Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems

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***Abstract*—We consider a downlink multi-cell multiple-input multiple-output (MIMO) interference broadcast channel (IBC) scenario using orthogonal frequency division multiplexing (OFDM) with multiple-user contending for space-frequency resources in a given scheduling instant. The problem is to determine the transmit precoders by the base stations (BSs) in a coordinated approach to minimize the total number of backlogged packets in the BSs, which are destined for the users in the system. Traditionally, it is solved using weighted sum rate maximization (WSRM) objective with the number of backlogged packets as the corresponding weights, *i.e*, longer the queue size, higher the priority. In contrast, we design the precoders jointly across the space-frequency resources by minimizing the total user queue deviations. The problem is nonconvex and therefore we employ successive convex approximation (SCA) technique to solve the problem by a sequence of convex subproblems using first order Taylor approximations. At first, we propose a centralized joint space-frequency resource allocation (JSFRA) solution using two different formulations by employing SCA technique, namely the sum rate formulation and the mean squared error (MSE) reformulation. We then introduce distributed precoder designs using primal and alternating directions method of multipliers method for the JSFRA solutions. Finally, we propose a practical distributed iterative precoder design based on MSE reformulation approach by solving the Karush-Kuhn-Tucker conditions with closed form expressions. Numerical results are used to compare the proposed algorithms with the existing solutions.**

***Index Terms*—Convex approximations, MIMO-IBC, MIMO-OFDM, Precoder design, SCA, WSRM.**

I. Introduction

In a network with multiple base stations (BSs) serving multiple-users (MUs), the main driving factor for the transmission are the packets waiting at each BS corresponding to the different users present in the network. These available packets are transmitted over the shared wireless resources subject to certain system limitations and constraints. We consider the problem of transmit precoder design over the space-frequency resources provided by the multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) framework in the downlink interference broadcast channel (IBC) to minimize the number of queued packets. Since the space-frequency resources are shared by multiple users associated with different BSs, it can be viewed as a resource allocation problem.

In general, the resource allocation problems are formulated by assigning a binary variable for each user to indicate the presence or the absence in a particular resource [1]. **In contrast, the linear transmit precoders, which are complex vectors, are implicitly used as decision variables, thereby avoiding the explicit modeling of binary decision variables. It solves two purposes. First, the formulation determines the transmission rate on a certain resource, and, secondly, by making the transmit beamformer of a particular user to be a zero vector, the corresponding user will not be scheduled on a certain resource. In this way, the soft decisions are used in the optimization problem and the hard decisions are made after the algorithm convergence**.

The queue minimizing precoder designs are closely related to the weighted sum rate maximization (WSRM) problem with additional rate constraints determined by the number of backlogged packets for each user in the system. The topics on MIMO IBC precoder design have been studied extensively with different performance criteria in the literature. Due to the nonconvex nature of the MIMO IBC precoder design problems, the successive convex approximation (SCA) method has become a powerful tool to deal with these problems [2]. For example, in [3], the nonconvex part of the objective has been linearized around an operating point in order to solve the WSRM problem in an iterative manner. Similar approach of solving the WSRM problem by using arithmetic-geometric inequality has been proposed in [4].

The connection between the achievable capacity and the mean squared error (MSE) for the received symbol by using the fixed minimum mean squared error (MMSE) receivers as shown in [5], [6] can also be used to solve the WSRM problem. In [6], [7], the WSRM problem is reformulated via MSE, casting the problem as a convex one for fixed linearization coefficients. In this way, the original problem is expressed in terms of the MSE weight, precoders, and decoders. Then the problem is solved using an alternating optimization method, i.e., finding a subset of variables while the remaining others are fixed. The MSE reformulation for the WSRM problem has also been studied in [8] by using the SCA to solve the problem in an iterative manner. Additional rate constraints based on the quality of service (QoS) requirements were included in the WSRM problem and solved via MSE reformulation in [9], [10].

The problem of precoder design for the MIMO IBC system are solved either by using a centralized controller or

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by using decentralized algorithms where each BS handles the corresponding subproblem independently with the limited information exchange with the other BSs via back-haul. The distributed approaches are based on primal, dual or alternating directions method of multipliers (ADMM) decomposition, which has been discussed in [11], [12]. In the primal decomposition, the so-called coupling interference variables are fixed for the subproblem at each BS to find the optimal precoders. The fixed interference are then updated by using the subgradient method as discussed in [13]. The dual and ADMM approaches control the distributed subproblems by fixing the *'interference price'* for each BS as detailed in [14].

By adjusting the weights in the WSRM objective properly, we can find an arbitrary rate-tuple in the rate region that maximizes the suitable objective measures. For example, if the weight of each user is set to be inversely proportional to its average data rate, the corresponding problem guarantees fairness on an average among the users. As an approximation, we may assign weights based on the current queue size of the users. More specifically, the queue states can be incorporated to traditional weighted sum rate objective $\sum_k w_k R_k$ by replacing the weight w_k with the corresponding queue state Q_k or its function, which is the outcome of minimizing the Lyapunov drift between the current and the future queue states [15], [16]. In backpressure algorithm, the differential queues between the source and the destination nodes are used as the weights scaling the transmission rate [17].

Earlier studies on the queue minimization problem were summarized in the survey paper [18], [19]. In particular, the problem of power allocation to minimize the number of backlogged packets was considered in [20] using geometric programming. Since the problem addressed in [20] assumed single antenna transmitters and receivers, the queue minimizing problem reduces to the optimal power allocation problem. In the context of wireless networks, the backpressure algorithm mentioned above was extended in [21] by formulating the corresponding user queues as the weights in the WSRM problem. Recently, the precoder design for the video transmission over MIMO system is considered in [22]. In this design, the MU-MIMO precoders are designed by the MSE reformulation as in [6] with the higher layer performance objective such as playback interruptions and buffer overflow probabilities.

Main Contributions: **In this paper, we design the precoders jointly across space-frequency resources by minimizing the total number of backlogged packets waiting at the BSs. The proposed design provides better control over the resource allocation strategy by the change of a variable. Since the transmissions are guided by the backlogged packets, the proposed formulation limits the resource allocation beyond the number of backlogged packets without additional rate constraint. Since the problem is nonconvex due to difference of convex (DC) constraints, we adopt SCA to solve by a sequence of convex subproblems using first order approximations.** Initially, we propose centralized joint space-frequency resource allocation (JSFRA) algorithms, which employs SCA for the nonconvex DC constraint. First method is by using the direct formulation and the second one is by using the MSE equivalence with the rate expression to solve for an optimal precoders. Then we propose distributed precoder designs based on the primal and the ADMM methods. Finally, we propose a iterative practical algorithm to decouple the precoder design across the coordinating BSs with limited information exchange by solving the Karush-Kuhn-Tucker (KKT) conditions for the MSE reformulation solution. **It is worth noting that the joint space-frequency channel matrix can be formed by stacking the channel of each sub-channel in a block-diagonal form for all users**.

The paper is organized as follows. In Section II, we introduce the system model and the problem formulation for the queue minimizing precoder design. The existing and the proposed centralized precoder designs are presented in Section III. The distributed solutions are provided in Section IV followed by the simulation results in Section V. Conclusions are drawn in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a downlink MIMO IBC scenario in an OFDM framework with N sub-channels and N_B BSs each equipped with N_T transmit antennas, serving in total K users each with N_R receive antennas. The set of users associated with BS b is denoted by $\mathcal{U}_b$ and the set $\mathcal{U}$ represents all users in the system, i.e., $\mathcal{U} = \underset{b\in\mathcal{B}}{\cup} \mathcal{U}_b$, where $\mathcal{B}$ is the set of indices of all coordinating BSs. Data for user k is transmitted from only one BS which is denoted by $b_k \in \mathcal{B}$. We denote by $\mathcal{N} = \{1, 2, \ldots, N\}$ the set of all sub-channel indices available in the system.

We adopt linear transmit beamforming technique at BSs. Specifically, the data symbols $d_{l,k,n}$ for user k on the l^{th} spatial stream over the sub-channel n is multiplied with beamformer $\mathbf{m}_{l,k,n} \in \mathbb{C}^{N_T\times 1}$ before being transmitted. In order to detect multiple spatial streams at the user terminal, receive beamforming vector $\mathbf{w}_{l,k,n}$ is employed for each user. Consequently, the received data symbol estimate corresponding to the l^{th} spatial stream over sub-channel n at user k is given by

$$\hat{d}_{l,k,n} = \mathbf{w}^{\rm H}_{l,k,n} \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}^{\rm H}_{l,k,n} \mathbf{n}_{k,n} + \mathbf{w}^{\rm H}_{l,k,n} \sum_{i\in\mathcal{U}\setminus\{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n}, \quad (1)$$

where $\mathbf{H}_{b,k,n} \in \mathbb{C}^{N_R\times N_T}$ is the channel between BS b and user k on sub-channel n, and $\mathbf{n}_{k,n} \sim \mathcal{CN}(0, N_0)$ is the additive noise vector for the user k on the n^{th} sub-channel and l^{th} spatial stream. In (1), $L = \text{rank}(\mathbf{H}_{b,k,n}) = \min(N_T, N_R)$ is the maximum number of spatial streams[1]. Assuming independent detection of data streams, we can write the signal-to-interference-plus-noise ratio (SINR) as

$$\gamma_{l,k,n} = \frac{\left|\mathbf{w}^{\rm H}_{l,k,n}\, \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n}\right|^2}{\tilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}^{\rm H}_{l,k,n} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2}, \quad (2)$$

[1]It can be easily extended for user specific streams L_k instead of using common L streams for all users. L streams are initialized but after solving the problem, only $L_{k,n} \leq L$ non-zero data streams are transmitted

where $\widetilde{N}_0 = N_0 \operatorname{tr}(\mathbf{w}_{l,k,n}\mathbf{w}_{l,k,n}^{\mathrm{H}})$ denotes the equivalent noise variance. **To reduce the overhead involved in feeding back the user channels, we consider a time division duplexing (TDD) system, which uses channel reciprocity.**

Let Q_k be the number of backlogged packets destined for the user k at a given scheduling instant. The queue dynamics of the user k are modeled using the Poisson arrival process with the average number of packet arrivals of $A_k = \mathbf{E}_i\{\lambda_k\}$ packets/bits, where $\lambda_k(i) \sim \operatorname{Pois}(A_k)$ represents the instantaneous number of packets arriving for the user k at the i^{th} time instant[2]. The total number of queued packets at the $(i+1)^{\text{th}}$ instant for the user k, denoted as $Q_k(i+1)$, is given by

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i)\Big]^+ + \lambda_k(i), \tag{3}$$

where $[x]^+ \equiv \max\{x, 0\}$ and t_k denotes the number of transmitted packets or bits for user k. At the i^{th} instant, transmission rate of the user k is given by

$$t_k(i) = \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}(i), \tag{4}$$

where $t_{l,k,n}$ denotes the number of transmitted packets or bits over l^{th} spatial stream on the n^{th} sub-channel. The maximum rate achieved over the (l,n) space-frequency resource is given by $t_{l,k,n} \le \log_2(1+\gamma_{l,k,n})$ for the signal-to-interference-plus-noise ratio (SINR) of $\gamma_{l,k,n}$[3]. Note that the units of t_k and Q_k are in bits defined per channel use.

B. Problem Formulation

To minimize the total number of backlogged packets, we consider minimizing the weighted ℓ_q-norm of the queue deviation given by

$$v_k = Q_k - t_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L}\log_2(1+\gamma_{l,k,n}), \tag{5}$$

where $\gamma_{l,k,n}$ is given by (2) and the optimization variables are the transmit precoders $\mathbf{m}_{l,k,n}$ and the receive beamformers $\mathbf{w}_{l,k,n}$.

