# Traffic Aware Precoder Design for Space Frequency Resource Allocation

Ganesh Venkatraman†, Antti Tölli, Le-Nam Tran and Markku Juntti
Email: {gvenkatr, antti.tolli, le.nam.tran, markku.juntti}@ee.oulu.fi

Centre for Wireless Communications (CWC),
Department of Communications Engineering (DCE),
University of Oulu, Oulu, FI-90014

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# Section 1

## Introduction

# Introduction and Motivation

- Complex algorithms are adopted to maximize throughput to satisfy the data requirements from higher layers
- Available wireless resources are to be utilized efficiently to minimize the backlogged packets
- Spatial and Frequency resources are exploited to empty the packets waiting at the BSs
- We discuss precoder designs for multiple users MIMO-OFDM setup to minimize the number of queued packets

Section 2

# System Model & Problem Formulation

# Symbols used

- OFDM system with $N$ sub-channels and $N_B$ BSs, each equipped with $N_T$ transmit antennas
- Let $K$ be the total number of users with $N_R$ antennas
- Let $\mathcal{B}$ and $\mathcal{U}$ denote the set of coordinating BSs and users in the system
- Let $L$ be the total available spatial streams for a user $k$, given by $\min(N_T, N_R)$

# System Model

- The $l$th spatial signal received on sub-channel $n$ of user $k$ is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n} \quad (1)$$

- where $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are transmit and receive beamformers corresponding to the $l$th spatial stream on the $n$th sub-channel of user $k$

# System Model

- $\mathbf{H}_{b_k,k,n} \in \mathbb{C}^{N_R \times N_T}$ denotes the channel between BS $b_k$ and user $k$
- $d_{l,k,n}$ and $n_{l,k,n}$ correspond to data symbol and equivalent noise on $l$th spatial stream of user $k$
- Using the above notations, the SINR seen by the $l$th spatial stream on the $n$th sub-channel for user $k$ is given by

$$\gamma_{l,k,n} = \frac{\left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2}{\grave{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2} \tag{2}$$

- where $\grave{N}_0 = \|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n}\|^2$

# Queueing Model

- Each user is associated with backlogged packets of size $Q_k$ packets.
- Queued packets $Q_k$ of each user follows dynamic equation at the $i$th instant as

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i)\Big]^+ + \lambda_k(i) \tag{3}$$

- where $t_k = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$ denotes the total number of transmitted packets corresponding to user $k$ in the previous $i$th instant
- $\lambda_k$ represents the fresh arrivals of user $k$ at BS $b_k$

# Problem Formulation

- Objective - to minimize the number of backlogged packets waiting at BSs
- Optimization variables - transmit precoders and receive beamformers
- MIMO-OFDM - scheduling of users across sub-channels is inherently performed by precoders

# Section 3

## Centralized Solutions

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Q-WSRM formulation is the result of minimizing the conditional Lyapunov drift[†]
- Q-WSRM formulation is also called as back pressure algorithm, since it acts greedily in minimizing the backlogged packets at each instant

$$\underset{t_{l,k,n}}{\text{minimize}} \quad \sum_{k\in\mathcal{U}} \left\{ Q_k(i)^2 - Q_k(i-1)^2 \right\},$$

- where $Q_k$ follows the dynamic Queue expression in (3)

[†] Neely, Michael J. "Stochastic network optimization with application to communication and queueing systems." Synthesis Lectures on Communication Networks 3.1 (2010): 1-211.

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Q-WSRM formulation, which is obtained by solving Lyapunov drift, is given by

$$\underset{t_{l,k,n}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} Q_k \left( \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right) \tag{4a}$$

$$\sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \leq Q_k \ / \ Q_{k,n} \tag{4b}$$

- Queue-Rate product is maximized
- Users with more number of backlogged packets are favored over good channel users

# JSFRA Formulation (SINR Relaxation)

- Centralized Design - precoders are designed by a controller, which are then used by all BSs in $\mathcal{B}$
- The objective used to design transmit precoders is

$$v_k = \left| Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right|^q \quad (5)$$

