Portfolio Theory
From Concept to Standard

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Page 1 Headline

NOTHING IS EASY

Industry is pushing MPT to limits----and beyond

MPT at 50: Still making noise, 50 years later:

see story on Page 3

Page 3 Story

The House That Harry Built

Modern Portfolio Theory at 50

By Joel Chernoff

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Markowitz's "Portfolio Selection": A Fifty-Year Retrospective

Mark Rubinstein

Editors Note: The Editor wishes to thank Mark Rubinstein for agreeing to prepare this retrospective…

What happened in 1952?

What was investment theory and practice like before then?

How is “Portfolio Theory” used in practice now?

How did this happen?

“Portfolio Theory” Before 1952

See:
Markowitz, Harry M. (1999)
The Early History of Portfolio Theory:
1600 – 1960 Financial Analyst Journal
July/August, 5-16

It is sometimes said that investors did not diversify much before 1952. Not correct. See:

Shakespeare "Merchant of Venice"

> My ventures are not in one bottom trusted,
> Nor to one place; nor is my whole estate
> Upon the fortune of this present year;
> Therefore, my merchandise makes me not sad.
> - Act I, Scene 1

Captain Long John Silver in Treasure Island

> I puts it all away, some here, some there,
> none too much anywheres, by reason of suspicion.
>
> Chapter 11. Robert Louis Stevenson

What was lacking in 1952 was an adequate theory covering:

- The effect of diversification when risks are correlated
- Risk/return tradeoff on the portfolio as a whole.

See: Williams [1938], Leavens [1945].

First, some terminology:

Standard Deviation (σ)

A measure of "dispersion".

Variance (V)

= (Std Dev)^2

Correlation (ρ)

A measure of how closely two random variables go up and down together.

Covariance (cov)

Correlation $_{i\ j}$ times Std Dev $_i$ times Std Dev $_j$

Importance of Covariance

An extreme example:

A security is likely to have a high return but has a small chance of going broke. Is a small investment in this security a reasonably safe bet?

Not if the remainder of the portfolio consists of similar bets all of which will go broke at the same time.

Law of the Average Covariance

See Markowitz [1959] Chapter 5

For an equal weighted portfolio, as the number of securities held increases Portfolio Variance approaches the Average Covariance

Example 1. For uncorrelated securities
Portfolio Variance approaches zero

Example 2. If all securities have the same variance V_S, and all distinct pairs of securities have the same correlation coefficient ρ, then

Portfolio Variance V_P approaches correlation ρ times Security Variance V_S

Portfolio Std Dev σ_P approaches square-root of correlation ρ times Security Std Dev σ_S

I.e.,	If ρ =	Then σ_P/σ_S =
	.25	.500
	.10	.316

Even with unlimited diversification!

Williams, John B. [1938], *The Theory of Investment Value*,
Harvard University Press, Cambridge, Mass.

Most of the ideas in Markowitz [1952] popped up one day in 1950 when Markowitz read Williams [1938]

Williams asserted:

The value of a stock should be the present value of its future dividends.

Markowitz thought:

Future is uncertain; must mean <u>expected</u> present value.

If one is only interested in the expected values of securities, then one must only be interested in the expected value of the portfolio.

One maximizes expected value by putting all resources into the single security with greatest expected value.

Diversification exists and is good.

See Wiesenberger.

Clearly investors (and investment companies) seek return, avoid risk. Let's measure these by expected value and standard deviation, like statisticians do.

Two criteria: let's draw a tradeoff curve.

Markowitz (Ph.D. candidate, Economics, University of Chicago; zero investment experience) was currently taking a course by Koopmans on **efficient** and **inefficient** allocation of resources. Hence efficient versus inefficient portfolios.

Portfolio return is a weighted sum of security returns. What is the formula for the variance of a weighted sum?

Formula supplied by Uspensky [1937]. Wow! Covariance's!

