

# Traffic Aware Precoder Design for Space Frequency Resource Allocation

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# Outline

## Introduction

## System Model & Problem Formulation

## Centralized Solutions

- Existing Q-WSRM Formulation

- JSFRA Formulation (SINR Relaxation)

- JSFRA Formulation (MSE Reformulation)

## Distributed Solutions

- Primal & ADMM based decompositions

- KKT based Distributed Solution

## Simulation Results

- SISO Example

- MIMO Examples

  - Centralized Solutions

  - Distributed Solutions

  - Effect of varying  $\ell_q$  norms

  - KKT Based Approach

- Time Correlated Fading Performance

## Conclusions

# Section 1

## Introduction

# Introduction and Motivation

- ▶ Complex algorithms are adopted to maximize the throughput to satisfy the data requirements of the higher layers
- ▶ Available wireless resources are to be utilized efficiently to minimize the backlogged packets
- ▶ Spatial and Frequency resources are exploited to empty the packets waiting at the BSs
- ▶ In this work, we discuss the precoder design for the multiple users MIMO-OFDM setup to minimize the number of queued packets

## Section 2

# System Model & Problem Formulation

# Symbols used

- ▶ OFDM system with  $N$  sub-channels and  $N_B$  base stations (BSs), each equipped with  $N_T$  transmit antennas
- ▶ Let  $K$  be the total number of users with  $N_R$  antennas
- ▶ Let  $\mathcal{B}$  and  $\mathcal{U}$  denote the set of coordinating BSs and users in the system
- ▶ The set of users belonging to BS  $b$  is denoted by  $\mathcal{U}_b \in \mathcal{U}$
- ▶ Let  $b_k \in \mathcal{B}$  denotes the BS serving the user  $k$
- ▶ Let  $L$  be the total available spatial streams for a user  $k$ , given by  $\min(N_T, N_R)$

# System Model

- The  $l$ th spatial signal received on sub-channel  $n$  of user  $k$  is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^H \mathbf{n}_{l,k,n} + \mathbf{w}_{l,k,n}^H \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^L \mathbf{m}_{j,i,n} d_{j,i,n} \quad (1)$$

- where  $\mathbf{m}_{l,k,n}$  and  $\mathbf{w}_{l,k,n}$  are transmit and receive beamformers corresponding to the  $l$ th spatial stream on the  $n$ th sub-channel of user  $k$

# System Model

- ▶  $\mathbf{H}_{b_k,k,n} \in \mathbb{C}^{N_R \times N_T}$  denotes the channel between BS  $b_k$  and user  $k$
- ▶  $d_{l,k,n}$  and  $n_{l,k,n}$  correspond to data symbol and equivalent noise on  $l$ th spatial stream of user  $k$
- ▶ Using the above notations, the SINR seen by the  $l$ th spatial stream on the  $n$ th sub-channel for user  $k$  is given by

$$\gamma_{l,k,n} = \frac{\left| \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2}{\hat{N}_0 + \sum_{(j,i) \neq (l,k)} \left| \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n} \right|^2} \quad (2)$$

- ▶ where  $\hat{N}_0 = \|\mathbf{w}_{l,k,n}^H \mathbf{n}_{l,k,n}\|^2$



# Queueing Model

- ▶ Each user is associated with backlogged packets of size  $Q_k$  packets.
- ▶ Queued packets  $Q_k$  of each user follows dynamic equation at the  $i$ th instant as

$$Q_k(i+1) = \left[ Q_k(i) - t_k(i) \right]^+ + \lambda_k(i) \quad (3)$$

- ▶ where  $t_k = \sum_{n=1}^N \sum_{l=1}^L t_{l,k,n}$  denotes the total number of transmitted packets corresponding to user  $k$  in the previous  $i$ th instant
- ▶  $\lambda_k$  represents the fresh arrivals of user  $k$  at BS  $b_k$

# Problem Formulation

- ▶ Objective is to transmit the queued packets waiting at BSs to corresponding users in the system.
- ▶ The available spatial and frequency resources are to be efficiently utilized to minimize the queued packets
- ▶ In order to achieve this, precoders are to be designed with certain objective that involves the backlogged packets as well
- ▶ Precoders can perform scheduling of users by providing zero powers to exclude from the resource elements

