08-03-2022 Introduction to double and change of order of integration

03 March 2022 11:07

Unit - I

→ Double Integration

→ change of order of integration

→ Change of polar Coordinates

→ Triple Integration

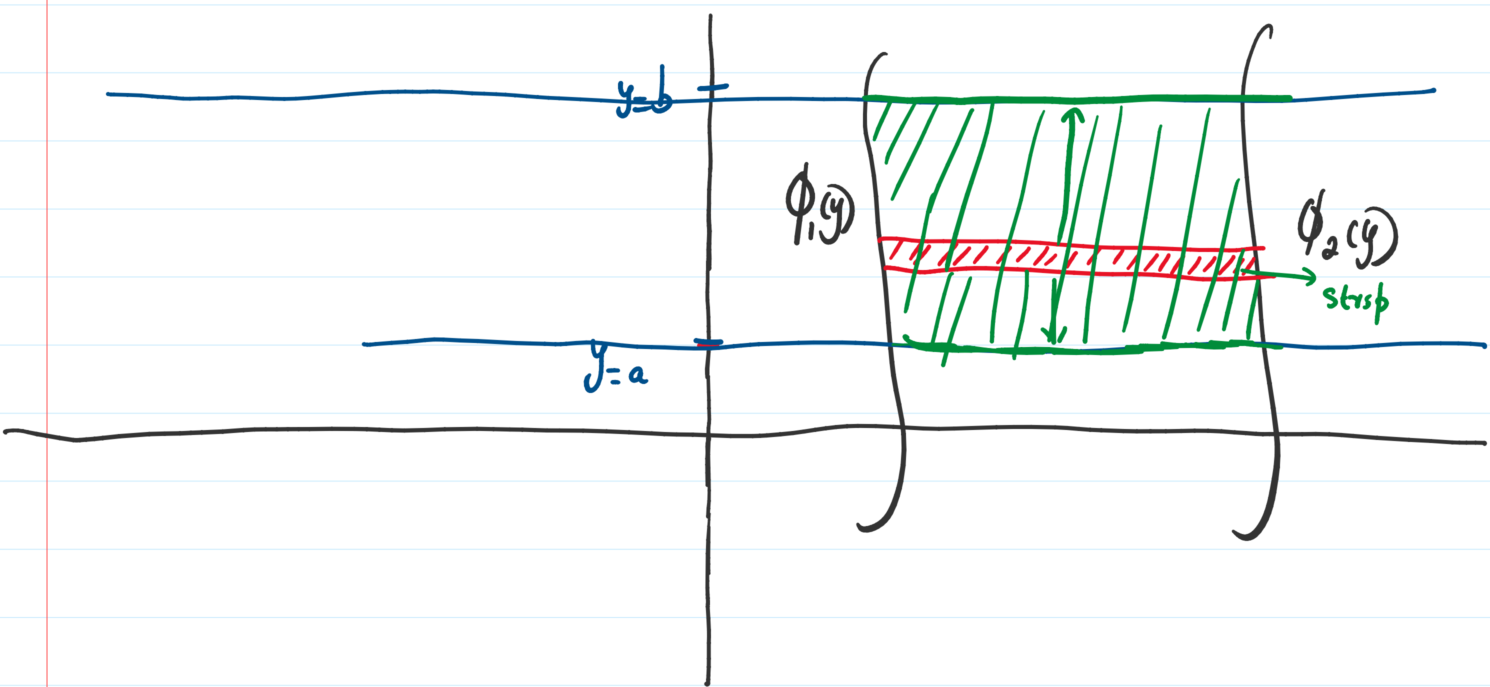
Double Integration :

length of the strip

horizontal strip

$$\int_{y=a}^{y=b} \underbrace{\int_{x=\phi_1(y)}^{x=\phi_2(y)} f(x,y)\, dx}_{\text{Inner integral}}\, dy$$

