric inequality has been proposed in [3].

The relation between the achievable capacity and the mean squared error (MSE) of the received symbol by using fixed minimum mean squared error (MMSE) receivers can be used to solve the WSRM problem [4]. In [5], [6], the WSRM problem is reformulated via MSE, casting the problem as a convex one for fixed linearization coefficients. In this way, the original problem is expressed in terms of the MSE weight, precoders, and decoders. Then the problem is solved using an alternating optimization method, i.e., finding a subset of variables while the remaining others are fixed. The MSE reformulation for the WSRM problem has also been studied in [7] by using the SCA to solve the problem in an iterative manner. Additional rate constraints based on the quality of service (QoS) requirements were included in the WSRM problem and solved via MSE reformulation in [8], [9].

The problem of precoder design for the MIMO IBC system are solved either by using a centralized controller or by using decentralized algorithms where each BS handles the corresponding subproblem independently with the limited

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†The authors are with the Centre for Wireless Communications (CWC), Department of Communications Engineering (DCE), University of Oulu, Oulu, FI-90014, (e-mail: {ganesh.venkatraman, antti.tolli, markku.juntti}@ee.oulu.fi)

‡The author is with the Department of Electronic Engineering, Maynooth University, Maynooth, Co. Kildare, Ireland, (e-mail:ltran@eeng.nuim.ie)

information exchange with the other BSs via back-haul. The distributed approaches are based on primal, dual or alternating directions method of multipliers (ADMM) decomposition, which has been discussed in [10], [11]. In the primal decomposition, the so-called coupling interference variables are fixed for the subproblem at each BS to find the optimal precoders. The fixed interference are then updated by using the subgradient method as discussed in [12]. The dual and ADMM approaches control the distributed subproblems by fixing the *'interference price'* for each BS as detailed in [13].

By adjusting the weights in the WSRM objective properly, we can find an arbitrary rate-tuple in the rate region that maximizes the suitable objective measures. For example, if the weight of each user is set to be inversely proportional to its average data rate, the corresponding problem guarantees fairness on an average among the users. As an approximation, we may assign weights based on the current queue size of the users. More specifically, the queue states can be incorporated to traditional weighted sum rate objective $\sum_k w_k R_k$ by replacing the weight w_k with the corresponding queue state Q_k or its function, which is the outcome of minimizing the Lyapunov drift between the current and the future queue states [14], [15]. In backpressure algorithm, the differential queues between the source and the destination nodes are used as the weights scaling the transmission rate [16].

Earlier studies on the queue minimization problem were summarized in the survey paper [17], [18]. In particular, the problem of power allocation to minimize the number of backlogged packets was considered in [19] using geometric programming. Since the problem addressed in [19] assumed single antenna transmitters and receivers, the queue minimizing problem reduces to the optimal power allocation problem. In the context of wireless networks, the backpressure algorithm mentioned above was extended in [20] by formulating the corresponding user queues as the weights in the WSRM problem. Recently, the precoder design for the video transmission over MIMO system is considered in [21]. In this design, the MU-MIMO precoders are designed by the MSE reformulation as in [5] with the higher layer performance objective such as playback interruptions and buffer overflow probabilities.

Main Contributions: In this paper, we design the precoders jointly across space-frequency resources by minimizing the total number of backlogged packets waiting at the BSs queues. Since the transmissions are guided by the backlogged packets, the proposed formulation limit the allocations beyond the number of backlogged packets without additional rate constraint. We adopt SCA to solve by a sequence of convex subproblems using first order approximations due to the nonconvexity imposed by the difference of convex (DC) constraints. Initially, we propose centralized joint space-frequency resource allocation (JSFRA) algorithms, which employs SCA for the nonconvex DC constraint. First method is by using the direct formulation and the second one is by using the MSE equivalence with the rate expression to solve for an optimal precoders. Then we propose distributed precoder designs based on the primal and the ADMM methods. Finally, an iterative practical algorithm is proposed to decouple the precoder design across the coordinating BSs with limited information exchange by solving the Karush-Kuhn-Tucker (KKT) conditions for the MSE reformulation. Note that the joint space-frequency channel matrix can be formed by stacking the channel of each sub-channel in a block-diagonal form for all users.

The paper is organized as follows. In Section II, we introduce the system model and the problem formulation for the queue minimizing precoder design. The existing and the proposed centralized precoder designs are presented in Section III. The distributed solutions are provided in Section IV followed by the simulation results in Section V. Conclusions are drawn in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a downlink MIMO IBC scenario in an OFDM framework with N sub-channels and N_B BSs each equipped with N_T transmit antennas, serving in total K users each with N_R receive antennas. The set of users associated with BS b is denoted by $\mathcal{U}_b$ and the set $\mathcal{U}$ represents all users in the system, i.e., $\mathcal{U} = \underset{b\in\mathcal{B}}{\cup}\mathcal{U}_b$, where $\mathcal{B}$ is the set of indices of all coordinating BSs. Data for user k is transmitted from only one BS which is denoted by $b_k \in \mathcal{B}$. We denote by $\mathcal{N} = \{1, 2, \ldots, N\}$ the set of all sub-channel indices available in the system.

We adopt linear transmit beamforming technique at BSs. Specifically, the data symbols $d_{l,k,n}$ for user k on the l^{th} spatial stream over the sub-channel n is multiplied with beamformer $\mathbf{m}_{l,k,n} \in \mathbb{C}^{N_T\times 1}$ before being transmitted. In order to detect multiple spatial streams at the user terminal, receive beamforming vector $\mathbf{w}_{l,k,n}$ is employed for each user. Consequently, the received data symbol estimate corresponding to the l^{th} spatial stream over sub-channel n at user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{n}_{k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}}\sum_{i\in\mathcal{U}\backslash\{k\}}\mathbf{H}_{b_i,k,n}\sum_{j=1}^{L}\mathbf{m}_{j,i,n}d_{j,i,n} \quad (1)$$

where $\mathbf{H}_{b,k,n} \in \mathbb{C}^{N_R\times N_T}$ is the channel between BS b and user k on sub-channel n, and $\mathbf{n}_{k,n} \sim \mathcal{CN}(0, N_0)$ is the additive noise vector for the user k on the n^{th} sub-channel and l^{th} spatial stream. In (1), $L = \text{rank}(\mathbf{H}_{b,k,n}) = \min(N_T, N_R)$ is the maximum number of spatial streams[1]. Assuming independent detection of data streams, we can write the signal-to-interference-plus-noise ratio (SINR) as

$$\gamma_{l,k,n} = \frac{\left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\,\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}\right|^2}{\widetilde{N}_0 + \sum_{(j,i)\neq(l,k)}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2} \quad (2)$$

where $\widetilde{N}_0 = N_0\,\text{tr}(\mathbf{w}_{l,k,n}\mathbf{w}_{l,k,n}^{\mathrm{H}})$ denotes the equivalent noise variance. To reduce the overhead involved in feeding back the user channels, we consider a time division duplexing (TDD) system, which uses channel reciprocity.

[1] It can be easily extended for user specific streams L_k instead of using common L streams for all users. L streams are initialized but after solving the problem, only $L_{k,n} \leq L$ non-zero data streams are transmitted

Let Q_k be the number of backlogged packets destined for the user k at a given scheduling instant. The queue dynamics of the user k are modeled using the Poisson arrival process with the average number of packet arrivals of $A_k = \mathbf{E}_i\{\lambda_k\}$ packets/bits, where $\lambda_k(i) \sim \text{Pois}(A_k)$ represents the instantaneous number of packets arriving for the user k at the i^{th} time instant[2]. The total number of queued packets at the $(i+1)^{\text{th}}$ instant for the user k, denoted as $Q_k(i+1)$, is given by

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i)\Big]^+ + \lambda_k(i) \tag{3}$$

where $[x]^+ \equiv \max\{x, 0\}$ and t_k denotes the number of transmitted packets or bits for user k. At the i^{th} instant, transmission rate of the user k is given by

$$t_k(i) = \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}(i) \tag{4}$$

where $t_{l,k,n}$ denotes the number of transmitted packets or bits over l^{th} spatial stream on the n^{th} sub-channel. The maximum rate achieved over the (l,n) space-frequency resource is given by $t_{l,k,n} \leq \log_2(1+\gamma_{l,k,n})$ for the signal-to-interference-plus-noise ratio (SINR) of $\gamma_{l,k,n}$[3]. Note that the units of t_k and Q_k are in bits defined per channel use.

B. Problem Formulation

To minimize the total number of backlogged packets, we consider minimizing the weighted ℓ_q-norm of the queue deviation objective as given by

$$v_k = Q_k - t_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L}\log_2(1+\gamma_{l,k,n}) \tag{5}$$

where $\gamma_{l,k,n}$ is given by (2) and the optimization variables are the transmit precoders $\mathbf{m}_{l,k,n}$ and the receivers $\mathbf{w}_{l,k,n}$.

Explicitly, the objective of the problem considered is given by $\sum_{k\in\mathcal{U}} a_k|v_k|^q$. With this objective function, the weighted queued packet minimization formulation is given by

$$\underset{\mathbf{m}_{l,k,n},\mathbf{w}_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{6a}$$

$$\text{subject to} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\text{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\text{H}}\right) \leq P_{\max}\forall b \tag{6b}$$

where $\tilde{v}_k \triangleq a_k^{1/q}v_k$ is the element of vector $\tilde{\mathbf{v}}$, and a_k is the weighting factor which is incorporated to control user priority based on their respective QoS. In (6b), BS specific sum power constraint for all sub-channels is considered.

For practical reasons, we impose a constraint on the maximum number of transmitted bits for the user k, since it is limited by the total number of backlogged packets available at the transmitter. As a result, the number of backlogged packets v_k for user k remaining in the system is given by

$$v_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L}\log_2(1+\gamma_{l,k,n}) \geq 0. \tag{7}$$

[2]The unit can either be packets or bits as long as the arrival and the transmission units are similar

[3]Upper bound is achieved by using Gaussian signaling

The above positivity constraint need to be satisfied by v_k to avoid the excessive allocation of the resources.

Before proceeding further, we show that the constraint in (7) is handled implicitly by the definition of norm ℓ_q in the objective of (6). Suppose that $t_k > Q_k$ for certain k at the optimum, i.e., $-v_k = t_k - Q_k > 0$. Then there exists $\delta_k > 0$ such that $-v'_k = t'_k - Q_k < -v_k$ where $t'_k = t_k - \delta_k$. Since $\|\tilde{\mathbf{v}}\|_q = \||\tilde{\mathbf{v}}|\|_q = \||-\tilde{\mathbf{v}}|\|_q$, this means that the newly created vector $\mathbf{t}'$ achieves a smaller objective which contradicts with the fact that the optimal solution has been obtained. The choice of the norm ℓ_q used in the objective function [17], [19] alters the priorities for the queue deviation function as follows

- ℓ_1 results in greedy allocation *i.e*, emptying the queue of users with good channel conditions before considering the users with worse channel conditions. As a special case, it is easy to see that (6) reduces to the WSRM problem when the queue size is large enough for all users.
- ℓ_2 prioritizes users with higher number of queued packets before considering the users with a smaller number of backlogged packets. For example, it could be more ideal for the delay limited scenario when the packet arrival rates of the users are similar, since the number of backlogged packets is proportional to the delay in the transmission following the Little's law [15].
- ℓ_∞ minimizes the maximum number of queued packets among users with the current transmission, thereby providing queue fairness by allocating the resources proportional to the number of backlogged packets.

III. Proposed Queue Minimizing Precoder Designs

In general, the precoder design for the MIMO OFDM problem is difficult due to its nonconvex nature. In addition, the objective of minimizing the number of the queued packets over space-frequency dimensions adds further complexity. Since the scheduling of users in each sub-channel attained by allocating zero transmit power over certain sub-channels, our solutions perform joint precoder design and user scheduling. Before discussing the proposed solutions, we consider the existing algorithm to minimize the number of backlogged packets with additional constraints required by the problem.

A. Queue Weighted Sum Rate Maximization (Q-WSRM) Formulation

The queue minimizing algorithms are discussed extensively in the networking literature to provide congestion-free routing between any two nodes in the network. One such algorithm is the *backpressure algorithm* [14]–[16]. It determines an optimal control policy in the form of rate or resource allocation for the nodes in the network by considering the differential backlogged packets between the source and the destination nodes. Even though the algorithm is primarily designed for the wired infrastructure, it can be extended to the wireless networks by designing the user rate variable t_k in accordance to the wireless network.