Explicitly, the objective of the problem considered is given by $\sum_{k\in\mathcal{U}} a_k |v_k|^q$. With this objective function, the weighted queued packet minimization formulation is given by

$$\underset{\mathbf{m}_{l,k,n},\mathbf{w}_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{5a}$$

$$\text{subject to} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \le P_{\max}, \forall b, \tag{5b}$$

where $\tilde{v}_k \triangleq a_k^{1/q} v_k$ is the element of vector $\tilde{\mathbf{v}}$, and a_k is the weighting factor which is incorporated to control user priority based on their respective QoS. In (5b), BS specific sum power constraint for all sub-channels is considered.

For practical reasons, we may impose a constraint that the maximum number of transmitted bits for the user k is limited by the total number of backlogged packets available at the transmitter. As a result, the number of backlogged packets v_k for user k remaining in the system is given by

$$v_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L}\log_2(1+\gamma_{l,k,n}) \ge 0. \tag{6}$$

The above positivity constraint need to be satisfied by v_k to avoid the excessive allocation of the resources.

Before proceeding further, we show that the constraint in (6) is handled implicitly by the definition of norm ℓ_q in the objective of (6). Suppose that $t_k > Q_k$ for certain k at the optimum, i.e., $-v_k = t_k - Q_k > 0$. Then there exists $\delta_k > 0$ such that $-v'_k = t'_k - Q_k < -v_k$ where $t'_k = t_k - \delta_k$. Since $\|\tilde{\mathbf{v}}\|_q = \||\tilde{\mathbf{v}}|\|_q = \||-\tilde{\mathbf{v}}|\|_q$, this means that the newly created vector $\mathbf{t}'$ achieves a smaller objective which contradicts with the fact that the optimal solution has been obtained. The choice of the norm ℓ_q used in the objective function [18], [20] alters the priorities for the queue deviation function as follows

- ℓ_1 results in greedy allocation *i.e*, emptying the queue of users with good channel conditions before considering the users with worse channel conditions. As a special case, it is easy to see that (6) reduces to the WSRM problem when the queue size is large enough for all users.
- ℓ_2 prioritizes users with higher number of queued packets before considering the users with a smaller number of backlogged packets. For example, it could be more ideal for the delay limited scenario when the packet arrival rates of the users are similar, since the number of backlogged packets is proportional to the delay in the transmission following the Little's law [16].
- ℓ_∞ minimizes the maximum number of queued packets among users with the current transmission, thereby providing queue fairness by allocating the resources proportional to the number of backlogged packets.

III. Proposed Queue Minimizing Precoder Designs

In general, the precoder design for the MIMO OFDM problem is difficult due to its nonconvex nature. In addition, the objective of minimizing the number of the queued packets over space-frequency dimensions adds further complexity. Since the scheduling of users in each sub-channel attained by allocating zero transmit power over certain sub-channels, our solutions perform joint precoder design and user scheduling. Before discussing the proposed solutions, we consider the existing algorithm to minimize the number of backlogged packets with additional constraints required by the problem.

A. Queue Weighted Sum Rate Maximization (Q-WSRM) Formulation

The queue minimizing algorithms are discussed extensively in the networking literature to provide congestion-free routing between any two nodes in the network. One such algorithm is the *backpressure algorithm* [15]–[17]. It determines an optimal control policy in the form of rate or resource allocation for the nodes in the network by considering the differential backlogged packets between the source and the destination nodes. Even though the algorithm is primarily designed for

[2]The unit can either be packets or bits as long as the arrival and the transmission units are similar

[3]Upper bound is achieved by using Gaussian signaling

the wired infrastructure, it can be extended to the wireless networks by designing the user rate variable t_k in accordance to the wireless network.

The queue weighted sum rate maximization (Q-WSRM) formulation extends the *backpressure algorithm* to the downlink MIMO-OFDM framework, in which the multiple BSs act as the source nodes and the user terminals as the receiver nodes. The control policy in the form of transmit precoders aims at minimizing the number of queued packets waiting in the BSs. In order to find the optimal strategy, we resort to the Lyapunov theory, which is predominantly used in the control theory to achieve system stability. Since at each time slot, the system is described by the channel conditions and the number of backlogged packets of each user, the Lyapunov function is used to provide a scalar measure, which grows large when the system moves toward the undesirable state. By following [16], the scalar measure for the queue stability is given by

$$\mathrm{L}\left[\mathbf{Q}(i)\right] = \frac{1}{2} \sum_{k \in \mathcal{U}} Q_k^2(i), \tag{7}$$

where $\mathbf{Q}(i) = [Q_1(i), Q_2(i), \ldots, Q_K(i)]^{\mathrm{T}}$ and $\frac{1}{2}$ is used for the convenience. It provides a scalar measure of congestion present in the system [16, Ch. 3].

To minimize the total number of backlogged packets for an instant i, the optimal transmission rate of all users are obtained by minimizing the Lyapunov function drift expressed as

$$\begin{aligned}\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] = \\ \frac{1}{2}\Big[\sum_{k \in \mathcal{U}} \Big([Q_k(i) - t_k(i)]^+ + \lambda_k(i)\Big)^2 - Q_k^2(i)\Big].\end{aligned} \tag{8}$$

In order to eliminate the nonlinear operator $[x]^+$, we bound the expression in (8) as

$$\leq \sum_{k \in \mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} + \sum_{k \in \mathcal{U}} Q_k(i) \left\{\lambda_k(i) - t_k(i)\right\}, \tag{9}$$

by using the following inequality

$$\left[\max(Q - t, 0) + \lambda)\right]^2 \leq Q^2 + t^2 + \lambda^2 + 2Q(\lambda - t). \tag{10}$$

The total number of backlogged packets at any given instant i is reduced by minimizing the conditional expectation of the Lyapunov drift expression (9) given the current number of queued packets $\boldsymbol{Q}(i)$ waiting in the system. The expectation is taken over all possible arrival and transmission rates of the users to obtain the optimal rate allocation strategy.

Now, the conditional Lyapunov drift, denoted by $\Delta(\boldsymbol{Q}(i))$, is given by the infimum over the transmission rate as

$$\inf_{\mathbf{t}} \quad \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}} \left\{\mathrm{L}\left[\mathbf{Q}(i+1)\right] - \mathrm{L}\left[\mathbf{Q}(i)\right] | \mathbf{Q}(i)\right\} \tag{10a}$$

$$\begin{aligned}\leq \; & \underbrace{\mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{ \sum_{k \in \mathcal{U}} \frac{\lambda_k^2(i) + t_k^2(i)}{2} | \mathbf{Q}(i)\Big\}}_{\leq B} + \sum_{k \in \mathcal{U}} Q_k(i) A_k(i) \\ & - \mathbb{E}_{\boldsymbol{\lambda},\mathbf{t}}\Big\{ \sum_{k \in \mathcal{U}} Q_k(i) t_k(i) | \mathbf{Q}(i)\Big\},\end{aligned} \tag{10b}$$

where the subscripts $\mathbf{t}$ and $\boldsymbol{\lambda}$ represents the vector formed by stacking the transmission and the arrival rate of all users in the system. Since the transmission and the arrival rates are bounded, the second order moments in the first term of (10b) can be bounded by a constant B without affecting the optimal solution of the problem [16]. The second term in (10b) follows from the Poisson arrival process.

The expression in (11) looks similar to the WSRM formulation if the weights in the WSRM problem are replaced by the number of backlogged packets corresponding to the users. The above discussed approach is extended for the wireless networks in [21], in which the queues are used as weights in the WSRM formulation to determine the transmit precoders. Since the expectation is minimized by minimizing the function inside, the Q-WSRM formulation is given by

$$\underset{\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}}{\text{maximize}} \quad \sum_{k \in \mathcal{U}} Q_k \left(\sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \right) \tag{10c}$$

$$\text{subject to.} \quad \sum_{n=1}^{N} \sum_{k \in \mathcal{U}_b} \sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n} \mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}, \forall b. \tag{10d}$$

In order to avoid the excessive allocation of the resources, we include an additional rate constraint $t_k \leq Q_k$ to address $[x]^+$ operation in (3). The rate constrained version of the Q-WSRM, denoted by Q-WSRM extended (Q-WSRME) problem for a cellular system, is given by with the additional constraint

$$\sum_{n=1}^{N} \sum_{l=1}^{L} \log_2(1 + \gamma_{l,k,n}) \leq Q_k, \forall k \in \mathcal{U}, \tag{11}$$

where the precoders are associated with $\gamma_{l,k,n}$ defined in (2). By using the number of queued packets as the weights, the resources can be allocated to the user with the more backlogged packets, which essentially does greedy allocation.

As a special case of the problem defined in (11), we can formulate the sum rate maximization problem by setting the weights in (10c) as unity, leading to the problem as in (11) with $Q_k = 1, \forall k \in \mathcal{U}$. This approach provides a greedy queue minimizing allocation as compared to Q-WSRME, since the resource allocation is driven by the channel conditions in comparison to the number of queued packets as in Q-WSRME. Note that in both formulations, the resources allocated to the users are limited by the number of backlogged packets with an explicit maximum rate constraint defined by (11).

B. JSFRA Scheme via SCA approach

The problem defined in (11) ignores the second order term arising from the Lyapunov drift minimization objective by limiting it to a constant value. **In fact, using $\ell_{q=2}$ in (5), we obtain the following objective**

$$\underset{t_k}{\textbf{minimize}} \sum_{k} v_k^2 = \underset{t_k}{\textbf{minimize}} \sum_{k} Q_k^2 - 2\, Q_k t_k + t_k^2, \tag{12}$$

which is equivalent to the objective defined in (11). Note that the equivalence can be seen either by discarding t_k^2 from (12) or when the total number of queued packets is significantly large for all users such that t_k^2 has no impact on the objective function.

By limiting t_k^2 with a constant value, the Q-WSRM formulation requires an explicit rate constraint (11) to avoid over-allocation of the available resources. In the proposed queue deviation formulation, the explicit rate constraint is not needed, since it is handled by the objective function (5) itself. It makes the problem simpler and allows us to employ efficient algorithms to distribute the precoder design problem across each BSs independently by exchanging minimal information exchange [12]. In contrast to the WSRM formulation, the JSFRA and the Q-WSRME problems include the sub-channels jointly to obtain an efficient allocation by identifying the optimal space-frequency resource for the users.

We now present an iterative algorithm to solve problem (6) by using alternating optimization technique in conjunction with successive convex approximation (SCA) [23]. To do this, first by using the SINR expression in (2), we equivalently reformulate problem (6) as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\\ \beta_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{12a}$$

$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}} \tag{12b}$$

$$\beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{12c}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}, \forall b. \tag{12d}$$

Since the equality of the SINR in (2) cannot be included as a constraint directly, we relaxed the SINR expression by using inequalities (12b) and (12c). Note that (12b) is an under estimator for SINR $\gamma_{l,k,n}$, and (12c) provides an upper bound for the total interference seen by user $k \in \mathcal{U}_b$, denoted by variable $\beta_{l,k,n}$. The constraints are tight when each BS objective is non zero at the optimal point, as discussed in Appendix A. Since the JSFRA formulation can be modeled as a WSRM problem, which is known to be NP-hard [24], it also belongs to the class of NP-hard problems.

