# Traffic Aware Precoder Design for Space Frequency Resource Allocation

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# Outline

Section 1

# Introduction

# Introduction and Motivation

- Complex algorithms are adopted to maximize throughput to satisfy the data requirements from higher layers
- Available wireless resources are to be utilized efficiently to minimize the backlogged packets
- Spatial and Frequency resources are exploited to empty the packets waiting at the BSs
- We discuss precoder designs for multi-user MIMO-OFDM setup to minimize the number of queued packets

Section 2

## System Model & Problem Formulation

# Symbols used

- OFDM system with $N$ sub-channels and $N_B$ BSs, each equipped with $N_T$ transmit antennas
- Let $K$ be the total number of users with $N_R$ antennas
- Let $\mathcal{B}$ and $\mathcal{U}$ denote the set of coordinating BSs and users in the system
- Let $L$ be the total available spatial streams for a user $k$, given by $\min(N_T, N_R)$

# System Model

- The $l$th spatial signal received on sub-channel $n$ of user $k$ is given by

$$\hat{d}_{l,k,n} = \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n} d_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n} + \mathbf{w}_{l,k,n}^{\mathrm{H}} \sum_{i \in \mathcal{U} \setminus \{k\}} \mathbf{H}_{b_i,k,n} \sum_{j=1}^{L} \mathbf{m}_{j,i,n} d_{j,i,n} \quad (1)$$

- where $\mathbf{m}_{l,k,n}$ and $\mathbf{w}_{l,k,n}$ are transmit and receive beamformers corresponding to the $l$th spatial stream on the $n$th sub-channel of user $k$

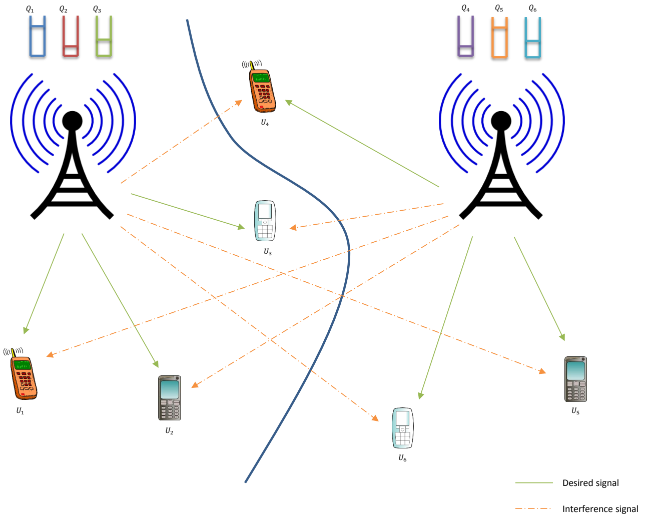
# System Model

- $\mathbf{H}_{b_k,k,n} \in \mathbb{C}^{N_R \times N_T}$ denotes the channel between BS $b_k$ and user $k$
- $d_{l,k,n}$ and $n_{l,k,n}$ correspond to data symbol and equivalent noise on $l$th spatial stream of user $k$
- Using the above notations, the SINR seen by the $l$th spatial stream on the $n$th sub-channel for user $k$ is given by

$$\gamma_{l,k,n} = \frac{\left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2}{\grave{N}_0 + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}\right|^2} \qquad (2)$$

- where $\grave{N}_0 = \left\|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{n}_{l,k,n}\right\|^2$

# System Model with User Queues



# Queueing Model

- Each user is associated with backlogged packets of size $Q_k$ packets.
- Queued packets $Q_k$ of each user follows dynamic equation at the $i$th instant as

$$Q_k(i+1) = \Big[Q_k(i) - t_k(i)\Big]^+ + \lambda_k(i) \tag{3}$$

- where $t_k = \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n}$ denotes the total number of transmitted packets corresponding to user $k$ in the previous $i$th instant
- $\lambda_k$ represents the fresh arrivals of user $k$ at BS $b_k$