Later in the book Williams addresses the issue of risk and uncertainty:

> "Whenever the value of a security is uncertain and has to be expressed in terms of probability, the correct value to choose is the mean value....The customary way to find the value of a risky security has always been to add a "premium for risk" to the pure interest rate, and then use the sum as the interest rate for discounting future receipts. In the case of the bond under discussion, which at 40 would yield 12 per cent to maturity, the "premium for risk" is 8 per cent when the pure interest rate is 4 per cent.
>
> Strictly speaking, however, there is no risk in buying the bond in question if its price is right. Given adequate diversification, gains on such purchases will offset losses, and a return at the pure interest rate will be obtained. Thus the net risk turns out to be nil. To say that a "premium for risk" is needed is really an elliptical way of saying that payment of the full face value of interest and principal is not to be expected on the average".
>
> pp.67,68.

Leavens, D. H. [1945], “Diversification of Planning”,
Trusts and Estates, 80, May, pp. 469-73.

An examination of some fifty books and articles on investment that have appeared during the last quarter of a century shows that most of them refer to the desirability of diversification. The majority, however, discuss it in general terms and do not clearly indicate why it is desirable.

Leavens illustrated the benefits of diversification on the assumption that risks are independent. However, the last paragraph of Leavens cautioned:

The assumption, mentioned earlier, that each security is acted upon by independent causes, is important, although it cannot always be fully met in practice. Diversification among companies in one industry cannot protect against unfavorable factors that may affect the whole industry; additional diversification among industries is needed for that purpose. Nor can diversification among industries protect against cyclical factors that may depress all industries at the same time.

Portfolio Theory
1952

Markowitz, Harry M. (1952)
Portfolio Selection, The Journal of Finance,
7(1), March, 77-91

Roy, A. D. (1952), *Safety First and the Holding of Assets"*
Econometrica, 20, pp.431-49

Markowitz, H. [1952], "Portfolio Selection",
The Journal of Finance, Vol. 7, No. 1, pp. 77-91.

Proposes mean (a.k.a. expected) return and variance of return on portfolio as a whole as criteria for portfolio selection.

Distinguishes between efficient and inefficient portfolios.

Proposes that means, variances and covariances of securities be estimated by a combination of statistical analysis and security analyst judgment (no details supplied), and the set of mean-standard deviation combinations implied by these be presented to investor for choice of desired risk-return combination.

Uses geometric analysis of 3- and 4-security examples to illustrate properties of efficient sets, assuming non-negative investments subject to budget constraint.

Computing procedures for large number of securities subject to various constraints presented in Markowitz [1956]

Roy, A.D. [1952], "Safety First and the Holding of Assets",
Econometrica, 20, pp. 431-49.

Proposes portfolio choice based on mean and variance of portfolio as a whole.

Specifically proposes choosing portfolio which maximizes

$$\frac{m-d}{\sigma}$$

Where m is mean return, σ is standard deviation and d is a fixed (for the analysis) disastrous return.

Why did Markowitz get a Nobel Prize and not Roy?

Differences between Markowitz [1952] and Roy [1952]??

