Matrices

1. Simple matrices
2. Determinants $\longrightarrow$ (2A) Rank of a matrix
3. System of eqns. $\longrightarrow$ Matrix inversion method, $X = A^{-1}B$

   $\longrightarrow$ Cramer's rule

   $\longrightarrow$ Jordan method

   $\longrightarrow$ Gauss elimination method, $A = IA$

4. Gauss Jordan method to inverse of a matrix
5. Linear dependence ~~&~~ of vectors.

$a_{50000,30002}$

$R_{50000} \rightarrow R_{50000} - KR_{30002}$

$C_{30002} \rightarrow C_{30002} - \lambda C_{50000}$

General Method $a_{ij} \rightarrow 0$

$R_i \rightarrow R_i - KR_j$

$C_j \rightarrow C_j - \lambda C_i$

Gauss Jordan Method to find inverse of matrix.

(a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$ Find A^{-1}.

Sol: $A = IA$

$$\Rightarrow \begin{array}{c} \\ a_{21} \\ a_{31} \\ a_{41} \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$\boxed{AA^{-1} = I}$$

Apply $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} A$$

Interchanging $R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} A$$

$R_2 \to \dfrac{R_2}{-2}$

$$\begin{array}{c} \begin{array}{cccc} & a_{12} & & \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \tfrac12 & 0 & 0 & -\tfrac12 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} \tfrac12 & 0 & 0 & \tfrac12 \\ \tfrac12 & 0 & 0 & -\tfrac12 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} A$$

$R_3 \rightarrow R_3/-2$

$R_4 \rightarrow R_4/-2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix} A$$

$$I = A^{-1} A$$

$$\therefore A^{-1} = \begin{bmatrix} -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{bmatrix}$$

(2) Find inverse of $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$

$A = IA$

$$\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_1 \longrightarrow R_1 - R_2$

$$\begin{bmatrix} -1 & -2 & -2 & 0 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_1 \rightarrow \frac{R_1}{-1}$

$$\begin{bmatrix} 1 & 2 & 2 & 0 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_2 \longrightarrow R_2 - 3R_1$

$R_3 \longrightarrow R_3 - 2R_1$

$R_4 \longrightarrow R_4 - 4R_1$

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$$\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & 14 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 4 & -4 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 + R_3$ or $\rightarrow R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & 14 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 5 & -4 & 1 & 0 \\ 2 & -2 & 1 & 0 \\ 4 & -4 & 0 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2$

$R_3 \rightarrow R_3 - R_2$

$R_4 \rightarrow R_4 + 3R_2$

$$\begin{bmatrix} 1 & 0 & 8 & 2 \\ 0 & 1 & -3 & -1 \\ & & & \\ & & & \end{bmatrix} = \begin{bmatrix} & & & \\ 5 & -4 & 1 & 0 \\ & & & \\ & & & \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -1 & 2 \\ 0 & -3 & 6 & 14 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 3 & -2 & 0 & 0 \\ 4 & -4 & 0 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2$

$R_4 \rightarrow R_4 + 3R_2$

$$\begin{bmatrix} 1 & 0 & 6 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 5 & -2 & 0 \\ 2 & -2 & 1 & 0 \\ 3 & -2 & 0 & 0 \\ 10 & -10 & 3 & 1 \end{bmatrix} A$$

$$R_3 \rightarrow (-1) R_3$$

$$\begin{bmatrix} 1 & 0 & 6 & 6 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 5 & -2 & 0 \\ 2 & -2 & 1 & 0 \\ -3 & 2 & 0 & 0 \\ 10 & -10 & 3 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 6R_3$$
$$R_2 \rightarrow R_2 + 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 13 & -7 & -2 & 0 \\ -4 & 2 & 1 & 0 \\ -3 & 2 & 0 & 0 \\ 10 & -10 & 3 & 1 \end{bmatrix} A$$

$$R_4 \rightarrow \frac{R_4}{5}$$

$$\begin{bmatrix} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 13 & -7 & -2 & 0 \\ -4 & 2 & 1 & 0 \\ -3 & 2 & 0 & 0 \\ 2 & -2 & 3/5 & 1/5 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 18R_4$$
$$R_2 \rightarrow R_2 + 7R_4$$
$$R_3 \rightarrow R_3 + 2R_4$$

$1+7\times\frac{3}{5}$

$1+\frac{21}{5}$

$36-5\times\frac{21}{5}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -23 & 29 & -64/5 & -18/5 \\ 10 & -12 & 26/5 & 7/5 \\ 1 & -2 & 6/5 & 2/5 \\ 2 & -2 & 3/5 & 1/5 \end{bmatrix} A$$

$\Rightarrow$ **Define Elementary Transformations**

sol Followings are called elementary operations transformations.

① Multiplying any row or column by a scalar.

For e.g.

$$R_5 \longrightarrow \underbrace{3}_{\text{scalar}}R_5$$

$$C_2 \rightarrow 7C_2$$

② Interchanging any two rows or columns.

For e.g. $R_3 \leftrightarrow R_1$

$C_3 \leftrightarrow C_2$

③ Adding or Subtracting elements of a row (or column) to k times corresponding elements of any other row (or columns).

e.g. $R_3 \rightarrow R_3 + kR_2$

$C_4 \rightarrow C_4 - kC_2$

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HOME WORK

Problems

① $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ ② $\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$

③ $\begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

Sol ① $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

$$A = IA$$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow (-1) R_1$$

$$\begin{bmatrix} 1 & 3 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - R_1$$
$$R_3 \rightarrow R_3 - 2R_1$$
$$R_4 \rightarrow R_4 + R_1$$

$$\begin{bmatrix} 1 & 3 & -3 & 1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2/-2$$

$$\begin{bmatrix} 1 & 3 & -3 & 1 \\ 0 & 1 & -1 & 1/2 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 3R_2$$
$$R_3 \rightarrow R_3 + 11R_2$$
$$R_4 \rightarrow R_4 - 4R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & -1 & 1/2 \\ 0 & 0 & -3 & 1/2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 \\ 7/2 & -11/2 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} A$$

$$R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & -1 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 7/2 & -11/2 & 1 & 0 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 + R_3$$
$$R_4 \rightarrow R_4 + 3R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 0 & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 & 1 \\ 1 & 2 & 0 & 1 \\ \frac{13}{2} & \frac{1}{2} & 1 & 3 \end{bmatrix} A$$

Apply $R_4 \to 2R_4$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 0 & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 13 & 1 & 2 & 6 \end{bmatrix} A$$

