

①

Unit - 1

KIPPLE SPAHNZ

①

## D.C. Circuits

Electricity play an important role in our day to day life.  
 Electricity is used for

- 1) Lighting (Lamps)
2. Heating (Heaters)
3. Cooking
4. Entertainment (radio + T.V.)
5. Transportation (electric traction)
6. Calculations (calculators)

Now-a-days, all our activities are depended upon electricity.

Electricity:-

The invisible energy which constitutes the flow of electrons in a closed circuit to do work is called electricity.

Modern Electron theory:-

In past, electricity were developed through experiments & by observing its behaviour.

Modern electron theory says that every matter consists of very small divisible particles called molecules. Molecules is further made up of very small particles called atoms.

An atom consists of following two main parts

- 1) Nucleus
- 2) Extra Nucleus

1) Nucleus:- The central part of an atom consists of protons & neutrons

A proton has +ve charge ( $1.602 \times 10^{-19}$  coulombs)  
 & neutron has no charge

Thus, the whole nucleus of an atom bears +ve charge

Extra Nucleus:- The outer part of an atom which contains only electrons. An electron has -ve charge ( $1.602 \times 10^{-19}$  coulombs) equal to that of proton.

Atomic Weight = no of protons + no of neutrons in the nucleus

Atomic Number = no of protons or electrons in an atom

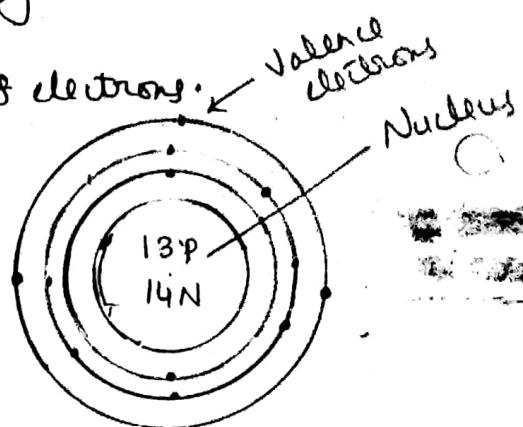
No of electrons in any orbit is given by =  $2n^2$

The one orbit cannot have more than 18 electrons.

Eg Aluminium (Al)

Atomic No = 13

Atomic weight = 27



No of protons = 13

No of neutrons =  $27 - 13 = 14$

At a time,  
Out of 3 electrons of the outermost orbit, only one electron  
is free to move from one atom to the other & is  
called free electron.

Nature of Electricity:-

Every matter is electrical in nature since it contains charge particles like protons & electrons.

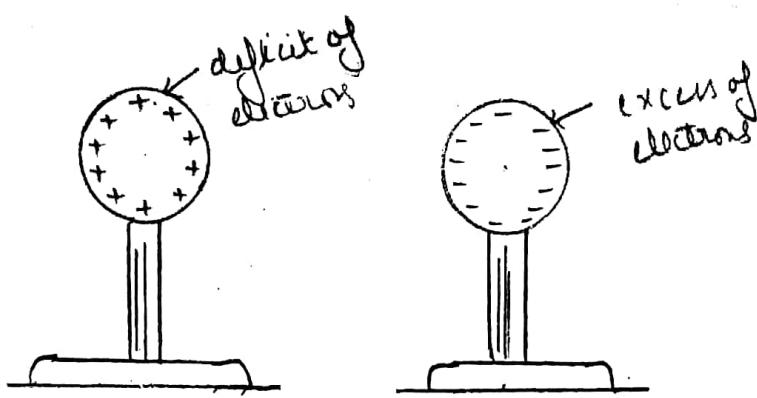
Therefore

- 1) Ordinarily, a body is neutral as it contains same number of protons & neutrons.
- 2) If some of electrons are removed from the body, then ②

(2)

a deficit of electrons & the body attains a positive charge.

- 3) If some of electrons are supplied to the body, there occurs excess of electrons & the body attains a negative charge.



A body is said to be charged +vely or -vely if it has deficit or excess of electrons from its normal share respectively.

### Unit of Charge

The practical unit of charge is coulomb.

One coulomb = charge on  $6.28 \times 10^{18}$  electrons.

### Free Electrons:-

The valence electrons which are loosely attached to the nucleus of an atom & can be loosely detached are called free electrons.

### Electrical potential:-

The capacity of charged body to do work is called electrical potential.