In order to find a tractable solution for (13), we note that the constraints (12d) are convex with involved variables. Thus, we only need to deal with (12b) and (12c). Towards this end, we resort to the alternating optimization (AO) technique by fixing the linear receivers, and finding the optimal transmit beamformers. For a fixed receivers $\mathbf{w}_{l,k,n}$, the problem now is to find the optimal transmit beamformers $\mathbf{m}_{l,k,n}$ which is still a challenging task. We note that for a fixed $\mathbf{w}_{l,k,n}$, (12c) can be written as a second-order cone (SOC) constraint. Thus, the difficulty is due to the non-convexity of the DC constraint in (12b). **To arrive at a tractable formulation, we adopt SCA to handle (12b) by replacing the original non-convex constraint by a series of convex constraints [23].** Let us define a function,

$$f(\mathbf{u}_{l,k,n}) \triangleq \frac{|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{l,k,n}\mathbf{m}_{l,k,n}|^2}{\beta_{l,k,n}},$$

where $\mathbf{u}_{l,k,n} \triangleq \{\mathbf{w}_{l,k,n}, \mathbf{m}_{l,k,n}, \beta_{l,k,n}\}$ is the vector which needs to be identified for the optimal allocation. Note that the function $f(\mathbf{u}_{l,k,n})$ is convex for a fixed $\mathbf{w}_{l,k,n}$, since it is in fact the ratio between a quadratic form of $\mathbf{m}_{l,k,n}$ over an affine function of $\beta_{l,k,n}$ [25]. **Note that constraint (12b) is in DC form and to solve the problem with a DC constraint, it is solved iteratively by linearizing the convex function $f(\mathbf{u}_{l,k,n})$ with the first order Taylor approximation around a fixed operating point $\tilde{\mathbf{u}}_{l,k,n}$ as discussed in [26], [27].**

For this purpose, let the real and imaginary component of the complex number $\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}$ be represented by

$$p_{l,k,n} \triangleq \Re\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{13a}$$

$$q_{l,k,n} \triangleq \Im\left\{\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right\} \tag{13b}$$

and hence $f(\mathbf{u}_{l,k,n}) = (p_{l,k,n}^2 + q_{l,k,n}^2)/\beta_{l,k,n}$[4]. Suppose that the current value of $p_{l,k,n}$ and $q_{l,k,n}$ at a specific iteration are $\tilde{p}_{l,k,n}$ and $\tilde{q}_{l,k,n}$, respectively. Using first order Taylor approximation around the local point $[\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}, \tilde{\beta}_{l,k,n}]^T$, we can approximate (12b) by the following linear inequality

$$2\frac{\tilde{p}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(p_{l,k,n} - \tilde{p}_{l,k,n}\right) + 2\frac{\tilde{q}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\left(q_{l,k,n} - \tilde{q}_{l,k,n}\right) + \frac{\tilde{p}_{l,k,n}^2 + \tilde{q}_{l,k,n}^2}{\tilde{\beta}_{l,k,n}}\left(1 - \frac{\beta_{l,k,n} - \tilde{\beta}_{l,k,n}}{\tilde{\beta}_{l,k,n}}\right) \geq \gamma_{l,k,n}. \tag{14}$$

In summary, for the fixed linear receivers $\mathbf{w}_{l,k,n}$ and the operating point $[\tilde{p}_{l,k,n}, \tilde{q}_{l,k,n}, \tilde{\beta}_{l,k,n}]^T$, the relaxed convex subproblem to find transmit beamformers is given by

$$\underset{\substack{\mathbf{m}_{l,k,n},\\ \gamma_{l,k,n},\beta_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{14a}$$

$$\text{subject to} \quad \beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{14b}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}, \forall b, \tag{14c}$$

$$\text{and (14).} \tag{14d}$$

Now, the optimal linear receivers for the fixed transmit precoders $\mathbf{m}_{j,i,n}\,\forall i \in \mathcal{U}$, $\forall n \in \mathcal{C}$ are obtained by minimizing (6) with respect to $\mathbf{w}_{l,k,n}$ as

$$\underset{\substack{\gamma_{l,k,n},\\ \mathbf{w}_{l,k,n},\beta_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}\|_q \tag{14e}$$

$$\text{subject to} \quad \beta_{l,k,n} \geq \widetilde{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2 \tag{14f}$$

$$\text{and (14).} \tag{14g}$$

Solving (15) using the KKT conditions, we obtain the following iterative expression for the receiver $\mathbf{w}_{l,k,n}^*$ as

$$\mathbf{A}_{l,k,n} = \sum_{(j,i)\neq(l,k)} \mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \tag{14h}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \left(\frac{\tilde{\beta}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)}}{\|\mathbf{w}_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\|^2}\right)\mathbf{A}_{l,k,n}^{-1}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}, \tag{14i}$$

where $\mathbf{w}_{l,k,n}^{(i-1)}$ is the receive beamformer from the previous

[4]Note that $p_{l,k,n}$ and $q_{l,k,n}$ are just symbolic notation and not the newly introduced optimization variables. In CVX [28], for example, we declare $p_{l,k,n}$ and $q_{l,k,n}$ with the '*expression*' qualifier

iteration, upon which the linear relaxation is performed for the nonconvex constraint in (15). **The optimal receiver $\mathbf{w}^{*}_{l,k,n}$ is obtained by either iterating (14i) until convergence or for fixed number of iterations. Note that the receiver has no explicit relation with the choice of ℓ_q norm used in the objective function. The dependency is implicitly implied by the transmit precoders $\mathbf{m}_{l,k,n}$, which in deed depend on the q value.**

It can be seen that the optimal receiver in (14i) is in fact a scaled version of the MMSE receiver, which is given by

$$\mathbf{R}_{l,k,n} = \sum_{i\in\mathcal{U}}\sum_{j=1}^{L} \mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\mathbf{m}_{j,i,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}^{\mathrm{H}} + N_0\,\mathbf{I}_{N_R} \quad (14j)$$

$$\mathbf{w}_{l,k,n} = \mathbf{R}_{l,k,n}^{-1}\,\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}. \quad (14k)$$

Since the scaling present in the optimal receiver (14i) has no impact on the received SINRs, the MMSE receiver in (14k) can also be used without compromising the performance or the convergence behavior.

The proposed subproblems in (15) and (15) are solved in an iterative manner by updating the operating point from the previous iteration. The iterative algorithm is referred to as queue minimizing JSFRA scheme with a per BS power constraint, and it is outlined in Algorithm 1. The iterative procedure repeats until the improvement on the objective is less than a predetermined tolerance parameter or the maximum number of iterations is reached. Instead of initializing $\mathbf{u}_{l,k,n}$ arbitrarily to a feasible point, transmit precoders can also be initialized with some feasible point $\tilde{\mathbf{m}}_{l,k,n}$, which is then used to find $\mathbf{u}_{l,k,n}$ as briefed in Algorithm 1. For a fixed receive beamformer $\mathbf{w}_{l,k,n}$, the SCA iteration is carried out until convergence or for the predefined iterations, say, $J_{\max}$ for the optimal transmit precoders $\mathbf{m}_{l,k,n}$. Next, the receive beamformers are updated based on either (14i) or (14k) using the fixed transmit precoders $\mathbf{m}_{l,k,n}$. This procedure is carried out until convergence of the queue deviation or for fixed number of iterations by $I_{\max}$ as outlined in Algorithm 1. The convergence proof is discussed in Appendix B

C. JSFRA Scheme via MSE Reformulation

In this, we solve the JSFRA problem by exploiting the equivalence between the MSE and the achievable sum rate for the receivers designed based on the MMSE criterion [5], [6]. The MSE $\epsilon_{l,k,n}$, for a data symbol $d_{l,k,n}$ is given by

$$\mathbb{E}\big[(d_{l,k,n} - \hat{d}_{l,k,n})^2\big] = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2 + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2 + \tilde{N}_0 = \epsilon_{l,k,n}, \quad (15)$$

where $\hat{d}_{l,k,n}$ is the estimate of the transmitted symbol. **Using the MMSE receive beamformer (14k) in the MSE expression (15) and in the SINR expression (2), we can arrive at the following relation between the MSE and the SINR as**

$$\epsilon_{l,k,n} = (1+\gamma_{l,k,n})^{-1}. \quad (16)$$

The above equivalence is valid only if the receivers are based on the MMSE criterion. Using the equivalence in (16), the WSRM objective can be reformulated as the weighted minimum mean squared error (WMMSE) equivalent to obtain the precoders for the MU-MIMO scenario as discussed in [6]–[8]. **Note that the receiver is invariably based on the MMSE criterion irrespective of the ℓ_q norm used in the objective function to obtain the optimal transmit precoders $\mathbf{m}_{l,k,n}$.**

Let $v'_k = Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}$ denote the queue deviation corresponding to user k and $\tilde{v}'_k \triangleq a_k^{1/q} v'_k$ represents the weighted equivalent. By using the relaxed MSE expression in (15), the problem in (6) can be expressed as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \quad (16a)$$

$$\text{subject to } t_{l,k,n} \leq -\log_2(\epsilon_{l,k,n}) \quad (16b)$$

$$\sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}\right|^2 + \tilde{N}_0 + \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2 \leq \epsilon_{l,k,n} \quad (16c)$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}, \forall b. \quad (16d)$$

The alternative MSE formulation given by (17) is nonconvex even for the fixed $\mathbf{w}_{l,k,n}$ due to the constraint (16b), which is in fact a DC constraint. We resort to the SCA approach [23] by relaxing the constraint by a sequence of convex subsets using first order Taylor series approximation around a fixed MSE point $\tilde{\epsilon}_{l,k,n}$ as

$$-\log_2(\tilde{\epsilon}_{l,k,n}) - \frac{(\epsilon_{l,k,n} - \tilde{\epsilon}_{l,k,n})}{\log(2)\,\tilde{\epsilon}_{l,k,n}} \geq t_{l,k,n}, \quad (17)$$

Using the above approximation for the rate constraint, the problem defined in (17) is solved for optimal transmit precoders $\mathbf{m}_{l,k,n}$, MSEs $\epsilon_{l,k,n}$, and the user rates over each sub-channel $t_{l,n,k}$ for a fixed receive beamformers. The optimization subproblem to find the transmit precoders for a fixed

Algorithm 1: Algorithm of JSFRA scheme

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}, \forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l \in \{1,2,\ldots,L\}$
Initialize: $i = 0$ and transmit precoders $\tilde{\mathbf{m}}_{l,k,n}$ randomly satisfying the total power constraint (5b)
update $\mathbf{w}_{l,k,n}, \mathbf{u}_{l,k,n}$ using (14k) and (14) with $\mathbf{m}_{l,k,n}$
repeat
  initialize $j = 0$
  repeat
    solve for the transmit precoders $\mathbf{m}_{l,k,n}$ using (15)
    update the constraint set (14) with $\mathbf{u}_{l,k,n}$ and $\mathbf{m}_{l,k,n}$ using (13)
    $j = j + 1$
  until *SCA convergence or* $j \geq J_{\max}$
  update the receive beamformers $\mathbf{w}_{l,k,n}$ using (15) or (14k) with the updated precoders $\mathbf{m}_{l,k,n}$
  $i = i + 1$
until *Queue convergence or* $i \geq I_{\max}$

receive beamformers $\mathbf{w}_{l,k,n}$ is given by

$$\underset{t_{l,k,n},\mathbf{m}_{l,k,n},\epsilon_{l,k,n}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}'\|_q \tag{17a}$$

$$\text{subject to} \quad (16c), (16d), \text{ and } (17). \tag{17b}$$

The optimal transmit precoders for a fixed receivers are obtained by solving the subproblem (18) iteratively by updating the fixed MSE point $\tilde{\epsilon}_{l,k,n}$ with $\epsilon_{l,k,n}$ from the previous iteration until termination as discussed in Section III-B.

D. Reduced Complexity Resource Allocation

The complexity of the JSFRA algorithm scales with the number of sub-channels considered. In addition, the iterations required for the algorithm convergence increases with the problem size. To overcome the complexity, we can adopt the distributed approaches presented in [11], [12] for sub-channel wise precoder design using primal or dual decomposition.