The queue weighted sum rate maximization (Q-WSRM) formulation extends the *backpressure algorithm* to the downlink MIMO-OFDM framework, in which the multiple BSs act as the source nodes and the user terminals as the receiver nodes. The control policy in the form of transmit precoders aims at minimizing the number of queued packets waiting in the BSs. In order to find the optimal strategy, we resort to the Lyapunov theory, which is predominantly used in the control theory to achieve system stability. Since at each time slot, the system is described by the channel conditions and the number of backlogged packets of each user, the Lyapunov function is used to provide a scalar measure, which grows large when the system moves toward the undesirable state. By following [15], the scalar measure for the queue stability is given by

$$\mathrm{L}\left[\mathbf{Q}(i)\right] = \frac{1}{2}\sum_{k\in\mathcal{U}} Q_k^2(i) \tag{8}$$

where $\mathbf{Q}(i) = [Q_1(i), Q_2(i), \ldots, Q_K(i)]^{\mathrm{T}}$ and $\frac{1}{2}$ is used for the convenience. It provides a scalar measure of congestion present in the system [15, Ch. 3].

To minimize the total number of backlogged packets for an instant i, the optimal transmission rate of all users are obtained by minimizing the Lyapunov function drift expressed as

$$\begin{aligned}&\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] = \\ &\quad \frac{1}{2}\Big[\sum_{k\in\mathcal{U}}\Big([Q_k(i) - t_k(i)]^+ + \lambda_k(i)\Big)^2 - Q_k^2(i)\Big].\end{aligned} \tag{9}$$

In order to eliminate the nonlinear operator $[x]^+$, we bound the expression in (9) as

$$\leq \sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} + \sum_{k\in\mathcal{U}} Q_k(i)\left\{\lambda_k(i) - t_k(i)\right\} \tag{10}$$

by using the following inequality

$$[\max(Q - t, 0) + \lambda)]^2 \leq Q^2 + t^2 + \lambda^2 + 2Q(\lambda - t). \tag{11}$$

The total number of backlogged packets at any given instant i is reduced by minimizing the conditional expectation of the Lyapunov drift expression (10) given the current number of queued packets $\boldsymbol{Q}(i)$ waiting in the system. The expectation is taken over all possible arrival and transmission rates of the users to obtain the optimal rate allocation strategy.

Now, the conditional Lyapunov drift, denoted by $\Delta(\boldsymbol{Q}(i))$, is given by the infimum over the transmission rate as

$$\inf_{\mathbf{t}} \quad \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\left\{\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] | \mathbf{Q}(i)\right\} \tag{12a}$$

$$\begin{aligned}\leq \quad & \underbrace{\mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{\sum_{k\in\mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} |\mathbf{Q}(i)\Big\}}_{\leq B} + \sum_{k\in\mathcal{U}} Q_k(i) A_k(i) \\ & - \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{\sum_{k\in\mathcal{U}} Q_k(i) t_k(i) |\mathbf{Q}(i)\Big\},\end{aligned} \tag{12b}$$

where the subscripts $\mathbf{t}$ and $\boldsymbol{\lambda}$ represents the vector formed by stacking the transmission and the arrival rate of all users in the system. Since the transmission and the arrival rates are bounded, the second order moments in the first term of (12b) can be bounded by a constant B without affecting the optimal solution of the problem [15]. The second term in (12b) follows from the Poisson arrival process.

The expression in (12) looks similar to the WSRM formulation if the weights in the WSRM problem are replaced by the number of backlogged packets corresponding to the users. The above discussed approach is extended for the wireless networks in [20], in which the queues are used as weights in the WSRM formulation to determine the transmit precoders. Since the expectation is minimized by minimizing the function inside, the Q-WSRM formulation is given by

$$\underset{\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} Q_k \left(\sum_{n=1}^{N}\sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n})\right) \tag{13a}$$

$$\text{subject to.} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \forall b. \tag{13b}$$

To avoid excessive allocation of the resources, we include an additional rate constraint $t_k \leq Q_k$ to address $[x]^+$ operation in (3). The rate constrained version of the Q-WSRM, denoted by Q-WSRM extended (Q-WSRME) problem for a cellular system, is given by with the additional constraints as

$$\sum_{n=1}^{N}\sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \leq Q_k \; \forall k \in \mathcal{U} \tag{14}$$

where the precoders are associated with $\gamma_{l,k,n}$ defined in (2). By using the number of queued packets as the weights, the resources can be allocated to the user with the more backlogged packets, which essentially does greedy allocation.

As a special case of the problem defined in (13), we can formulate the sum rate maximization problem by setting the weights in (13a) as unity, leading to the problem as in (13) with $Q_k = 1, \forall k \in \mathcal{U}$. This approach provides a greedy queue minimizing allocation as compared to Q-WSRME, since the resource allocation is driven by the channel conditions in comparison to the number of queued packets as in Q-WSRME. Note that in both formulations, the resources allocated to the users are limited by the number of backlogged packets with an explicit maximum rate constraint defined by (14).

B. JSFRA Scheme via SCA approach

The problem defined in (13) ignores the second order term arising from the Lyapunov drift minimization objective by limiting it to a constant value. In fact, using $\ell_{q=2}$ in (5), we obtain the following objective

$$\underset{t_k}{\text{minimize}} \sum_{k} v_k^2 = \underset{t_k}{\text{minimize}} \sum_{k} Q_k^2 - 2\,Q_k t_k + t_k^2 \tag{15}$$

which is similar to the objective in (13). It is achieved either by removing t_k^2 from (15) or when the total number of queued packets is significantly large for all users such that t_k^2 has no impact on the objective function.

By limiting t_k^2 with a constant value, the Q-WSRM formulation requires an explicit rate constraint (14) to avoid over-allocation of the available resources. In the proposed queue deviation formulation, the explicit rate constraint is not needed, since it is handled by the objective function (5) itself. It makes the problem simpler and allows us to employ efficient algorithms to distribute the precoder design problem across each BSs independently by exchanging minimal information exchange [11]. In contrast to the WSRM formulation, the

JSFRA and the Q-WSRME problems include the sub-channels jointly to obtain an efficient allocation by identifying the optimal space-frequency resource for the users.

We present an iterative algorithm to solve (6) by using alternating optimization technique in conjunction with successive convex approximation (SCA) [22]. The problem is to determine the transmit precoders $\mathbf{m}_{l,k,n}$ and the receive beamformers $\mathbf{w}_{l,k,n}$ to minimize the total number of backlogged packets in the system. The SINR expression in (2) cannot be used to formulate the problem directly due to the equality constraint of (2). Using additional variables, we relax the expression (2) by inequality constraints to solve (6) as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\\ \beta_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{16a}$$

$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \tag{16b}$$

$$\beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{16c}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}\ \forall b. \tag{16d}$$

The SINR expression in (2) is relaxed by the inequalities (16b) and (16c) in the above formulation. Note that (16b) is an under estimator for SINR $\gamma_{l,k,n}$, and (16c) provides an upper bound for the total interference seen by user $k \in \mathcal{U}_b$, denoted by the variable $\beta_{l,k,n}$. The constraints are tight when each BS objective is non zero at the optimal point, as discussed in Appendix A. Since the JSFRA formulation can be modeled as a WSRM problem, which is known to be NP-hard [23], it also belongs to the class of NP-hard problems.

In order to find a tractable solution for (16), we note that (16d) is the only convex constraint with the involved variables. Thus, we only need to deal with (16b) and (16c). We resort to the alternating optimization (AO) technique by fixing the linear receivers, to solve for the transmit beamformers. For a fixed receivers $\mathbf{w}_{l,k,n}$, the problem now is to find the optimal transmit beamformers $\mathbf{m}_{l,k,n}$ which is still a challenging task. We note that for a fixed $\mathbf{w}_{l,k,n}$, (16c) can be written as a second-order cone (SOC) constraint. Thus, the difficulty is due to the non-convexity of the DC constraint in (16b). Let us define a function,

$$f(\mathbf{u}_{l,k,n}) \triangleq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \tag{17}$$

where $\mathbf{u}_{l,k,n} \triangleq \{\mathbf{w}_{l,k,n}, \mathbf{m}_{l,k,n}, \beta_{l,k,n}\}$ is the vector which needs to be identified for the optimal allocation. Note that the function $f(\mathbf{u}_{l,k,n})$ is convex for a fixed $\mathbf{w}_{l,k,n}$, since it is in fact the ratio between a quadratic form of $\mathbf{m}_{l,k,n}$ over an affine function of $\beta_{l,k,n}$ [24]. The nonconvex set defined by the DC constraint (16b) can be decomposed by a series of convex subsets by linearizing the convex function $f(\mathbf{u}_{l,k,n})$ with its first order Taylor approximation around a fixed operating point $\tilde{\mathbf{u}}_{l,k,n}$ [25], [26], also called as SCA approach in [22]. By using the reduced convex subset for (16b), the problem defined in (16) can be solved at each operating point iteratively.

For this purpose, let the real and imaginary component of the complex number $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}$ be represented by

$$p_{l,k,n} \triangleq \Re\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{18a}$$

$$q_{l,k,n} \triangleq \Im\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{18b}$$

and hence $f(\mathbf{u}_{l,k,n}) = (p_{l,k,n}^2 + q_{l,k,n}^2)/\beta_{l,k,n}$[4]. Suppose that the current value of $p_{l,k,n}$ and $q_{l,k,n}$ at a specific iteration are $\tilde{p}_{l,k,n}$ and $\tilde{q}_{l,k,n}$, respectively. Using first order Taylor approximation around the local point $[\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}, \tilde{\beta}_{l,k,n}]^T$, we can approximate (16b) by the following linear inequality

$$2\frac{\tilde{p}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(p_{l,k,n}-\tilde{p}_{l,k,n}\right) + 2\frac{\tilde{q}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(q_{l,k,n}-\tilde{q}_{l,k,n}\right) + \frac{\tilde{p}_{l,k,n}^2+\tilde{q}_{l,k,n}^2}{\tilde{\beta}_{l,k,n}}\left(1-\frac{\beta_{l,k,n}-\tilde{\beta}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\right) \geq \gamma_{l,k,n}. \tag{19}$$

In summary, for the fixed linear receivers $\mathbf{w}_{l,k,n}$ and the operating point $[\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}, \tilde{\beta}_{l,k,n}]^T$, the relaxed convex subproblem to find transmit beamformers is given by

$$\underset{\substack{\mathbf{m}_{l,k,n},\\ \gamma_{l,k,n},\beta_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{20a}$$

$$\text{subject to} \quad \beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{20b}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}\ \forall b \tag{20c}$$

$$\text{and (19).} \tag{20d}$$

Now, the optimal linear receivers for the fixed transmit precoders $\mathbf{m}_{j,i,n}\ \forall i \in \mathcal{U}$, $\forall n \in \mathcal{C}$ are obtained by minimizing (16) with respect to $\mathbf{w}_{l,k,n}$ as

$$\underset{\substack{\gamma_{l,k,n},\\ \mathbf{w}_{l,k,n},\beta_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{21a}$$

$$\text{subject to} \quad \beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{21b}$$

$$\text{and (19).} \tag{21c}$$

Solving (21) using the KKT conditions, we obtain the following iterative expression for the receiver $\mathbf{w}_{l,k,n}^{o}$ as

$$\mathbf{A}_{l,k,n} = \sum_{(j,i)\neq(l,k)} \mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{22a}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \left(\frac{\tilde{\beta}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)}}{\|\mathbf{w}_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\|^2}\right)\mathbf{A}_{l,k,n}^{-1}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n} \tag{22b}$$

where $\mathbf{w}_{l,k,n}^{(i-1)}$ is the receive beamformer from the previous iteration, upon which the linear relaxation is performed for the nonconvex constraint in (21). The optimal receiver $\mathbf{w}_{l,k,n}^{o}$ is obtained by either iterating (22b) until convergence or for fixed number of iterations. Note that the receiver has no explicit relation with the choice of ℓ_q norm used in the objective function. The dependency is implicitly implied by the transmit precoders $\mathbf{m}_{l,k,n}$, which in deed depend on the q value.

It can be seen that the optimal receiver in (22b) is in fact a

[4]Note that $p_{l,k,n}$ and $q_{l,k,n}$ are just symbolic notation and not the newly introduced optimization variables. In CVX [27], for example, we declare $p_{l,k,n}$ and $q_{l,k,n}$ with the '*expression*' qualifier

scaled version of the MMSE receiver, which is given by

$$\mathbf{R}_{l,k,n} = \sum_{i\in\mathcal{U}}\sum_{j=1}^{L}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \quad (23a)$$

$$\mathbf{w}_{l,k,n} = \mathbf{R}_{l,k,n}^{-1}\,\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}. \quad (23b)$$

Since the scaling present in the optimal receiver (22b) has no impact on the received SINRs, the MMSE receiver in (23b) can also be used without compromising the performance or the convergence behavior.