outer integral



Find the Value of integral ... nd or right side ... l.h.b

Find the Value of integral

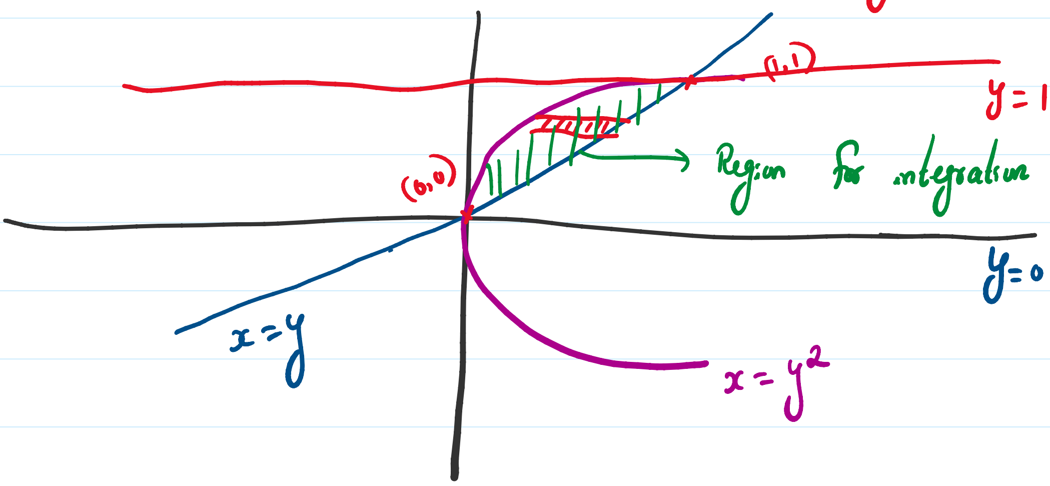
$x=y \rightarrow$ upper bound or right side

$\rightarrow$ horizontal strip

$$\int_{y=0}^{1}\int_{x=y^2}^{x=y} xy \; \underline{\underline{dx}} \; dy$$

$x=y^2 \rightarrow$ lower bound or left side

Solution :- First we have to identify the region



$x=y$

$x=y^2$

$x=x^2$

$x^2-x=0$

$x(x-1)=0$

$x=0\,,\; x=1$

$x=0 \Rightarrow y=0$

$x=1 \Rightarrow y=1$

$$\int_{y=0}^{1}\boxed{\int_{x=y^2}^{y} xy\,dx}\,dy \rightarrow \text{Integrate with respect to 'x' treating 'y' as constant}$$

