Reviewers Comments & Authors Replies

Manuscript No.	Paper T-SP-18051-2014, submitted to *"IEEE Transactions on Signal Processing"*
Title	"Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems"
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The authors are grateful to the associate editor for giving us an additional opportunity to revise and correct our convergence analysis. In this regard, we would like to give a special thank to Review 3 who pointed out the issues with our convergence analysis of the proposed solutions. In this revision, we have mainly focused on providing a more mathematically rigorous proof of the convergence analysis of the proposed iterative algorithms. Towards this end the following changes have been made to the revised manuscript.

1. We have restated the convergence results of the centralized algorithms in this revision, claiming that any accumulation point (i.e., limit point) is a stationary point. The proof is essentially based on the convergence of a subsequence of iterates. We have rewritten Appendix A-D according to these changes. In particular, to discuss the convergence of the sequence of iterates generated by the centralized method, we use a unified superscript index t to refer to the step index of both alternating optimization (AO) and successive convex approximation (SCA) procedures (*i.e.*, index t denotes SCA and AO update step (k, i)). Then, by using index t, we denote iterate $\mathbf{x}^t$ as a vector formed by stacking the minimizer obtained by solving (48) at iteration t as $\mathbf{x}^t = \mathbf{z}_k^{(i)}$, and let $\{\mathbf{x}^t\}$ be the sequence of minimizers obtained at each iteration. With these notations, we have revised our convergence discussions in Appendices A-C and A-D.
2. We have updated the convergence of distributed algorithms in a similar way.

We have also revised some parts of text to support the changes mentioned above and to further improve the readability. Specifically, the following parts of the revision have been rewritten.

- Page 6, paragraph after (28): We have elaborated the discussions on showing that the feasible set is bounded.
- Page 8, last paragraph in Section IV-B: We have highlighted the conditions under which the overall objective sequence generated by the distributed methods will converge.
- Last sentence after (51) on page 14: We have emphasized the convergence of objective sequence only in each AO step and not the iterates.
- Last sentence after (60) on page 15: We have rewritten the convergence claim on the sequence of iterates generated by the Algorithm 2 similar to that of centralized method.

In what follows, the comments are listed, each followed immediately by the corresponding reply from the authors. The reviewers questions in the revised manuscript are highlighted in blue color and the authors responses are presented in black. Unless otherwise stated, all the numbered items (figures, equations, references, citations, etc) in this response letter refer to the revised manuscript. All revisions in the manuscript are highlighted in blue color.

Response to first reviewer comments

The reviewer's concerns have been addressed and the manuscript can be published as it is.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.

Response to second reviewer comments

I have no further comments.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.

Response to third reviewer comments

The authors have modified the manuscript taken into account the reviewers' comments. However, the proof of convergence still lacks of convincing argumentation to make this paper suitable for publication. Two main aspects are arguable as follows:

Response: We thank the reviewer for pointing out the mistake in our previous convergence proof. Based on the reviewer's comments and suggestions, we have rewritten Appendix A-D using subsequence convergence instead of claiming the sequence convergence. We hope that our current changes will be convincing enough to make the convergence proof complete.

1. When discussing the convergence of the SCA algorithm (Appendix A-C), the authors claim that the SCA algorithm converges because the objective function is strictly decreasing and the minimizer in each iteration is unique. However, this is in general not enough to guarantee the convergence of the whole sequence. Instead, one can only claim that every limit point of the whole sequence is a stationary point, or globally optimal solution if the problem is convex. As an example, simply consider the gradient projection algorithm, where each subproblem is strongly convex and the objective function is strictly decreasing but the whole sequence may not converge. The authors are referred to Bertsekas' book: Nonlinear Programming.

   Response: We thank the reviewer for pointing out that the sequence of iterates generated by the SCA algorithm in each AO step need not be convergent. Based on the reviewer's reference to the gradient projection algorithm in Nonlinear Programming textbook, we have realized that our earlier convergence proof is not mathematically rigorous. We apologize for our emphasis on the convergence of iterates in our previous manuscript. As suggested by the reviewer, stationarity of limit points of the whole sequence of iterates is now addressed in Appendix A-D by using the following statements.