As an alternative sub-optimal solution, we present queue minimizing spatial resource allocation (SRA), which designs the precoders independently across the sub-channels in a sequential manner using the unserviced number of packets after each sub-channel allocation. The BS transmit power is fixed across the sub-channel n as $P_{\max,n}$. The power sharing can be equal or based on predetermined pattern as in partial frequency reuse across the sub-channels. After fixing the power and the backlogged packets for a sub-channel i, precoders are determined using the JSFRA algorithm presented in Section III-B or III-C for each sub-channel in a sequential manner. Once the precoders and the corresponding rates are identified, the unserviced backlogged packets for the next sub-channel $i+1$ is updated for user k as

$$Q_{k,i+1} = \max\left\{Q_k - \sum_{j=1}^{i}\sum_{l=1}^{L} t_{l,k,j}, 0\right\}, \ \forall\, k \in \mathcal{U}, \tag{18}$$

where $t_{l,k,j}$ denotes the rate corresponding to the user k on the j^{th} sub-channel and l^{th} spatial stream and the initial values are given as $Q_{k,1} = Q_k, \forall k$. Note that the proposed scheme is sensitive to the order of the sub-channel selection due to the sequential precoder design for each sub-channel. However, the SRA approach provides faster convergence in contrast to the JSFRA formulation due to the substantial reduction in the optimization variables for each sub-channel problem. As the number of user increases, the SRA formulation will be insusceptible to the sub-channel ordering due to the channel hardening and the availability of users with all possible channel vectors.

IV. Distributed Solutions

The distributed precoder designs for the proposed JSFRA scheme are discussed in this section. The convex formulation in (15) or (18) requires a centralized controller to perform the precoder design for all users belonging to the coordinating BSs. In order to design the precoders independently at each BS with the minimal information exchange via backhaul, iterative decentralization methods are addressed. In particular, the primal decomposition and the ADMM based dual decomposition approaches are considered.

Let us consider the convex subproblem with the fixed receive beamformers $\mathbf{w}_{l,k,n}$ presented in (15) based on the Taylor series approximation for the nonconvex constraint. The following discussions are equally valid for the MSE based solution outlined in (18) as well. Since the objective of (15) can be decoupled across each BS, the centralized problem can be equivalently written as

$$\underset{\gamma_{l,k,n},\mathbf{m}_{l,k,n},\beta_{l,k,n}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}} \|\tilde{\mathbf{v}}_b\|_q \tag{18a}$$

$$\text{subject to} \quad (14b) - (14d), \tag{18b}$$

where $\tilde{\mathbf{v}}_b$ denotes the vector of weighted queue deviation corresponding to users $k \in \mathcal{U}_b$.

To begin with, let $\bar{\mathcal{B}}_b$ denote the set $\mathcal{B}\backslash\{b\}$ and $\bar{\mathcal{U}}_b$ represents the set $\mathcal{U}\backslash\mathcal{U}_b$. Following similar approaches presented in [13], [14], the coupling constraint (14b) or (16c) can be expressed by grouping the interference contribution from each BS in $\mathcal{B}$ as

$$\widetilde{N}_0 + \sum_{j=1,j\neq l}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,k,n}|^2 + \sum_{b\in\bar{\mathcal{B}}_{b_k}} \zeta_{l,k,n,b} + \sum_{i\in\mathcal{U}_{b_k}\backslash\{k\}}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n}, \tag{19}$$

where $\zeta_{l,k,n,b}$ is the total interference caused by the transmission of BS b to user $k \in \mathcal{U}_{b_k}$ in the spatial stream l and sub-channel n. It is given by the following upper bound as

$$\zeta_{l,k,n,b} \geq \sum_{i\in\mathcal{U}_b}\sum_{j=1}^{L} |\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_i,k,n}\mathbf{m}_{j,i,n}|^2, \forall b \in \bar{\mathcal{B}}_{b_k}. \tag{20}$$

The decentralization is achieved by decomposing the original convex problem in (19) by a parallel iterative subproblems coordinated by either primal or dual decomposition update. The coupling variables are updated in each iteration by exchanging limited information among the subproblems. Before proceeding further, let $\bar{\boldsymbol{\zeta}}_b$ be the vector formed by stacking interference terms (20) from the neighboring BSs to the users of BS b and $\hat{\boldsymbol{\zeta}}_b$ be the stacked interference terms caused by BS b to all users in the neighboring BSs $\bar{\mathcal{B}}_b$, represented as

$$\bar{\boldsymbol{\zeta}}_b = \left[\zeta_{l,k,n,\bar{\mathcal{B}}_b(1)}, \dots, \zeta_{l,k,n,\bar{\mathcal{B}}_b(|\bar{\mathcal{B}}_b|)}\right]^{\mathrm{T}}, \forall k \in \mathcal{U}_b, \tag{20a}$$

$$\hat{\boldsymbol{\zeta}}_b = \left[\zeta_{l,\bar{\mathcal{U}}_b(1),n,b}, \zeta_{l,\bar{\mathcal{U}}_b(2),n,b}, \dots, \zeta_{l,\bar{\mathcal{U}}_b(|\bar{\mathcal{U}}_b|),n,b}\right]^{\mathrm{T}}. \tag{20b}$$

Let us define the vector $\boldsymbol{\zeta}_b$, formed by stacking the interference terms corresponding to the BS b as

$$\boldsymbol{\zeta}_b = \left[\hat{\boldsymbol{\zeta}}_b^{\mathrm{T}}, \bar{\boldsymbol{\zeta}}_b^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{21}$$

Since the decentralization solution is an iterative procedure, we represent the i^{th} iteration index as $x^{(i)}$. Let $\boldsymbol{\zeta}_b(b_k)$

denote the interference terms corresponding to the BS b_k in BS b as

$$\boldsymbol{\zeta}_b(b_k) = \left[\zeta_{l,\mathcal{U}_b(1),n,b_k}, \ldots, \zeta_{l,\mathcal{U}_b(|\mathcal{U}_b|),n,b_k}\right]. \quad (22)$$

Now, to decouple the problem in (19), the BS specific vector $\boldsymbol{\zeta}_b$ in (21), which include all interference terms relevant for the transmission of BS b, can either be fixed or treated as a variable in each iteration in accordance to the decomposition method. Since the BS specific precoders are solved independently, we assume local channel information and queues are available at each BS b_k together with the cross channel knowledge $\mathbf{H}_{b_k,k,n}, \forall k \in \bar{\mathcal{U}}_{b_k}$ through uplink sounding and the receive beamformers $\mathbf{w}_{l,k,n}$.

A. Primal Decomposition

In primal decomposition, the convex problem in (19) is solved for the optimal transmit precoders in an iterative manner for a fixed BS specific interference terms $\boldsymbol{\zeta}_{b_k}$ using master-slave model [13]. The slave subproblem is solved in each BS for the optimal transmit precoders only for the associated users by assuming fixed interference terms $\boldsymbol{\zeta}_{b_k}^{(i)}$ in each i^{th} iteration. Upon finding the optimal associated transmit precoders by each slave subproblems, the master problem is used to update the BS specific interference terms $\boldsymbol{\zeta}_{b_k}^{(i+1)}$ for the next iteration by using dual variables corresponding to the interference constraint (19) as discussed in [13]. In this manner, the interference variables are updated until the global consensus is obtained. The primal approach is similar to the minimum power precoder design presented in [13]. Note that the master problem treats $\boldsymbol{\zeta}_b$ as a variable and the slave subproblems assumes it to be a constant for each iteration to find the transmit precoders.

B. Alternating Directions Method of Multipliers (ADMM)

In this section, we discuss the ADMM approach to decouple the precoder design across multiple BSs to solve the convex problem in (19). The ADMM is preferred over the dual decomposition (DD) approach in [14] for its robustness and improved convergence behavior [12]. In contrast to the primal decomposition, the ADMM approach relaxes the interference constraints by including in the objective function of each subproblem with a penalty pricing [11], [12]. Similar decomposition for the precoder design in the minimum power context is considered in [29].

Using the formulation presented in [12], [29], we can write the BS b specific ADMM subproblem for the i^{th} iteration as

$$\underset{\substack{\gamma_{l,k,n},\mathbf{m}_{l,k,n}\\ \beta_{l,k,n},\boldsymbol{\zeta}_b}}{\text{minimize}} \quad \|\tilde{\mathbf{v}}_b\|_q + \boldsymbol{\nu}_b^{(i)\,\mathrm{T}}\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i)}\right) + \frac{\rho}{2}\left\|\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i)}\right\|^2 \quad (22a)$$

$$\text{subject to} \quad \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max} \quad (22b)$$

$$\sum_{\bar{b}\in\bar{\mathcal{B}}_b}\zeta_{l,k,n,\bar{b}} + \sum_{\{\bar{l},\bar{k}\}\neq\{l,k\}}|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b,k,n}\mathbf{m}_{\bar{l},\bar{k},n}|^2 + \tilde{N}_0 \leq \beta_{l,k,n}, \quad (22c)$$

$$\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L}|\mathbf{w}_{\bar{l},\bar{k},n}^{\mathrm{H}}\mathbf{H}_{b,\bar{k},n}\mathbf{m}_{l,k,n}|^2 \leq \zeta_{\bar{l},\bar{k},n,b}, \forall \bar{k}\in\bar{\mathcal{U}}_b, \forall n \quad (22d)$$

$$\text{and (14),} \quad (22e)$$

where $\boldsymbol{\zeta}_b^{(i)}$ denotes the interference vector updated from the earlier iteration and $\boldsymbol{\nu}_b^{(i)}$ represents the dual vector corresponding to the equality constraint at the i^{th} iteration as

$$\boldsymbol{\zeta}_b = \boldsymbol{\zeta}_b^{(i)}. \quad (23)$$

Upon solving (23) for $\boldsymbol{\zeta}_b \forall b$ in the i^{th} iteration, the next iterate is updated by exchanging the corresponding interference terms between two BSs b and b_k as

$$\boldsymbol{\zeta}_{b_k}(b)^{(i+1)} = \boldsymbol{\zeta}_b(b_k)^{(i+1)} = \frac{\boldsymbol{\zeta}_b(b_k) + \boldsymbol{\zeta}_{b_k}(b)}{2}. \quad (24)$$

The dual vector for the next iteration is updated by using subgradient to maximize the dual objective as

$$\boldsymbol{\nu}_b^{(i+1)} = \boldsymbol{\nu}_b^{(i)} + \rho\left(\boldsymbol{\zeta}_b - \boldsymbol{\zeta}_b^{(i+1)}\right), \quad (25)$$

where the step size parameter ρ is chosen in accordance with [12]. The above iteration is performed until convergence or for certain accuracy in the variation of the objective value between two consecutive updates. The distributed precoder design using ADMM approach is shown in Algorithm 2.