58

(3)

$$\text{Electric potential} = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$$

$$V = \frac{W}{Q}$$

Unit of electrical potential is Volts or Joules/coulomb.

Defn- A body is said to have an electric potential of 1 Volt if 1 Joule of work is done to charge the body to 1 coulomb.

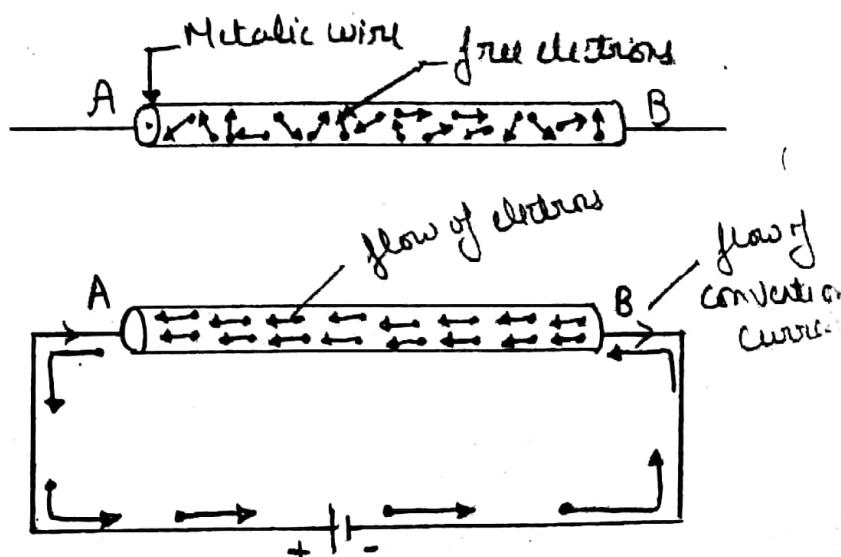
Potential difference-

The difference in electric potential of the two charged bodies is called Potential difference.

Unit of potential difference is Volts.

Electric Current-

In metallic wire, a large number of electrons are available which move from one atom to the other at random.



When an electric potential is applied across the metallic wire, the loosely attached free electrons start moving towards the two terminals of the cell.

Thus, a continuous flow of electrons in an electric circuit is called electric current.

(4)

(3)

Aq:-

Current is rate of flow of electrons i.e. charge flowing per second.

$$I = \frac{Q}{t}$$

The unit of current is ampere (A).

E.m.f (electromotive force) & potential difference -

E.m.f is the force that causes an electric current to flow in an electric circuit.

In fig it is not a force but it is energy.

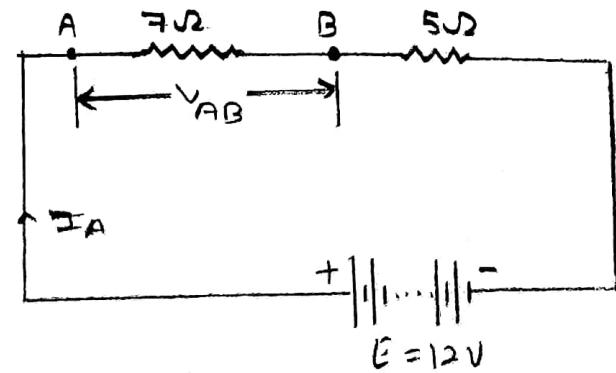
E.m.f :- The amount of energy supplied by the source to each coulomb of charge is known as emf

Potential difference :- The amount of energy used by one coulomb of charge in moving from one point to the other is known as potential difference between two points.

In fig as shown on R.H.S.

A battery has emf of 12 Volts

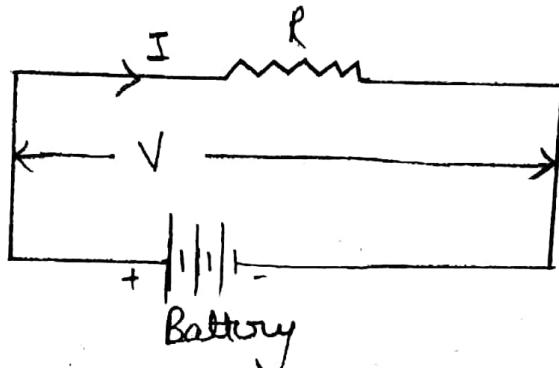
The potential diff b/w A + B is  
7 Volts



Resistance - The opposition offered to flow of electric current is called resistance.

