Reviewers Comments & Authors Replies

Manuscript No.	Paper T-SP-18051-2014, submitted to *"IEEE Transactions on Signal Processing"*
Title	"Traffic Aware Resource Allocation Schemes for Multi-Cell MIMO-OFDM Systems"
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The authors are grateful to the associate editor for being considerate enough to provide us with this additional opportunity to revise and correct our convergence analysis. We thank all the reviewers for their valuable comments on the manuscript, which have been greatly helpful to improve the paper quality. Based on the comments, we have revised our manuscript to ensure that all the reviewers comments are addressed properly and also verified the technical correctness of the convergence proof. We have also modified some parts of text to improve the readability. In summary, the revised manuscript includes the following changes.

1. We have rewritten Appendix A-D to discuss the stationarity of limit points of the sequence of beamformer iterates generated by the centralized algorithms.
2. We have updated the convergence of distributed algorithm to reflect the same.

In what follows, the comments are listed, each followed immediately by the corresponding reply from the authors. The reviewers questions in the revised manuscript are highlighted in blue color and the authors responses are presented in black. Unless otherwise stated, all the numbered items (figures, equations, references, citations, etc) in this response letter refer to the revised manuscript. All revisions in the manuscript are highlighted in blue color.

List of Changes:

- Page 6, paragraph after (28)
- Page 8, last paragraph in Section IV-B
- Page 12, paragraph following itemized requirements in Appendix A
- Last sentence after (51) on page 14
- Last paragraph in Appendix A-C
- Appendix A-D is rewritten using subsequence convergence
- Last sentence after (60) on page 15

In order to discuss the convergence of the sequence of iterates generated by the centralized scheme in Algorithm 1, we use an unified superscript index t to represent both alternating optimization (AO) index i and successive convex approximation (SCA) indexing k. By doing so, we represent iterate $\mathbf{x}^t$ as the solution variable produced by the iterative algorithm in every SCA k and AO step i. Using this notation, we have revised our convergence discussions in Appendices A-C and A-D.

Response to first reviewer comments

The reviewer's concerns have been addressed and the manuscript can be published as it is.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.

Response to second reviewer comments

I have no further comments.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.

Response to third reviewer comments

The authors have modified the manuscript taken into account the reviewers' comments. However, the proof of convergence still lacks of convincing argumentation to make this paper suitable for publication. Two main aspects are arguable as follows:

Response: We thank the reviewer for pointing out the mistake in our previous convergence proof. Based on the reviewer's comments and suggestions, we have rewritten Appendix A-D using subsequence convergence instead of claiming the sequence convergence. We hope that our current changes will be convincing enough to make the convergence proof complete.

1. When discussing the convergence of the SCA algorithm (Appendix A-C), the authors claim that the SCA algorithm converges because the objective function is strictly decreasing and the minimizer in each iteration is unique. However, this is in general not enough to guarantee the convergence of the whole sequence. Instead, one can only claim that every limit point of the whole sequence is a stationary point, or globally optimal solution if the problem is convex. As an example, simply consider the gradient projection algorithm, where each subproblem is strongly convex and the objective function is strictly decreasing but the whole sequence may not converge. The authors are referred to Bertsekas' book: Nonlinear Programming.

   Response: We thank the reviewer for pointing out that the sequence of iterates generated by the SCA algorithm in each AO step need not be convergent. Based on the reviewer's reference to the gradient projection algorithm in Nonlinear Programming textbook, we have realized that our earlier convergence proof is not mathematically rigorous. We apologize for our emphasis on the convergence of iterates in our previous manuscript. As suggested by the reviewer, stationarity of limit points of the whole sequence of iterates is now addressed in Appendix A-D by using the following statements.

   - The sequence of iterates generated by the algorithm is bounded, therefore, we can only claim that there exist at least one subsequence that converges to a limit point of the original sequence.
   - Using a convergent subsequence of the original sequence of iterates, we then discuss that every limit point is a stationary point of nonconvex problem in Appendix A-D.