The proposed subproblems in (20) and (21) are solved in an iterative manner by updating the operating point from the previous iteration. The iterative algorithm is referred to as queue minimizing JSFRA scheme with a per BS power constraint, and it is outlined in Algorithm 1. The iterative procedure repeats until the improvement on the objective is less than a predetermined tolerance parameter or the maximum number of iterations is reached. Instead of initializing $\mathbf{u}_{l,k,n}$ arbitrarily to a feasible point, transmit precoders can also be initialized with some feasible point $\tilde{\mathbf{m}}_{l,k,n}$, which is then used to find $\mathbf{u}_{l,k,n}$ as briefed in Algorithm 1. For a fixed receive beamformer $\mathbf{w}_{l,k,n}$, the SCA iteration is carried out until convergence or for the predefined iterations, say, $J_{\max}$ for the optimal transmit precoders $\mathbf{m}_{l,k,n}$. Next, the receive beamformers are updated based on either (22b) or (23b) using the fixed transmit precoders $\mathbf{m}_{l,k,n}$. This procedure is carried out until convergence of the queue deviation or for fixed number of iterations by $I_{\max}$ as outlined in Algorithm 1. The convergence proof is discussed in Appendix B.

Algorithm 1: Algorithm of JSFRA scheme

Input: $a_k,\ Q_k,\ \mathbf{H}_{b,k,n},\ \forall b\in\mathcal{B},\forall k\in\mathcal{U},\forall n\in\mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}\forall l\in\{1,2,\ldots,L\}$
Initialize: $i=0$ and transmit precoders $\tilde{\mathbf{m}}_{l,k,n}$ randomly satisfying the total power constraint (6b)
update $\mathbf{w}_{l,k,n},\mathbf{u}_{l,k,n}$ using (23b) and (19) with $\mathbf{m}_{l,k,n}$
repeat
  initialize $j=0$
  repeat
    solve for the transmit precoders $\mathbf{m}_{l,k,n}$ using (20)
    update the constraint set (19) with $\mathbf{u}_{l,k,n}$ and $\mathbf{m}_{l,k,n}$ using (18)
    $j=j+1$
  until *SCA convergence or* $j\geq J_{\max}$
  update the receive beamformers $\mathbf{w}_{l,k,n}$ using (21) or (23b) with the updated precoders $\mathbf{m}_{l,k,n}$
  $i=i+1$
until *Queue convergence or* $i\geq I_{\max}$

C. JSFRA Scheme via MSE Reformulation

In this, we solve the JSFRA problem by exploiting the equivalence between the MSE and the achievable sum rate for the receivers designed based on the MMSE criterion [4], [5]. The MSE $\epsilon_{l,k,n}$, for a data symbol $d_{l,k,n}$ is given by

$$\mathbb{E}\big[(d_{l,k,n}-\hat{d}_{l,k,n})^2\big] = \big|1-\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\big|^2 + \sum_{(j,i)\neq(l,k)}\big|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\big|^2 + \widetilde{N}_0 = \epsilon_{l,k,n} \quad (24)$$

where $\hat{d}_{l,k,n}$ is the estimate of the transmitted symbol. Using the MMSE receive beamformer (23b) in the MSE expression (24) and in the SINR expression (2), we can arrive at the following relation between the MSE and the SINR as

$$\epsilon_{l,k,n} = (1+\gamma_{l,k,n})^{-1}. \quad (25)$$

The above equivalence is valid only if the receivers are based on the MMSE criterion. Using the equivalence in (25), the WSRM objective can be reformulated as the weighted minimum mean squared error (WMMSE) equivalent to obtain the precoders for the MU-MIMO scenario as discussed in [5]–[7]. Note that the receiver is invariably based on the MMSE criterion irrespective of the ℓ_q norm used in the objective function to obtain the optimal transmit precoders $\mathbf{m}_{l,k,n}$.

Let $v'_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}$ denote the queue deviation corresponding to user k and $\tilde{v}'_k \triangleq a_k^{1/q} v'_k$ represents the weighted equivalent. By using the relaxed MSE expression in (24), the problem in (6) can be expressed as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \quad (26a)$$

$$\text{subject to } t_{l,k,n} \leq -\log_2(\epsilon_{l,k,n}) \quad (26b)$$

$$\sum_{(j,i)\neq(l,k)}\big|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\big|^2 + \widetilde{N}_0 + \big|1-\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\big|^2 \leq \epsilon_{l,k,n} \quad (26c)$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}\ \forall b. \quad (26d)$$

The alternative MSE formulation given by (26) is non-convex even for the fixed $\mathbf{w}_{l,k,n}$ due to the constraint (26b), which is in fact a DC constraint. We resort to the SCA approach [22] by relaxing the constraint by a sequence of convex subsets using first order Taylor series approximation around a fixed MSE point $\tilde{\epsilon}_{l,k,n}$ as

$$-\log_2(\tilde{\epsilon}_{l,k,n}) - \frac{(\epsilon_{l,k,n}-\tilde{\epsilon}_{l,k,n})}{\log(2)\,\tilde{\epsilon}_{l,k,n}} \geq t_{l,k,n} \quad (27)$$

Using the above approximation for the rate constraint, the problem defined in (26) is solved for optimal transmit precoders $\mathbf{m}_{l,k,n}$, MSEs $\epsilon_{l,k,n}$, and the user rates over each sub-channel $t_{l,n,k}$ for a fixed receive beamformers. The optimization subproblem to find the transmit precoders for a fixed receive beamformers $\mathbf{w}_{l,k,n}$ is given by

$$\underset{t_{l,k,n},\mathbf{m}_{l,k,n},\epsilon_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \quad (28a)$$

$$\text{subject to} \quad (26c), (26d), \text{and } (27). \quad (28b)$$

The optimal transmit precoders for a fixed receivers are obtained by solving the subproblem (28) iteratively by updating the fixed MSE point $\tilde{\epsilon}_{l,k,n}$ with $\epsilon_{l,k,n}$ from the previous iteration until termination as discussed in Section III-B. The convergence analysis follows the discussions in Appendix B.

D. Reduced Complexity Spatial Resource Allocation (SRA)

The complexity of the JSFRA algorithm scales quickly with the number of sub-channels, since the complexity of the interior point method, which is used to solve the problem, increases quickly with the problem size. Thus, we can use the decomposition methods presented in [10], [11] to overcome this complexity by designing precoders for each sub-channels independently with minimal information exchange.

As an alternative sub-optimal solution, we present queue minimizing spatial resource allocation (SRA), which solves for the precoders using JSFRA formulation only for a specific sub-channel i with a fixed transmit power $P_{\max,i}$. The sharing of power can be equal or based on a predetermined pattern as in partial frequency reuse for the sub-channels as given by

$$\sum_{i=1}^{N} P_{\max,i} = P_{\max}. \tag{29}$$

Even though N sub-channels are present at any given scheduling instant, precoders are computed for each sub-channel in a sequential manner with the sub-channel specific total power constraint $P_{\max,i}$ and the backlogged packets. Let $Q_{k,i}$ be the number of backlogged packets associated with user k before solving for the precoders specific to the sub-channel i. Since the precoder design is sequential, *i.e*, the precoders are designed for sub-channels $[0, i-1]$ before i^{th} sub-channel, the number of backlogged packets for the initial sub-channel is initialized as $Q_{k,1} = Q_k$. The queues associated with the consecutive sub-channels are given by

$$Q_{k,i+1} = \max\left(Q_k - \sum_{j=1}^{i}\sum_{l=1}^{L} t_{l,k,j}, 0\right) \, \forall\, k \in \mathcal{U} \tag{30}$$

where $t_{l,k,j}$ denotes the rate corresponding to the user k on the j^{th} sub-channel and l^{th} spatial stream. Note that the proposed scheme is sensitive to the order of the sub-channel selection due to the sequential precoder design for each sub-channel. However, the SRA approach provides faster convergence in contrast to the JSFRA formulation due to the substantial reduction in the optimization variables for each sub-channel problem. As the user count increases, the SRA formulation will be insusceptible to the sub-channel ordering due to the multi-user diversity.

IV. Distributed Solutions

The distributed precoder designs for the proposed JSFRA scheme are discussed in this section. The convex formulation in (20) or (28) requires a centralized controller to perform the precoder design for all users belonging to the coordinating BSs. In order to design the precoders independently at each BS with the minimal information exchange via backhaul, iterative decentralization methods are addressed. In particular, the primal decomposition and the ADMM based dual decomposition approaches are considered.

Let us consider the convex subproblem with the fixed receive beamformers $\mathbf{w}_{l,k,n}$ presented in (20) based on the Taylor series approximation for the nonconvex constraint. The following discussions are equally valid for the MSE based solution outlined in (28) as well. Since the objective of (20) can be decoupled across each BS, the centralized problem can be equivalently written as

$$\underset{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\beta_{l,k,n}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}} \|\tilde{\mathbf{v}}_b\|_q \tag{31a}$$

$$\text{subject to} \quad (20\text{b}) - (20\text{d}) \tag{31b}$$

where $\tilde{\mathbf{v}}_b$ denotes the vector of weighted queue deviation corresponding to users $k \in \mathcal{U}_b$.

To begin with, let $\bar{\mathcal{B}}_b$ denote the set $\mathcal{B}\backslash\{b\}$ and $\bar{\mathcal{U}}_b$ represents the set $\mathcal{U}\backslash\mathcal{U}_b$. Following similar approach presented in [12], [13], the coupling constraint (20b) or (26c) can be expressed by grouping the interference from each BS in $\mathcal{B}$ as

$$\widetilde{N}_0 + \sum_{j=1,j\neq l}^{L} |\mathbf{w}_{l,k,n}^{\text{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,k,n}|^2 + \sum_{b\in\bar{\mathcal{B}}_{b_k}} \zeta_{l,k,n,b} + \sum_{i\in\mathcal{U}_{b_k}\backslash\{k\}}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\text{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n} \tag{32}$$

where $\zeta_{l,k,n,b}$ is the total interference caused by the transmission of BS b to user $k \in \mathcal{U}_{b_k}$ in the spatial stream l and sub-channel n. It is given by the following upper bound as

$$\zeta_{l,k,n,b} \geq \sum_{i\in\mathcal{U}_b}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\text{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \, \forall b \in \bar{\mathcal{B}}_{b_k}. \tag{33}$$

The decentralization is achieved by decomposing the original convex problem in (31) by a parallel iterative subproblems coordinated by either primal or dual decomposition update. The coupling variables are updated in each iteration by exchanging limited information among the subproblems. Before proceeding further, let $\bar{\boldsymbol{\zeta}}_b$ be the vector formed by stacking interference terms (33) from the neighboring BSs to the users of BS b and $\hat{\boldsymbol{\zeta}}_b$ be the stacked interference terms caused by BS b to all users in the neighboring BSs $\bar{\mathcal{B}}_b$, represented as

$$\bar{\boldsymbol{\zeta}}_b = \left[\zeta_{l,k,n,\bar{\mathcal{B}}_b(1)}, \ldots, \zeta_{l,k,n,\bar{\mathcal{B}}_b(|\bar{\mathcal{B}}_b|)}\right]^{\text{T}}, \forall k \in \mathcal{U}_b \tag{34a}$$

$$\hat{\boldsymbol{\zeta}}_b = \left[\zeta_{l,\bar{\mathcal{U}}_b(1),n,b}, \zeta_{l,\bar{\mathcal{U}}_b(2),n,b}, \ldots, \zeta_{l,\bar{\mathcal{U}}_b(|\bar{\mathcal{U}}_b|),n,b}\right]^{\text{T}}. \tag{34b}$$

Let us define the vector $\boldsymbol{\zeta}_b$, formed by stacking the interference terms corresponding to the BS b as

$$\boldsymbol{\zeta}_b = \left[\hat{\boldsymbol{\zeta}}_b^{\text{T}}, \bar{\boldsymbol{\zeta}}_b^{\text{T}}\right]^{\text{T}}. \tag{35}$$

Since the decentralization solution is an iterative procedure, we represent the i^{th} iteration index as $x^{(i)}$. Let $\boldsymbol{\zeta}_b(b_k)$ denote the interference terms corresponding to the BS b_k in BS b as

$$\boldsymbol{\zeta}_b(b_k) = \left[\zeta_{l,\mathcal{U}_b(1),n,b_k}, \ldots, \zeta_{l,\mathcal{U}_b(|\mathcal{U}_b|),n,b_k}\right]. \tag{36}$$

To decentralize the problem in (31), the BS specific vector $\boldsymbol{\zeta}_b$ in (35), which are relevant for the BS b, can either be fixed or treated as a variable in accordance to the decomposition method. To decouple the precoder design across BSs, the equivalent downlink channel $\mathbf{w}_{l,k,n}^{\text{H}}\mathbf{H}_{b,k,n}, \forall k \in \mathcal{U}$ are to be known at each BS b through the precoded uplink pilots from all the users in the system, where the precoders are the MMSE receiver $\mathbf{w}_{l,k,n}$ evaluated at the user. Similarly to update the MMSE receivers at each user k, the equivalent channels $\mathbf{H}_{b,k,n}\mathbf{m}_{l,k',n}, \forall k' \in \mathcal{U}_b, \forall b \in \mathcal{B}$ are to be known through the user specific precoded downlink pilots precoded with the updated transmit beamformers $\mathbf{m}_{l,k,n}, \forall k \in \mathcal{B}$ evaluated at the BS b by using the equivalent downlink channel that includes the updated MMSE receivers of all the users, as in [28].