Unit of resistance is ohm ( $\Omega$ )

Ohm's law -



Ohm's law states that current flowing between any two points of conductor is directly proportional to the potential difference across them, provided physical conditions (i.e. temp) do not change.

Mathematically

$$I \propto V$$

$$\text{or } \frac{V}{I} = \text{constant}$$

$$\frac{V}{I} = R$$

$$V = IR$$

$$\text{or } I = \frac{V}{R}$$

Limitations -

1) Ohm's law cannot be applied to the circuit consisting of transistors or diode tubes because such elements are not bilateral i.e. they behave in different way when direction of flow of current is

reversed as in case of diodes.

2: Ohm's law cannot be applied to the circuits consisting of non-linear elements such as electric arc, powdered carbon for silicon carbide, the relationship b/w current & voltage is given as  $V = kI^m$ ,  $k$  &  $m$  are constants &  $m$  is less than unity.

### Law of Resistance:-

The resistance of a wire depends upon

1) It is directly proportional to its length

$$R \propto L \quad \textcircled{1}$$

2) It is inversely proportional to its area of cross-section

$$R \propto \frac{1}{A} \quad \textcircled{2}$$

3) It depends upon the nature of the material of which the wire is made.

4) It also depends upon the temperature of the wire

Combining  $\textcircled{1}$  &  $\textcircled{2}$

$$R \propto \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

$\rho$  is called resistivity of the wire

Resistivity depends upon the nature of material of the wire

Resistivity

$$\rho = \frac{RA}{L}$$

Ω

Unit of resistivity is Ωm

7

### Specific Resistance-

The specific resistance of a material is defined as the resistance of the material having specific dimensions i.e. one meter length + one square meter as area of cross-section.

Conductance- The ease to the flow of current is called conductance.

The conductance is reciprocal to the resistance

$$G = \frac{1}{R}$$

Unit of conductance is mho

Conductivity- It is property or nature of material due to which it allows current to flow through it

$$R = \rho \frac{l}{A}$$

$$\rho = \frac{RA}{l}$$

$$\sigma =$$

(8)

(5)

## Numericals on Ohm's law

Ques:-1 A wire of length 1 m at a resistance  $2\Omega$ . Obtain its resistance if specific resistance double, diameter is doubled length is made three times of first.

Soln:- Given

$$l_1 = 1 \text{ m}, \quad R_1 = 2\Omega, \quad P_2 = 2P_1, \quad d_2 = 2d_1$$

$$\text{&} \quad l_2 = 3l_1$$

$$R_1 = \frac{P_1 l_1}{A_1} \quad \text{--- (1)}$$

$$R_1 = \frac{4P_1 l_1}{\pi d_1^2} \quad \text{--- (i)}$$

$$R_2 = \frac{P_2 l_2}{A_2} \quad \text{--- (2)}$$

$$R_2 = \frac{4P_2 l_2}{\pi d_2^2} \quad \text{--- (ii)}$$

Divide (i) by (ii)

$$\frac{R_1}{R_2} = \frac{\frac{4P_1 l_1}{\pi d_1^2}}{\frac{4P_2 l_2}{\pi d_2^2}} \times \frac{\pi d_2^2}{\pi d_1^2}$$

$$\frac{R_1}{R_2} = \frac{\frac{P_1 l_1}{d_1^2}}{\frac{(2P_1)(3l_1)}{(2d_1)^2}}$$

$$\frac{R_1}{R_2} = \frac{\frac{P_1 l_1}{d_1^2}}{\frac{2P_1}{2} \times \frac{3l_1}{4d_1^2}}$$

$$\frac{R_1}{R_2} = \frac{4}{6} = \frac{2}{3}$$

67

$$R_2 = \frac{3}{2} \times R_1 = \frac{3}{2} \times 2 = 3\Omega$$

(9)

Ques 1.2 There are two wires A & B of same material. A is 20 times longer than B and has one fifth of the cross-section as that of B. If the resistance of A is 10 ohm. What is the resistance of B?