   - The sequence of iterates generated by the algorithm is bounded, therefore, we can claim that there exist at least one subsequence of original sequence that converges to a limit point of original sequence.
   - Using a convergent subsequence of original sequence of iterates, we then discuss that every limit point is a stationary point of nonconvex problem in Appendix A-D.

   We have made changes in Appendices A-C and A-D to address the concerns raised by the reviewer.

2. Even under the assumption that the inner loop SCA converges, there are doubts with respect to the convergence of the alternating optimization algorithm described by (54). The authors cite [27] to justify convergence, but it is not straightforward to claim that the algorithmic model considered in this manuscript is the same as in [27]. More specifically, let us consider the variable x (so it is consistent with the authors' notation in Appendix A-D). In the algorithmic model of [27], at iteration $t+1$, all elements of $\mathbf{x}$ are updated simultaneously, based on $\mathbf{x}^t$. But in the authors' model, the update of $\mathbf{x}$ is performed in two phases: in the first phase, only the elements of $\mathbf{m}$ and γ in $\mathbf{x}$ are updated based on $\mathbf{x}^t$. In the second phase, the remaining elements of $\mathbf{x}$ are updated, i.e. $\mathbf{w}$ and γ, based on both $\mathbf{x}^t$ and the elements that have been updated in the first phase. This is not a trivial difference and thus the direct application of the conclusion from [27] in the current context cannot be taken for granted. Due to the open concerns after several revision rounds, it is recommended to reject the paper.

   Response: We understand the reviewer's concern on using reference [R1] to our problem involving both SCA and AO updates. In each iteration, the proposed method solves only a subset of optimization variables by keeping others fixed, therefore, we cannot use reference [R1] directly as suggested by the reviewer. Even though we have revised our convergence proof based on a convergent subsequence without referring to [R1], nevertheless, we clarify our usage of [R1] in our previous manuscript for the convergence of iterates.

- We refer to [R2] for a similar usage of reference [R1] while discussing the convergence of AO method used in designing tight frames by solving matrix nearness problem. The reference [R1] used to discuss the convergence of iterates generated by the AO method in [R2] by referring AO update as a composition of two sub-algorithms, wherein each sub-algorithm updates only a subset of optimization variables. It has been discussed comprehensively in [R2, Appendices I and II]. In particular, the discussion on referring AO as two sub-algorithms is presented in [R2, Appendix I-F].
- However, we apologize for the incomplete usage of [R1, Theorem 3.1] in our previous manuscript to claim the convergence of beamformer iterates. In [R1, Theorem. 3.1], it has been shown that if the strict monotonicity of the objective sequence is ensured and the iterates are bounded, then *either the sequence of iterates converges or the accumulation points of* $\{\mathbf{x}^t\}$ *is a continuum.*
- Therefore, our previous statement claiming guaranteed convergence of iterates is not correct, as pointed out by the reviewer.
- A similar approach of restricting the solution set with a proximal operator to ensure strict monotonicity of the objective sequence is presented in [R1, Section 4] under restrictive mapping title. Therefore, even if we use the discussions from reference [R1], we can only claim that the sequence of iterates $\{\mathbf{x}^t\}$ converges to a continuum of accumulation points, and every limit point is a stationary point.

We have revised Appendix A-D titled *"Stationarity of Limit Points"* to discuss the convergence of the sequence of iterates. We have revised our statement as every limit point of the sequence of iterates generated by the centralized algorithm is a stationary point.

R1. R.R. Meyer, “Sufficient Conditions for the Convergence of Monotonic Mathematical Programming Algorithms,” *Journal of Computer and System Sciences*, vol. 12, no. 1, pp. 108-121, 1976

R2. J. Tropp, I. Dhillon, R. Heath, and T. Strohmer, “Designing structured tight frames via an alternating projection method,” *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 188209, Jan. 2005.

Response to fourth reviewer comments

This reviewer's concerns have been addressed, and this manuscript is now deemed fit for publication.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.