# Problem Formulation

- Objective - to minimize the number of backlogged packets waiting at BSs
- Optimization variables - transmit precoders and receive beamformers
- MIMO-OFDM - scheduling of users across sub-channels is inherently performed by precoders

# Section 3

## Centralized Solutions

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Q-WSRM formulation is the result of minimizing the conditional Lyapunov drift[1]
- Q-WSRM formulation is also called as back pressure algorithm, since it acts greedily in minimizing the backlogged packets at each instant

$$\underset{t_{l,k,n}}{\text{minimize}} \quad \sum_{k \in \mathcal{U}} \left\{ Q_k(i)^2 - Q_k(i-1)^2 \right\},$$

- where $Q_k$ follows the dynamic Queue expression in (3)

[1]Neely, Michael J. "Stochastic network optimization with application to communication and queueing systems." Synthesis Lectures on Communication Networks 3.1 (2010): 1-211.

# Queue-Weighted Sum Rate Maximization (Q-WSRM)

- Q-WSRM formulation, which is obtained by solving Lyapunov drift, is given by

$$\underset{t_{l,k,n}}{\text{maximize}} \quad \sum_{k\in\mathcal{U}} Q_k \left( \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right) \tag{4a}$$

$$\sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \leq Q_k \;/\; Q_{k,n} \tag{4b}$$

- Queue-Rate product is maximized
- Users with more number of backlogged packets are favored over good channel users

# JSFRA Formulation (SINR Relaxation)

- Centralized Design - precoders are designed by a controller, which are then used by all BSs in $\mathcal{B}$
- The objective used to design transmit precoders is

$$v_k = \left| Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} t_{l,k,n} \right|^q \quad (5)$$

- To generalize the objective, we use $\tilde{v}_k \triangleq a_k^{\frac{1}{q}} v_k$, where $a_k$ is arbitrary weights used control the priorities
- Exponent $q$ plays different role based on the value it assumes
  - $\ell_{q=1}$ results in greedy allocation
  - $\ell_{q=2}$ ideal for the delay or buffer size limited scenarios
  - $\ell_{q=\infty}$ provides fair resource allocation in each transmission instant

# JSFRA Formulation (SINR Relaxation)

- Optimization problem with queue difference objective is nonconvex due to the constraint

$$\underset{\mathbf{m}_{l,k,n}}{\text{minimize}} \quad \left| Q_k - \sum_{n=1}^{N} \sum_{l=1}^{L} \log\left(1 + \gamma_{l,k,n}\right) \right|^q \tag{6a}$$

$$\text{subject to} \quad \gamma_{l,k,n} \leq \frac{\left| \mathbf{w}_{l,k,n}^H \, \mathbf{H}_{b_k,k,n} \, \mathbf{m}_{l,k,n} \right|^2}{\beta_{l,k,n}} \tag{6b}$$

$$\grave{N}_0 + \sum_{(j,i)\neq(l,k)} |\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_j,k,n} \mathbf{m}_{j,i,n}|^2 \leq \beta_{l,k,n} \tag{6c}$$

- The nonconvex constraints are approximated by sequence of convex subsets and solved iteratively by successive convex approximation (SCA) method
- Receive beamformers are designed by the MMSE receivers using the converged transmit precoders

# JSFRA Formulation (MSE Reformulation)

- Alternatively, we solve the queue minimization problem by utilizing the relation between the MSE and the SINR as

$$\epsilon_{l,k,n} = (1 + \gamma_{l,k,n})^{-1} \tag{7}$$

- Equivalence is valid only when the receivers are designed with the mean squared error (MSE) objective, *i.e.*, using MMSE receivers
- Problem involves nonconvex constraint

$$\begin{aligned} t_{l,k,n} &\leq -\log_2(\epsilon_{l,k,n}) &\quad (8a)\\ \epsilon_{l,k,n} &= \mathbb{E}\left[(d_{l,k,n} - \hat{d}_{l,k,n})^2\right] = \left|1 - \mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_k,k,n} \mathbf{m}_{l,k,n}\right|^2 \\ &\quad + \sum_{(j,i)\neq(l,k)} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b_i,k,n} \mathbf{m}_{j,i,n}\right|^2 + \grave{N}_0 &\quad (8b) \end{aligned}$$