$$\text{Soln} - R_2 = R_1 = R$$

$$L_1 = 20L_2$$

$$A_1 = \frac{1}{5} A_2$$

$$R_1 = 1 \text{ ohm}$$

$$R_1 = \frac{R_1 L_1}{A_1} \quad \dots \quad (1)$$

$$R_2 = \frac{R_2 L_2}{A_2} \quad \dots \quad (2)$$

Divide (1) by (2)

$$\begin{aligned} \frac{R_1}{R_2} &= \frac{\rho_1 L_1 \times A_2}{\rho_1 L_1 \times \rho_2 L_2} \\ &= \frac{1 \times 20L_2 \times A_2}{\frac{1}{5} A_2 \times 10 L_2} \end{aligned}$$

$$\frac{R_1}{R_2} = \frac{100}{1}$$

$$R_2 = \frac{R_1}{100} = \frac{1}{100} \text{ ohm}$$

## Electric Circuit:-

The path for flow of electric current is called electric circuit. The electric circuit is just an arrangement of electrical energy sources & various circuit elements such as  $R$ ,  $L$  &  $C$  are connected in series, parallel or series parallel combinations.

### Circuit Elements

- 1) Active & passive elements
- 2) Unilateral & bilateral elements
- 3) Linear & Non-linear elements
- 4) lumped & distributed "

#### i. Active & passive elements:-

(i) The elements which supplies energy to the circuit or network are called active elements.

All energy sources such as batteries or generators are active elements.

(ii) The elements which receives the energy are called passive elements.

e.g.  $R$ ,  $L$  &  $C$

#### 2) Unilateral & Bilateral elements:-

(i) The elements which conducts the current in one direction only are called Unilateral elements.

Such as diodes, vacuum tubes, Rectifiers etc.

(ii) Bilateral elements:- The elements which conduct current in both directions are called bilateral elements such as resistors.

### 3) Linear & non linear elements:-

(i) The elements which have V-I characteristics as straight line are called linear elements. e.g resistors.

(ii) The elements which do <sup>not</sup> follow V-I characteristics as straight line are non-linear elements e.g diodes & transistors.

## Dependence

### ④ Bumped & distributed elements:-

The elements in which action takes place simultaneously are bumped elements such as resistor, inductors & capacitors. These elements are smaller in size.

The elements in which action for a given cause is not occurring simultaneously at the same instant but it is distributed along its length are called distributed elements such as transmission lines which is having distributed resistance.

(ii) Bilateral elements:- The elements which conduct current in both directions are called bilateral elements such as resistors.

### 3) Linear & non linear elements:-

- (i) The elements which have V-I characteristics as straight line are called linear elements of resistors.
- (ii) The elements which do <sup>not</sup> follow V-I characteristics as straight line are non-linear elements of diodes & transistors.

### Elements

#### ④ Bumped & distributed elements:-

The elements in which action takes place simultaneously are lumped elements such as resistor, inductors & capacitors. These elements are smaller in size.

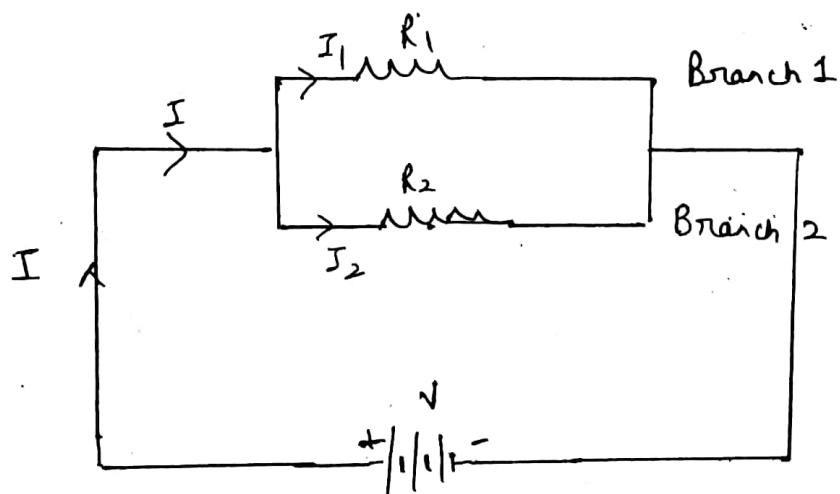
The elements in which action for a given cause is not occurring simultaneously at the same instant but it is distributed are called distributed elements such as transmission lines which is having distributed resistance along its length.

(2)

D.C. circuit :- The closed path for flow of direct current is called d.c. circuit.