## Section 3

### Centralized Solutions

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- ▶ Q-WSRM formulation is the result of minimizing the conditional Lyapunov drift <sup>1</sup>
- ▶ Q-WSRM formulation is also called as back pressure algorithm, since it acts greedily in minimizing the backlogged packets at each instant

$$\underset{t_{l,k,n}}{\text{minimize}} \quad \sum_{k \in \mathcal{U}} \{Q_k(i)^2 - Q_k(i-1)^2\},$$

- ▶ where  $Q_k$  follows the dynamic Queue expression in (3) and  $t_k = \sum_{n=1}^N \sum_{l=1}^L t_{l,k,n}$

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Upon solving the Lyapunov drift expression, we obtain the Q-WSRM formulation as

$$\underset{t_{l,k,n}}{\text{maximize}} \quad \sum_{k \in \mathcal{U}} Q_k \left( \sum_{n=1}^N \sum_{l=1}^L t_{l,k,n} \right) \quad (4a)$$

$$\sum_{n=1}^N \sum_{l=1}^L t_{l,k,n} \leq Q_k / Q_{k,n} \quad (4b)$$

- Performs better in cell-edge scenarios
- Complexity can be reduced if precoders are designed for each sub-channel independently

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Complexity can be reduced if precoders are designed for each sub-channel independently
- Coupling across sub-channels is obtained by the queues, which are updated after evaluating the rate from previously chosen sub-channels as

$$Q_{k,n} = \max \left\{ Q_k - \sum_{j=1}^{n-1} \sum_{l=1}^L t_{l,k,j}, 0 \right\}, \forall k \in \mathcal{U}$$

# JSFRA Formulation (SINR Relaxation)

- ▶ Precoders are designed by a centralized controller, which are then used by all BSs in  $\mathcal{B}$
- ▶ The objective used to design transmit precoders is

$$v_k = \left| Q_k - \sum_{n=1}^N \sum_{l=1}^L t_{l,k,n} \right|^q$$

- ▶ To generalize the objective, we use  $\tilde{v}_k \triangleq a_k v_k$ , where  $a_k$  is arbitrary weights used control the priorities
- ▶ Exponent  $q$  plays different role based on the value it assumes
  - ▶  $q=1$  results in greedy allocation
  - ▶  $q=2$  ideal for the delay or buffer size limited scenarios
  - ▶  $q=\infty$  provides fair resource allocation in each transmission instant

# JSFRA Formulation (SINR Relaxation)

- Now, the precoder design problem is given as

$$\underset{t_{l,k,n}, \mathbf{M}_{k,n}, \mathbf{W}_{k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \quad (5a)$$

$$\text{subject to} \quad t_{l,k,n} \leq \log_2(1 + \gamma_{l,k,n}) \quad (5b)$$

$$\gamma_{l,k,n} \leq \frac{\left| \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2}{\beta_{l,k,n}} \triangleq f(\tilde{\mathbf{u}}_{l,k,n}), \quad (5c)$$

$$\beta_{l,k,n} \geq \dot{N}_0 + \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^H \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2, \quad (5d)$$

$$\sum_{n=1}^N \sum_{k \in \mathcal{U}_b} \text{tr}(\mathbf{M}_{k,n} \mathbf{M}_{k,n}^H) \leq P_{\max}, \forall b \quad (5e)$$

- where  $\tilde{\mathbf{u}}_{l,k,n} \triangleq \{\mathbf{w}_{l,k,n}^H, \mathbf{H}_{b_k,k,n}, \mathbf{m}_{l,k,n}, \beta_{l,k,n}\}$



# JSFRA Formulation (SINR Relaxation)

- ▶ To maximize the received SINR,  $\mathbf{w}_{l,k,n}$  are modeled with the MMSE receivers
- ▶ The JSFRA formulation in (5) is nonconvex due to the constraint

defined by (5c) as  $\gamma_{l,k,n} \leq \frac{\left| \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2}{\beta_{l,k,n}}$

- ▶ In order to solve the problem in (5), we use successive convex approximation (SCA) approach for the constraint defined by (5c)
- ▶ Let,  $f(\tilde{\mathbf{u}}_{l,k,n}) = (p_{l,k,n}^2 + q_{l,k,n}^2)/\beta_{l,k,n}$ , where

$$p_{l,k,n} \triangleq \Re \left\{ \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right\}, \quad (6a)$$

$$q_{l,k,n} \triangleq \Im \left\{ \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right\} \quad (6b)$$