Apply $R_2 \to R_2 - \left(\frac{1}{2}\right)R_4$

$R_1 \to R_1 + \left(\frac{1}{2}\right)R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 2 & 1 & 3 \\ -6 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ 13 & 1 & 2 & 6 \end{bmatrix} A$$

$$I = A^{-1}A$$

$$A^{-1} = \begin{bmatrix} 7 & 2 & 1 & 3 \\ -6 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ 13 & 1 & 2 & 6 \end{bmatrix}$$

(2) $\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$

Sol $A = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$

$$A = IA$$

$$\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2$$

$9-2\times5$
$9-10$
$7-2(-1)$
$7+2$
$3-2(4)$
$3-8$

$$\begin{bmatrix} -1 & 9 & -5 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow (-1)R_1$$

$0-3(2)$

$$\begin{bmatrix} 1 & -9 & 5 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_2 \rightarrow R_2 - 5R_1$
$R_3 \rightarrow R_3 - 3R_1$

$-1-5(-9)$
$-1+45$
$4-5(5)$
$4-25$
$4-3(-9)$
$4+27$
$1-3(5)$
$1-15$

$$\begin{bmatrix} 1 & -9 & 5 \\ 0 & 44 & -21 \\ 0 & 31 & -14 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 5 & -9 & 0 \\ 3 & -6 & 1 \end{bmatrix} A$$

$R_2 \to R_2/44$

$$\begin{bmatrix} 1 & -9 & 5 \\ 0 & 1 & -21/44 \\ 0 & 31 & -14 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 5/44 & -9/44 & 0 \\ 3 & -6 & 1 \end{bmatrix} A$$

$-6-3$

-264

$1-31$

$R_3 \to R_3 - 31R_2$

$R_1 \to R_1 + 9R_2$

$$\begin{bmatrix} 1 & 0 & \frac{51}{44} \\ 0 & 1 & \frac{-21}{44} \\ 0 & 0 & \frac{35}{44} \end{bmatrix} = \begin{bmatrix} 1/44 & 7/44 & 0 \\ 5/44 & -9/44 & 0 \\ \frac{-23}{44} & \frac{15}{44} & 1 \end{bmatrix} A$$

$R_3 \to R_3 \left(\frac{44}{35}\right)$

$$\begin{bmatrix} 1 & 0 & 51/44 \\ 0 & 1 & -21/44 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/44 & 7/44 & 0 \\ 5/44 & -9/44 & 0 \\ -23/35 & 3/7 & 44/35 \end{bmatrix} A$$

Apply $R_1 \to R_1 - \frac{51}{44} R_3$

$R_2 \to R_2 + \frac{21}{44} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ & & \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ -23/35 & 3/7 & 44/35 \end{bmatrix} A$$

(3) $\begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

Sol

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = IA$$

$$\begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & -1 \\ 4 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

Apply $R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 4R_1$

2(3)

(-1)

3)

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & -7 \\ 0 & 3 & -11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -4 \end{bmatrix} A$$

Apply $R_2 \rightarrow R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & -1 & 4 \\ 0 & 3 & -11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 2 \\ 1 & 0 & -4 \end{bmatrix} A$$

Apply $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -4 \\ 0 & 3 & -11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & -2 \\ 1 & 0 & -4 \end{bmatrix} A$$

Apply $R_1 \rightarrow R_1 + R_2$
$R_3 \rightarrow R_3 - 3R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -2 \\ -2 & 3 & 2 \end{bmatrix} A$$

Apply $R_2 \rightarrow R_2 + 4R_3$
$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ -7 & 11 & 6 \\ -2 & 3 & 2 \end{bmatrix} A$$

$$I = A^{-1} A$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ -7 & 11 & 6 \\ -2 & 3 & 2 \end{bmatrix}$$

(4) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$$A = IA$$

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

-3 1 0 2

Apply $R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} A$$

Apply $R_3 \longrightarrow R_3 - R_2$

$R_4 \longrightarrow R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & -3 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} A$$

NORMAL FORM - can have rectangular matrix
By applying elementary transformations, a matrix can be converted one of the following forms.

① I_r i.e. identity matrix of order r.

② $\begin{bmatrix} I_p & 0 \end{bmatrix}$ ③ $\begin{bmatrix} I_p \\ 0 \end{bmatrix}$

④ $\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$

Working rule to find Rank using Normal form.
~~Rank~~

No. of non-zero rows in normal form is equal rank of matrix.

Ques Convert to normal form and find rank.

$$\begin{bmatrix} 3 & -2 & -1 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -3 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

Sol Let $A = \begin{bmatrix} 3 & -2 & -1 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -3 \\ 0 & 1 & 2 & 2 \end{bmatrix}$

Apply $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -2 & -3 & -3 \\ 0 & 2 & 2 & 1 \\ 3 & -2 & -1 & -1 \\ 0 & 1 & 2 & 2 \end{bmatrix} =$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & -2 & -3 & -3 \\ 0 & 2 & 2 & 1 \\ 0 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 4R_4$

$$\begin{bmatrix} 1 & -2 & -3 & -3 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$\to a_{42}$

$$\begin{bmatrix} 1 & -2 & -3 & -3 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \rightarrow C_2 + 2C_1$
$C_3 \rightarrow C_2 + 3C_1$
$C_4 \rightarrow C_4 + 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{C_2 \rightarrow}$ $R_3 \rightarrow R_3 - 2R_2$

$$\begin{array}{cl} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} \rightarrow a_{23} \\ \rightarrow a_{24} \\ \\ \\ \end{array} \end{array}$$

$C_3 \rightarrow C_3 - 2C_2$
$C_4 \rightarrow C_4 - 2C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_3 \to R_3/-2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - \frac{3}{2} C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{It is } \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

Since normal form of the matrix contains three non-zero rows, so rank of A is 3. i.e. $\rho(A) = 3$

Que - Convert to normal form: $\begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Sol $A = \begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Apply $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 4 & 1 & 2 \\ 5 & -13 & -4 & -7 \\ 2 & -6 & -2 & -3 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \to (-1)R_1$

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 5 & -13 & -4 & -7 \\ 2 & -6 & -2 & -3 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 - 5R_1$

a_{12}

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 7 & 1 & 3 \end{bmatrix}$$

Apply $C_2 \to C_2 + 4C_1$
$C_3 \to C_3 + C_1$
$C_4 \to C_4 + 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 7 & 1 & 3 \end{bmatrix}$$

Apply $R_3 \to R_3 - 2R_2$
$R_4 \to R_4 - 7R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Apply $R_4 \to (-1)R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_4$
$R_3 \to R_3 + 4R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is I_4

Since normal form of matrix contains four non-zero rows, so rank of A is 4 . i.e. $\rho(A) = 4$

⇒ Result → If A is a matrix of order $m \times n$, then there exists two non-singular sq. matrices (whose determinent is non zero) P of order $m \times m$ & Q of order $n \times n$.

such that PAQ is normal form of A

Then $A^{-1} = QP$

Qu $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ Find P & Q such that PAQ is normal also find rank of A.