- To generalize the objective, we use $\tilde{v}_k \triangleq a_k^{\frac{1}{q}} v_k$, where $a_k$ is arbitrary weights used control the priorities
- Exponent $q$ plays different role based on the value it assumes
  - $\ell_{q=1}$ results in greedy allocation
  - $\ell_{q=2}$ ideal for the delay or buffer size limited scenarios
  - $\ell_{q=\infty}$ provides fair resource allocation in each transmission instant

# JSFRA Formulation (SINR Relaxation)

- Optimization problem with queue difference objective is nonconvex due to the constraint

$$\underset{\mathbf{m}_{l,k,n}}{\text{minimize}} \quad \left| Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} \log\left(1 + \gamma_{l,k,n}\right) \right|^q \tag{6a}$$

$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{\left| \mathbf{w}_{l,k,n}^H \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} \right|^2}{\beta_{l,k,n}} \tag{6b}$$

$$\grave{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n} \tag{6c}$$

- The nonconvex constraints are approximated by sequence of convex subsets and solved iteratively by successive convex approximation (SCA) method
- Receive beamformers are designed by the MMSE receivers using the converged transmit precoders

# JSFRA Formulation (MSE Reformulation)

- Alternatively, we solve the queue minimization problem by utilizing the relation between the MSE and the SINR as

$$\epsilon_{l,k,n} = (1 + \gamma_{l,k,n})^{-1} \tag{7}$$

- Equivalence is valid only when the receivers are designed with the mean squared error (MSE) objective, *i.e.*, using MMSE receivers
- Problem involves nonconvex constraint

$$\begin{aligned} t_{l,k,n} &\leq -\log_2(\epsilon_{l,k,n}) && \text{(8a)} \\ \epsilon_{l,k,n} &= \mathbb{E}\left[(d_{l,k,n} - \hat{d}_{l,k,n})^2\right] = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2 \\ &\quad + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}\right|^2 + \grave{N}_0 && \text{(8b)} \end{aligned}$$

# JSFRA Formulation (MSE Reformulation)

- The nonconvex constraint is approximated by a sequence of convex constraints, which is performed using SCA technique
- The iterative procedure is carried out until convergence or for suitable number of iterations
- The above reformulation works only with the MMSE receiver

# Section 4

## Distributed Solutions

# Distributed Methods

- Small System - centralized approach is viable, provided channel remains constant for multiple transmission slots
- However, overhead involved in the centralized design scales up significantly as the network size grows
- Distributed approaches based on primal decomposition or ADMM can be used to reduce the signaling requirements
- Signaling involved in the design of precoders are only scalar interference variables
- Approximated convex subproblem in each SCA step is performed via distributed methods

# Primal Decomposition Method

- Precoder design is based on master-slave approach
- Interference to the neighboring BS users are bounded by a scalar variable, treated as a constant in subproblems
- The interference thresholds are determined by the master problem and used in each slave subproblem constraint as

$$\zeta_{l,k,n,b} \geq \sum_{i \in \mathcal{U}_b} \sum_{j=1}^{L} \left| \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b,k,n} \mathbf{m}_{j,i,n} \right|^2 \ \forall b \in \bar{\mathcal{B}}_{b_k}. \quad (9)$$

# ADMM based Decomposition Method

- The ADMM is superior to other distributed schemes in terms of the convergence speed
- ADMM includes an additional quadratic term

$$\|v_k\|_q + \sum_{l,k,n} \nu_{l,k,n}^{(j)} \left(\zeta_b - \zeta_b^{(j)}\right) + \frac{\rho}{2}\|\zeta_b - \zeta_b^{(j)}\|^2 \qquad (10)$$

in objective, where $\zeta_b^{(j)}$ is global consensus variable

- $\zeta_{l,k,n,b}$ in (9) is treated as an optimization variable in ADMM
- Consensus variables are updated upon exchanging corresponding local $\zeta_{l,k,n,b}$'s among coordinating BSs