## JSFRA Formulation (SINR Relaxation)

- ▶ Now, the first order Taylor approximation is used for the function  $f(\tilde{\mathbf{u}}_{l,k,n})$  around an arbitrary point  $\tilde{\mathbf{u}}_{l,k,n}^{(i-1)}$
- ▶ With this approximation, the problem in (5) can be solved using the well known solvers for the optimal precoders and  $\tilde{\mathbf{u}}_{l,k,n}$
- ▶ Once the precoders  $\mathbf{m}_{l,k,n}$  are evaluated,  $\mathbf{w}_{l,k,n}$  are updated using the MMSE receivers
- ▶ The local point  $\tilde{\mathbf{u}}_{l,k,n}^{(i-1)}$  is updated with the current optimal point  $\tilde{\mathbf{u}}_{l,k,n}^{(i)}$
- ▶ With the updated precoders and  $\tilde{\mathbf{u}}_{l,k,n}^{(i)}$ , optimization is carried out in an iterative manner until convergence

# JSFRA Formulation (MSE Reformulation)

- In this approach, we utilize the relation between the MSE and the SINR as  $\epsilon_{l,k,n} = (1 + \gamma_{l,k,n})^{-1}$
- Equivalence is valid only when the receivers are designed with the mean squared error (MSE) objective, *i.e.*, using MMSE receivers

$$\mathbb{E}[(d_{l,k,n} - \hat{d}_{l,k,n})^2] = |1 - \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}|^2 + \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^H \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2 + \hat{N}_0 = \epsilon_{l,k,n} \quad (7)$$

# JSFRA Formulation (MSE Reformulation)

- Using the above reformulation, we can formulate the JSFRA problem as

$$\begin{array}{ll} \text{minimize} & \|\tilde{\mathbf{v}}'\|_q \\ & t_{l,k,n}, \mathbf{m}_{l,k,n}, \\ & \epsilon_{l,k,n}, \mathbf{w}_{l,k,n} \end{array} \quad (8a)$$

$$\text{subject to} \quad t_{l,k,n} \leq -\log_2(\epsilon_{l,k,n}) \quad (8b)$$

$$\begin{aligned} & \sum_{(j,i) \neq (l,k)} |\mathbf{w}_{l,k,n}^H \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2 + \dot{N}_0 \\ & + |1 - \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}|^2 \leq \epsilon_{l,k,n} \end{aligned} \quad (8c)$$

$$\sum_{n=1}^N \sum_{k \in \mathcal{U}_b} \sum_{l=1}^L \text{tr}(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^H) \leq P_{\max} \quad \forall b. \quad (8d)$$

# JSFRA Formulation (MSE Reformulation)

- ▶ The nonconvex constraint in (8b) is approximated by a sequence of convex constraints
- ▶ It is achieved by using SCA technique as earlier
- ▶ The iterative procedure is performed until convergence or for suitable number of iterations
- ▶ The above reformulation works only with the MMSE receiver

## Section 4

# Distributed Solutions

# Distributed Methods

- ▶ When the system size is small, centralized approach is viable if the channel remains constant for multiple transmission slots
- ▶ However, the overhead involved in the centralized design scales up significantly as the network size grows
- ▶ Distributed schemes based on primal decomposition or ADMM can be used to reduce the signaling requirements
- ▶ The overhead involved in the design of precoders are only scalar interference variables
- ▶ Only the convex approximated subproblem in each SCA step is performed via distributed approaches

# Primal Decomposition Method

- Precoder design is performed by a master-slave approach
- Interference created to the neighboring BS users are bounded by a scalar variable
- The interference thresholds are determined by the master problem, which is performed at each BS with the signaling exchange

$$\zeta_{l,k,n,b} \geq \sum_{i \in \mathcal{U}_b} \sum_{j=1}^L |\mathbf{w}_{l,k,n}^H \mathbf{H}_{b,k,n} \mathbf{m}_{j,i,n}|^2 \quad \forall b \in \bar{\mathcal{B}}_{b_k}. \quad (9)$$

- Each BS subproblem includes (9) as an additional interference constraint in the respective optimization problem