⇒ A^{-1} could not exist as it is rect. matrix.

Sol $A = IAI$

⇒ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Apply $R_2 \to R_2 + 2R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~Apply~~ $\cancel{C_2 \to C_2 - 2C_1}$

$\cancel{C_3 \to C_3 + C_1}$

$\cancel{C_4 \to C_4 - 3C_1}$

$$\cancel{\begin{bmatrix} & & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 1 & \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$C_3 \to C_3 + C_1$

$C_4 \to C_4 + 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 2 & 1 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \leftrightarrow C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow (-1) R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -3 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$C_3 \rightarrow C_3 + 2C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -3 & -1 & 1 \end{bmatrix}}_{P} A \underbrace{\begin{bmatrix} 1 & -3 & -5 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}}_{Q}$$

$\rho(A) = 2$

It is $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & -3 & -5 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

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(1) Find the inverse of following using Gauss Jordon method.

(i) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

(2) Find the rank of matrices after reducing to normal form.

(i) $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

(3) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrix, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Also find A^{-1}, (if it exists)

means first find its determinant

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(1) (i) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Sol $A = IA$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

Apply $R_2 \to R_2/2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$\cancel{R_2}$ $C_2 \to C_2 - C_1$
$C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1/2 & 1/2 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

Apply $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1/2 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix} A$$

Apply $R_3 \to R_3/-4$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 \end{bmatrix} A$$

Apply $R_2 \to R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ 1/4 & 5/4 & 3/4 \\ 1/4 & 1/4 & 1/4 \end{bmatrix} A$$

$$I = A^{-1} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & -3 \\ 1/4 & 5/4 & 3/4 \\ 1/4 & 1/4 & 1/4 \end{bmatrix}$$

(ii) $A = \begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Sol: $A = IA$

$$\begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 4 & 1 & 2 \\ 5 & -13 & -4 & -7 \\ 2 & -6 & -2 & -3 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Now apply $R_1 \to (-1)R_1$

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 5 & -13 & -4 & -7 \\ 2 & -6 & -2 & -3 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_2 \to R_2 - 5R_1$
$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 0 & 7 & 1 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 5 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & -4 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 7 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix} A$$

Apply $R_1 \to R_1 + 4R_2$
$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 - 7R_2$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & -2 \\ 0 & 1 & 5 & -7 \end{bmatrix} A$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 5 & -7 \\ 1 & 0 & 2 & -2 \end{bmatrix} A$$

Apply $R_4 \to (-1)R_4$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 5 & -7 \\ -1 & 0 & -2 & 2 \end{bmatrix} A$$

Apply $R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 5 & -7 \\ -1 & 0 & -2 & 2 \end{bmatrix} A$$

Apply $R_1 \to R_1 + 2R_4$
$R_2 \to R_2 - R_4$
$R_3 \to R_3 + 4R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -4 & 1 & -3 & 1 \\ -1 & 0 & -2 & 2 \end{bmatrix} A$$

$$I = A^{-1}A$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -4 & 0 & -3 & 1 \\ -1 & 0 & -2 & 2 \end{bmatrix}$$

(2) $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$

Sol Apply $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & -2 \\ 0 & -5 & 7 \end{bmatrix}$$

3(4)
12
-3(5)
15

Apply $C_2 \to C_2 - 4C_1$
$C_3 \to C_3 - 5C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -5 & 7 \end{bmatrix}$$

Apply $R_2 \to R_2/-2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -5 & 7 \end{bmatrix}$$

Apply $R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 12 \end{bmatrix}$$

Apply $R_3 \to R_3/12$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is I_3

As normal form of matrix contains three non-zero rows, ∴ rank of A is 3,
i.e. $\rho(A) = 3$

(ii) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Sol,

Apply $R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$1-2(-2)$, $1+4$, $1-2(-4)$, $-1+8$, $1-3(-1)$, $1+3$, $3-3(-2)$, $3+6$, $-2-3(-4)$, $-2+12$, $3-6(-1)$, $3+6$, $0-6(-2)$, $7-6(-4)$, $-7+24$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

Apply $C_2 \to C_2 + C_1$
$C_3 \to C_3 + 2C_1$
$C_4 \to C_4 + 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

Apply $R_3 \to R_3 - 4R_2$
$R_4 \to R_4 - 9R_2$

$9-4\ldots$, $9+2\ldots$, $10-4\ldots$, $10+1\ldots$, $12-9\ldots$, $12+54\ldots$, $17-9\ldots$, $17+2\ldots$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

Apply $R_3 \to R_3/33$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

$-3+6\times\frac{2}{3}$, $-3+4$, $44-66\times\frac{22}{\ldots}$

Apply $R_2 \to R_2 + 6R_3$
$R_4 \to R_4 - 66R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \leftarrow a_{24} \\ \leftarrow a_{34} \\ \\ \end{matrix}$$

Apply $C_4 \to C_4 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply $C_4 \to C_4 - \frac{2}{3}C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It is I_3. $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

As normal form contains three non-zero rows, $\therefore$ rank of matrix is 3

i.e. $\rho(A) = 3$

(3) $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$A = IAI$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1$

$-3(-1)$
$+3$
$-3(-1)$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply ~~R_2~~ $C_2 \to C_2 + C_1$
$C_3 \to C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2/2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ -1 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $C_3 \to C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ -5 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = 2$

It is $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ -1 & -2 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^{-1} = QP$

Echelon form:- $\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \\ 0 & 0 & 0 & j \end{bmatrix}$

$\rho(A) =$ no. of non-zero rows in Echelon form

Ques For e.g.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 6 & 8 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

They are related to each other

$R_3 \rightarrow R_3 - \frac{1}{2} R_2$

$\rho(A) = 2$

Ques Find rank of

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 1 & 0 & -1 & 1 & 1 \\ 3 & -1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 2 & 9 \\ 3 & 1 & 0 & 3 & 9 \end{bmatrix}$$

Sol

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 + R_1$

$R_5 \rightarrow R_5 - 3R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & -2 & 2 & -3 & -8 \\ 0 & -7 & 10 & -12 & -28 \\ 0 & 3 & -3 & 6 & 18 \\ 0 & -5 & 9 & -9 & -18 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_4$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & 1 & -1 & 3 & 10 \\ 0 & -7 & 10 & -12 & -28 \\ 0 & 3 & -3 & 6 & 18 \\ 0 & -5 & 9 & -9 & -18 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 7R_2$