Algorithm 2: Distributed JSFRA scheme using ADMM

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}$, $\forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l$
Initialize: $i = 0$ and $\mathbf{m}_{l,k,n}$ randomly satisfying total power constraint (22b)
update $\mathbf{w}_{l,k,n}$ with (14k) and $\tilde{\mathbf{u}}_{l,k,n}$ using (12c) and (13)
initialize the interference vectors $\boldsymbol{\zeta}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \forall b \in \mathcal{B}$
initialize the dual vectors $\boldsymbol{\nu}_b^{(0)} = \mathbf{0}^{\mathrm{T}}, \forall b \in \mathcal{B}$
foreach *BS* $b \in \mathcal{B}$ **do**
  repeat
    initialize $j = 0$
    repeat
      solve for $\mathbf{m}_{l,k,n}$ and $\boldsymbol{\zeta}_b$ with (23) using $\boldsymbol{\zeta}_b^{(j)}$
      exchange $\boldsymbol{\zeta}_b$ among BSs in $\mathcal{B}$
      update interference vector $\boldsymbol{\zeta}_b^{(j+1)}$ using (24)
      update dual variables in $\boldsymbol{\nu}_b^{(j+1)}$ using (25)
      $j = j + 1$
    until *convergence or* $j \geq J_{\max}$
    downlink precoded pilot transmission with $\mathbf{m}_{l,k,n}$
    update $\mathbf{w}_{l,k,n}$ and notify all BSs in $\mathcal{B}$ using uplink precoded pilots [30]
    update $\tilde{\mathbf{u}}_{l,k,n}$ using (12c) and (13) for SCA point or $\tilde{\epsilon}_{l,k,n}$ using (16c) for MSE operating point
    $i = i + 1$
  until *convergence or* $i \geq I_{\max}$
end

C. Convergence Analysis for Distributed Algorithms

The convergence of the distributed algorithm outlined in Algorithm 2 follows the same discussion in Appendix B if the subproblem (19) converge to the centralized solution. Since the subproblem (19) is convex, each BS specific

slave subproblem is also convex for a fixed interference vector $\zeta_{b_k}^{(i)}$ [11]. The master subproblem in the primal decomposition uses subgradient to update the coupling interference vectors in consensus with the objective function, it is guaranteed to converge to the centralized solution as the iteration $i \to \infty$ [2] for a diminishing step size. It can be seen that the subproblem (19) satisfies Slater's constraint qualification by having non empty interior and bounded due to the total power constraint for the transmit precoders.

To prove the convergence of the ADMM approach, we use the argument presented in [31] Proposition 4.2. If the problem is written as

$$\underset{\mathbf{x},\mathbf{z}}{\text{minimize}} \quad G(\mathbf{x}) + H(\mathbf{y}) \tag{25a}$$
$$\text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{z} \tag{25b}$$
$$\mathbf{x} \in \mathcal{C}_1, \mathbf{z} \in \mathcal{C}_2, \tag{25c}$$

following conditions are required for the convergence if ADMM is used.

- **G, H should be convex**
- **$\mathcal{C}_1, \mathcal{C}_2$ should be a convex set and bounded**
- **$\mathbf{A}^{\mathrm{H}}\mathbf{A}$ should be invertible.**

Note that the equality constraint (25b) is identical to (23) used in the ADMM subproblem (23). It is evident from the equality constraint (23) that $\mathbf{A} = \mathbf{I}$, which is an identity matrix and is invertible. The objective function G, H are ℓ_q norm in (19) are convex and the set defined by the constraints of the problem (19) are all convex sets and has a nonempty interior. The feasibility of the interior point is verified by having a non zero precoder for only one user. Therefore, by following [31] Proposition 4.2, it can be seen that the ADMM approach converges to the centralized solution as $i \to \infty$.

D. Decomposition via KKT Conditions for MSE Formulation

In this section, we discuss an alternative way to decentralize the precoder design across the coordinating BSs in $\mathcal{B}$ based on the MSE reformulation method discussed in Section III-C. In contrast to Section IV-A and IV-B, the problem is solved using the KKT conditions in which the transmit precoders, receive beamformers and the subgradient updates are performed at the same instant to minimize the global queue deviation objective with few number of iterations. The proposed methods in this section provide algorithms that can be of practical importance owing to the limited signaling requirements. **We consider a TDD system due to the unquantized channel state information at the transmitter**. Similar work has been considered for the WSRM problem with minimum rate constraints in [9], [10]. Since the formulation in [9], [10] are similar to the Q-WSRME scheme with an additional maximum rate constraint (11), it requires explicit dual variables to handle the maximum rate constraint, thereby making the problem difficult to solve in an iterative manner.

In the proposed JSFRA formulation, the maximum rate constraints are implicitly handled by the objective function without the need of explicit constraints. However, the KKT conditions cannot be formulated due to the non-differential objective function. The non-differentiability is due to the absolute value operator present in the norm function. In order to make the objective function differentiable, we consider the following two cases for which the absolute operator can be ignored without affecting the optimal solution, namely,

- when the exponent q is even, or
- when the number of backlogged packets of each user is large enough, *i.e*, $Q_k \gg \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}$ to ignore the absolute operator, which means also ignoring the queues in the first place as well.

With the assumption of either one of the above conditions to be true, the problem in (18) can be written as

$$\underset{\substack{t_{l,k,n},\mathbf{m}_{l,k,n},\\ \epsilon_{l,k,n},\mathbf{w}_{l,k,n}}}{\text{minimize}} \quad \sum_{b\in\mathcal{B}}\sum_{k\in\mathcal{U}_b} a_k \left(Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}\right)^q \tag{25d}$$

subject to

$$\alpha_{l,k,n} : \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}\right|^2 + \tilde{N}_0 + \sum_{(x,y)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}\right|^2 \leq \epsilon_{l,k,n} \tag{25e}$$

$$\sigma_{l,k,n} : \log_2(\tilde{\epsilon}_{l,k,n}) + \frac{(\epsilon_{l,k,n} - \tilde{\epsilon}_{l,k,n})}{\log(2)\tilde{\epsilon}_{l,k,n}} \leq -t_{l,k,n} \tag{25f}$$

$$\delta_b : \sum_{n=1}^{N}\sum_{k\in\mathcal{U}_b}\sum_{l=1}^{L} \operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right) \leq P_{\max}, \forall b, \tag{25g}$$

where $\alpha_{l,k,n}, \sigma_{l,k,n}$ and δ_b are the dual variables corresponding to the constraints defined in (25e), (25f) and (25g).

The problem in (26) is solved using the KKT expressions, which are obtained by the derivative of the Lagrangian function w.r.t the primal and the dual variables, complementary slackness conditions, and the primal, dual feasibility requirements as shown in Appendix C. Upon solving, we obtain the iterative solution as

$$\mathbf{m}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L} \alpha_{y,x,n}^{(i-1)}\mathbf{H}_{b_k,x,n}^{\mathrm{H}}\mathbf{w}_{y,x,n}^{(i-1)}\mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)}\mathbf{H}_{b_k,x,n} + \delta_b\mathbf{I}_{N_T}\Bigg)^{-1} \alpha_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)} \tag{25h}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \Bigg(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L} \mathbf{H}_{b_x,k,n}\mathbf{m}_{y,x,n}^{(i)}\mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_x,k,n}^{\mathrm{H}} + \mathbf{I}_{N_R}\Bigg)^{-1} \mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}^{(i)}. \tag{25i}$$

$$\epsilon_{l,k,n}^{(i)} = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_k,k,n}\mathbf{m}_{l,k,n}^{(i)}\right|^2 + N_0\left\|\mathbf{w}_{l,k,n}^{(i)}\right\|^2 + \sum_{(x,y)\neq(l,k)}\left|\mathbf{w}_{l,k,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_y,k,n}\mathbf{m}_{x,y,n}^{(i)}\right|^2 \tag{25j}$$

$$t_{l,k,n}^{(i)} = -\log_2(\epsilon_{l,k,n}^{(i-1)}) - \frac{\left(\epsilon_{l,k,n}^{(i)} - \epsilon_{l,k,n}^{(i-1)}\right)}{\log(2)\,\epsilon_{l,k,n}^{(i-1)}} \tag{25k}$$

$$\sigma_{l,k,n}^{(i)} = \left[\frac{a_k\,q}{\log(2)}\left(Q_k - \sum_{n=1}^{N}\sum_{l=1}^{L} t_{l,k,n}^{(i)}\right)^{(q-1)}\right]^{+} \tag{25l}$$

$$\alpha_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i-1)} + \rho\left(\frac{\sigma_{l,k,n}^{(i)}}{\epsilon_{l,k,n}^{(i)}} - \alpha_{l,k,n}^{(i-1)}\right) \tag{25m}$$

Since the dual variables $\alpha^{(i)}$ and $\sigma^{(i)}$ are interdependent in (26), one has to be fixed to optimize for the other. So, $\alpha^{(i)}$ is fixed to evaluate $\sigma^{(i)}$ using (26). **At each iteration, the dual variables $\alpha^{(i)}$ are linearly interpolated with any point between the fixed iterate $\alpha^{(i-1)}$ and $\frac{\sigma^{(i)}}{\epsilon^{(i)}}$ using a step size $\rho \in (0,1)$. The choice of ρ depends on the system model and it affects the convergence behavior of the algorithm. It is used to reduce the oscillations in the objective function when $\sigma^{(i)}$ is negative due to over allocation.**

When the allocated rate $t_k^{(i-1)}$ is greater than the number of queued packets Q_k for a user k, the corresponding dual variable $\sigma^{(i)}$ will be negative and due to the projection operator $[x]^+$ in (25l), it will be zero, thereby forcing $\alpha_k^{(i)} < \alpha_k^{(i-1)}$ as in (25m). Once the $\alpha_k^{(i)}$ is reduced, the precoder weight in (25h) is lowered to make the rate $t_k^{(i)} < t_k^{(i-1)}$ eventually. **Since ρ depend on the system size, we fix the value of $\rho = 0.1$ in our simulations irrespective of the system, which affects the convergence behavior marginally.**

The KKT expressions in (26) are solved in an iterative manner by initializing the transmit and the receive beamformers $\mathbf{m}_{l,k,n}, \mathbf{w}_{l,k,n}$ with the single user beamforming and the MMSE vectors. The dual variable α's are initialized with ones to have equal priorities to all the users in the system. Then the transmit and the receive beamformers are evaluated using the expressions in (26). The transmit precoder in (25h) depends on the BS specific dual variable δ_b, which can be found by bisection search satisfying the total power constraint (25g). Note that the fixed SCA operating point is given by $\tilde{\epsilon}_{l,k,n} = \epsilon_{l,k,n}^{(i-1)}$, which is considered in the expression (26).

To devise an algorithm for a practical implementation, we assume the cross channels $\mathbf{H}_{b,k,n}, \forall k \in \bar{\mathcal{U}}_b$ and the receive beamformers $\mathbf{w}_{l,k,n}$ of all users in the system are known through uplink signaling. We extend the decentralization methods discussed in [30], for the current problem as follows. After receiving the updated transmit precoders from all BSs in $\mathcal{B}$, each user evaluates the MMSE receiver in (25i) and notify them to the BSs via uplink precoded pilots. On receiving pilot signals, BSs update the MSE in (15) as

$$\epsilon_{l,k,n}^{(i)} = 1 - \mathbf{w}_{l,k,n}^{(i)\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}^{(i)}. \tag{26}$$

Using the current MSE value, $t_{l,k,n}^{(i)}, \sigma_{l,k,n}^{(i)}$, and $\alpha_{l,k,n}^{(i)}$ are evaluated using (25k), (25l) and (25m), and the updated dual variables $\alpha_{l,k,n}$ are exchanged between the BSs to evaluate the transmit precoders $\mathbf{m}_{l,k,n}^{(i+1)}$ for the next iteration. The SCA operating point is also updated with the current MSE value.

To avoid the back-haul exchanges between BSs, as an alternative approach, users may perform all processing required and BSs will update the precoders based on the feedback information from the users. Upon receiving the transmit precoders from BSs, each user will update the receive beamformer $\mathbf{w}_{l,k,n}$, the MSE $\epsilon_{l,k,n}$, and the dual variables $\lambda_{l,k,n}$ and $\alpha_{l,k,n}$. The updated $\alpha_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are notified to the BSs using two separate precoded uplink pilot symbols with $\tilde{\mathbf{w}}_{l,k,n}^{(i)} = \sqrt{\alpha_{l,k,n}^{(i)}} \mathbf{w}_{l,k,n}^{*(i)}$ and $\bar{\mathbf{w}}_{l,k,n}^{(i)} = \alpha_{l,k,n}^{(i)} \mathbf{w}_{l,k,n}^{*(i)}$ as the precoders. On receiving the precoded uplink pilots, each BS use the effective channel $\mathbf{H}_{b,k,n}^{\mathrm{T}} \tilde{\mathbf{w}}_{l,k,n}^{(i)}$ and $\mathbf{H}_{b,k,n}^{\mathrm{T}} \bar{\mathbf{w}}_{l,k,n}^{(i)}$ in (25h) to update the transmit precoders, where $\mathbf{x}^*$ is the complex conjugate of $\mathbf{x}$. Finally, Algorithm 3 outlines the distributed precoder design using the KKT based MSE reformulated JSFRA problem.