- Series circuit
- Parallel circuit

Circuits in parallel circuits :-



According to ohm's law

$$V = I_1 R_1 \quad \text{in branch 1}$$

$$V = I_2 R_2 \quad " " 2$$

$$V = I_1 R_1 = I_2 R_2$$

Total voltage in the circuit is  $V = IR$  (ohm's law)

$$IR = I_1 R_1 = I_2 R_2$$

Total resistance in parallel circuit

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

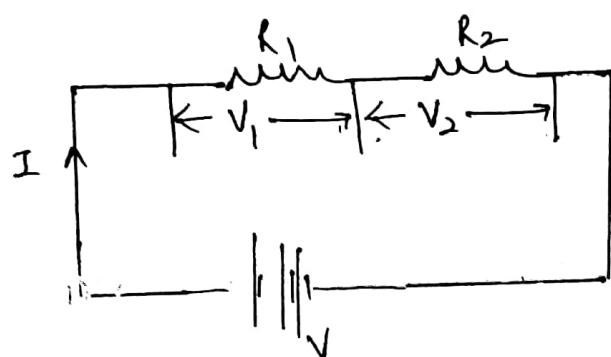
$$I_1 R_1 = I_2 R_2 = I \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

Ten

$$I_1 = \frac{R_2}{R_1 + R_2} (I)$$

$$I_2 = \frac{R_1}{R_1 + R_2} (I)$$

Voltage in Series circuit :-



According to ohm's law

Current in resistance  $R_1$  is

$$I = \frac{V_1}{R_1} \quad \text{--- (1)}$$

Current in resistance  $R_2$  is

$$I = \frac{V_2}{R_2} \quad \text{--- (2)}$$

But value of  $I$  is given  
in (1) & (2)

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = I \quad \text{--- (3)}$$

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V}{R_1 + R_2}$$

Total current in circuit is

$$I = \frac{V}{R} \quad \text{--- (4)}$$

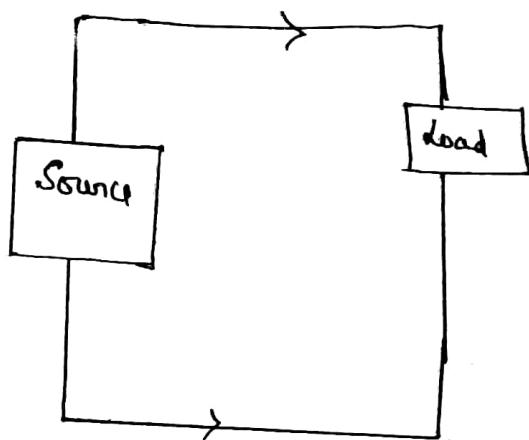
$$V_1 = \frac{R_1}{R_1 + R_2} (V)$$

$$V_2 = \frac{R_2}{R_1 + R_2} (V)$$

Total resistance of circuit is  $R = R_1 + R_2$

## Voltage & Current source -

Circuit, To deliver electrical energy to the electrical source, a source is required + a load is connected to source as shown in fig



The source may be d.c. (direct current) source or an a.c. source

D.C. source:- Any source that produces direct voltage continuously + has ability to deliver direct current is called d.c. source  
eg batteries + generators

A.C. source:- Any source that produces alternating voltage continuously + has ability to deliver the alternating current is called A.C. source

eg alternators + oscillators + signal generators

(a)

## Independent & dependent sources

There are two types of sources - voltage source & current source. Sources can be either independent or dependent upon some other quantities.

### Dependent

#### Independent Voltage source:-

which is not affected by any other quantities.

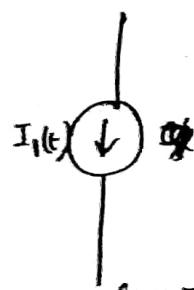
The source ~~works~~ maintains voltage (d.c or a.c) on

The voltage does not depend on other voltage or current in the circuit.

#### Symbol for voltage & current source



Independent  
~ Voltage source



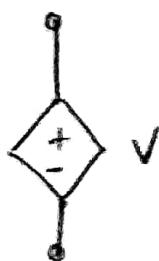
current source

eg batteries or  
generators (Voltage source)