$$= \int_{y=0}^{1} y\left[\frac{x^2}{2}\right]_{y^2}^{y} dy$$

$$= \int^{1} y\,\left[\,y^2 - y^4\,\right] dy$$

$$= \int_{y=0} \frac{y}{2} \left[y^2 - y^4 \right] dy$$

$$= \frac{1}{2} \int_{y=0}^{1} \left[y^3 - y^5 \right] dy$$

$$= \frac{1}{2} \left[\frac{y^4}{4} - \frac{y^6}{6} \right]_0^1$$

$$= \frac{1}{2} \left[\frac{1}{4} - \frac{1}{6} - 0 \right]$$

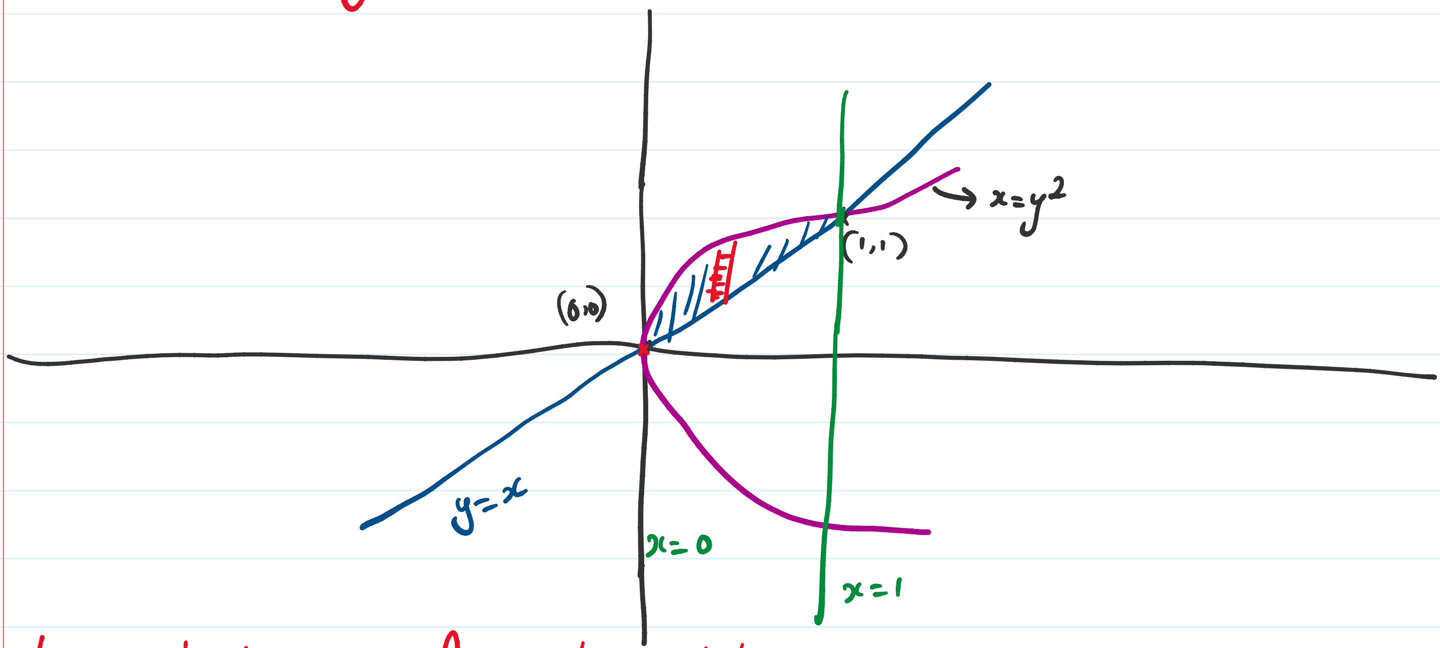
$$= \frac{1}{2} \left[\frac{3-2}{12} \right]$$

$$= \frac{1}{24}$$

The Same integration by Change of order of integration.

$$\int_{y=0}^{1} \int_{x=y^2}^{x=y} xy \underbrace{dx\, dy}_{\text{given order}} = \int_{x=0}^{x=1} \int_{y=x}^{y=\sqrt{x}} xy \underbrace{dy\, dx}_{\text{vertical strip}}$$

To change the order from $dx\,dy \to dy\,dx$.



Lower bound of the Vertical strip $y=x$

upper bound of the Vertical strip $y^2=x \quad y=\pm\sqrt{x}$

$y=+\sqrt{x}$ $\because$ we are in the first Cordinat

$$\int_{x=0}^{1}\int_{y=x}^{y=\sqrt{x}} xy \, dy\, dx$$

$$= \int_{x=0}^{1} x \int_{y=x}^{\sqrt{x}} y\, dy \, dx$$

$$= \int_{x=0}^{1} x \left[\frac{y^2}{2}\right]_x^{\sqrt{x}} dx$$

$$= \int_{x=0}^{1} \frac{x}{2}\left[(\sqrt{x})^2 - x^2\right] dx$$

$$= \frac{1}{2}\int^{1} x\,(x - x^2)\, dx$$

$$= \frac{1}{2} \int_{x=0}^{1} x\,(x - x^2)\, dx$$

$$= \frac{1}{2} \int_{x=0}^{1} (x^2 - x^3)\, dx$$

$$= \frac{1}{2} \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{4} - 0 \right]$$

$$= \frac{1}{2} \left[\frac{4-3}{12} \right]$$

$$= \frac{1}{24}$$

Home Work

1) Evaluate $\int_0^2 \int_0^1 4xy \, dx\, dy$

ii) Evaluate $\iint (x^2+y^2)\, dx\, dy$ over the area of the triangle whose vertices are $(0,1)$ $(1,1)$ $(1,2)$

iii) Change the order and evaluate $\int_{x=0}^{x=\infty} \int_{y=x}^{y=\infty} \frac{e^{-y}}{y}\, dy\, dx$