Algorithm 3: KKT approach for the JSFRA scheme

Input: a_k, Q_k, $\mathbf{H}_{b,k,n}$, $\forall b \in \mathcal{B}$, $\forall k \in \mathcal{U}, \forall n \in \mathcal{N}$
Output: $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n} \forall l \in \{1, 2, \ldots, L\}$
Initialize: $i = 1$, $\mathbf{w}_{l,k,n}^{(0)}, \tilde{\epsilon}_{l,k,n}$ randomly, dual variables $\alpha_{l,k,n}^{(0)} = 1$, and $I_{\max}$ for certain value
foreach *BS* $b \in \mathcal{B}$ **do**
- initialize $i = 0$
- **repeat**
  - update $\mathbf{m}_{l,k,n}^{(i)}$ using (25h), and perform downlink transmission
  - find $\mathbf{w}_{l,k,n}^{(i)}$ using (25i) at each user
  - evaluate $\epsilon_{l,k,n}^{(i)}$, $t_{l,k,n}^{(i)}$, $\sigma_{l,k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ using (25j) and (25k), (25l) and (25m) at each user with the updated $\mathbf{w}_{l,k,n}^{(i)}$
  - using precoded uplink pilots, $\mathbf{m}_{l,k,n}^{(i)}$ and $\alpha_{l,k,n}^{(i)}$ are notified to all BSs in $\mathcal{B}$
  - $i = i + 1$
- **until** *until convergence or* $i \geq I_{\max}$

end

The Algorithm 3 outlines a practical way of implementing the transmit precoders in a distributed manner using over-the-air (OTA) signaling of transmit precoders and the receive beamformers for certain iterations before the actual transmission of data is performed. Unlike primal decomposition (PD) or ADMM approach, all variables are updated at once, *i.e*, the SCA point of $\epsilon^{(i-1)}$, AO update of $\mathbf{w}_{l,k,n}$ and the dual variable α using subgradient update, it is not guaranteed to obtain the same point as that of centralized solutions. It can be viewed as a primal decomposition approach in which the master-slave iterations is performed only once before updating other variables. Since each primal subproblem yields a feasible solution, the monotonic behavior is obtained by this algorithm. If the step size $\rho < 1$, the algorithm will converge by using the arguments made for controlling overallocation problem and the nonincreasing objective function, since the earlier iterates are in fact the operating point for the current iteration.

V. Simulation Results

The simulations carried out in this work consider the path loss varying uniformly across all users in the system with the channels drawn from the *i.i.d.* samples. The queues are generated based on the Poisson process with the average values specified in each section presented.

A. Centralized Solutions

We discuss the performance of the centralized algorithms in Section III for some system configurations. To begin with, we consider a single cell single-input single-output (SISO) model operating at 10 dB signal-to-noise ratio (SNR) with $K = 3$ users sharing $N = 3$ sub-channel resources. The number of packets waiting at the transmitter for each user is given by $Q_k = 4, 8$ and 4 bits, respectively.

Table I tabulates the channel seen by the users over each sub-channel followed by the rates assigned by three different algorithms, Q-WSRME allocation, JSFRA approach and the band-wise Q-WSRM scheme using the WMMSE design [7]. The performance metric used for the comparison is the total number of backlogged bits left over at each slot after the allocation, which is denoted as $\chi = \sum_{k=1}^{K} [Q_k - t_k]^+$. Even though $\mathcal{U}(1)$ and $\mathcal{U}(3)$ has equal number of backlogged packets of $Q_1 = Q_3 = 4$ bits, user $\mathcal{U}(3)$ is scheduled in the first sub-channel due to the better channel condition. In contrast, the JSFRA approach assigns the first user on the first sub-channel, which reduces the total number of backlogged packets waiting at the transmitter. The rate allocated for $\mathcal{U}(2)$ on the second sub-channel is higher in JSFRA scheme compared to the other schemes. It is due to the efficient allocation of the total power shared across the sub-channels.

For a MIMO framework, we consider a system with $N = 3$ sub-channels and $N_B = 3$ BSs, each equipped with $N_T = 4$ transmit antennas operating at 10dB SNR, serving $|\mathcal{U}_b| = 3$ users each. The path loss between the BSs and the users are uniformly generated from $[0, -3]$ dB and the association is made by selecting the BS with the lowest path loss component. Fig. 1(a) shows the performance of the centralized schemes for a single receive antenna system. The total number of queued packets for Fig. 1(a) is given by $Q_k = [14, 15, 14, 8, 12, 9, 12, 11, 11]$ bits and for Fig. 1(b) is $Q_k = [9, 12, 8, 12, 5, 4, 10, 8, 5]$ bits respectively.

The performance of the centralized algorithms are compared in terms of the total number of residual bits remaining in the system after each SCA update in Fig. 1. The Q-WSRM algorithm is not optimal due to the problem of over-allocation when the number of queued packets are few in number. In contrast, the Q-WSRME algorithm provides more favorable allocation by including the explicit rate constraint to avoid the over-allocation. It can be seen that the JSFRA algorithms converges to a final point for all formulations.

For both scenarios in Fig. 1, the Q-WSRME performs marginally inferior to the JSFRA algorithms due to the weights used in the algorithm. The performance loss is attributed to the fact that the Q-WSRME algorithm favors the users with the large number of backlogged packets as compared to the users with better channel conditions. Fig. 1(b) compares the algorithms for $N_R = 2$ receive antenna case. In all figures, the receivers are updated along with the SCA update instants *i.e*, $J_{\max} = 1$ in Algorithm 1. It is also noted that the performance degradation by performing the group update is very minimal. Since the receiver minimizes the objective for the fixed transmit precoders, the convergence is monotonic as can be seen from the figures.

The behavior of the JSFRA algorithm for different exponents q is outlined in the Table II for the users located at the cell-edge of the system employing $N_T = 4$ transmit antennas. It is evident that the JSFRA algorithm minimizes the total number of queued bits for the ℓ_1 norm compared to the ℓ_2 norm, which is shown in the column displaying the total number of left over packets χ in bits. The ℓ_∞ norm provides fair allocation of the resources by making the left over packets to be equal for all users to $\chi_k = 3.58$ bits.

B. Distributed Solutions

The performance of the distributed algorithms are compared using the total number of backlogged packets after each SCA update points. Fig. 2 compares the performance of the algorithms for the system configuration $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ with $N_T = 4$ transmit antennas at the BSs. Each BS serves $|\mathcal{U}_b| = 4$ users in a coordinated manner to reduce the total number of backlogged packets at each BS. The total number of queued packets assumed for both figures is $Q_k = [5, 7, 9, 11, 8, 12, 5, 4]$ bits. As pointed out in Section IV, the performance and the convergence speed of the distributed algorithms are susceptible to the step size used in the subgradient update. Due to the fixed interference levels in the primal approach, it may lead to infeasible solutions if the initial or any intermediate update is not feasible.

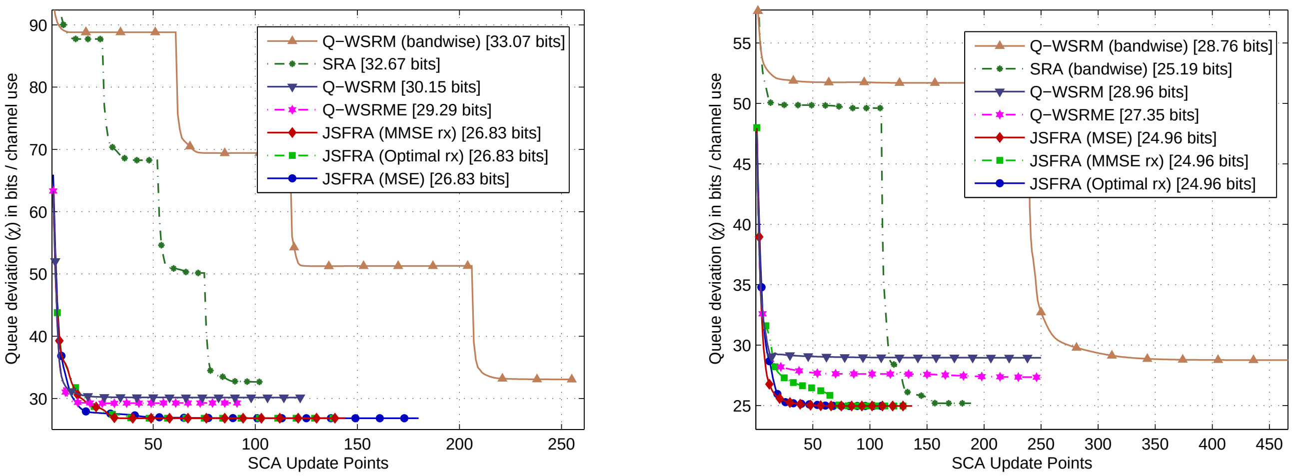
Fig. 2 plots the performance of the primal and the ADMM solutions for the JSFRA scheme using the SCA and by MSE relaxation at each SCA point. In between the SCA updates, the primal or the ADMM scheme is performed for $J_{\max} = 20$ iterations to exchange the respective coupling variables. In Fig. 2, the total number of backlogged packets at each SCA points are plotted without the inner loop iterations of $J_{\max}$ times for the primal or the dual variables convergence. It can be seen from Fig. 2 that the distributed algorithms approach the centralized performance by exchanging minimal information between the coordinating BSs.

Fig. 3 compares the performance of the centralized and the KKT algorithm in Section IV-D for different exponents by plotting the total number of backlogged packets at each SCA update point. The ℓ_1 norm JSFRA scheme provides better performance over other schemes due to the greedy objective. The KKT approach for ℓ_1 norm is not defined due to the non-differentiability of the objective as discussed in the Section IV-D. If used for ℓ_1 norm, the problem of over-allocation will not affect the dual variables $\sigma_{l,k,n}$ and $\alpha_{l,k,n}$ since the queue deviation is raised to the power zero in (25l), which will always be equal to one. A heuristic method based on subdifferential calculus in [2] is proposed in Fig. 3 by assigning zero for $\sigma_{l,k,n}$ when the queue deviation is negative, *i.e*, $Q_k - t_k < 0$. It is required to address the problem of over-allocation in the ℓ_1 norm for dropping the absolute value operator from the objective function. It can be seen that the heuristic method oscillates near the stationary point with the deviation determined by the factor ρ used in (25m).