eg Semiconductor devices  
eg diode or transistor  
current source

#### Dependent Voltage / Current source:-

The voltage does depend on another voltage or current in the circuit

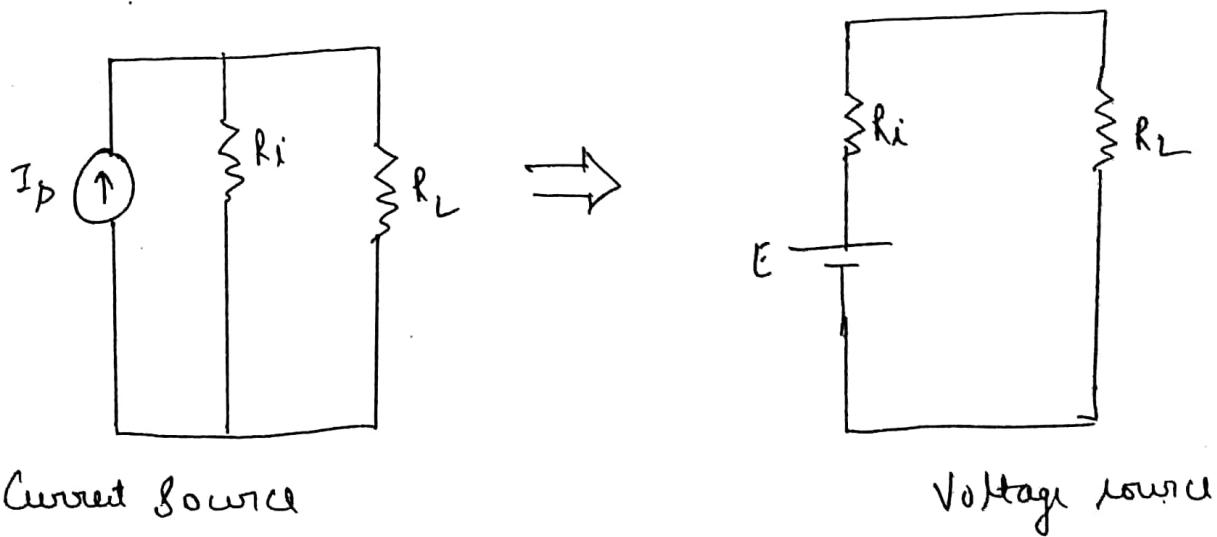
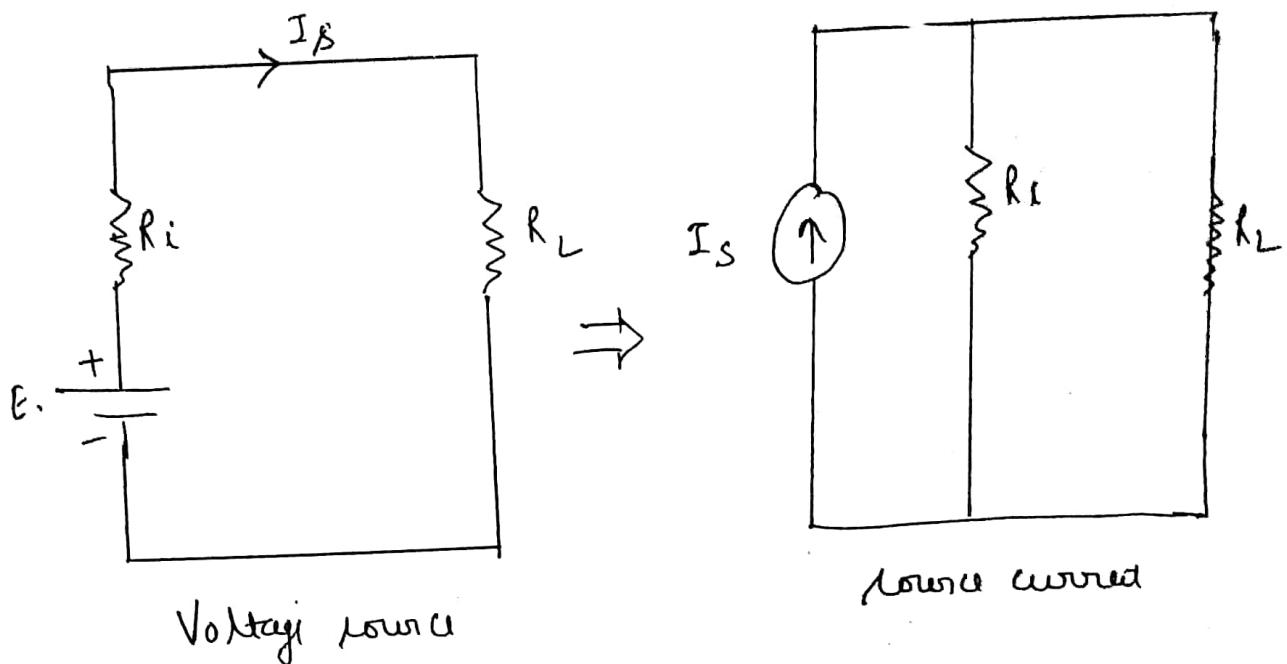