# JSFRA Formulation (MSE Reformulation)

- The nonconvex constraint is approximated by a sequence of convex constraints, which is performed using SCA technique
- The iterative procedure is carried out until convergence or for suitable number of iterations
- The above reformulation works only with the MMSE receiver

# Centralized Solutions

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{2, 3, 9, 4, 2\}$

# Time Correlated Fading - Centralized Performance

Figure: System Model - $\{N, N_B, K, N_R\} = \{3, 2, 8, 1\}$ after 250 transmissions

Section 4

# Distributed Solutions

# Distributed Methods

- Small System - centralized approach is viable, provided channel remains constant for multiple transmission slots
- However, overhead involved in the centralized design scales up significantly as the network size grows
- Distributed approaches based on primal decomposition or **ADMM!** can be used to reduce the signaling requirements
- Signaling involved in the design of precoders are only scalar interference variables
- Approximated convex subproblem in each SCA step is performed via distributed methods

# Primal Decomposition Method

- Precoder design is based on master-slave approach
- Interference to the neighboring BS users are bounded by a scalar variable, treated as a constant in subproblems
- The interference thresholds are determined by the master problem and used in each slave subproblem constraint as

$$\zeta_{l,k,n,b} \geq \sum_{i \in \mathcal{U}_b} \sum_{j=1}^{L} \left|\mathbf{w}_{l,k,n}^{\mathrm{H}} \mathbf{H}_{b,k,n} \mathbf{m}_{j,i,n}\right|^2 \; \forall b \in \bar{\mathcal{B}}_{b_k}. \tag{9}$$

# ADMM based Decomposition Method

- The **ADMM!** is superior to other distributed schemes in terms of the convergence speed
- **ADMM!** includes an additional quadratic term

$$\|v_k\|_q + \sum_{l,k,n} \nu_{l,k,n}^{(j)} \left(\zeta_b - \zeta_b^{(j)}\right) + \frac{\rho}{2}\|\zeta_b - \zeta_b^{(j)}\|^2 \quad (10)$$

in objective, where $\zeta_b^{(j)}$ is global consensus variable

- $\zeta_{l,k,n,b}$ in (9) is treated as an optimization variable in **ADMM!**
- Consensus variables are updated upon exchanging corresponding local $\zeta_{l,k,n,b}$'s among coordinating BSs

# KKT based Distributed Solution

- Decentralization methods involve significant signaling exchanges via backhaul
- Overhead is large for multi-antenna receivers - iterative design should reduce the backlogged packets significantly in first few iterations
- To achieve that, we design precoders by solving the Karush-Kuhn-Tucker (KKT) equations of the JSFRA problem via MSE reformulation
- Group update of all involved optimization variables - to speed up the convergence of precoder design

# Performance of Distributed Solutions

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{3, 2, 8, 4, 1\}$

# Performance of KKT based Approach

Figure: System Model - $\{N, N_B, K, N_T, N_R\} = \{5, 2, 8, 4, 1\}$

Section 5

# Time Correlated Fading Performance

# Time Correlated Precoder Design - Introduction

- So far, we have discussed few methods to design precoders in a distributed manner
- Now, we discuss a practical approach to design precoders with minimal backhaul usage
- To discuss further, we consider KKT based design for following reasons
  - Due to closed form solution, each SCA update requires only single exchange of coupling variables
  - Moreover, transmit and receive precoders expressions can be formed by using precoded pilot transmissions
- We adopt Bi-directional training (BiT) to update transmit and receive beamformers [2]