A. Primal Decomposition

In primal decomposition, the convex problem in (31) is solved for the optimal transmit precoders in an iterative manner for a fixed BS specific interference terms $\boldsymbol{\zeta}_{b_k}$ using master-slave model [12]. The slave subproblem is solved in each BS for the optimal transmit precoders only for the associated users by assuming fixed interference terms $\boldsymbol{\zeta}_{b_k}^{(i)}$ in each i^{th} iteration. Upon finding the optimal associated transmit precoders by each slave subproblems, the master problem is used to update the BS specific interference terms $\boldsymbol{\zeta}_{b_k}^{(i+1)}$ for the next iteration by using dual variables corresponding to the interference constraint (32) as discussed in [12]. In this manner, the interference variables are updated until the global consensus is obtained. The primal approach is similar to the minimum power precoder design presented in [12]. Note that the master problem treats $\boldsymbol{\zeta}_b$ as a variable and the slave subproblems assumes it to be a constant for each iteration to find the transmit precoders.

B. Alternating Directions Method of Multipliers (ADMM)

In this section, we discuss the ADMM approach to decouple the precoder design across multiple BSs to solve the convex problem in (31). The ADMM is preferred over the dual decomposition (DD) approach in [13] for its robustness and improved convergence behavior [11]. In contrast to the primal decomposition, the ADMM approach relaxes the interference constraints by including in the objective function of each subproblem with a penalty pricing [10], [11]. Similar decomposition for the precoder design in the minimum power context is considered in [29].

Using the formulation presented in [11], [29], we can write the BS b specific ADMM subproblem for the i^{th} iteration as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n}\\ \beta_{l,k,n},\boldsymbol{\zeta}_b}}{\text{minimize}} \ \|\tilde{\mathbf{v}}_b\|_q + \boldsymbol{\nu}_b^{(i)\,\mathrm{T}}\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i)}\right) + \frac{\rho}{2}\left\|\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i)}\right\|^2 \quad (37a)$$

$$\text{subject to} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \le P_{\max} \quad (37b)$$

$$\sum_{\bar{b}\in\bar{\mathcal{B}}_b}\zeta_{l,k,n,\bar{b}} + \sum_{\{\bar{l},\bar{k}\}\neq\{l,k\}}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{\bar{l},\bar{k},n}|^2 + \tilde{N}_0 \le \beta_{l,k,n} \quad (37c)$$

$$\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}|\mathbf{w}_{\bar{l},\bar{k},n}^{\mathrm{H}}\mathbf{H}_{b,\bar{k},n}\mathbf{m}_{l,k,n}|^2 \le \zeta_{\bar{l},\bar{k},n,b} \ \forall \bar{k}\in\bar{\mathcal{U}}_b \ \forall n \quad (37d)$$

$$\text{and } (19) \quad (37e)$$

where $\boldsymbol{\zeta}_b^{(i)}$ denotes the interference vector updated from the earlier iteration and $\boldsymbol{\nu}_b^{(i)}$ represents the dual vector corresponding to the equality constraint at the i^{th} iteration as

$$\boldsymbol{\zeta}_b = \boldsymbol{\zeta}_b^{(i)}. \quad (38)$$

Upon solving (37) for $\boldsymbol{\zeta}_b \forall b$ in the i^{th} iteration, the next iterate is updated by exchanging the corresponding interference terms between two BSs b and b_k as

$$\boldsymbol{\zeta}_{b_k}(b)^{(i+1)} = \boldsymbol{\zeta}_b(b_k)^{(i+1)} = \frac{\boldsymbol{\zeta}_b(b_k) + \boldsymbol{\zeta}_{b_k}(b)}{2}. \quad (39)$$

The dual vector for the next iteration is updated by using subgradient to maximize the dual objective as

$$\boldsymbol{\nu}_b^{(i+1)} = \boldsymbol{\nu}_b^{(i)} + \rho\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i+1)}\right) \quad (40)$$

where the step size parameter ρ is chosen in accordance with [11], which depend on the system model under consideration. The convergence rate of the distributed algorithm is susceptible to the choice of the step size parameter ρ. For our simulation models, we consider the step size parameter $\rho = 2$. The above iteration is performed until convergence or for certain accuracy in the variation of the objective value between two consecutive updates. The distributed precoder design using ADMM approach is shown in Algorithm 2. The convergence analysis of the distributed algorithms are discussed in Appendix C.

Algorithm 2: Distributed JSFRA scheme using ADMM

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}$, $\forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l$
Initialize: $i = 0$ and $\mathbf{m}_{l,k,n}$ randomly satisfying total power constraint (37b)
update $\mathbf{w}_{l,k,n}$ with (23b) and $\tilde{\mathbf{u}}_{l,k,n}$ using (16c) and (18)
initialize the interference vectors $\boldsymbol{\zeta}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \forall b \in \mathcal{B}$
initialize the dual vectors $\boldsymbol{\nu}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \forall b \in \mathcal{B}$
foreach *BS* $b \in \mathcal{B}$ **do**
  repeat
    initialize $j = 0$
    repeat
      solve for $\mathbf{m}_{l,k,n}$ and $\boldsymbol{\zeta}_b$ with (37) using $\boldsymbol{\zeta}_b^{(j)}$
      exchange $\boldsymbol{\zeta}_b$ among BSs in $\mathcal{B}$
      update interference vector $\boldsymbol{\zeta}_b^{(j+1)}$ using (39)
      update dual variables in $\boldsymbol{\nu}_b^{(j+1)}$ using (40)
      $j = j + 1$
    until *convergence or* $j \ge J_{\max}$
    downlink precoded pilot transmission with $\mathbf{m}_{l,k,n}$
    update $\mathbf{w}_{l,k,n}$ and notify all BSs in $\mathcal{B}$ using uplink precoded pilots [28]
    update $\tilde{\mathbf{u}}_{l,k,n}$ using (16c) and (18) for SCA point or $\tilde{\epsilon}_{l,k,n}$ using (26c) for MSE operating point
    $i = i + 1$
  until *convergence or* $i \ge I_{\max}$
end

C. Decomposition via KKT Conditions for MSE Formulation

In this section, we discuss an alternative way to decentralize the precoder design across the coordinating BSs in $\mathcal{B}$ based on the MSE reformulation method discussed in Section III-C. In contrast to Section IV-A and IV-B, the problem is solved using the KKT conditions in which the transmit precoders, receive beamformers and the subgradient updates are performed at the same instant to minimize the global queue deviation objective with few number of iterations. The proposed methods in this section provide algorithms that can be of practical importance owing to the limited signaling requirements. We consider an idealized TDD system due to the knowledge of complete channel information at the transmitter. Similar work has been considered for the WSRM problem with minimum rate constraints in [8], [9]. Since the formulation in [8], [9] are similar to the Q-WSRME scheme with an additional maximum rate constraint (14), it requires explicit dual variables to handle the maximum rate constraint, thereby making the problem difficult to solve in an iterative manner.

In the proposed JSFRA formulation, the maximum rate constraints are implicitly handled by the objective function without the need of explicit constraints. However, the KKT conditions cannot be formulated due to the non-differential objective function. The non-differentiability is due to the absolute value operator present in the norm function. In order to make the objective function differentiable, we consider the following two cases for which the absolute operator can be ignored without affecting the optimal solution, namely,

- when the exponent q is even, or
- when the number of backlogged packets of each user is large enough, *i.e*, $Q_k \gg \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}$ to ignore the absolute operator and queues in the first place as well.

With the assumption of either one of the above conditions to be true, the problem in (28) can be written as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}}\sum_{k\in\mathcal{U}_b} a_k \left(Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}\right)^q \tag{41a}$$

subject to

$$\alpha_{l,k,n} : \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2 + \tilde{N}_0 + \sum_{(x,y)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}\right|^2 \leq \epsilon_{l,k,n} \tag{41b}$$

$$\sigma_{l,k,n} : \log_2(\tilde{\epsilon}_{l,k,n}) + \frac{(\epsilon_{l,k,n} - \tilde{\epsilon}_{l,k,n})}{\log(2)\tilde{\epsilon}_{l,k,n}} \leq -t_{l,k,n} \tag{41c}$$

$$\delta_b : \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}\ \forall b \tag{41d}$$

where $\alpha_{l,k,n}, \sigma_{l,k,n}$ and δ_b are the dual variables corresponding to the constraints defined in (41b), (41c) and (41d).

The problem in (41) is solved using the KKT expressions, obtained by the derivative of the Lagrangian function w.r.t the primal and the dual variables, complementary slackness, and the primal, dual feasibility requirements as shown in Appendix D. Upon solving, we obtain the iterative solution as

$$\mathbf{m}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L} \alpha_{y,x,n}^{(i-1)}\mathbf{H}_{b_k,x,n}^{\mathrm{H}}\mathbf{w}_{y,x,n}^{(i-1)}\mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)}\mathbf{H}_{b_k,x,n} + \delta_b\mathbf{I}_{N_T}\Bigg)^{-1} \alpha_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)} \tag{42a}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L} \mathbf{H}_{b_x,k,n}\mathbf{m}_{y,x,n}^{(i)}\mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_x,k,n}^{\mathrm{H}} + \mathbf{I}_{N_R}\Bigg)^{-1} \mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}^{(i)} \tag{42b}$$

$$\epsilon_{l,k,n}^{(i)} = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}^{(i)}\right|^2 + N_0\left\|\mathbf{w}_{l,k,n}^{(i)}\right\|^2 + \sum_{(x,y)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}^{(i)}\right|^2 \tag{42c}$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{\left(\epsilon_{l,k,n}^{(i)} - \epsilon_{l,k,n}^{(i-1)}\right)}{\log(2)\,\epsilon_{l,k,n}^{(i-1)}} \tag{42d}$$

$$\sigma_{l,k,n}^{(i)} = \left[\frac{a_k\,q}{\log(2)}\left(Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}^{(i)}\right)^{(q-1)}\right]^+ \tag{42e}$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)} + \rho\left(\frac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}} - \alpha_{l,k,n}^{(i-1)}\right) \tag{42f}$$

Since the dual variables $\alpha^{(i)}$ and $\sigma^{(i)}$ are interdependent in (42), one has to be fixed to optimize for the other. So, $\alpha^{(i)}$ is fixed to evaluate $\sigma^{(i)}$ using (42). At each iteration, the dual variables $\alpha^{(i)}$ are linearly interpolated with any point between the fixed iterate $\alpha^{(i-1)}$ and $\frac{\sigma^{(i)}}{\epsilon^{(i)}}$ using a step size $\rho \in (0,1)$. The choice of ρ depends on the system model and it affects the convergence behavior. It reduces the oscillations in the objective function when $\sigma^{(i)}$ is negative due to over allocation.

When the allocated rate $t_k^{(i-1)}$ is greater than the number of queued packets Q_k for a user k, the corresponding dual variable $\sigma^{(i)}$ will be negative and due to the projection operator $[x]^+$ in (42e), it will be zero, thereby forcing $\alpha_k^{(i)} < \alpha_k^{(i-1)}$ as in (42f). Once the $\alpha_k^{(i)}$ is reduced, the precoder weight in (42a) is lowered to make the rate $t_k^{(i)} < t_k^{(i-1)}$ eventually. The choice of ρ is susceptible to the system model under consideration, which affects the convergence speed of the iterative algorithm. In all simulations, the step size ρ is fixed to 0.1 irrespectively.