The objective values are mentioned in the legend for all the schemes and the objective of the ℓ_2 norm is not the same as that of the ℓ_1 norm used for plotting. For simulations, we update all variables in (26) at once at each iteration, *i.e*, $J_{\max} = 1$, which is well justified for the practical implementations due to the signaling overheads. The ℓ_2 norm for the JSFRA and the KKT approach achieves nearly the same value of 6.62 with different χ, due to the limited number of

TABLE I
SUB-CHANNEL-WISE LISTING OF CHANNEL GAINS AND RATE ALLOCATIONS BY DIFFERENT ALGORITHMS FOR A SCHEDULING INSTANT

Users	Queued Packets	Channel Gains			Q-WSRME approach (modified ***backpressure***)			JSFRA Scheme			Q-WSRM band Alloc Scheme		
		SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3	SC-1	SC-2	SC-3
1	4	1.71	0.53	0.56	0	0	0	4.0	0	0	0	0	0
2	8	0.39	1.41	1.03	0	4.88	3.11	0	5.49	0	0	4.39	3.53
3	4	2.34	1.26	2.32	4.0	0	0	0	0	4.0	5.81	0	0
Remaining backlogged packets (χ)					3.92 bits			2.51 bits			5.89 bits		



(a). System Model $\{N, N_B, K, N_T, N_R\} = \{4, 3, 9, 4, 1\}$ (b). System Model $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

Fig. 1. Total number of backlogged packets χ present in the system after each SCA updates

TABLE II
NUMBER OF BACKLOGGED BITS ASSOCIATED WITH EACH USER FOR A SYSTEM $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$.

q	user indices								χ
	1	2	3	4	5	6	7	8	
1	15.0	3.95	5.26	8.95	7.0	11.9	12.0	9.7	25.15
2	11.2	3.9	10.76	10.65	10.27	9.68	8.77	5.9	27.77
∞	11.4	4.4	10.4	10.4	10.4	8.4	8.4	6.4	28.68
Q_k	15.0	8.0	14.0	14.0	14.0	12.0	12.0	10.0	

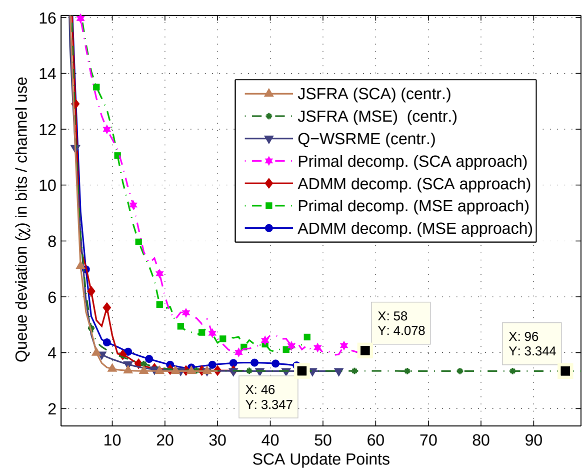


Fig. 2. Convergence behaviour of the centralized and the distributed algorithms for a system $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$

iterations for the dual variable convergence between each SCA update. Fig. 3 also shows the effect of dropping the squared rate variable from the objective in the Q-WSRME scheme compared to the ℓ_2 norm which includes it. By dropping it, the Q-WSRME scheme minimizes the number of queued packets in a prioritized manner based on the respective queues. On contrary, the ℓ_2 norm allocate rates to the users with the higher number of queued packets before addressing the users with the smaller number of queued packets.

C. Average Backlogged Packets Over Slots

We discuss performance of the JSFRA algorithm for different values of ℓ_q over multiple time slots. It is compared with the existing Q-WSRME scheme by varying the average arrival rate A_k of all users. Fig. V-C demonstrates the performance of the centralized algorithms for different ℓ_q values. The horizontal axis indicates the average number of arrivals A_k in bits per user and it is constant for all users in the system. The vertical axis corresponds to the average of number of backlogged packets left in the system after each time slot. Even though A_k's are constant for all users, the instantaneous arrivals are random and is based on Poisson arrival process. We considered a 4×1 MIMO system with $N = 4$ sub-channels and $N_B = 2$ BSs. The path loss is modeled as a uniform random variable $[0, -3]$ dB with the maximum SINR seen by any user is 6 dB.

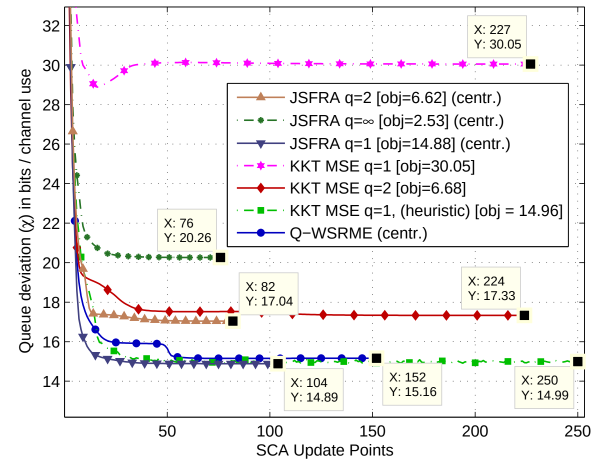


Fig. 3. Impact of varying q in the total number of backlogged packets after each SCA update for a system $\{N, N_B, K, N_R\} = \{5, 2, 8, 1\}$

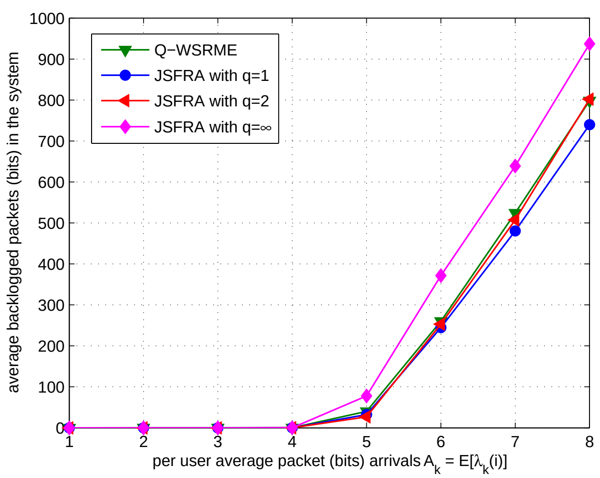


Fig. 4. System $\{N, N_B, K, N_R\} = \{4, 2, 12, 1\}$

The average is performed over 100 **time slots.**

The performance of the JSFRA scheme using ℓ_2 **and Q-WSRME approach are similar in the average number of residual packets after each transmission slot. Note that the additional rate constraints in the Q-WSRME scheme is the reason for the equivalence. Both Q-WSRM and Q-WSRME performs similar to** ℓ_2 **JSFRA scheme when the arrival rates are significantly greater than the actual transmissions. It can be seen from Fig. V-C that the average number of backlogged packets are noticeably less for the** ℓ_1 **JSFRA formulation due to the greedy resource allocation by serving users with better channel conditions. Fig V-C shows that the** ℓ_∞ **JSFRA scheme performs worst in terms of the average number of backlogged packets due to the fairness constraints.**

VI. Conclusions

In this paper, we addressed the problem of allocating downlink space-frequency resources to the users in a multi-cell MIMO IBC system using OFDM. The resource allocation is considered as a joint space-frequency precoder design problem since the allocation of a resource to a user is obtained by a non-zero precoding vector. We proposed the JSFRA scheme by relaxing the nonconvex DC constraint by a sequence of convex subsets using SCA for designing the precoders to minimize the total number of user queued packets. Additionally, an alternative MSE relaxation approach is also proposed by using SCA to address the nonconvex DC constraints for a fixed MMSE receivers. We also proposed distributed precoder designs for the JSFRA problem using primal and ADMM methods.Finally, we proposed a practical iterative algorithm to obtain the precoders in a decentralized manner by solving the KKT conditions of the MSE reformulated JSFRA method. The proposed iterative algorithm requires few iterations and limited signaling exchange between the coordinating BSs to obtain the efficient precoders for a given number of iterations. Numerical results are used to compare the performance of the proposed algorithms with the existing solutions. The distributed precoder design for the time correlated fading will be considered in future.

Appendix A
Tightness of SINR relaxation

For the constraints (12b) **and** (12c) **to be active, there should be at least one user in each BS with enough backlogged packets that cannot be served with the given power budget. On the other hand, to make the constraints active in all cases, the objective of the JSFRA formulation should be regularized with the transmit power without affecting the solution as**

$$\|\tilde{\mathbf{v}}\|_q + \varphi \sum_{k\in\mathcal{U}} \sum_{n=1}^{N} \sum_{l=1}^{L} \operatorname{tr}\left(\mathbf{m}_{l,k,n}\mathbf{m}_{l,k,n}^{\mathrm{H}}\right),$$

where $\varphi \approx 0$**. Note that the modified objective will relax the power constraint by making the constraints** (12b) **and** (12c) **active at the final solution.**

Appendix B
Convergence Analysis of Centralized Algorithm

Let us express the JSFRA problem in (13) **and** (17) **as**

$$\begin{aligned} \underset{\mathbf{x},\mathbf{y},\mathbf{z}}{\textbf{minimize}} \quad & f(\mathbf{x},\mathbf{y},\mathbf{z}) && (26a)\\ \textbf{subject to} \quad & h(\mathbf{z}) - g_0(\mathbf{x},\mathbf{y}) \le 0 && (26b)\\ & g_1(\mathbf{x},\mathbf{y}) \le 0, && (26c)\\ & g_2(\mathbf{x}) \le 0, && (26d) \end{aligned}$$

wher g_2, f **are convex functions and** h **is a linear function. Let** g_0, g_1 **are convex functions only on** x **or** y **as the variable but not on both. Note that the** (26b) **correspond to the constraints in** (12b) **and** (16b) **and** (26c) **corresponds to the constraints in** (12c) **and** (16c)**. Other convex constraints are addressed by the constraint** (26d)**. With this, the feasible set of the problem** (27) **is given by**

$$\mathcal{F} = \{\mathbf{x},\mathbf{y},\mathbf{z} | h(\mathbf{z}) - g_0(\mathbf{x},\mathbf{y}) \le 0, g_1(\mathbf{x},\mathbf{y}) \le 0, g_2(\mathbf{x}) \le 0\}$$

In order to solve the problem, we resort to AO by fixing a block of varibles and optimize for others. In the problem (27), even after fixing the variable y, the problem is nonconvex due to the DC constraint in (26b). In order to solve the problem after fixing the variable y, we adopt the SCA approach presented in [26], [27], [32] by relaxing the nonconvex set by a sequence of convex subsets. Since the proposed method involves two level of iterations, we denote the AO iteration index by a superscript (i) and the DC constraint relaxations by a subscript k. Let $\mathcal{X}_k^{(i)}$ be a feasible set for the i^{th} AO iteration and the k^{th} SCA point for a fixed y and for a fixed x, the set is represented as $\mathcal{Y}_k^{(i)}$. Since the SCA iterations are performed until convergence, let $\mathrm{x}_*^{(i)}$ denotes the converged point of x in the i^{th} AO iteration. For the sake of clarity, we define the optimal value of z obtained for the i^{th} AO iterate for the fixed y variable as $\mathrm{z}_{*|y}^{(i)}$.

