Refinement Types:

Future of typing, now

Mistral Contrastin

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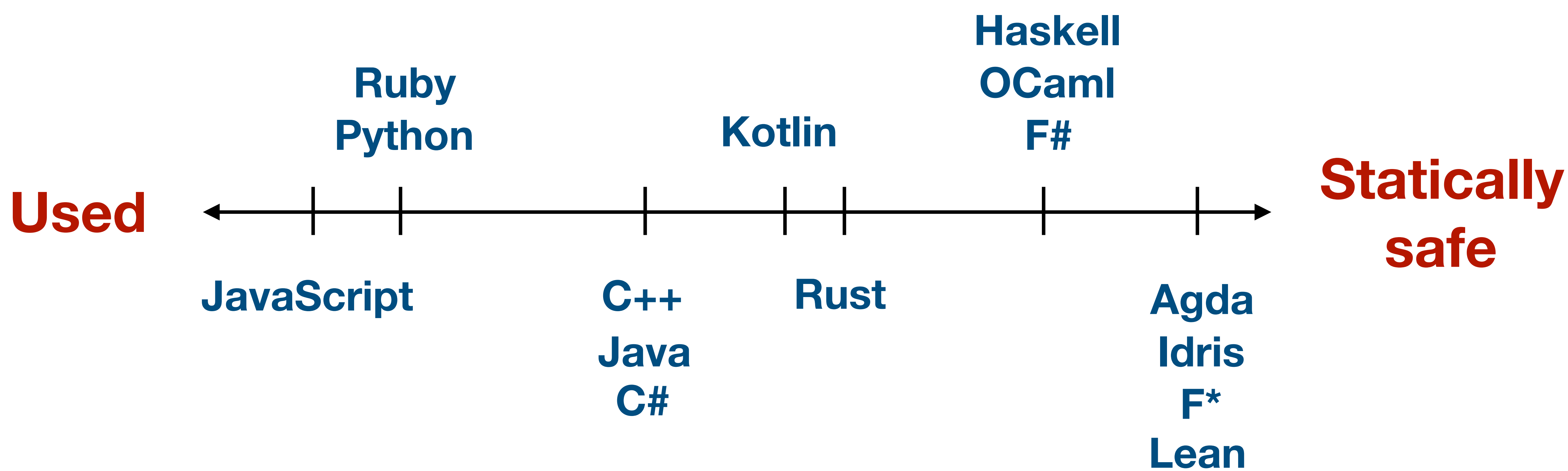
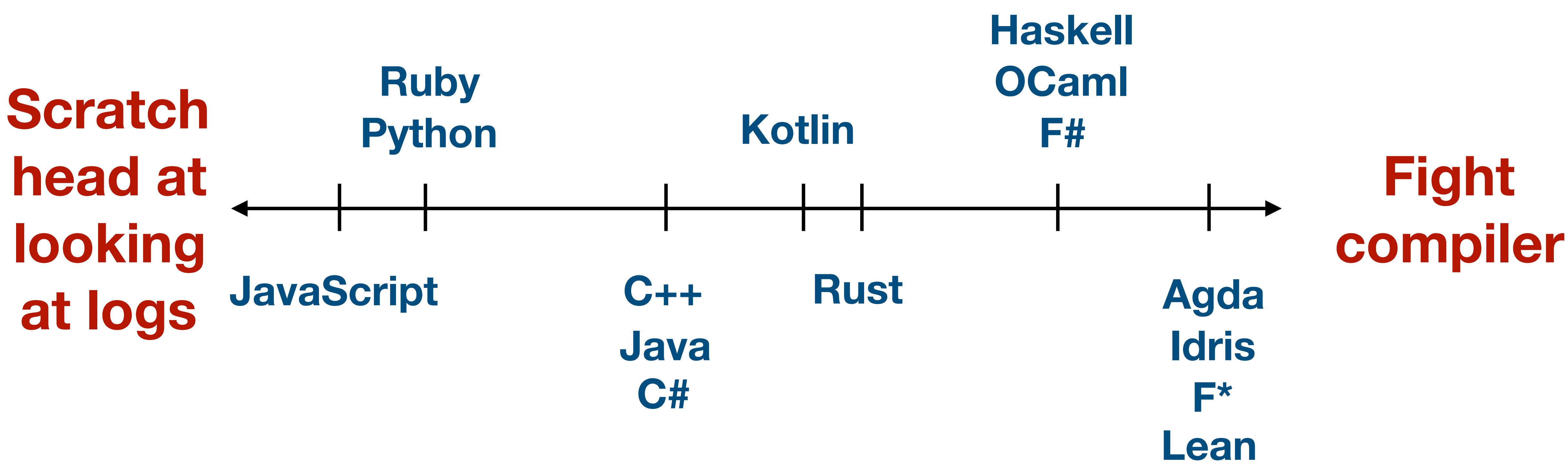
Safer you are, lonelier you get



Safety is hard work



Traditional types are rigid

- Every type is different

  `List α ≠ NonEmpty α ≠ Singleton α`

- ```
  max :: NonEmpty Int -> Int
  max = …

  max (xs :: List Int)          -- type error
  max (xs :: NonEmpty  Int) -- type checks
  max (xs :: Singleton Int) -- type error
  ```

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  ```

Refinements are fluid

- Refinement types are related with **subtyping**

  List α
  ⊆
  NonEmpty α = { v:List α | length v > 0 }
  ⊆
  Singleton α = { v:List α | length v = 1 }

- ```
  max (xs :: List Int)       -- type error
  max (xs :: NonEmpty  Int) -- type checks
  max (xs :: Singleton Int) -- type checks
  ```
  ```

Type checker with a grade school degree

What does it take to type check `max (xs :: Singleton Int)`?

Prove…

```
Singleton Int ⊆ NonEmpty Int
```

$\Leftrightarrow$

```
  { v:List α | len v = 1 }
⊆ { v:List α | len v > 0 }
```

$\Leftrightarrow$

```
(len v = 1) ⇒ (len v > 0)
```

$\Leftrightarrow$

```
1 > 0
```

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Satisfiability Modulo Theories can solve 1>0

- SMT decides on certain logical formulae
  - Boolean logic, e.g., $x \land y \Rightarrow x \lor y$
  - Theory of linear arithmetic, e.g., $x + y > 0 \Rightarrow 2\,x > -3y$
  - Theory of arrays, e.g., $arr[x \mapsto 42][x \mapsto 10] \neq 42$
  - Theory of uninterpreted functions,
    e.g., $f(g(x)) = f\,(y)$
  - …

Type checking is now context sensitive

```
if (2 * x > 2)
    then …
    else …
```

$x > 1$

$x \leq 1$

$$(-498 \times -1307) + 601$$

Abstract interpretation

$\times \rightarrow \times^{\#}$

$\times^{\#}$	0	+	-
0	0	0	0
+	0	+	-
-	0	-	+

$+ \rightarrow +^{\#}$

$+^{\#}$	0	+	-
0	0	+	-
+	+	+	?
-	-	?	-

LiquidHaskell

Demo

github.com/madgen/refinement-types-seminar