The KKT expressions in (42) are solved in an iterative manner by initializing the transmit and the receive beamformers $\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}$ with the single user beamforming and the MMSE vectors. The dual variable α's are initialized with ones to have equal priorities to all the users in the system. Then the transmit and the receive beamformers are evaluated using the expressions in (42). The transmit precoder in (42a) depends on the BS specific dual variable δ_b, which can be found by bisection search satisfying the total power constraint (41d). Note that the fixed SCA operating point is given by $\tilde{\epsilon}_{l,k,n} = \epsilon_{l,k,n}^{(i-1)}$, which is considered in the expression (42).

To devise an algorithm for a practical implementation, we assume the cross channels $\mathbf{H}_{b,k,n}, \forall k \in \bar{\mathcal{U}}_b$ and the receive beamformers $\mathbf{w}_{l,k,n}$ of all users in the system are known through uplink signaling. We extend the decentralization methods discussed in [28], for the current problem as follows. After receiving the updated transmit precoders from all BSs in $\mathcal{B}$, each user evaluates the MMSE receiver in (42b) and notify them to the BSs via uplink precoded pilots. On receiving pilot signals, BSs update the MSE in (24) as

$$\epsilon_{l,k,n}^{(i)} = 1 - \mathbf{w}_{l,k,n}^{(i)\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}^{(i)}. \tag{43}$$

Using the current MSE value, $t_{l,k,n}^{(i)}, \sigma_{l,k,n}^{(i)}$, and $\alpha_{l,k,n}^{(i)}$ are evaluated using (42d), (42e) and (42f), and the updated dual variables $\alpha_{l,k,n}$ are exchanged between the BSs to evaluate the transmit precoders $\mathbf{m}_{l,k,n}^{(i+1)}$ for the next iteration. The SCA operating point is also updated with the current MSE value.

To avoid the back-haul exchanges between BSs, as an alternative approach, users may perform all processing required and BSs will update the precoders based on the feedback information from the users. Upon receiving the transmit precoders from BSs, each user will update the receive beamformer $\mathbf{w}_{l,k,n}$, the MSE $\epsilon_{l,k,n}$, and the dual variables $\lambda_{l,k,n}$ and $\alpha_{l,k,n}$. The updated $\alpha_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are notified to the BSs using two separate precoded uplink pilot symbols with $\tilde{\mathbf{w}}^{(i)}_{l,k,n} = \sqrt{\alpha^{(i)}_{l,k,n}}\mathbf{w}^{*(i)}_{l,k,n}$ and $\bar{\mathbf{w}}^{(i)}_{l,k,n} = \alpha^{(i)}_{l,k,n}\mathbf{w}^{*(i)}_{l,k,n}$ as the precoders. On receiving the precoded uplink pilots, each BS use the effective channel $\mathbf{H}^{\mathrm{T}}_{b,k,n}\tilde{\mathbf{w}}^{(i)}_{l,k,n}$ and $\mathbf{H}^{\mathrm{T}}_{b,k,n}\bar{\mathbf{w}}^{(i)}_{l,k,n}$ in (42a) to update the transmit precoders, where $\mathbf{x}^*$ is the complex conjugate of $\mathbf{x}$. Finally, Algorithm 3 outlines the distributed precoder design using the KKT based MSE reformulated JSFRA problem.

Algorithm 3: KKT approach for the JSFRA scheme

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}$, $\forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l \in \{1, 2, \ldots, L\}$
Initialize: $i = 1$, $\mathbf{w}^{(0)}_{l,k,n}$, $\tilde{\epsilon}_{l,k,n}$ randomly, dual variables $\alpha^{(0)}_{l,k,n} = 1$, and $I_{\max}$ for certain value
foreach *BS* $b \in \mathcal{B}$ **do**
  initialize $i = 0$
  repeat
    update $\mathbf{m}^{(i)}_{l,k,n}$ using (42a), and perform downlink transmission
    find $\mathbf{w}^{(i)}_{l,k,n}$ using (42b) at each user
    evaluate $\epsilon^{(i)}_{l,k,n}$, $t^{(i)}_{l,k,n}$, $\sigma^{(i)}_{l,k,n}$ and $\alpha^{(i)}_{l,k,n}$ using (42c) and (42d), (42e) and (42f) at each user with the updated $\mathbf{w}^{(i)}_{l,k,n}$
    using precoded uplink pilots, $\mathbf{m}^{(i)}_{l,k,n}$ and $\alpha^{(i)}_{l,k,n}$ are notified to all BSs in $\mathcal{B}$
    $i = i + 1$
  until *until convergence or* $i \geq I_{\max}$
end

Algorithm 3 outlines a practical way of updating the transmit and the receive beamformers by using over-the-air (OTA) signaling with the precoded pilots. Unlike primal decomposition (PD) or ADMM schemes, all variables are updated at once, *i.e*, the SCA update of $\epsilon^{(i-1)}$, AO update of $\mathbf{w}_{l,k,n}$ and the dual variable update of α using subgradient. It is not guaranteed to obtain the same point as that of the centralized problem. The algorithm in (42) is identical to (28), if the receivers $\mathbf{w}_{l,k,n}$ and the MSE operating point $\epsilon^{(i-1)}_{l,k,n}$ are fixed to find the optimal transmit precoders $\mathbf{m}_{l,k,n}$ and the dual variable $\alpha_{l,k,n}$. Note that it requires four nested loops to obtain the centralized solution, namely, the receive beamformer loop, MSE operating point loop, dual variable update loop and the bisection method to find the transmit precoders. To avoid this, the proposed method performs group update of all variables at once to obtain the transmit and the receive beamformers with the limited number of iterations, thus it achieves improved speed of convergence. Even though the convergence is not theoretically guaranteed, it converges in all numerical experiments considered in Section V-B. If the step size $\rho < 1$, the algorithm for ℓ_2 norm will converge by using the arguments on controlling overallocation and the nonincreasing objective function, since the earlier iterates are the operating point for the current iteration.

V. Simulation Results

The simulations carried out in this work consider the path loss (PL) varying uniformly across all users in the system with the channels drawn from the *i.i.d.* samples. The queues are generated based on the Poisson process with the average values specified in each section presented.

A. Centralized Solutions

We discuss the performance of the centralized algorithms in Section III for some system configurations. To begin with, we consider a single cell single-input single-output (SISO) model operating at 10 dB signal-to-noise ratio (SNR) with $K = 3$ users sharing $N = 3$ sub-channel resources. The number of packets waiting at the transmitter for each user is given by $Q_k = 4, 8$ and 4 bits, respectively.

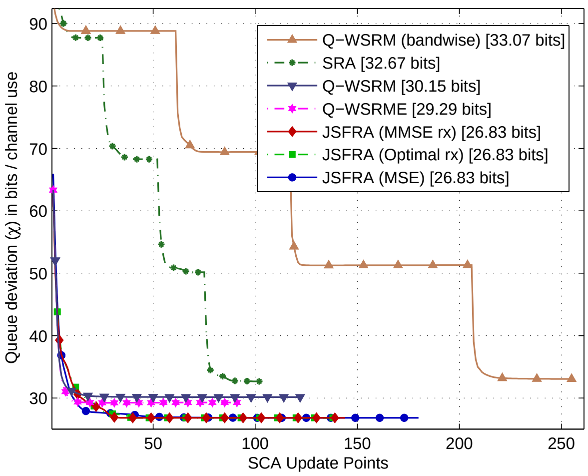
Table I tabulates the channel seen by the users over each sub-channel followed by the rates assigned by three different algorithms, Q-WSRME allocation, JSFRA approach and the band-wise Q-WSRM scheme using the WMMSE design [6]. The performance metric used for the comparison is the total number of backlogged bits left over at each slot after the allocation, which is denoted as $\chi = \sum_{k=1}^{K} [Q_k - t_k]^+$. Even though $\mathcal{U}(1)$ and $\mathcal{U}(3)$ has equal number of backlogged packets of $Q_1 = Q_3 = 4$ bits, user $\mathcal{U}(3)$ is scheduled in the first sub-channel due to the better channel condition. In contrast, the JSFRA approach assigns the first user on the first sub-channel, which reduces the total number of backlogged packets waiting at the transmitter. The rate allocated for $\mathcal{U}(2)$ on the second sub-channel is higher in JSFRA scheme compared to the other schemes. It is due to the efficient allocation of the total power shared across the sub-channels.

For a MIMO scenario, we consider a system with $N = 3$ sub-channels and $N_B = 3$ BSs, each equipped with $N_T = 4$ transmit antennas operating at 10dB SNR, serving $|\mathcal{U}_b| = 3$ users each. The PL between the BSs and the users are uniformly generated from $[0, -3]$ dB and the associations are made by selecting the BS with the lowest PL component. Fig. 1(a) shows the performance of the centralized schemes for a single receive antenna system. The total number of queued packets for Fig. 1(a) is given by $Q_k = [14, 15, 14, 8, 12, 9, 12, 11, 11]$ bits and for Fig. 1(b) is $Q_k = [9, 12, 8, 12, 5, 4, 10, 8, 5]$ bits respectively.

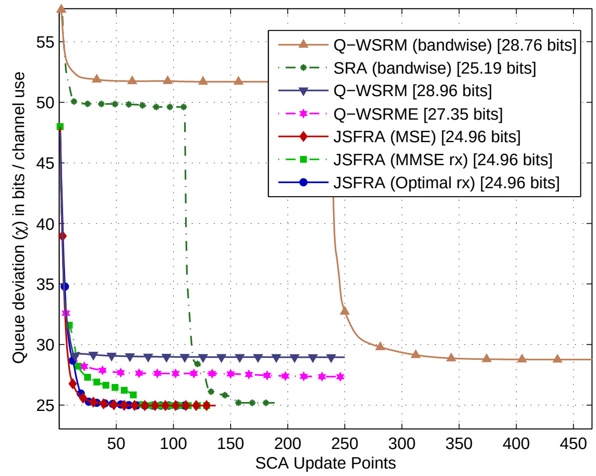
The performance of the centralized algorithms are compared in terms of the total number of residual bits remaining in the system after each SCA update in Fig. 1. The Q-WSRM algorithm is not optimal due to the problem of over-allocation when the number of queued packets are few in number. In contrast, the Q-WSRME algorithm provides more favorable allocation by including the explicit rate constraint to avoid the over-allocation. It can be seen that the JSFRA algorithms converges to a final point for all formulations.

TABLE I
SUB-CHANNEL-WISE LISTING OF CHANNEL GAINS AND RATE ALLOCATIONS BY DIFFERENT ALGORITHMS FOR A SCHEDULING INSTANT

Users	Queued Packets	Channel Gains			Q-WSRME approach (modified *backpressure*)			JSFRA Scheme			Q-WSRM band Alloc Scheme		
		SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3
1	4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
2	8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
3	4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining backlogged packets (χ)					3.92 bits			2.51 bits			5.89 bits		



(a). System Model $\{N, N_B, K, N_T, N_R\} = \{4, 3, 9, 4, 1\}$



(b). System Model $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

Fig. 1. Total number of backlogged packets χ present in the system after each SCA updates using $\ell_1 (q = 1)$ norm for JSFRA schemes

TABLE II
NUMBER OF BACKLOGGED BITS ASSOCIATED WITH EACH USER FOR A SYSTEM $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$.

q	user indices								χ
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
∞	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
Q_k	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	

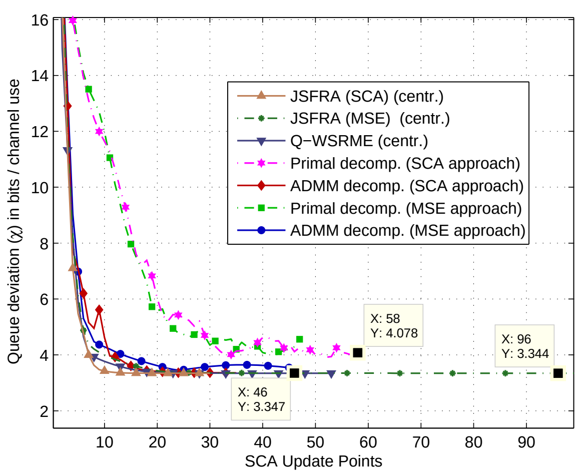


Fig. 2. Convergence of the centralized and the distributed algorithms for a system $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ using ℓ_1 norm for JSFRA schemes

For both scenarios in Fig. 1, the Q-WSRME performs marginally inferior to the JSFRA algorithms due to the weights used in the algorithm. The performance loss is attributed to the fact that the Q-WSRME algorithm favors the users with the large number of backlogged packets as compared to the users with better channel conditions. Fig. 1(b) compares the algorithms for $N_R = 2$ receive antenna case. In all figures, the receivers are updated along with the SCA update instants *i.e*, $J_{\max} = 1$ in Algorithm 1. It is also noted that the degradation by performing combined update is marginal, since the receiver minimizes the objective for a fixed transmit precoders thereby leading to a monotonic convergence.