$R_4 \rightarrow R_4 - 3R_2$

$R_5 \rightarrow R_5 + 5R_2$

$-12+21$

$-28+70$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & 1 & -1 & 3 & 10 \\ 0 & 0 & 3 & 9 & 42 \\ 0 & 0 & 0 & -3 & -12 \\ 0 & 0 & 4 & 6 & 32 \end{bmatrix}$$

$R_5 \rightarrow R_5/2$

$R_4 \rightarrow R_4/-3$

Apply $R_3 \rightarrow R_3/3$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & 1 & -1 & 3 & 10 \\ 0 & 0 & 1 & 3 & 14 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & 3 & 16 \end{bmatrix}$$

$16-28$

Apply $R_5 \rightarrow R_5 - 2R_3$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & 1 & -1 & 3 & 10 \\ 0 & 0 & 1 & 3 & 14 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -3 & -12 \end{bmatrix}$$

$R_5 \rightarrow R_5 + 3R_4$

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 0 & 1 & -1 & 3 & 10 \\ 0 & 0 & 1 & 3 & 14 \\ 0 & 0 & 0 & +1 & +4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since no. of non-zero rows in Echelon form is 4.

$\rho(A) = 4$

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COLUMN OPERATIONS ARE NOT ALLOWED

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SYSTEM OF EQNS. → No column operations are allowed

Four methods
1. Matrix inversion method.
2. Cramer's rule.
3. Gauss elimination method
4. Gauss jordan method.

Gauss elimination & Gauss ~~elim~~ Jordan Method

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

Augmented matrix

$$[A:B] = \left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

Rules for consistency of system :-

1. If rank of A i.e. $\rho(A) = \rho(A:B) =$ no of unknowns or variables

then The system is consistent & has unique soln.

2. If $\rho(A) \neq \rho(A:B)$ ~~< no of unknowns~~

then system is inconsistent & it does not have a soln.

3. If $\rho(A) = \rho(A:B) <$ no of unknowns then the system is consistent. It has infinitly many solns.

Note → $\rho(A) \leq \rho(A:B)$, bcuz A is contained in A:B.

Gauss Elimination method:-

① Convert the system to matrix form.
② Reduce the matrix to Echelon form.
③ Apply back substitution. (applies from last row)

Gauss Jordan Method :- Modification of above method.

① Convert the system to matrix form.
② Reduce the matrix to Normal form:
modification ③ Apply back substitution.

Ques Solve
$$x - 3y + z = 1$$
$$2x + y - 4z = -1$$
$$6x - 7y + 8z = 7$$
by (i) Gauss elimination method.
(ii) Gauss Jordan method.

Sol (i) Gauss elimination method

$$[A:B] = \left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 2 & 1 & -4 & -1 \\ 6 & -7 & 8 & 7 \end{array}\right]$$

1+6

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 6R_1$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 0 & 7 & -6 & -3 \\ 0 & 11 & 2 & 1 \end{array}\right]$$

Apply $R_2 \to R_2/7$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 0 & 1 & -6/7 & -3/7 \\ 0 & 11 & 2 & 1 \end{array}\right]$$

Apply $R_3 \to R_3 - 11R_2$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 0 & 1 & -6/7 & -3/7 \\ 0 & 0 & 80/7 & 40/7 \end{array}\right]$$

$\rho(A) = 3$, $\rho(A:B) = 3$

no of unknowns $= 3$

$\therefore \rho(A) = \rho(A:B) =$ no of unknowns

System is consistent.

Thus system has unique soln.

Applying back Substitution :-

From R_3, $0.x + 0.y + \frac{80}{7}z = \frac{40}{7}$

$\therefore z = \frac{1}{2}$

From R_2 $0.x + 1.y - \frac{6}{7}z = -\frac{3}{7}$

$y - \frac{6}{7} \times \frac{1}{2} = -\frac{3}{7}$

Multiply by 7

$7y - 3 = -3$

$7y = -3+3$

$7y = 0$

$y = 0$

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From R_1, $1x - 3y + z = 1$

$$x - 3(0) + \frac{1}{2} = 1$$

$$x = \frac{1}{2}$$

$$\boxed{\begin{array}{l} x = \frac{1}{2} \\ y = 0 \\ z = \frac{1}{2} \end{array}}$$

(ii) <u>Gauss Jordan Method</u>

$$[A:B] = \left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 2 & 1 & -4 & -1 \\ 6 & -7 & 8 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 6R_1$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 0 & 7 & -6 & -3 \\ 0 & 11 & 2 & 1 \end{array}\right]$$

$R_2 \to R_2/7$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 1 \\ 0 & 1 & -6/7 & -3/7 \\ 0 & 11 & 2 & 1 \end{array}\right]$$

Apply $R_3 \to R_3 - 11R_2$

Scanned by CamScanner

$$\begin{bmatrix} 1 & -3 & 1 & : & 1 \\ 0 & 1 & -6/7 & : & -3/7 \\ 0 & 0 & 80/7 & : & 40/7 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -11/7 & : & -2/7 \\ 0 & 1 & -6/7 & : & -3/7 \\ 0 & 0 & 80/7 & : & 40/7 \end{bmatrix}$$

$R_3 \rightarrow \frac{R_3}{80/7}$, $R_1 \rightarrow 7R_1$, $R_2 \rightarrow 7R_2$

$$\begin{bmatrix} 7 & 0 & -11 & : & -2 \\ 0 & 7 & -6 & : & -3 \\ 0 & 0 & 1 & : & 1/2 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 11R_3$
$R_2 \rightarrow R_2 + 6R_3$

$$\begin{bmatrix} 7 & 0 & 0 & : & 7/2 \\ 0 & 7 & 0 & : & 0 \\ 0 & 0 & 1 & : & 1/2 \end{bmatrix}$$

$R_1 \rightarrow R_1/7$
$R_2 \rightarrow R_2/7$

$$\begin{bmatrix} 1 & 0 & 0 & : & 1/2 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 1 & : & 1/2 \end{bmatrix}$$

$\rho(A) = 3$, $\rho(A:B) = 3$
no of unknowns $= 3$
$\rho(A) = \rho(A:B) =$ no of unknowns
System is consistent
$\therefore$ system has unique soln.