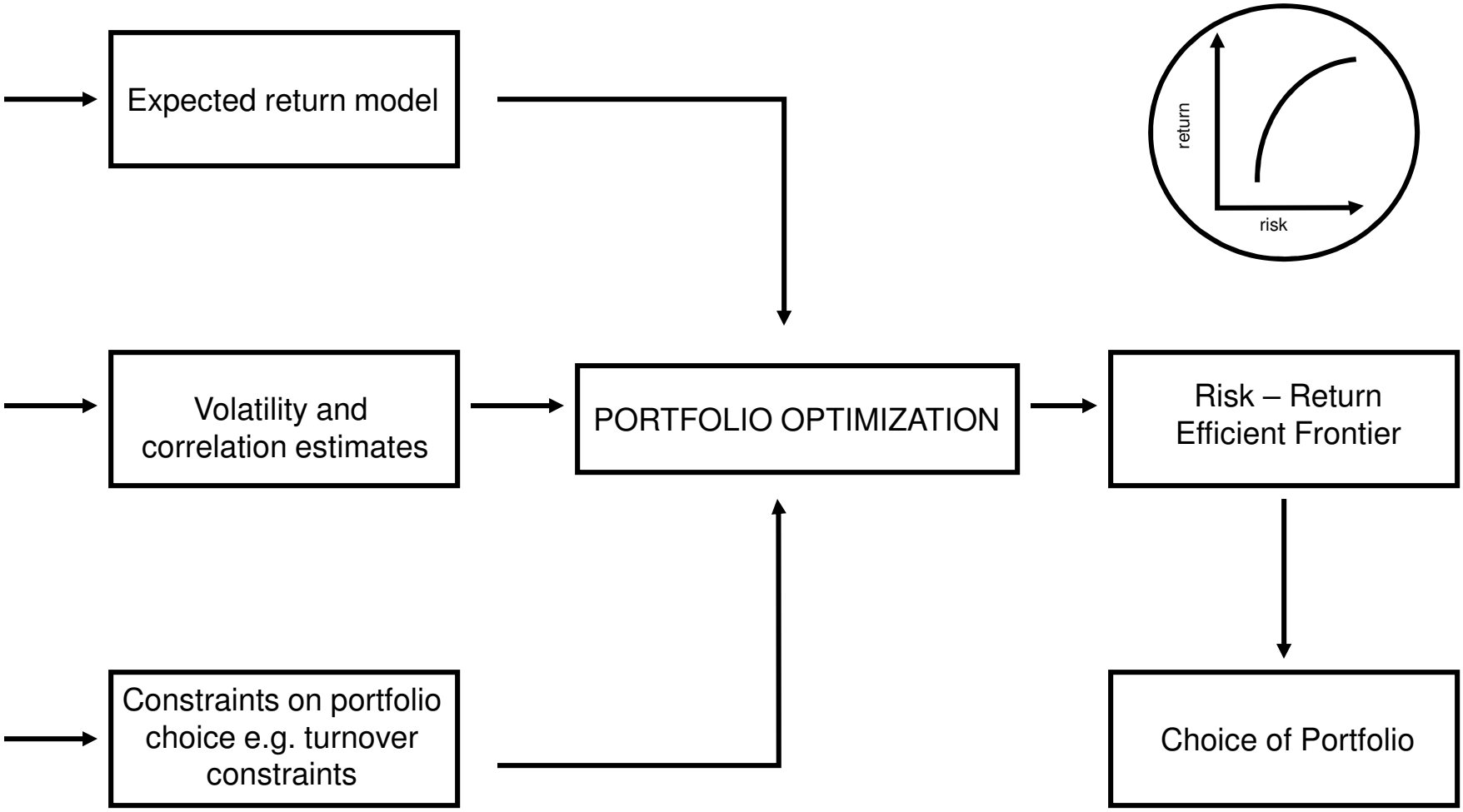
More likely: visibility to Nobel Committee in 1990

Roy [1952] is Roy's first and last paper in finance

Markowitz also wrote Markowitz [1959, 1987] and assorted articles.

Portfolio Theory in Practice
A Half-Century Later

MPT Investment Process



Uses of Portfolio Theory in Investment Management

1. **Full use** (by some quantitative managers)

Expected returns

Usually by a proprietary expected return model

Covariances

Historical or factor model

Constraints

Upper bounds

Turnover

Liquidity

Some rule for picking portfolio from efficient frontier.

2. Top down, asset class, analysis

a. Select asset classes

e.g., large cap stocks, small cap stocks,
long term bonds, etc.

b. Review history (e.g., Ibbotson)

c. Make estimates

e.g., will equity premium be as high in the next
few years as it has been "recently"?

d. Generate frontier

e. Pick portfolio-perhaps use Monte Carlo to help see consequences.

f. Implement

Index funds
Active managers

3. Risk Control and Attribution Analysis

a. Arrive at current or proposed portfolio in any manner

b. Beforehand, determine exposures to factors using model of covariance developed for MPT

c. Afterwards, ascribe performance to good/bad style bets versus good/bad selection within style.

The difference between 1952 and now

Infrastructure

Data
- Return Series
- Other

Models of
- Expected Return
- Covariance

Optimizers

Hand-holders, etc.

Now

Not everyone uses portfolio theory

But it *is* widely used:

- in whole or in part
- by itself or with other techniques
- at an aggregate or detailed level
- for institutions or families

In particular ...

The Use of Mathematical/Computer-Based Analysis

The secret of MPT's success

IT WORKS