The behavior of the JSFRA algorithm for different exponents q is outlined in the Table II for the users located at the cell-edge of the system employing $N_T = 4$ transmit antennas. It is evident that the JSFRA algorithm minimizes the total number of queued bits for the ℓ_1 norm compared to the ℓ_2 norm, which is shown in the column displaying the total number of left over packets χ in bits. The ℓ_∞ norm provides fair allocation of the resources by making the left over packets to be equal for all users to $\chi_k = 3.58$ bits.

B. Distributed Solutions

The distributed algorithms are compared using the total number of backlogged packets after each SCA update points. Fig. 2 compares the performance of the algorithms for the configuration $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ with $N_T = 4$

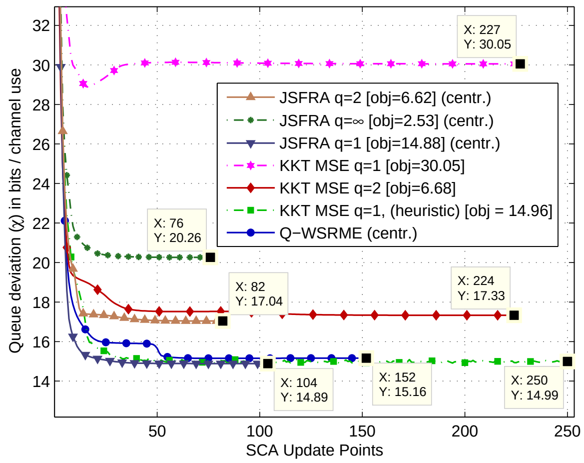


Fig. 3. Impact of varying q in the total number of backlogged packets after each SCA update for a system $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$

transmit antennas at the BSs with the PL varies uniformly between $[0, -6]$ dB. Each BS serves $|\mathcal{U}_b| = 4$ users in a coordinated manner to reduce the number of backlogged packets at each BS. The number of queued packets is given by $Q_k = [5, 7, 9, 11, 8, 12, 5, 4]$ bits. As discussed in Section IV, the performance and the convergence speed of the distributed algorithms are susceptible to the step size used in the subgradient update. Due to the fixed interference levels in the primal approach, it may lead to infeasible solutions if the initial or any intermediate update is not feasible.

Fig. 2 plots the performance of the primal and the ADMM solutions for the JSFRA scheme using the SCA and by MSE relaxation at each SCA point. In between the SCA updates, the primal or the ADMM scheme is performed for $J_{\max} = 20$ iterations to exchange the respective coupling variables. In Fig. 2, the total number of backlogged packets at each SCA points are plotted without the inner loop iterations of $J_{\max}$ times for the primal or the dual variables convergence. It can be seen from Fig. 2 that the distributed algorithms approach the centralized performance by exchanging minimal information between the coordinating BSs.

Fig. 3 compares the performance of the centralized and the KKT algorithm in Section IV-C for different exponents with $Q_k = [9, 16, 14, 16, 9, 13, 11, 12]$ bits and the PL varies uniformly between $[0, -3]$ dB. The ℓ_1 norm JSFRA scheme provides better performance over other schemes due to the greedy objective. The KKT approach for ℓ_1 norm is not defined due to the non-differentiability of the objective as discussed in the Section IV-C. If used for ℓ_1 norm, the problem of over-allocation will not affect the dual variables $\sigma_{l,k,n}$ and $\alpha_{l,k,n}$ since the queue deviation is raised to the power zero in (42e), which will always be equal to one. A heuristic method based on subdifferential calculus in [30] is proposed in Fig. 3 by assigning zero for $\sigma_{l,k,n}$ when the queue deviation is negative, *i.e*, $Q_k - t_k < 0$. It is required to address the problem of over-allocation in the ℓ_1 norm for dropping the absolute value operator from the objective function. It can be seen that the heuristic method oscillates near the stationary point with the deviation determined by the factor ρ used in (42f).

The objective values are mentioned in the legend for all the schemes and the objective of the ℓ_2 norm is not the same as that of the ℓ_1 norm used for plotting. For simulations, we update all variables in (42) at once at each iteration, *i.e*, $J_{\max} = 1$, which is well justified for the practical implementations due to the signaling overheads. The ℓ_2 norm for the JSFRA and the KKT approach achieves nearly the same value of 6.62 with different χ, due to the limited number of iterations for the dual variable convergence between each SCA update. Fig. 3 also shows the effect of dropping the squared rate variable from the objective in the Q-WSRME scheme compared to the ℓ_2 norm which includes it. By dropping it, the Q-WSRME scheme minimizes the number of queued packets in a prioritized manner based on the respective queues. On contrary, the ℓ_2 norm allocate rates to the users with the higher number of queued packets before addressing the users with the smaller number of queued packets.

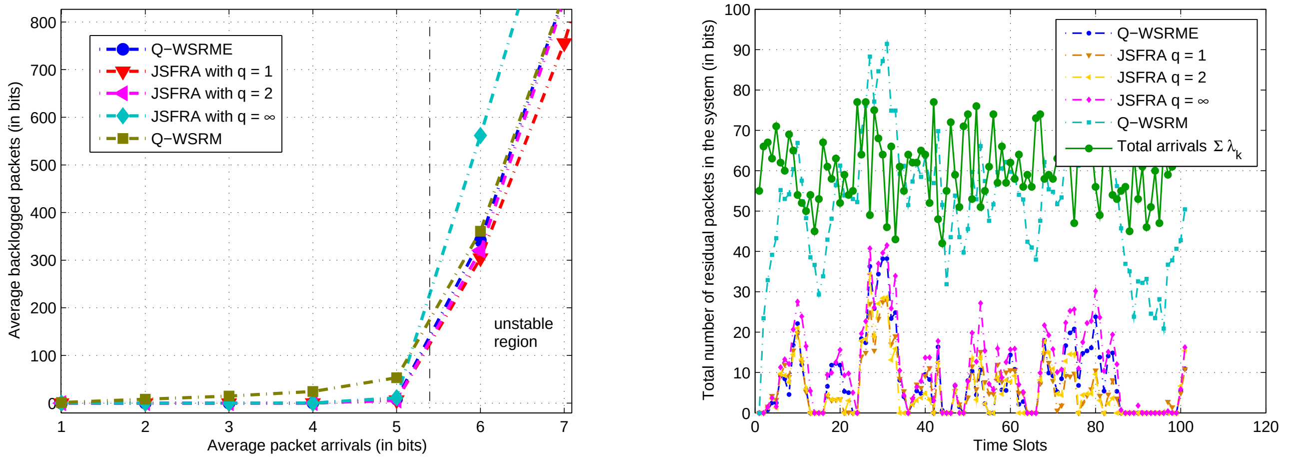
C. Queuing Analysis over Multiple Transmission Slots

We discuss the performance of the JSFRA algorithm for different ℓ_q over multiple transmission slots. It is compared with the existing Q-WSRME scheme by varying the average arrival rate A_k of all users. Fig. 4 plots the performance of the centralized algorithms for different ℓ_q values. Even though A_k's are constant for all users, the instantaneous arrivals are random and it follows Poisson process. We considered a 4×1 MIMO system with $N = 4$ sub-channels and $N_B = 2$ BSs. The PL is modeled as a uniform random variable $[0, -3]$ dB.

Fig. 4(a) compares various schemes with the average number of backlogged packets present in the system after each transmission slot. The performance of the JSFRA scheme using ℓ_2 and Q-WSRME approach are similar in the average number of residual packets after each transmission slot. Note that the additional rate constraints in the Q-WSRME scheme is the reason for the equivalence. Both Q-WSRM and Q-WSRME performs similar to ℓ_2 JSFRA scheme when the arrival rates are greater than the actual transmissions. Fig. 4(a) and Fig. 4(b) shows that the number of backlogged packets are noticeably less for the ℓ_1 JSFRA scheme due to the greedy allocation of serving users with better channel. Fig. 4 highlights the ℓ_∞ JSFRA scheme performs equally good compared to other objectives when the system is in the stable region. On contrary, the performance is inferior in the unstable region, since the fair allocation is not effective when the number of backlogged packets is large for all users.

VI. Conclusions

In this paper, we addressed the problem of allocating downlink space-frequency resources to the users in a multi-cell MIMO IBC system using OFDM. The resource allocation is considered as a joint space-frequency precoder design problem since the allocation of a resource to a user is obtained by a non-zero precoding vector. We proposed the JSFRA scheme by relaxing the nonconvex DC constraint by a sequence of convex subsets using SCA for designing the precoders to minimize



(a). Average backlogged packets in the system after 100 transmission instants

(b). Total backlogged packets at each transmission slot for $A_k = 5$ bits

Fig. 4. Time analysis of the Queue dynamics for a system $\{N, N_B, K, N_R\} = \{4, 2, 12, 1\}$

the total number of user queued packets. Additionally, an alternative MSE relaxation approach is also proposed by using SCA to address the nonconvex DC constraints for a fixed MMSE receivers. We also proposed distributed precoder designs for the JSFRA problem using primal and ADMM methods.Finally, we proposed a practical iterative algorithm to obtain the precoders in a decentralized manner by solving the KKT conditions of the MSE reformulated JSFRA method. The proposed iterative algorithm requires few iterations and limited signaling exchange between the coordinating BSs to obtain the efficient precoders for a given number of iterations. Numerical results are used to compare the performance of the proposed algorithms with the existing solutions. The distributed design for the time correlated fading will be considered in future.

Appendix A
Tightness of SINR Relaxation

For the constraints (16b) and (16c) to be active, there should be at least one user in each BS with enough backlogged packets that cannot be served with the given power budget. On the other hand, to make the constraints active in all cases, the objective of the JSFRA formulation should be regularized with the transmit power without affecting the solution as

$$\|\tilde{\mathbf{v}}\|_q + \varphi \sum_{k \in \mathcal{U}} \sum_{n=1}^{N} \sum_{l=1}^{L} \operatorname{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \tag{44}$$

where $\varphi \approx 0$. Note that the modified objective will relax the power constraint by making the constraints (16b) and (16c) active at the final solution.

Appendix B
Convergence Proof for Centralized Algorithm

To prove the convergence of the transmit, receive beamformers, and the objective values of the centralized algorithms in (16) and (26), we need to show that the following conditions are satisfied by the centralized formulations.

(a) The function should be coercive and bounded below
(b) The feasible set should be a compact set
(c) The sequence of the objective values should be strictly decreasing in each iteration
(d) Uniqueness of the minimizer, *i.e*, the transmit and the receive beamformers should be unique in each iteration.

Using [30, Prop. A.8], the existence of a global minimizer in the feasible set can be guaranteed if the conditions (a) and (b) are satisfied. Since the feasible set is not fixed in each iteration, we require additional conditions (c) and (d) to prove the global convergence of the objective function and the corresponding arguments namely, the transmit and the receive precoders. Assuming the conditions (a), (b) and (c) are satisfied, using [31, Th. 3.14], we can show that the bounded monotonically decreasing objective value sequence has a unique minimum. Finally, we show the limiting point of the iterative algorithm is indeed a stationary point for the problem in (16) and (26).

A. Bounded Objective Function and Compact Feasible Set

The feasible set of the problem (16) and (26) are bounded and closed, which can be verified by the total power constraint on the transmit precoders (16d), and therefore, a compact set.

The minimum value of the norm operator in the objective is zero, *i.e*, $\|x\|_q > -\infty$, therefore it is bounded below. The objective function is Lipschitz continuous over the feasible set, and therefore, it is bounded from above as well, since the feasible set is bounded. The objective function in (16) and (26) is continuous and approaches ∞ as $\mathbf{t}_k \to \infty$, therefore it is coercive. Using conditions (a) and (b), we can guarantee the existence of a minimizer to the problem in (16) and (26).