Let us consider the order for AO by optimizing variable x before optimizing the variable y. Without affecting the convergence proof, let us consider the variable y is fixed for the AO i with the optimal value achieved from the previous iteration $i-1$ as $\mathrm{y}_*^{(i-1)}$. In order to find the optimal value of x for the SCA iteration k, we linearize the nonconvex function g_0 using previous SCA iterate of x as

$$\begin{aligned}\hat{g}_o(\mathrm{x},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)}) &= g_0(\mathrm{x}_k^{(i)},\mathrm{y}_*^{(i-1)}) \\ &\quad + \nabla g_0(\mathrm{x}_k^{(i)},\mathrm{y}_*^{(i-1)})^{\mathrm{T}}(\mathrm{x}-\mathrm{x}_k^{(i)}). \quad (27)\end{aligned}$$

Using (27), the convex subproblem for i^{th} AO iteration and k^{th} SCA point for the variable x and z is given by

$$\begin{aligned}&\underset{\mathrm{x},\mathrm{z}}{\text{minimize}} && f(\mathrm{x},\mathrm{y}_*^{(i-1)},\mathrm{z}) && (28a)\\ &\text{subject to} && h(\mathrm{z})-\hat{g}_0(\mathrm{x},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)})\le 0 && (28b)\\ &&& g_1(\mathrm{x},\mathrm{y}_*^{(i-1)})\le 0, && (28c)\\ &&& g_2(\mathrm{x})\le 0, && (28d)\end{aligned}$$

Let the set defined by the problem in (28) be represented as $\mathcal{X}_k^{(i)}\subset\mathcal{F}$. In order to prove the convergence of the convex subproblem (28) for a fixed $\mathrm{y}=\mathrm{y}_*^{(i-1)}$ operating at $\mathrm{x}_k^{(i)}$, let us assume that (28) yields $\mathrm{x}_{k+1}^{(i)}$ and $\mathrm{z}_{k+1}^{(i)}$ as the solution at the k^{th} iteration. To show $\mathrm{x}_{k+1}^{(i)}$ and $\mathrm{z}_{k+1}^{(i)}$ minimizes the objective function ans also feasible, let us assume that the point $\mathrm{x}_k^{(i)}\in\mathcal{X}_k^{(i)}$, which is feasible for (28). Since the function $g_0(\mathrm{x},\mathrm{y}_*^{(i-1)})$ is linearized at $\mathrm{x}_k^{(i)}$, it satisfies

$$h(\mathrm{z})-g_0(\mathrm{x},\mathrm{y}_*^{(i-1)})\le h(\mathrm{z})-\hat{g}_0(\mathrm{x},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)})\le 0, \quad (29)$$

since $g_0(\mathrm{x},\mathrm{y}_*^{(i-1)})\le\hat{g}_0(\mathrm{x},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)}),\forall\mathrm{x}\in\mathcal{X}_k^{(i)}$. Now, $\mathrm{x}_{k+1}^{(i)}$ and $\mathrm{z}_{k+1}^{(i)}$ are the optimal and feasible point for the k^{th} subproblem (28), it satisfies

$$\begin{aligned}h(\mathrm{z}_{k+1}^{(i)})-g_0(\mathrm{x}_{k+1}^{(i)},\mathrm{y}_*^{(i-1)}) &\le h(\mathrm{z}_{k+1}^{(i)})-\hat{g}_0(\mathrm{x}_{k+1}^{(i)},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)})\\ &\le h(\mathrm{z}_k^{(i)})-\hat{g}_0(\mathrm{x}_k^{(i)},\mathrm{y}_*^{(i-1)};\mathrm{x}_k^{(i)})\le 0. \quad (30)\end{aligned}$$

Using (30), we can prove that the solution $\mathrm{x}_{k+1}^{(i)}$ and $\mathrm{z}_{k+1}^{(i)}$ are feasible, since the initial point of $\mathrm{x}=\mathrm{x}_*^{(i-1)}$ was chosen to be feasible from the earlier AO iteration $i-1$. In order to prove the convergence of the objective, using (30), we can see that $\mathcal{X}_0^{(i)}\subseteq\cdots\subseteq\mathcal{X}_{k-1}^{(i)}\subseteq\mathcal{X}_k^{(i)}\subset\mathcal{F}$. Since the feasible set of the problem (28) includes the feasible sets from the earlier iteration, we arrive at

$$\begin{aligned}f(\mathrm{x}_0^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_0^{(i)}) &\ge f(\mathrm{x}_k^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_k^{(i)})\\ &\ge f(\mathrm{x}_{k+1}^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_{k+1}^{(i)})\ge f(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_{*|y}^{(i)}). \quad (31)\end{aligned}$$

Thus the sequence $f(\mathrm{x}_k^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_k^{(i)})$ is nonincreasing and converges to a critical point. Note that feasible point $(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_{*|y}^{(i)})$ need not be a stationary point of the problem (27), since it is the minimizer only in the feasible set $\mathcal{X}_*^{(i)}\subset\mathcal{F}$, which depends on x and z only.

Once the solution is found for a fixed y, we fix x as $\mathrm{x}_*^{(i)}$ and optimize for y. Even after treating x as a constant, the problem is still nonconvex due to the DC constraint. Following similar approach, we can find the minimizer $\mathrm{y}_k^{(i)}$ and $\mathrm{z}_k^{(i)}$ for the convex subproblem (28) at each iteration k. Note that $\mathrm{z}_k^{(i)}$ is reused since the variable x is fixed for the i^{th} AO iteration. The convergence and the nonincreasing behavior of the problem follows similar arguments as above. Now, the optimal solution of the converged subproblems with y as variable are $\mathrm{y}_*^{(i)}$ and $\mathrm{z}_*^{(i)}$. Note that the solution point $(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i)},\mathrm{z}_{*|x}^{(i)})$ is the unique minimizer in the set $\mathcal{Y}_*^{(i)}$.

Finally, to prove the global convergence of the objective, we need to show the nonincreasing behavior of the objective function between each AO update, *i.e*,

$$f(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i)},\mathrm{z}_{*|x}^{(i)})\le f(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_{*|y}^{(i)}).$$

Let us consider an AO iteration i in which the optimal value for x and z are obtained as $\mathrm{x}_*^{(i)}$ and $\mathrm{x}_{*|y}^{(i)}$ using fixed $\mathrm{y}=\mathrm{y}_*^{(i-1)}$. In order to find $\mathrm{y}_*^{(i)}$, we fix x as $\mathrm{x}_*^{(i)}$ and optimize for y. Since the problem (27) is nonconvex even after fixing x, we have to linearize the convex function in the DC constraint (26b) around some fixed operating point of y. Since we know that $\mathrm{y}_*^{(i-1)}$ is already a feasible point for y along with $\mathrm{x}_*^{(i)}$, linearization is performed around this feasible point. Using the inequality in (30), we can show that $\{\mathrm{y}_*^{(i-1)},\mathrm{x}_*^{(i)},\mathrm{z}_{*|y}^{(i)}\}\in\mathcal{Y}_0^{(i)}$. Now the optimization is performed to find the optimal y using the relaxed subproblem (28), the optimal solution $\mathrm{z}_0^{(i)}$ for the initial iteration after AO update follows

$$f(\mathrm{x}_*^{(i)},\mathrm{y}_0^{(i)},\mathrm{z}_0^{(i)})\le f(\mathrm{x}_*^{(i)},\mathrm{y}_*^{(i-1)},\mathrm{z}_{*|x}^{(i)}),$$

since the feasible set includes the earlier optimal points $\{\mathrm{y}_*^{(i-1)},\mathrm{x}_*^{(i)}\}$ as the operating point for AO and SCA iteration. Note that it is not possible to say $\mathcal{X}_*^{(i)}\subseteq\mathcal{Y}_0^{(i)}$, since the set is nonconvex on x,y, but $\{\mathrm{y}_*^{(i-1)},\mathrm{x}_*^{(i)},\mathrm{z}_{*|y}^{(i)}\}\in\{\mathcal{X}_*^{(i)}\cap\mathcal{Y}_0^{(i)}\}$. Now by using induction, we can show that the global problem converges to a feasible point of the

nonconvex problem (27) in a nonincreasing manner[5].

To show the converged point of the iterative algorithm is in fact the stationary point of the nonconvex problem (27), it must satisfy the KKT conditions of the nonconvex problem. Since the converged point is the minimizer for the iterative algorithm such that

$$f(\mathbf{x}_*^{(i)}, \mathbf{y}_*^{(i)}, \mathbf{z}_{*|x}^{(i)}) = f(\mathbf{x}_*^{(i+1)}, \mathbf{y}_*^{(i)}, \mathbf{z}_{*|y}^{(i+1)}) = f(\mathbf{x}_*^{(i+1)}, \mathbf{y}_*^{(i+1)}, \mathbf{z}_{*|x}^{(i+1)}), \quad (32)$$

the solution is inside the feasible set $\mathcal{F}$ and $(\mathbf{x}_*^{(i+1)}, \mathbf{y}_*^{(i+1)}, \mathbf{z}_{*|x}^{(i+1)})$ is the minimizer of the objective function f_0 in the feasible set $\mathcal{X}_*^{(i+1)} \subset \mathcal{F}$. Using the discussions in [23], we can easily show that the feasible point $\mathcal{F}$ and $(\mathbf{x}_*^{(i+1)}, \mathbf{y}_*^{(i+1)}, \mathbf{z}_{*|x}^{(i+1)})$, which is the minimizer in the local neighborhood, is a stationary point of the non convex problem in (27) satisfying the constraint qualifications and the KKT expressions for the set $\mathcal{X}_*^{(i+1)} \subset \mathcal{F}$. The non differentiability of the objective function in (13) and (17) requires the subdifferential set of the objective function to include $0 \in \partial f_0(\mathbf{z}_*)$ to satisfy the KKT conditions. The monotonic decrease in the objective is still valid if the variables x, y and z are updated together, since the update in y increases the objective function for a fixed point x and z.

The uniqueness of the convex subproblems (28) is required for the convergence of AO [33]. For the problem (15), the uniqueness of the transmit precoders are guaranteed by the constraint (14), which can be written as

$$\gamma_{l,k,n} + \tilde{\beta}_{l,k,n}^{-2} |\tilde{\mathbf{H}}_{b_k,k,n} \tilde{\mathbf{m}}_{l,k,n}|^2 (\beta_{l,k,n} - \tilde{\beta}_{l,k,n}) - \tilde{\beta}_{l,k,n}^{-1} \tilde{\mathbf{m}}_{l,k,n}^{\mathrm{H}} \tilde{\mathbf{H}}_{b_k,k,n}^{\mathrm{H}} \tilde{\mathbf{H}}_{b_k,k,n} (\mathbf{m}_{l,k,n} - \tilde{\mathbf{m}}_{l,k,n}) \leq 0, \quad (33)$$

where $\tilde{\mathbf{H}}_{b_k,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \tilde{\mathbf{H}}_{b_k,k,n}$ and for the (18) by

$$|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}|^2 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2 + \widetilde{N}_0 \leq \epsilon_{l,k,n}. \quad (34)$$

Using these arguments, we can claim that the proposed JSFRA centralized solution achieves a stationary point of the original nonconvex problem with the nonincreasing objective value at each iteration.

Appendix C
KKT conditions for MSE approach

In order to solve for an iterative precoder design algorithm, the KKT expressions for the problem in (26) are obtained by differentiating the Lagrangian by assuming the equality constraint for (25e) and (25f). At the stationary points, following conditions are satisfied.

$$\nabla_{\epsilon_{l,k,n}} : -\alpha_{l,k,n} + \frac{\sigma_{l,k,n}}{\tilde{\epsilon}_{l,k,n}} = 0 \quad (34a)$$

$$\nabla_{t_{l,k,n}} : -q\, a_k \Big(Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}\Big)^{(q-1)} + \frac{\sigma_{l,k,n}}{\log_2(e)} = 0 \quad (34b)$$

$$\nabla_{\mathbf{m}_{l,k,n}} : \sum_{y\in\mathcal{U}} \sum_{x=1}^{L} \alpha_{x,y,n} \mathbf{H}_{b_k,y,n}^{\mathrm{H}} \mathbf{w}_{x,y,n} \mathbf{w}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_k,y,n} \mathbf{m}_{l,k,n} + \delta_b \mathbf{m}_{l,k,n} = \alpha_{l,k,n} \mathbf{H}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n}, \quad (34c)$$

$$\nabla_{\mathbf{w}_{l,k,n}} : \sum_{(x,y)\neq(l,k)} \mathbf{H}_{b_y,k,n} \mathbf{m}_{x,y,n} \mathbf{m}_{x,y,n}^{\mathrm{H}} \mathbf{H}_{b_y,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n} + \mathbf{I}_{N_R} \mathbf{w}_{l,k,n} = \mathbf{H}_{b_k,k,n}\, \mathbf{m}_{l,k,n}. \quad (34d)$$

In addition to the primal constraints given in (25e), (25f) and (25g), the complementary slackness criterion must also be satisfied at the stationary point. Upon solving the above expressions in (35) with the complementary slackness conditions, we obtain the iterative algorithm to determine the transmit and the receive beamformers as shown in (26).

[5]Note that the objective is monotonically decreasing even if the SCA subproblem (28) is terminated after predetermined number iterations or for certain accuracy.

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