# ADMM based Decomposition Method

- ▶ The alternating directions method of multipliers (ADMM) is superior to other distributed schemes in terms of the convergence speed
- ▶ The ADMM includes an additional quadratic term in the objective as  $\|\zeta_b - \zeta_b^{(j)}\|^2$ , where  $\zeta_b^{(j)}$  is global consensus variable
- ▶ Unlike primal decomposition method,  $\zeta_{l,k,n,b}$  in (9) is treated as an optimization variable in ADMM
- ▶ The consensus variables are updated as

$$\zeta_{b_k}(b)^{(j+1)} = \zeta_b(b_k)^{(j+1)} = \frac{\zeta_b(b_k) + \zeta_{b_k}(b)}{2}. \quad (10)$$

- ▶ where  $\zeta_{b_k}(b)$  denotes the entries corresponding to BS  $b$  in BS  $b_k$

# KKT based Distributed Solution

- ▶ The decentralization methods considered so far involve considerable signaling exchanges via backhaul
- ▶ However, when the users are equipped with multiple receive antennas, the overhead requirement is significantly large
- ▶ Since the signaling requirements are large, the iterative algorithm should design efficient precoders in few number of iterations to reduce the backlogged packets
- ▶ In order to achieve that, we design an iterative procedure based on solving the Karush-Kuhn-Tucker (KKT) equations for the JSFRA problem via MSE reformulation
- ▶ Unlike the earlier schemes, we perform the group update of all the involved optimization variables to speed up the convergence of precoder design

# KKT based Distributed Solution

$$\mathbf{m}_{l,k,n}^{(i)} = \left( \sum_{x \in \mathcal{U}} \sum_{y=1}^L \alpha_{y,x,n}^{(i-1)} \mathbf{H}_{b_k,x,n}^H \mathbf{w}_{y,x,n}^{(i-1)} \mathbf{w}_{y,x,n}^{H(i-1)} \mathbf{H}_{b_k,x,n} + \delta_b \mathbf{I}_{N_T} \right)^{-1} \alpha_{l,k,n}^{(i-1)} \mathbf{H}_{b_k,k,n}^H \mathbf{w}_{l,k,n}^{(i-1)}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \left( \sum_{x \in \mathcal{U}} \sum_{y=1}^L \mathbf{H}_{b_x,k,n} \mathbf{m}_{y,x,n}^{(i)} \mathbf{m}_{y,x,n}^{H(i)} \mathbf{H}_{b_x,k,n}^H + N_0 \mathbf{I}_{N_R} \right)^{-1} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)} \quad (11a)$$

$$\epsilon_{l,k,n}^{(i)} = \left| 1 - \mathbf{w}_{l,k,n}^{H(i)} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)} \right|^2 + \sum_{(x,y) \neq (l,k)} \left| \mathbf{w}_{l,k,n}^{H(i)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n}^{(i)} \right|^2 + \dot{N}_0 \quad (11b)$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{(\epsilon_{l,k,n}^{(i)} - \epsilon_{l,k,n}^{(i-1)})}{\log(2) \epsilon_{l,k,n}^{(i-1)}} \quad (11c)$$

$$\sigma_{l,k,n}^{(i)} = \left[ \frac{a_k q}{\log(2)} \left( Q_k - \sum_{n=1}^N \sum_{l=1}^L t_{l,k,n}^{(i)} \right)^{(q-1)} \right]^+ \quad (11d)$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)} + \rho^{(i)} \left( \frac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}} - \alpha_{l,k,n}^{(i-1)} \right) \quad (11e)$$

## Section 5

### Simulation Results

# SISO Example

- Let us consider a simple example with  $N_T = N_R = 1$ ,  $N = 3$  sub-channels, and  $K = 3$  users

**Table:** Sub-channel-wise listing of channel gains and rate allocations

$Q_k$	$\mathbf{H}_{b_k,k}$			(A)			(B)			(C)		
	1	2	3	1	2	3	1	2	3	1	2	3
4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining packets ( $\chi$ )				3.92 bits			2.51 bits			5.89 bits		

- $\zeta = \sum_{k=1}^K [Q_k - t_k]^+$
- $\zeta = 5.89$  bits for Q-WSRM (C),  $\zeta = 3.92$  bits for Q-WSRME scheme (A) and,  $\zeta = 2.51$  bits for JSFRA scheme (B)