B. Monotonicity

Let us express the centralized problem in (16) and (26) as

$$\underset{\mathbf{m},\mathbf{w},\boldsymbol{\gamma}}{\text{minimize}} \qquad f(\mathbf{m}, \mathbf{w}, \boldsymbol{\gamma}) \tag{45a}$$

$$\text{subject to} \quad h(\boldsymbol{\gamma}) - g_0(\mathbf{m}, \mathbf{w}) \leq 0 \tag{45b}$$
$$g_1(\mathbf{m}, \mathbf{w}) \leq 0 \tag{45c}$$
$$g_2(\mathbf{m}) \leq 0 \tag{45d}$$

where g_2, f are convex and h is a linear function. Let g_0, g_1 be convex in either $\mathbf{m}$ or $\mathbf{w}$ but not on both. The constraint in (45b) corresponds to (16b) or (26b) and the constraint (45c) correspond to (16c) or (26c). Other convex constraints are addressed by (45d) and the feasible set of (45) is given by

$$\mathcal{F} = \{\, \mathbf{m}, \mathbf{w}, \boldsymbol{\gamma} \mid \quad h(\boldsymbol{\gamma}) - g_0(\mathbf{m}, \mathbf{w}) \leq 0, \\ g_1(\mathbf{m}, \mathbf{w}) \leq 0, g_2(\mathbf{m}) \leq 0\}. \tag{46}$$

To solve (45), we adopt AO by fixing a block of varibles and optimize for others [32]. In (45), even after fixing the variable $\mathbf{w}$, the problem is nonconvex due to the DC constraint (45b). We adopt SCA presented in [25], [26], [33] by relaxing the nonconvex set by a sequence of convex subsets. Since it involves two nested iterations, we denote the AO iteration index by a superscript (i) and the DC constraint relaxations by a subscript k. Let $\mathcal{X}_k^{(i)}$ be the feasible set for the i^{th} AO iteration and the k^{th} SCA point for a fixed $\mathbf{w}$ and $\mathcal{Y}_k^{(i)}$ denotes the feasible set for a fixed $\mathbf{m}$. Since the SCA iterations are performed until convergence, let $\mathbf{m}_*^{(i)}$ denote the converged point of $\mathbf{m}$ in the i^{th} AO iteration. Let $\boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}$ be the optimal value of $\boldsymbol{\gamma}$ obtained in the i^{th} AO iterate for a fixed $\mathbf{w}$.

To begin with, let us consider the variable $\mathbf{w}$ is fixed for the AO i with the optimal value achieved from the previous iteration $i-1$ as $\mathbf{w}_*^{(i-1)}$. In order to solve for $\mathbf{m}$ in the SCA iteration k, we linearize the nonconvex function g_0 using previous SCA iterate of $\mathbf{m}$ as

$$\hat{g}_o(\mathbf{m}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) = g_0(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}) \\ + \nabla g_0(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)})^{\mathrm{T}}(\mathbf{m} - \mathbf{m}_k^{(i)}). \tag{47}$$

Using (47), the convex subproblem for i^{th} AO iteration and k^{th} SCA point for the variable $\mathbf{m}$ and $\boldsymbol{\gamma}$ is given by

$$\underset{\mathbf{m}, \boldsymbol{\gamma}}{\text{minimize}} \quad f(\mathbf{m}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}) \tag{48a}$$
$$\text{subject to} \quad h(\boldsymbol{\gamma}) - \hat{g}_0(\mathbf{m}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \leq 0 \tag{48b}$$
$$g_1(\mathbf{m}, \mathbf{w}_*^{(i-1)}) \leq 0 \tag{48c}$$
$$g_2(\mathbf{m}) \leq 0. \tag{48d}$$

Let the feasible set defined by the problem in (48) be represented as $\mathcal{X}_k^{(i)} \subset \mathcal{F}$. In order to prove the convergence of the convex subproblem (48) for a fixed $\mathbf{w} = \mathbf{w}_*^{(i-1)}$ operating at $\mathbf{m}_k^{(i)}$, let us consider that (48) yields $\mathbf{m}_{k+1}^{(i)}$ and $\boldsymbol{\gamma}_{k+1}^{(i)}$ as the solution for the k^{th} iteration. Note that the point $\mathbf{m}_{k+1}^{(i)}$ and $\boldsymbol{\gamma}_{k+1}^{(i)}$, which minimizes the objective function, is also feasible for (48) using the following inequality

$$h(\boldsymbol{\gamma}_{k+1}^{(i)}) - g_0(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}) \leq -\hat{g}_0(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \\ + h(\boldsymbol{\gamma}_{k+1}^{(i)}) \leq h(\boldsymbol{\gamma}_k^{(i)}) - \hat{g}_0(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}; \mathbf{m}_k^{(i)}) \leq 0. \tag{49}$$

Using (49), we can prove that the solution $\mathbf{m}_{k+1}^{(i)}$ and $\boldsymbol{\gamma}_{k+1}^{(i)}$ are feasible, since the initial point of $\mathbf{m} = \mathbf{m}_*^{(i-1)}$ was chosen to be feasible from the earlier AO iteration $i-1$. At each SCA iteration, the feasible set includes the optimal point from the previous iteration as $\{\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k+1}^{(i)}\} \in \mathcal{X}_{k+1}^{(i)} \subset \mathcal{F}$, thereby, leading to the monotonic decrease in the objective values [26], [33], [34] as

$$f(\mathbf{m}_0^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_0^{(i)}) \geq f(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)}) \\ \geq f(\mathbf{m}_{k+1}^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{k+1}^{(i)}) \geq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}). \tag{50}$$

Thus the sequence $f(\mathbf{m}_k^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_k^{(i)})$ is nonincreasing and approaches limiting point as $k \to \infty$. Note that feasible point $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}\}$ need not be a stationary point of (45), since it is the minimizer over the set $\mathcal{X}_*^{(i)} \subset \mathcal{F}$, for a fixed $\mathbf{w}$.

Once the solution is found for a fixed $\mathbf{w}$, we fix $\mathbf{m}$ as $\mathbf{m}_*^{(i)}$ and optimize for $\mathbf{w}$. Even after treating $\mathbf{m}$ as a constant, the problem is still nonconvex due to the DC constraint. Following similar approach, we can find the minimizer $\mathbf{w}_k^{(i)}$ and $\boldsymbol{\gamma}_k^{(i)}$ for a similar convex subproblem (48) at each iteration k. Note that $\boldsymbol{\gamma}_k^{(i)}$ is reused since the variable $\mathbf{m}$ is fixed for the i^{th} AO iteration. The convergence and the nonincreasing behavior of the problem follows similar arguments as above[5]. Now, the optimal solution of the converged subproblems with $\mathbf{w}$ as variable are $\mathbf{w}_*^{(i)}$ and $\boldsymbol{\gamma}_*^{(i)}$. Note that the limiting point $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i)}, \boldsymbol{\gamma}_{*|\mathbf{m}}^{(i)}\}$ is the unique minimizer in the set $\mathcal{Y}_*^{(i)}$.

Finally, to prove the global convergence of the iterative algorithm, we need to show the nonincreasing behavior of the objective function between each AO update, *i.e*,

$$f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i)}, \boldsymbol{\gamma}_{*|\mathbf{m}}^{(i)}) \leq f(\mathbf{m}_*^{(i)}, \mathbf{w}_0^{(i)}, \boldsymbol{\gamma}_0^{(i)}) \\ \leq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}). \tag{51}$$

Let us consider an AO iteration i in which the optimal value for $\mathbf{m}$ and $\boldsymbol{\gamma}$ are obtained as $\mathbf{m}_*^{(i)}$ and $\boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}$ using fixed $\mathbf{w} = \mathbf{w}_*^{(i-1)}$. To find $\mathbf{w}_0^{(i)}$, we fix $\mathbf{m}$ as $\mathbf{m}_*^{(i)}$ and optimize for $\mathbf{w}$. Since we linearize the convex function in (45b), the fixed operating point is also included in the feasible set $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}\} \in \mathcal{Y}_0^{(i)}$ using (49). Using this, we can show the monotonicity of the objective value sequence as

$$f(\mathbf{m}_*^{(i)}, \mathbf{w}_0^{(i)}, \boldsymbol{\gamma}_0^{(i)}) \leq f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{m}}^{(i)}). \tag{52}$$

The AO update follows $\{\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i-1)}, \boldsymbol{\gamma}_{*|\mathbf{w}}^{(i)}\} \in \{\mathcal{X}_*^{(i)} \cap \mathcal{Y}_0^{(i)}\}$.

C. Convergence of the Beamformer iterates

Let $\mathbf{x} \triangleq [\mathbf{m}, \mathbf{w}, \boldsymbol{\gamma}]$ be the stacked vector of the optimization variables and $\mathcal{A}$ be a point-to-set mapping algorithm from $\mathcal{F}$ into the nonempty subsets of $\mathcal{F}$, *i.e*, $\mathbf{x}^{(i+1)} \in \mathcal{A}(\mathbf{x}^{(i)})$. The set of accumulation points in the compact set $\mathcal{F}$ be denoted by $\mathcal{F}^*$ and the objective function be f is closed and continuous. Let $\{\mathbf{x}^{(i)}\}$ be the sequence of iterates generated by the algorithm $\mathcal{A}$ and the objective function f. If the mapping is strictly monotonic and uniformly compact, then the following conditions hold [35] and [36, Theorem 3.1].

(i) all accumulation points will be fixed points,

[5]Note that we can also use the MMSE receiver in (23b) instead of performing the SCA updates until convergence for the optimal receiver

(ii) $f(\mathbf{x}^{(i)}) \to f(\mathbf{x}^*)$, where $\mathbf{x}^*$ is a fixed point,

(iii) $\mathbf{x}^*$ is a regular point, *i.e* $\|\mathbf{x}^{(i)} - \mathbf{x}^{(i+1)}\| \to 0$.

The mapping $\mathcal{A}$ for the iterative algorithm is given by

$$\mathcal{A}(\mathbf{x}^{(i)}) : \underset{\mathbf{x}}{\operatorname{argmin}}\ f(\mathbf{x}; \mathbf{x}^{(i)}),\ \text{subject to } \mathbf{x} \in \mathcal{F} \tag{53}$$

where $\mathbf{x}^{(i)}$ belongs to the set of equally valid iterates $\mathcal{A}(\mathbf{x}^{(i)})$ in the i^{th} iteration[6]. The mapping is strictly monotonic on $\mathcal{F}$ with respect to the objective function as $\mathbf{x}' \in \mathcal{A}(\mathbf{x})$ implies $f(\mathbf{x}') < f(\mathbf{x})$ whenever $\mathbf{x}$ is not a fixed point, *i.e*, $\mathbf{x} \notin \mathcal{F}^*$, as discussed in Appendix B-B. The sequence of iterates $\{\mathbf{x}^{(i)}\}$ will have at least one accumulation point, which follows from the compactness of the set [35]. The sequence $\{\mathbf{x}^{(i)}\}$ converges to the set of fixed point $\mathcal{F}^*$ for any feasible starting point $\mathbf{x}^{(0)} \in \mathcal{F}$, therefore the mapping is uniformly compact.

To show the set of fixed points $\mathcal{F}^*$ are the minimizer for the objective function f, we can rely on the strict monotonicity of the objective function for each iteration. Since $f(\mathcal{A}(\mathbf{x}^{(i)})) < f(\mathbf{x}^{(i)})$, the objective decreases monotonically for each iteration and the equality achieves when $\mathcal{A}(\mathbf{x}^*) = \mathbf{x}^*$, which is the fixed point. Therefore, the fixed points in $\mathcal{F}^*$ are the minimizer for the objective function f over the set $\mathcal{F}$.

If the uniqueness of the iterates are guaranteed, then the algorithm will find a unique solution at each iteration as $\mathbf{x}^{(i+1)} = \mathcal{A}(\mathbf{x}^{(i)})$ and the accumulation point is unique. The convergence of the iterates to a single fixed point is guaranteed by the discussions in [35], [36]. Even though the algorithm finds a single fixed point, all fixed points in $\mathcal{F}^*$ are equally valid as a minimizer for the function f in (45), since the SINR in (2) is invariant to the unitary rotations on the beamformers.

D. Uniqueness

The uniqueness of the transmit precoders and the receive beamformers for each convex subproblem is achieved by the constraint (19) for the problem (20) and (26c) for the MSE reformulation in (28). The receive beamformers $\mathbf{w}_{l,k,n}$ in (23b) are also unique for the given set of transmit precoders, since the convex subproblems in (20) and (28) are susceptible to the transmit precoder phase rotations, *i.e*, unitary transformations.