Apply back substitution

From R_3, $z = 1/2$

From R_2, $y = 0$

From R_1, $x = 1/2$

$$\boxed{\begin{aligned} x &= 1/2 \\ y &= 0 \\ z &= 1/2 \end{aligned}}$$

~~if we apply column operation only interchange of columns is al~~

HOME WORK

Ques $3x - 4y + 2z = -1$
$2x + 3y + 5z = 7$
$x + z = 2$

by (i) Gauss elimination method.
(ii) Gauss Jordan method

Sol (i) Gauss elimination method:-

$$[A:B] = \left[\begin{array}{ccc:c} 3 & -4 & 2 & -1 \\ 2 & 3 & 5 & 7 \\ 1 & 0 & 1 & 2 \end{array}\right]$$

Apply $R_1 \leftrightarrow R_3$

$$= \left[\begin{array}{ccc:c} 1 & 0 & 1 & 2 \\ 2 & 3 & 5 & 7 \\ 3 & -4 & 2 & -1 \end{array}\right]$$

Apply $R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$

$-1-6$

$$\left[\begin{array}{ccc:c} 1 & 0 & 1 & 2 \\ 0 & 3 & 3 & 3 \\ 0 & -4 & -1 & -7 \end{array}\right]$$

Apply $R_2 \to R_2/3$

$$\begin{bmatrix} 1 & 0 & 1 & : & 2 \\ 0 & 1 & 1 & : & 1 \\ 0 & -4 & -1 & : & -7 \end{bmatrix}$$

Apply $R_3 \to R_3 + 4R_2$

$$\begin{bmatrix} 1 & 0 & 1 & : & 2 \\ 0 & 1 & 1 & : & 1 \\ 0 & 0 & 3 & : & -3 \end{bmatrix}$$

$\rho(A) = 3$, $\rho(A:B) = 3$, no of unknowns $= 3$

as $\rho(A) = \rho(A:B) =$ no of unknowns

Thus system is consistent.

Thus system has unique soln.

Apply back substitution :-

From R_3, $0.x + 0.y + 3.z = -3$

$3z = -3$

$z = -1$

From R_2, $0.x + 1.y + 1.z = 1$

$1.y + -1 = 1$

$y = 2$

From R_1, $1.x + 0.y + 1.z = 2$

$x - 1 = 2$

$x = 3$

$$\boxed{\begin{aligned} x &= 3 \\ y &= 2 \\ z &= -1 \end{aligned}}$$

(ii) Gauss Jordan Method :-

$$[A:B] = \left[\begin{array}{ccc|c} 3 & -4 & 2 & -1 \\ 2 & 3 & 5 & 7 \\ 1 & 0 & 1 & 2 \end{array}\right]$$

Apply $R_1 \leftrightarrow R_3$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 2 & 3 & 5 & 7 \\ 3 & -4 & 2 & -1 \end{array}\right]$$

Apply $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 3 & 3 & 3 \\ 0 & -4 & -1 & -7 \end{array}\right]$$

$R_2 \to R_2/3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -4 & -1 & -7 \end{array}\right]$$

Apply $R_3 \to R_3 + 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & -3 \end{array}\right]$$

Apply $R_3 \to R_3/3$

$$\begin{bmatrix} 1 & 0 & 1 & : & 2 \\ 0 & 1 & 1 & : & 1 \\ 0 & 0 & 1 & : & -1 \end{bmatrix}$$

Apply $R_1 \to R_1 - R_3$
$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & -1 \end{bmatrix}$$

Apply Back Substitution

From R_3, $\boxed{z = -1}$

From R_2, $\boxed{y = 1}$

From R_1, $\boxed{x = 3}$

Que
$$2x + 6y = -11$$
$$6x + 20y - 6z = -3$$
$$6y - 18z = -1$$
using Gauss elimination method?

Sol ⇒ $[A:B] = \begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 6 & 20 & -6 & : & -3 \\ 0 & 6 & -18 & : & -1 \end{bmatrix}$

$20-18$
$6-3(0)$
$-3-3(-11)$
$-3+33$

Apply $R_2 \to R_2 - 3R_1$

$$= \begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 0 & 2 & -6 & : & 30 \\ 0 & 6 & -18 & : & -1 \end{bmatrix}$$

Apply $R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 0 & 2 & -6 & : & 30 \\ 0 & 0 & 0 & : & -91 \end{bmatrix}$$

$6 \quad -6$
$-18 \quad -3(30)$
$-1 \quad -90$
-1

$\rho(A) = 2 \quad , \quad \rho(A:B) = 3$

$\therefore \rho(A) \neq \rho(A:B)$

$\therefore$ system is inconsistent.

It does not have soln.

Que

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

Solve by Gauss Jordan method.

Soln

$$[A:B] = \begin{bmatrix} 2 & 1 & 2 & 1 & : & 6 \\ 6 & -6 & 6 & 12 & : & 36 \\ 4 & 3 & 3 & -3 & : & -1 \\ 2 & 2 & -1 & 1 & : & 10 \end{bmatrix}$$

Apply $R_1 \to R_1/2$

$$\begin{bmatrix} 1 & 1/2 & 1 & 1/2 & : & 3 \\ 6 & -6 & 6 & 12 & : & 36 \\ 4 & 3 & 3 & -3 & : & -1 \\ 2 & 2 & -1 & 1 & : & 10 \end{bmatrix}$$

Apply $R_2 \to R_2 - 6R_1$
$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 2R_1$

$$\left[\begin{array}{cccc:c} 1 & 1/2 & 1 & 1/2 & 3 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{array}\right]$$

Apply $R_2 \to R_2/-9$

$$\left[\begin{array}{cccc:c} 1 & 1/2 & 1 & 1/2 & 3 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{array}\right]$$

Apply $R_1 \to R_1 - \frac{1}{2}R_2$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - R_2$

$$\left[\begin{array}{cccc:c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & -3 & 1 & 6 \end{array}\right]$$

Apply $R_3 \to (-1)R_3$

$$\left[\begin{array}{cccc:c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 4 & 11 \\ 0 & 0 & -3 & 1 & 6 \end{array}\right]$$

Apply $R_1 \to R_1 - R_3$
$R_4 \to R_4 + 3R_3$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -3 & -7 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 4 & 11 \\ 0 & 0 & 0 & 13 & 39 \end{array}\right]$$

Apply $R_4 \to R_4/13$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -3 & -7 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 4 & 11 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Apply $R_1 \to R_1 + 3R_4$
$R_2 \to R_2 + R_4$
$R_3 \to R_3 - 4R_4$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$\rho(A) = 4$, $\rho(A:B) = 4$
no of unknowns $= 4$

as $\rho(A) = \rho(A{:}B) =$ no. of unknowns
Thus system is consistent & has unique soln.