Dependent voltage source



Dependent current source

## Source transformation

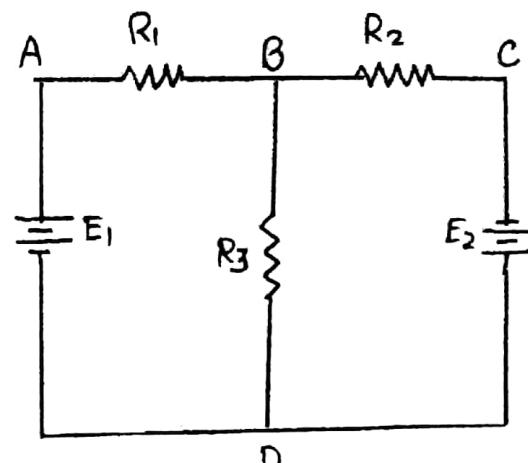


## Network Terminology:-

1) Electric Network - is interconnection of electric components

(e.g. batteries, resistors, inductors & capacitors)

2) Electric circuit - is network consisting of closed conducting path through which an electric current either flows or intended to flow.



3) Active Elements - The elements which supply energy to the circuit. In fig  $E_1$  &  $E_2$  are active elements

4) Passive Elements - The elements which receive the energy. In fig on R.H.S  $R_1$ ,  $R_2$  &  $R_3$  are passive elements

5) Passive Network - An electric network which does not contain any source of emf.

6) Active Network - An electric network which contains one or more than one source of emf.

7) Node - is a point where two or more circuit elements are joined. In fig A, B, C and D are nodes.

8) Junction - is a point in the network where three or more circuit elements are joined. It is a point where current is divided.

In fig B & D are junctions

67

9) Loop - The closed path of a network. e.g. ABDA, BCDB and ABCD

10) Mesh - The elementary form of loop which cannot be further divided is called mesh. e.g. ABDA and BCDB

11) Branch - Part of a network which lies between two junction points. In fig DAN, ACB and BD are the three branches.

19

## Kirchoff's Law

Kirchoff's First Law:- This law relates the currents flowing through the circuit; it is known as Kirchoff's current law (KCL)

Kirchoff's Current Law (KCL):- States that the algebraic sum of current meeting at a point or junction is zero.

Mathematically  $\sum I = 0$

Incoming currents as  $+v_c$

Outgoing currents as  $-v_c$

$$-I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

$$I_2 + I_3 = I_1 + I_4 + I_5$$

Sum of incoming currents = Sum of outgoing currents.

Kirchoff's Second Law:- This law relates

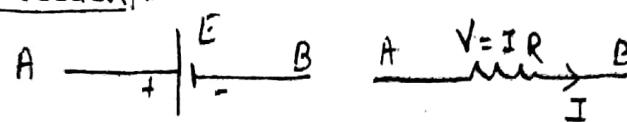
to voltage in a closed circuit of an electric network. It is known as Kirchoff's Voltage Law.

## Kirchoff's Voltage Law (KVL):-

In a closed circuit, the algebraic sum of all voltage drops is zero.

Mathematically  $\sum E + \sum V = 0$

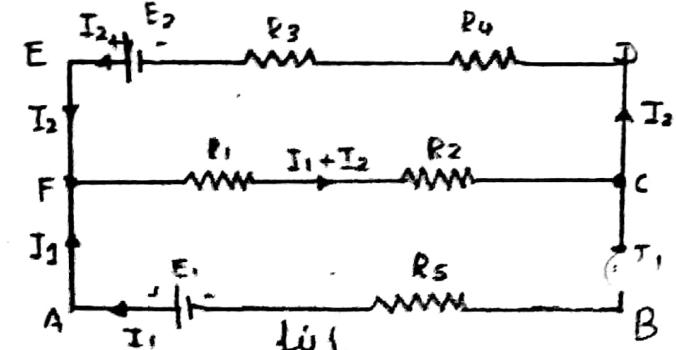
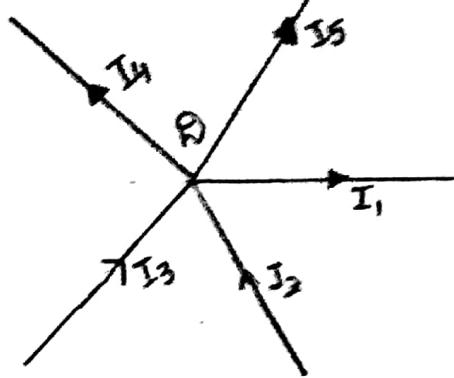
### Sign Convention:-