When the objective function is zero, the uniqueness of the transmit precoders are not guaranteed by using (19) and (26c) constraints, since they are not active at the optimal solution of the subproblems. To obtain unique set of transmit precoders when the objective is zero even for a single BS, we can regularize the objective with the transmit power as discussed in Appendix A without affecting the optimal value.

E. Stationary Point

Finally, to show the stationarity of the fixed points in $\mathcal{F}^*$, it must satisfy the KKT conditions of (45). As $i \to \infty$, it satisfies

$$\begin{aligned} f(\mathbf{m}_*^{(i)}, \mathbf{w}_*^{(i)}, \boldsymbol{\gamma}_{*|x}^{(i)}) &= f(\mathbf{m}_*^{(i+1)}, \mathbf{w}_*^{(i)}, \boldsymbol{\gamma}_{*|y}^{(i+1)}) \\ &= f(\mathbf{m}_*^{(i+1)}, \mathbf{w}_*^{(i+1)}, \boldsymbol{\gamma}_{*|x}^{(i+1)}) \end{aligned} \tag{54}$$

[6]Index i denotes the AO iteration to update the vector $\mathbf{x}$, which involves two SCA iterations to be performed until convergence, *i.e*, one for the variable $\mathbf{m}$ by keeping $\mathbf{w}$ fixed and another for the variable $\mathbf{w}$ by fixing $\mathbf{m}$ as constant

where $\{\mathbf{m}_*^{(i+1)}, \mathbf{w}_*^{(i+1)}, \boldsymbol{\gamma}_{*|x}^{(i+1)}\} \in \mathcal{F}^*$ is the fixed point and the minimizer of the objective function f in the feasible set $\mathcal{X}_*^{(i+1)} \subset \mathcal{F}$. Using the discussions in [22], we can show that the feasible point $\{\mathbf{m}_*^{(i+1)}, \mathbf{w}_*^{(i+1)}, \boldsymbol{\gamma}_{*|x}^{(i+1)}\}$, which is the minimizer in its local neighborhood, is a stationary point of (45) satisfying the constraint qualifications and the KKT expressions over $\mathcal{X}_*^{(i+1)} \subset \mathcal{F}$. The non differentiability of the objective function in (16) and (26) requires $\mathbf{0}^{\mathrm{T}} \in \partial f_0(\boldsymbol{\gamma}_*)$ to satisfy the KKT conditions, where $\partial f_0(\boldsymbol{\gamma}_*)$ denotes the subdifferential set. Using the above arguments, we can show that the JSFRA schemes reaches a stationary point of the original nonconvex problem.

Appendix C
Convergence Proof for Distributed Algorithms

The convergence of the distributed algorithm outlined in Algorithm 2 follows the same discussion in Appendix B if the subproblem (31) converge to the centralized solution. Since the subproblem (31) is convex, each BS specific slave subproblem is also convex for a fixed interference vector $\boldsymbol{\zeta}_{b_k}^{(i)}$ [10]. The master subproblem in the primal decomposition uses subgradient to update the coupling interference vectors in consensus with the objective function, it is guaranteed to converge to the centralized solution as the iteration $i \to \infty$ [30] for a diminishing step size. It can be seen that the subproblem (31) satisfies Slater's constraint qualification by having non empty interior and bounded due to the total power constraint for the transmit precoders.

To prove the convergence of the ADMM approach, we use the discussions in [37, Prop. 4.2] by writing the problem as

$$\begin{aligned} \underset{\mathbf{x},\mathbf{z}}{\text{minimize}} \quad & G(\mathbf{x}) + H(\mathbf{y}) && \text{(55a)} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{z} && \text{(55b)} \\ & \mathbf{x} \in \mathcal{C}_1, \mathbf{z} \in \mathcal{C}_2 && \text{(55c)} \end{aligned}$$

following conditions are required for the ADMM convergence.

- G, H should be convex
- $\mathcal{C}_1, \mathcal{C}_2$ should be a convex set and bounded
- $\mathbf{A}^{\mathrm{H}}\mathbf{A}$ should be invertible.

Note that the equality constraint (55b) is identical to (38) used in the ADMM subproblem (37). It is evident from the equality constraint (38) that $\mathbf{A} = \mathbf{I}$, which is an identity matrix and is invertible. The objective function G, H are ℓ_q norm in (31) are convex and the set defined by the constraints of the problem (31) are all convex sets and has a nonempty interior. The feasibility of the interior point is verified by having a non zero precoder for only one user. Therefore, by following [37, Prop. 4.2], it can be seen that the ADMM approach converges to the centralized solution as $i \to \infty$. The distributed algorithms also converges to a stationary point of the original nonconvex problem as that of the centralized algorithms by following the discussions on Appendix B, if the primal or the ADMM updates are iterated until convergence.

Appendix D
KKT conditions for MSE approach

In order to solve for an iterative precoder design algorithm, the KKT expressions for the problem in (41) are obtained by differentiating the Lagrangian by assuming the equality constraint for (41b) and (41c). At the stationary points, following conditions are satisfied.

$$\nabla_{\epsilon_{l,k,n}} : -\alpha_{l,k,n} + \frac{\sigma_{l,k,n}}{\tilde{\epsilon}_{l,k,n}} = 0 \tag{56a}$$

$$\nabla_{t_{l,k,n}} : -q\, a_k \Big(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \Big)^{(q-1)} + \frac{\sigma_{l,k,n}}{\log_2(e)} = 0 \tag{56b}$$

$$\begin{aligned} \nabla_{\mathbf{m}_{l,k,n}} : & \sum_{y \in \mathcal{U}} \sum_{x=1}^{L} \alpha_{x,y,n} \mathbf{H}^{\mathrm{H}}_{b_k,y,n} \mathbf{w}_{x,y,n} \mathbf{w}^{\mathrm{H}}_{x,y,n} \mathbf{H}_{b_k,y,n} \mathbf{m}_{l,k,n} \\ & + \delta_b \mathbf{m}_{l,k,n} = \alpha_{l,k,n} \mathbf{H}^{\mathrm{H}}_{b_k,k,n} \mathbf{w}_{l,k,n} \end{aligned} \tag{56c}$$

$$\begin{aligned} \nabla_{\mathbf{w}_{l,k,n}} : & \sum_{(x,y) \neq (l,k)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n} \mathbf{m}^{\mathrm{H}}_{x,y,n} \mathbf{H}^{\mathrm{H}}_{b_y,k,n} \mathbf{w}_{l,k,n} \\ & + \mathbf{I}_{N_R} \mathbf{w}_{l,k,n} = \mathbf{H}_{b_k,k,n}\ \mathbf{m}_{l,k,n}. \end{aligned} \tag{56d}$$

In addition to the primal constraints given in (41b), (41c) and (41d), the complementary slackness criterion must also be satisfied at the stationary point. Upon solving the above expressions in (56) with the complementary slackness conditions, we obtain the iterative algorithm to determine the transmit and the receive beamformers as shown in (42).

References

[1] E. Matskani, N. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, "Convex approximation techniques for joint multiuser downlink beamforming and admission control," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2682–2693, July 2008.

[2] C. Ng and H. Huang, "Linear Precoding in Cooperative MIMO Cellular Networks with Limited Coordination Clusters," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1446–1454, December 2010.

[3] L. N. Tran, M. Hanif, A. Tölli, and M. Juntti, "Fast Converging Algorithm for Weighted Sum Rate Maximization in Multicell MISO Downlink," *IEEE Signal Processing Letters*, vol. 19, no. 12, pp. 872–875, 2012.

[4] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and Sum-MSE Minimization," *IEEE Transactions on Signal Processing*, vol. 55, no. 11, pp. 5436–5446, Nov 2007.

[5] S. S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4792–4799, 2008.

[6] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331–4340, sept. 2011.

[7] M. Hong, Q. Li, Y.-F. Liu, and Z.-Q. Luo, "Decomposition by successive convex approximation: A unifying approach for linear transceiver design in interfering heterogeneous networks," 2012. [Online]. Available: http://arxiv.org/abs/1210.1507

[8] J. Kaleva, A. Tölli, and M. Juntti, "Primal decomposition based decentralized weighted sum rate maximization with QoS constraints for interfering broadcast channel," in *IEEE 14th Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*. IEEE, 2013, pp. 16–20.

[9] ——, "Decentralized beamforming for weighted sum rate maximization with rate constraints," in *24th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC Workshops)*. IEEE, 2013, pp. 220–224.

[10] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1439–1451, 2006.

[11] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

[12] H. Pennanen, A. Tölli, and M. Latva-Aho, "Decentralized coordinated downlink beamforming via primal decomposition," *IEEE Signal Processing Letters*, vol. 18, no. 11, pp. 647–650, 2011.

[13] A. Tölli, H. Pennanen, and P. Komulainen, "Decentralized minimum power multi-cell beamforming with limited backhaul signaling," *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 570–580, 2011.

[14] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936–1948, Dec 1992.

[15] M. Neely, *Stochastic network optimization with application to communication and queueing systems*, ser. Synthesis Lectures on Communication Networks. Morgan & Claypool Publishers, 2010, vol. 3, no. 1.

[16] L. Georgiadis, M. J. Neely, and L. Tassiulas, *Resource allocation and cross-layer control in wireless networks*. Now Publishers Inc, 2006.

[17] R. A. Berry and E. M. Yeh, "Cross-layer wireless resource allocation," *IEEE Signal Processing Magazine*, vol. 21, no. 5, pp. 59–68, 2004.

[18] M. Chiang, S. Low, A. Calderbank, and J. Doyle, "Layering as Optimization Decomposition: A Mathematical Theory of Network Architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–312, Jan 2007.

[19] K. Seong, R. Narasimhan, and J. Cioffi, "Queue proportional scheduling via geometric programming in fading broadcast channels," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1593–1602, 2006.

[20] P. C. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, "Resource allocation for cross-layer utility maximization in wireless networks," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 6, pp. 2790–2809, 2011.

[21] F. Zhang and V. Lau, "Cross-Layer MIMO Transceiver Optimization for Multimedia Streaming in Interference Networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 5, pp. 1235–1244, March 2014.

[22] B. R. Marks and G. P. Wright, "A General Inner Approximation Algorithm for Nonconvex Mathematical Programs," *Operations Research*, vol. 26, no. 4, pp. 681–683, 1978.

[23] Z.-Q. Luo and S. Zhang, "Dynamic Spectrum Management: Complexity and Duality," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 57–73, Feb 2008.

[24] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[25] T. Lipp and S. Boyd, "Variations and extensions of the convex-concave procedure," 2014.

[26] G. R. Lanckriet and B. K. Sriperumbudur, "On the convergence of the concave-convex procedure," in *Advances in neural information processing systems*, 2009, pp. 1759–1767.

[27] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.0 beta," http://cvxr.com/cvx, Sep. 2013.

[28] P. Komulainen, A. Tölli, and M. Juntti, "Effective CSI Signaling and Decentralized Beam Coordination in TDD Multi-Cell MIMO Systems," *IEEE Transactions on Signal Processing*, vol. 61, no. 9, pp. 2204–2218, 2013.

[29] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Distributed Robust Multicell Coordinated Beamforming With Imperfect CSI: An ADMM Approach," *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 2988–3003, June 2012.

[30] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Athena Scientific, sep 1999.

[31] W. Rudin, *Principles of Mathematical Analysis*. McGraw-Hill New York, 1964, vol. 3.

[32] J. C. Bezdek and R. J. Hathaway, "Some notes on alternating optimization," in *Advances in Soft Computing AFSS*. Springer, 2002, pp. 288–300.

[33] G. Scutari, F. Facchinei, L. Lampariello, and P. Song, "Distributed Methods for Constrained Nonconvex Multi-Agent Optimization – Part I: Theory." [Online]. Available: http://arxiv.org/abs/1410.4754v1

[34] T. D. Quoc and M. Diehl, "Sequential convex programming methods for solving nonlinear optimization problems with DC constraints," 2011. [Online]. Available: /http://arxiv.org/abs/1107.5841v1

[35] W. Zangwill, *Nonlinear programming: a unified approach*, ser. Prentice-Hall international series in management. Prentice-Hall, 1969.

[36] R. Meyer, "Sufficient Conditions for the Convergence of Monotonic Mathematical Programming Algorithms," *Journal of computer and system sciences*, vol. 12, no. 1, pp. 108–121, 1976.

[37] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and distributed computation: numerical methods*. Prentice hall Englewood Cliffs, NJ, 1989, vol. 23.