[2] P. Komulainen, A. Tölli & M. Juntti, Effective CSI Signaling and Decentralized Beam Coordination in TDD Multi-Cell MIMO Systems, IEEE Transactions on Signal Processing, vol. 61, no. 9, pp. 2204 – 2218, May 2013

# Time Correlated Precoder Design - Strategy A

- To devise **OTA!** (**OTA!**) based iterative algorithm - let us consider transmit and receive beamformer expressions

$$\mathbf{m}_{l,k,n}^{(i)} = \left(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L}\alpha_{y,x,n}^{(i-1)}\mathbf{H}_{b_k,x,n}^{\mathrm{H}}\mathbf{w}_{y,x,n}^{(i-1)}\mathbf{w}_{y,x,n}^{\mathrm{H}\,(i-1)}\mathbf{H}_{b_k,x,n} + \delta_b\mathbf{I}_{N_T}\right)^{-1}\alpha_{l,k,n}^{(i-1)}\mathbf{H}_{b_k,k,n}^{\mathrm{H}}\mathbf{w}_{l,k,n}^{(i-1)}$$

$$\mathbf{w}_{l,k,n}^{(i)} = \left(\sum_{x\in\mathcal{U}}\sum_{y=1}^{L}\mathbf{H}_{b_x,k,n}\mathbf{m}_{y,x,n}^{(i)}\mathbf{m}_{y,x,n}^{\mathrm{H}\,(i)}\mathbf{H}_{b_x,k,n}^{\mathrm{H}} + N_0\mathbf{I}_{N_R}\right)^{-1}\mathbf{H}_{b_k,k,n}\,\mathbf{m}_{l,k,n}^{(i)}$$

- Note that transmit precoders $\mathbf{m}_{l,k,n}$ depend on $\mathbf{H}_{b_k,i,n}^{\mathrm{T}}\mathbf{u}_{l,i,n}$, *i.e.*, effective uplink channel
- Similarly, receive beamformers $\mathbf{w}_{l,k,n}$ depend on $\mathbf{H}_{b_k,i,n}\mathbf{m}_{l,k,n}$, *i.e.*, effective downlink channel

# Time Correlated Precoder Design - Strategy A

- To update receive beamformers at the user terminals, each BS transmits precoded downlink pilots orthogonally by using $\mathbf{m}_{l,k,n}^{(i)}$
- Upon receiving effective channel, each user update the respective receive beamformer
- Similarly, to update transmit beamformers at BSs, each user transmits two precoded pilot transmission, namely, $\sqrt{\alpha_{l,k,n}^{(i-1)}}\mathbf{w}_{l,k,n}^{*}$ and $\alpha_{l,k,n}\mathbf{w}_{l,k,n}^{*}$
- Upon receiving two uplink precoded pilots, each BS then evaluates the respective transmit beamformers of all users
- The above iterative procedure is performed until convergence or for predetermined number of updates - Strategy A

# Bi-directional Frame Structure

- To perform iterative **OTA!** based update, we consider **TDD!** (**TDD!**) frame format
- Using **OTA!** transmissions, effective channel can be conveyed to BSs and users by precoded pilots

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# Time Correlated Precoder Design - Strategy B

- To speed up the convergence of iterative algorithm, we update the precoders of desired users locally by fixing neighboring BSs transmit beamformers
- To do so, in addition to the signaling requirements in strategy A, inter-cell interference covariance seen by desired users is also fed back to serving BS, *i.e.*,

$$\mathbf{Z}_{l,k,n} = \sum_{i \in \mathcal{U} \setminus \mathcal{U}_b} \mathbf{H}_{b_i,i,n} \mathbf{m}_{l,i,n} \mathbf{m}_{l,i,n}^{\mathrm{H}} \mathbf{H}_{b_i,i,n}^{\mathrm{H}} + N_0 \mathbf{I}_{N_R}. \tag{11}$$