   We have made changes in Appendices A-C and A-D to address the concerns raised by the reviewer.

2. Even under the assumption that the inner loop SCA converges, there are doubts with respect to the convergence of the alternating optimization algorithm described by (54). The authors cite [27] to justify convergence, but it is not straightforward to claim that the algorithmic model considered in this manuscript is the same as in [27]. More specifically, let us consider the variable x (so it is consistent with the authors' notation in Appendix A-D). In the algorithmic model of [27], at iteration $t+1$, all elements of $\mathbf{x}$ are updated simultaneously, based on $\mathbf{x}^t$. But in the authors' model, the update of $\mathbf{x}$ is performed in two phases: in the first phase, only the elements of $\mathbf{m}$ and γ in $\mathbf{x}$ are updated based on $\mathbf{x}^t$. In the second phase, the remaining elements of $\mathbf{x}$ are updated, i.e. $\mathbf{w}$ and γ, based on both $\mathbf{x}^t$ and the elements that have been updated in the first phase. This is not a trivial difference and thus the direct application of the conclusion from [27] in the current context cannot be taken for granted. Due to the open concerns after several revision rounds, it is recommended to reject the paper.

   Response: We understand the reviewer's concern on using [R1] to our problem involving both SCA and AO updates. In each iteration, the proposed method solves only a subset of optimization variables by keeping others fixed, therefore, we cannot use [R1] directly as suggested by the reviewer. Even though we have revised our convergence proof based on a convergent subsequence without referring to [R1], nevertheless, we clarify our usage of [R1] in our previous manuscript for the convergence of iterates.

   - We refer to [R2] for a similar usage of [R1] while discussing the convergence of AO method used in designing tight frames by solving matrix nearness problem. The reference [R1] used to discuss the convergence of iterates generated by the AO method in [R2] by referring AO update as a composition of two sub-algorithms, wherein each sub-algorithm updates only a subset of optimization variables. It has been discussed comprehensively in [R2, Appendices I and II]. In particular, the discussion on referring AO as two sub-algorithms is presented in [R2, Appendix I-F].

- However, we apologize for the incomplete usage of [R1, Theorem 3.1] in our previous manuscript to claim the convergence of beamformer iterates. In [R1, Theorem. 3.1], it has been shown that if the strict monotonicity of the objective sequence is ensured and the iterates are bounded, then *either the sequence of iterates converges or the accumulation points of* $\{\mathbf{x}^t\}$ *is a continuum.*
- Therefore, our previous claim on the guaranteed convergence of iterates is not correct, as pointed out by the reviewer.
- Similar approach of restricting the solution set with a proximal operator to ensure strict monotonicity of the objective sequence is presented in [R1, Section 4] under restrictive mapping section. Therefore, even if we use the discussions in [R1], we can only claim that the sequence of iterates $\{\mathbf{x}^t\}$ converges to a continuum of accumulation points, and every limit point is a stationary point.

We have revised Appendix A-D titled *"Stationarity of Limit Points"* to discuss the convergence of the sequence of iterates. We have revised our claim as every limit point of the sequence of iterates generated by the centralized algorithm is a stationary point.

R1. R.R. Meyer, "Sufficient Conditions for the Convergence of Monotonic Mathematical Programming Algorithms," *Journal of Computer and System Sciences*, vol. 12, no. 1, pp. 108-121, 1976

R2. J. Tropp, I. Dhillon, R. Heath, and T. Strohmer, "Designing structured tight frames via an alternating projection method," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 188209, Jan. 2005.

Response to fourth reviewer comments

This reviewer's concerns have been addressed, and this manuscript is now deemed fit for publication.

Reply: We thank the reviewer for all constructive comments and recommending the revised manuscript for the publication. Moreover, we have modified the convergence analysis with additional technical materials to address some missing details in the convergence of beamformer iterates.