Apply back substitution

From R_4 ~~$0.x_1 + 0.x_2 + 0x_3 + 1x_4 = 3$~~

$\boxed{x_4 = 3}$

From R_3 $\boxed{x_3 = -1}$

From R_2 $\boxed{x_2 = 1}$

From R_1 $\boxed{x_1 = 2}$

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= 1 \\ x_3 &= -1 \\ x_4 &= 3 \end{aligned}}$$

Ques

$$3x_1 + 2x_3 + 2x_4 = 0$$
$$-x_1 + 7x_2 + 4x_3 + 9x_4 = 0$$
$$7x_1 - 7x_2 - 5x_4 = 0$$

using Gauss elimination method

Soln

~~$$[A{:}B] = \begin{bmatrix} 3 & 2 & 2 & 2 & : & 0 \\ -1 & 7 & 4 & 9 & : & 0 \\ 7 & -7 & 0 & -5 & : & 0 \end{bmatrix}$$~~

$$[A{:}B] = \begin{bmatrix} 3 & 0 & 2 & 2 & : & 0 \\ -1 & 7 & 4 & 9 & : & 0 \\ 7 & -7 & 0 & -5 & : & 0 \end{bmatrix}$$

Apply $R_1 \leftrightarrow R_2$

$$\cancel{[A:B] = \left[\begin{array}{cccc|c} 1 & 0 & 2/3 & 2/3 & 0 \\ -1 & 7 & 4 & 9 & 0 \\ 7 & -7 & 0 & -5 & 0 \end{array}\right]}$$

~~Apply~~ $\cancel{R_2 \to R_2 + R_1}$
$\cancel{R_3 \to R_3 - 7R_1}$

$$\left[\begin{array}{cccc|c} -1 & 7 & 4 & 9 & 0 \\ 3 & 0 & 2 & 2 & 0 \\ 7 & -7 & 0 & -5 & 0 \end{array}\right]$$

Apply $R_2 \to R_2 + 3R_1$ $\quad -7+49$
$R_3 \to R_3 + 7R_1$ $\quad -5+63$

$$\left[\begin{array}{cccc|c} 1 & +7 & +4 & +9 & 0 \\ 0 & 21 & 14 & 29 & 0 \\ 0 & 42 & 28 & 58 & 0 \end{array}\right]$$

Apply $R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 7 & 4 & 9 & 0 \\ 0 & 21 & 14 & 29 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$, $\rho(A:B) = 2$

no of unknowns $= 4$

$\rho(A) = \rho(A:B) <$ no of unknowns

Then system has infinitily many soln.

Let $x_3 = a$, $x_4 = b$

Apply back substitution

From R_2,

$$0.x_1 + 21x_2 + 14x_3 + 29x_4 = 0$$

$$\Rightarrow \quad 21x_2 + 14a + 29b = 0$$

$$\Rightarrow \quad x_2 = \frac{-14a - 29b}{21}$$

From R_1, $-x_1 + 7x_2 + 4x_3 + 9x_4 = 0$

$$\Rightarrow \quad x_1 = 7x_2 + 4x_3 + 9x_4$$

$$x_1 = 7\left(\frac{-14a - 29b}{21}\right) + 4a + 9b$$

$\cancel{x_1 = \frac{-2a-2b}{21}}$ $\quad x_1 = \frac{-2a-2b}{3}$

Soln

$$\boxed{\begin{aligned} x_1 &= \frac{-2a-2b}{3} \\ x_2 &= \frac{-14a-29b}{21} \\ x_3 &= a \\ x_4 &= b \end{aligned}}$$

Que $5x + 3y + 7z = 4$
$3x + 26y + 2z = 9$ using Gauss elimination method.
$7x + 2y + 10z = 5$

Sol

$$[A:B] = \left[\begin{array}{ccc:c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

Apply $R_1 \rightarrow R_1/5$

$$\left[\begin{array}{ccc:c} 1 & 3/5 & 7/5 & 4/5 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

Apply $R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - 7R_1$

$(3/5)$

$$\left[\begin{array}{ccc:c} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 121/5 & -11/5 & 33/5 \\ 0 & -11/5 & 1/5 & -3/5 \end{array}\right]$$

$\frac{\cancel{33}^{\,3}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{121}_{\,11}}$

Apply $R_2 \rightarrow R_2 \times \frac{5}{121}$

$$\left[\begin{array}{ccc:c} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 1 & -1/11 & 3/11 \\ 0 & -11/5 & 1/5 & -3/5 \end{array}\right]$$

Apply $R_3 \rightarrow R_3 + \frac{11}{5} R_2$

$$\left[\begin{array}{ccc:c} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 1 & -1/11 & 3/11 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$, $\rho(A:B) = 2$, no of unknowns $= 3$

Let $z = a$

Apply back sustitution

From R_2, $0.x + 1.y - \frac{1}{11}z = \frac{3}{11}$

$$y - \frac{1}{11}z = \frac{3}{11}$$

$$y = \frac{3}{11} + \frac{1}{11}a$$

$$y = \frac{3+a}{11}$$

From R_1, $1.x + \frac{3}{5}.y + \frac{7}{5}z = \frac{4}{5}$

$$x = \frac{4}{5} - \frac{7z}{5} - \frac{3y}{5}$$

$$x = \frac{4}{5} - \frac{7a}{5} - \frac{3}{5}\left(\frac{3+a}{11}\right)$$

$$x = \frac{4}{5} - \frac{7a}{5} - \frac{1}{5}\left(\frac{9+3a}{11}\right)$$

$$x = \frac{44 - 77a - 9 - 3a}{55}$$

$$x = \frac{35 - 80a}{55}$$

Que For what values of k does the system

$$x+y+z=1$$
$$2x+y+4z=k$$
$$4x+y+10z=k^2$$

has a soln.? Solve in each case.

Sol $[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array}\right]$ Gauss elimination method.

Apply $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & -3 & 6 & k^2-4 \end{array}\right]$$

Apply $R_3 \to R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{array}\right]$$

$k^2-4-3(k-2)$
$k^2-4-3k+6$
k^2-3k+2

$\rho(A)=2$, $\rho(A:B)=3$, no of unknowns $=3$

$\therefore$ The system is consistent

$\Rightarrow k^2-3k+2=0$
$k=1,2$

Case I

when $k=1$

$$[A:B] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So the system has infinitely many solns.