- Since inter-cell interference covariance is not required explicitly, effective whitened channel is enough to evaluate precoders
- The whitening filter $\mathbf{Q}_{l,k,n}$ is given by $\mathbf{Z}_{l,k,n}^{-1} = \mathbf{Q}_{l,k,n}^{\mathrm{H}} \mathbf{Q}_{l,k,n}$
- Whitened channel is constructed at BS by transmitting columns of $\mathbf{Q}_{l,k,n}$ orthogonally from each user

# Time Correlated Precoder Design - Strategy B

- Upon receiving $\mathbf{Q}_{l,k,n}$, each BS can construct whitened channel as $\bar{\mathbf{H}}_{b_k,k} = \mathbf{Q}_{l,k,n}\mathbf{H}_{b_k,k}$
- The BS specific local iteration $j$ to update transmit and receive beamformer is given as

$$\mathbf{m}_{l,k,n}^{(j)} = \Big( \sum_{x \in \mathcal{U}_b} \sum_{y=1}^{L} \alpha_{y,x,n}^{(j-1)} \bar{\mathbf{H}}_{b_k,x,n}^{\mathrm{H}} \mathbf{w}_{y,x,n}^{(j-1)} \mathbf{w}_{y,x,n}^{\mathrm{H}\,(j-1)} \bar{\mathbf{H}}_{b_k,x,n} + \delta_b \mathbf{I}_{N_T} \Big)^{-1} \alpha_{l,k,n}^{(j-1)} \bar{\mathbf{H}}_{b_k,k,n}^{\mathrm{H}} \mathbf{w}_{l,k,n}^{(j-1)}$$

$$\mathbf{w}_{l,k,n}^{(j)} = \Big( \sum_{x \in \mathcal{U}_b} \sum_{y=1}^{L} \bar{\mathbf{H}}_{b_x,k,n} \mathbf{m}_{y,x,n}^{(j)} \mathbf{m}_{y,x,n}^{\mathrm{H}\,(j)} \bar{\mathbf{H}}_{b_x,k,n}^{\mathrm{H}} + \mathbf{I}_{N_R} \Big)^{-1} \bar{\mathbf{H}}_{b_k,k,n}\, \mathbf{m}_{l,k,n}^{(j)}$$

- The above iterative procedure is performed until convergence or for predetermined number of updates - Strategy B

# Summary - Signaling Requirement for OTA based Updates

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Figure: **TDD!** frame structure with BiT signaling

# Assumptions and Evaluation Model

- Every BS and user terminal uses orthogonal pilots in UL and DL **OTA!** signaling
- For simplicity, pilot transmissions used to convey the equivalent channel information in one BiT iteration - consume $\eta$ resource share.[3]
- Under this assumption, the effective rate by considering the signaling overhead is given as

$$\tilde{t}_{l,k,n} = (1 - I_{\max}\,\eta) \times t_{l,k,n} \tag{12}$$

- Total number of backlogged packets is evaluated as - $\chi = \sum_{k=1}^{K} [Q_k - \tilde{t}_k]^+$
- In all simulations, we consider $\eta = 1\%$

[3] In practice, the performance depends on the amount of available pilots and the size of coherence block.

# Average Backlogged Packets - Distributed Design

Figure: Average backlogged packets for $\{N, N_B, K, N_T, N_R\} = \{3, 2, 12, 4, 2\}$ evaluated over 250 slots with $f_d T_s \approx 0.1$

# Average Backlogged Packets - Distributed Design

Figure: Average backlogged packets for $\{N, N_B, K, N_T, N_R\} = \{3, 2, 12, 4, 2\}$ evaluated over 250 slots with $f_d T_s \approx 0.01$

Section 6

# Conclusions

# Conclusions

- We studied cross layer problem of designing transmit and receive beamformers based on the number of residual packets
- Since the problem is nonconvex, we solve the problem iteratively by solving convex subproblems in each iteration
- We also proposed a practical way of implementing the precoder design in a distributed manner by solving the KKT expressions
- Extensions of the proposed work in time-correlated fading scenario with limited number of information exchange cycle is also presented

Questions !