# Performance Comparison with Number of Residual Packets

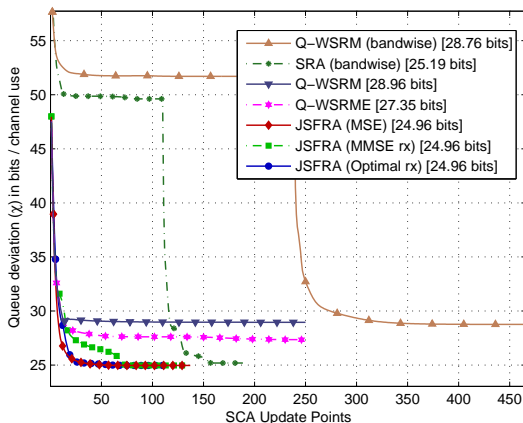


Figure: System Model -  $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

# Performance Comparison with Number of Residual Packets

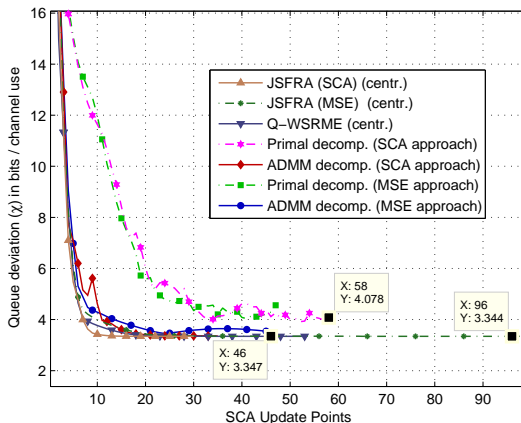


Figure: System Model -  $\{N, N_B, K, N_T, N_R\} = \{3, 2, 8, 4, 1\}$

# Effect on Residual Packets with different $\ell_q$ norms

**Table:** Number of backlogged bits associated with each user for a system  $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$ .

$q$	user indices								$\chi$
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
$\infty$	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
$Q_k$	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	



# Performance Comparison with Number of Residual Packets

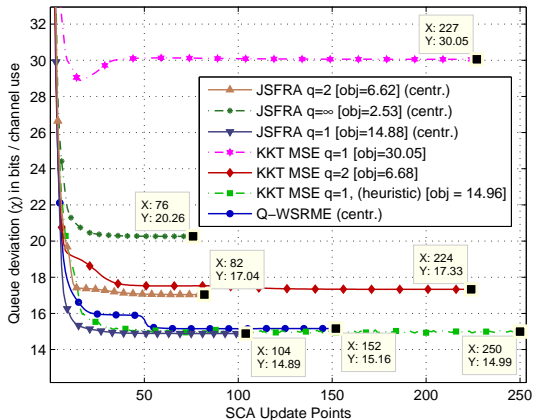


Figure: System Model -  $\{N, N_B, K, N_T, N_R\} = \{5, 2, 8, 4, 1\}$

# Average Residual Packets after each Transmission Slot

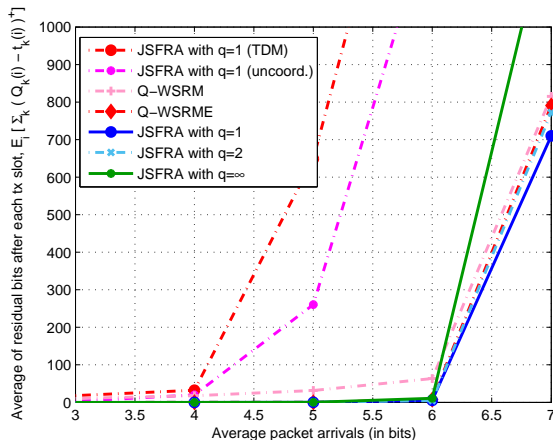


Figure: System Model -  $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$  after 250 transmissions

## Section 6

### Conclusions

# Conclusions

- ▶ We discussed the problem of wireless resource allocation to minimize backlogged packets in an efficient way
- ▶ The proposed approach uses SCA method by using linear approximation for the nonconvex constraint
- ▶ We also addressed different distributed methods for the precoder design across each BSs with minimal information exchange
- ▶ An iterative algorithm for the JSFRA scheme using MSE reformulation is also studied

# Questions !