~~Soln.~~ $\rho(A)=2$, $\rho(A:B)=2$, no of unknowns $=3$

$\rho(A)=2$ $\rho(A:B)$ < no of unknowns

Let $z=a$

Apply back substitution

From R_2, $0.x-1.y+2.z=-1$

$$-y+2a=-1$$

$$y=2a+1$$

From R_1; $1.x+1.y+1.z=1$

$$x+y+z=1$$

$$x+2a+1+a=1$$

$$x=-3a$$

Soln. $\boxed{\begin{array}{l} x=-3a \\ y=2a+1 \\ z=a \end{array}}$

Case-II

When $k=2$

$$[A:B] \longrightarrow \begin{bmatrix} 1 & 1 & 1 & : 1 \\ 0 & -1 & 2 & : 0 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

4-6+2

Thus system has infinitely many solns.

$\rho(A)=2$, $\rho(A:B)=2$, no of unknowns $=3$

Let $z=a$

Apply back substitution

From R_2, $-y + 2z = 0$
$y = 2a$

From R_1, $x + y + z = 1$
$\Rightarrow x + 2a + a = 1$
$x = 1 - 3a$

$$\boxed{\begin{array}{l} x = 1-3a \\ y = 2a \\ z = a \end{array}}$$

Q.4 For what value of λ & μ does the system

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

has (i) no soln.
(ii) infinitely many solns.
(iii) Unique soln.

Sol $[A:B] = \left[\begin{array}{ccc:c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$

Apply $R_1 \to R_1/2$

$$\left[\begin{array}{ccc:c} 1 & 3/2 & 5/2 & 9/2 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

Apply $R_2 \to R_2 - 7R_1$
$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 3/2 & 5/2 & 9/2 \\ 0 & -15/2 & -39/2 & -47/2 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

$\rho(A) = 3$, $\rho(A|B) = 3$, no of unknowns $= 3$

$\rho(A) = \rho(A:B) =$ no of unknowns

$\therefore$ System is consistent & has unique soln.

(i) For No soln.

$\rho(A) \neq \rho(A:B)$

$\Rightarrow \lambda - 5 = 0$ but $\mu - 9 \neq 0$

$\lambda = 5$ and $\mu \neq 9$

(ii) For infinitely many solns.

$\rho(A) = \rho(A:B) <$ no of unknowns

i.e. $\lambda - 5 = 0$ & $\mu - 9 = 0$

$\lambda = 5$, $\mu = 9$

(iii) for UNIQUE Soln.

$\rho(A) = \rho(A:B) =$ no of unknowns

$\lambda - 5 \neq 0$ & μ can have any value

Qus- $x+y+z=6$
$x+2y+3z=10$
$x+2y+\lambda z=\mu$:

for what values of λ & μ does the system has (i) No soln.
(ii) Unique soln.
(iii) Infinitely many soln.

Sol $[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$

Apply $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$

Apply $R_3 \to R_3 - R_2$

$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$

(i) For No soln.

$\rho(A) \neq \rho(A:B)$

$\Rightarrow \lambda-3=0$ & $\mu-10 \neq 0$

$\lambda=3$ & $\mu \neq 10$

(ii) For unique soln.

$\rho(A) = \rho(A|B) =$ no of unknowns

$\lambda - 3 \neq 0$ & μ can have any value

(iii) For infinitely many solns.

$\rho(A) = \rho(A:B) <$ no of unknowns

i.e. $\lambda - 3 = 0$ & $\mu - 10 = 0$

$\Rightarrow \lambda = 3$ & $\mu = 10$

⇒ CRAMER'S RULE :-

$a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

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$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

① If $\Delta \neq 0$, then the system is consistent and has unique soln.

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

② If $\Delta = 0$, also $\Delta_1 = \Delta_2 = \Delta_3 = 0$, then system has infinitely many solns.

③ If $\Delta = 0$, but atleast one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero, then the system has no soln.

PROBLEMS :-

Que

$$2x + 6y = -11$$
$$6x + 20y - 6z = -3$$
$$6y - 18z = -1$$

Solve by cramer's Rule :

Sol

$$\Delta = \begin{vmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{vmatrix}$$

$$\Delta = 2(-360+36) - 6(-108-0) + 0$$

$$\Delta = 2(-324) + 6(108) = 0$$

$$\Delta_1 = \begin{vmatrix} -11 & 6 & 0 \\ -3 & 20 & -6 \\ -1 & 6 & -18 \end{vmatrix}$$

$$= -11(-360+36) - 6(54-6) + 0$$

$$= -11(-324) - 6(48)$$

$$= +3564 - 288 = 3276 \neq 0$$

Since $\Delta = 0$, but $\Delta_1 \neq 0$, thus system is inconsistent & it has no soln.

Que Solve :-

$$x - 3y + z = -1$$
$$2x + y - 4z = -1$$
$$6x - 7y + 8z = 7$$

Solve by Cramer's rule

$$\Delta = \begin{vmatrix} 1 & -3 & 1 \\ 2 & 1 & -4 \\ 6 & -7 & 8 \end{vmatrix}$$

$$\Delta = 1(8-28) + 3(16+24) + 1(-14-6)$$

$$\Delta = -20 + 3(40) - 20$$

$$\Delta = 80$$

Since $\Delta \neq 0$, i.e. Δ is non-zero, $\therefore$ the system has unique soln.

$$\Delta_1 = \begin{vmatrix} -1 & -3 & 1 \\ -1 & 1 & -4 \\ 7 & -7 & 8 \end{vmatrix}$$

$$= -1(8-28) + 3(-8+28) + 1(7-7)$$

$$\Delta_1 = 20 + 60 = 80$$

$$x = \frac{\Delta_1}{\Delta} = \frac{80}{80} = 1$$

$$\Delta_2 = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & -4 \\ 6 & 7 & 8 \end{vmatrix}$$

$$= 1(-8+28) + 1(16+24) + 1(14+6)$$

$$= 20 + 40 + 20 = 80$$

$$y = \frac{\Delta_2}{\Delta} = \frac{80}{80} = 1$$

$$\Delta_3 = \begin{vmatrix} 1 & -3 & -1 \\ 2 & 1 & -1 \\ 6 & -7 & 7 \end{vmatrix}$$

$$= 1(7-7) + 3(14+6) - 1(-14-6)$$

$$= 0 + 3(20) + 20$$

$$= 60 + 20 = 80$$

$$z = \frac{\Delta_3}{\Delta} = \frac{80}{80} = 1$$

Solution $\boxed{\begin{matrix} x = 1 \\ y = 1 \\ z = 1 \end{matrix}}$

Que-

$$2x - y + z = 4$$
$$x + 3y + 2z = 12$$
$$3x + 2y + 3z = 16$$

Solve by Cramer's Rule

Sol^n

$$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$$= 2(9-4) + 1(3-6) + 1(2-9)$$