# KKT based Distributed Solution

- Decentralization methods involve significant signaling exchanges via backhaul
- Overhead is large for multi-antenna receivers - iterative design should reduce the backlogged packets significantly in first few iterations
- To achieve that, we design precoders by solving the Karush-Kuhn-Tucker (KKT) equations of the JSFRA problem via MSE reformulation
- Group update of all involved optimization variables - to speed up the convergence of precoder design

Section 5

# Simulation Results

# Centralized Solutions

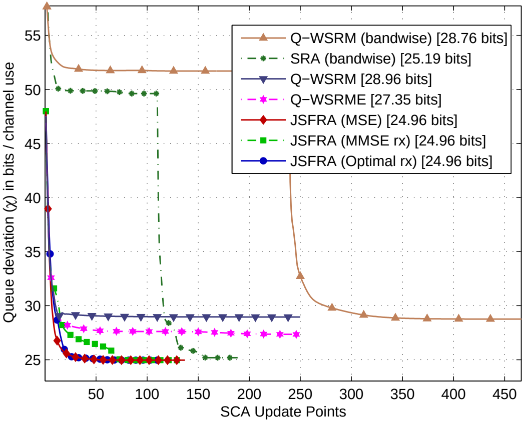


Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

# Distributed Solutions

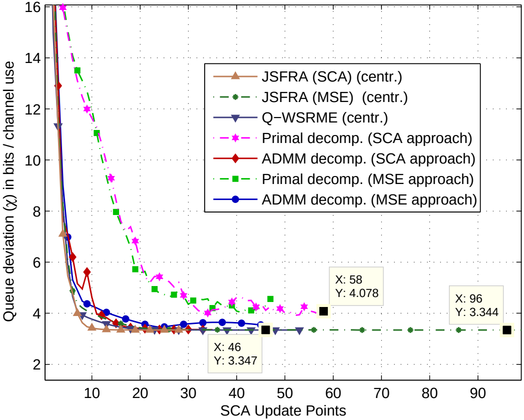


Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{3, 2, 8, 4, 1\}$

# Performance of KKT based Approach

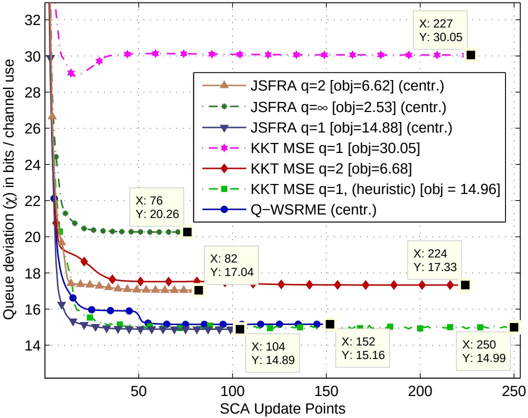


Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{5, 2, 8, 4, 1\}$

# Time Correlated Fading Performance

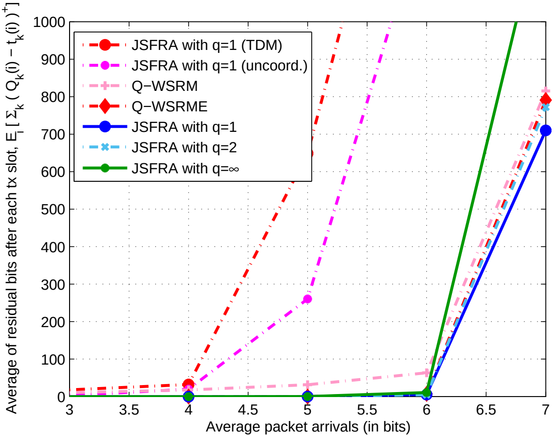


Figure: System Model - $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ after 250 transmissions

# Section 6

## Conclusions

# Conclusions

- We studied cross layer problem of designing transmit and receive beamformers based on the number of residual packets
- Since the problem is nonconvex, we solve the problem iteratively by solving convex subproblems in each iteration
- We also proposed a practical way of implementing the precoder design in a distributed manner by solving the KKT expressions
- Extensions of the proposed work in time-correlated fading scenario with limited number of information exchange cycle is in progress

# Questions !