$$= 2(5) + 1(-3) + 1(-7)$$

$$= 10 - 3 - 7 = 0$$

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$$\Delta_1 = \begin{vmatrix} 4 & -1 & 1 \\ 12 & 3 & 2 \\ 16 & 2 & 3 \end{vmatrix}$$

$$= 4(9-4) + 1(36-32) + 1(24-48)$$

$$= 4(5) + 1(4) + 1(-24)$$

$$= 20 + 4 - 24 = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 12 & 2 \\ 3 & 16 & 3 \end{vmatrix}$$

$$= 2(36-32) - 4(3-6) + 1(16-36)$$

$$= 2(4) - 4(-3) + 1(-20)$$

$$= 8 + 12 - 20 = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 4 \\ 1 & 3 & 12 \\ 3 & 2 & 16 \end{vmatrix}$$

$$\Delta_3 = 2(48-24) + 1(16-36) + 4(2-9)$$

$$\Delta_3 = 2(24) + 1(-20) + 4(-7)$$

$$\Delta_3 = 48 - 20 - 28 = 0$$

Since $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 0$

$\therefore$ the system has infinitely many soln.

From I^{st} & II^{nd} eqn.

$$2x - y = 4 - z$$
$$x + 3y = 12 - 2z$$

$$D = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7 \neq 0$$

$$D_1 = \begin{vmatrix} 4-z & -1 \\ 12-2z & 3 \end{vmatrix}$$

$$= 12 - 3z + 12 - 2z = 24 - 5z$$

$$D_2 = \begin{vmatrix} 2 & 4-z \\ 1 & 12-2z \end{vmatrix}$$

$$= 24 - 4z - 4 + z = 20 - 3z$$

Let $z = k$

$$x = \frac{D_1}{D} = \frac{24-5z}{7} = \frac{24-5k}{7}$$

$$y = \frac{D_2}{D} = \frac{20-3z}{7} = \frac{20-3k}{7}$$

$$z = k$$

Que Solve by Cramer's Rule:-

$$2x + y + 2z + t = 6$$
$$6x - 6y + 6z + 12t = 36$$
$$4x + 3y + 3z - 3t = -1$$
$$2x + 2y - z + t = 10$$

Sol

$$\Delta = \begin{vmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} -6 & 6 & 12 \\ 3 & 3 & -3 \\ 2 & -1 & 1 \end{vmatrix} - 1\begin{vmatrix} 6 & 6 & 12 \\ 4 & 3 & -3 \\ 2 & -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 6 & -6 & 12 \\ 4 & 3 & -3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$- 1\begin{vmatrix} 6 & -6 & 6 \\ 4 & 3 & 3 \\ 2 & 2 & -1 \end{vmatrix}$$

$$= 2\left[-6(3-3) - 6(3+6) + 12(-3-6)\right]$$

$$-1\left[6(3-3) - 6(4+6) + 12(-4-6)\right]$$

$$+2\left[6(3+6) + 6(4+6) + 12(8-6)\right]$$

$$-1\left[6(-3-6) + 6(-4-6) + 6(8-6)\right]$$

$$= 2[0 - 54 - 108] - 1[0 - 60 - 120] + 2[54 + 60 + 24]$$

$$-1[-54 - 60 + 12] = -324 + 180 + 276 + 102$$

$$= 234$$

Matrix Inversion Method

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$AX = B$, where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|} \text{adj}\cdot A$$

Co-factor of $a_{ij} = (-1)^{i+j}$ (Minor of a_{ij})

co-factor of $a_{22} = (-1)^{2+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$$C(A) = \begin{bmatrix} C(a_{11}) & C(a_{12}) & C(a_{13}) \\ C(a_{21}) & C(a_{22}) & C(a_{23}) \\ C(a_{31}) & C(a_{32}) & C(a_{33}) \end{bmatrix}$$

$$\text{adj}\cdot A = [C(A)]^T$$

PROBLEMS :-

Qus- Solve by matrix inversion method

$x-3y+z=-1$
$2x+y-4z=-1$
$6x-7y+8z=7$

Sol

$$A=\begin{bmatrix}1 & -3 & 1\\ 2 & 1 & -4\\ 6 & -7 & 8\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad B=\begin{bmatrix}-1\\ -1\\ 7\end{bmatrix}$$

$$AX=B$$

$$|A| = 1(8-28)+3(16+24)+1(-14-6)$$
$$|A| = -20+120-20$$
$$|A| = 80 \neq 0$$

$\begin{array}{r}24\\ +16\\ \hline 40\end{array}$

$\therefore A^{-1}$ exists

$$a_{11} = (-1)^{1+1}\begin{vmatrix}1 & -4\\ -7 & 8\end{vmatrix} = (1)(8-28) = -20$$

$$a_{12} = (-1)^{1+2}\begin{vmatrix}2 & -4\\ 6 & 8\end{vmatrix} = (-1)(16+24) = -40$$

$$a_{13} = (-1)^{1+3}\begin{vmatrix}2 & 1\\ 6 & -7\end{vmatrix} = (1)(-14-6) = -20$$

$a_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 1 \\ -7 & 8 \end{vmatrix} = (-1)(-24+7) = 17$

$a_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 6 & 8 \end{vmatrix} = (1)(8-6) = 2$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -3 \\ 6 & -7 \end{vmatrix} = (-1)(-7+18) = -11$

$a_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 1 \\ 1 & -4 \end{vmatrix} = (1)(12-1) = 11$

$a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} = (-1)(-4-2) = 6$

$a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = (1)(1+6) = 7$

$$C(A) = \begin{bmatrix} -20 & -40 & -20 \\ 17 & 2 & -11 \\ 11 & 6 & 7 \end{bmatrix}$$

$$\text{adj. } A = [C(A)]^T = \begin{bmatrix} -20 & -40 & -20 \\ 17 & 2 & -11 \\ 11 & 6 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} -20 & 17 & 11 \\ -40 & 2 & 6 \\ -20 & -11 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}.A$$

$$A^{-1} = \frac{1}{80}\begin{bmatrix} -20 & 17 & 11 \\ -40 & 2 & 6 \\ -20 & -11 & 7 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{80}\begin{bmatrix} -20 & 17 & 11 \\ -40 & 2 & 6 \\ -20 & -11 & 7 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ 7 \end{bmatrix}$$

$$= \frac{1}{80}\begin{bmatrix} 20 - 17 + 77 \\ 40 - 2 + 42 \\ 20 + 11 + 49 \end{bmatrix}$$

$$= \frac{1}{80}\begin{bmatrix} 80 \\ 80 \\ 80 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{matrix} x = 1 \\ y = 1 \\ z = 1 \end{matrix}}$$

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