Tracing branch A to B,  $E$  is  $-v_c$  ] If we are moving in same direction  
Tracing branch B to A,  $E$  is  $+v_c$  ] as that of current - fall in voltage drop.

Now apply KVL in fig 1 on R.H.S

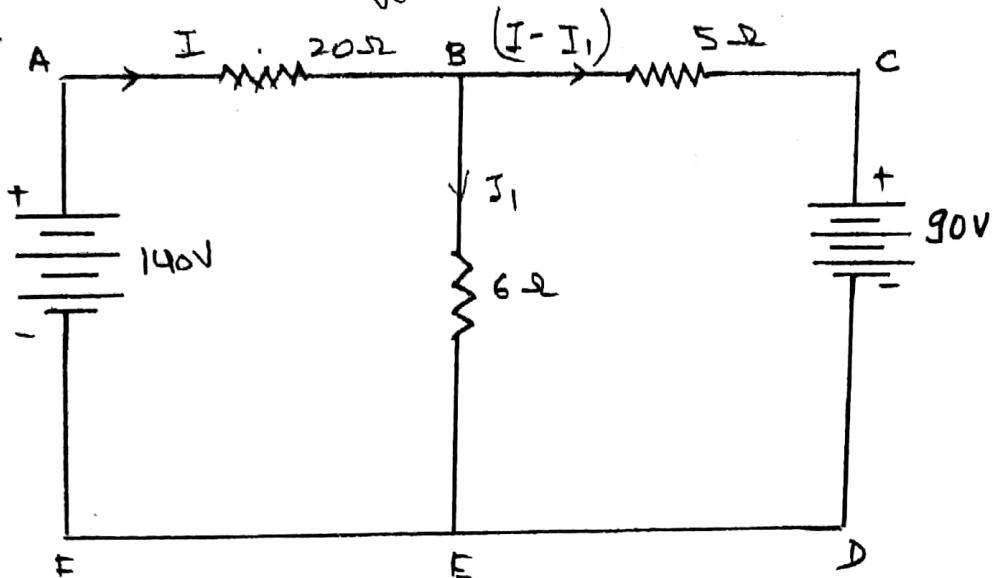
$$\text{In Loop AFCBA} \quad -R_1(I_1 + I_2) - R_2(I_1 + I_3) - I_1 R_5 + E_1 = 0 \\ \text{on } E_1 = R_1 I_1 + r_1 + R_2 I_2 + r_2 + I_1 R_5 \quad (1)$$



Ques:-1

In the circuit  
as shown in fig. 140V

Calculate current  
& power in the  
6 Ω resistor.

Solution:

~~Closed~~  
In loop ABF A

$$-20I - 6I_1 + 140 = 0$$

$$-20I + 6I_1 = 140$$

$$10I + 3I_1 = 70 \quad \text{--- (i)}$$

In closed loop BCDEB

$$-5(I - I_1) - 90 + 6I_1 = 0$$

$$-5I + 5I_1 + 6I_1 = 90$$

$$-5I + 11I_1 = 90 \quad \text{--- (ii)}$$

Multiply (ii) by 6

$$\cancel{10I} + 3I_1 = 70$$

$$\cancel{-10I_1} + \cancel{22I_1} = 180$$

$$25I_1 = 250$$

$$I_1 = \frac{250}{25} = 10A$$

68

$$\begin{aligned}\text{Power in } 6\Omega \text{ resistor} &= I_1^2 R \\ &= (10)^2 \times 6 \\ &= 600 \text{ watts}\end{aligned}$$

(21)

Ques:- Find the value of R and the current through it in the circuit shown in fig when the current is zero in branch OA.

Solution:-

Apply KVL in loop AOB

$$-4I_2 + 1I_1 = 0$$

$$I_1 = +4I_2 \quad \text{--- (1)}$$

In loop ABCA

$$I_1 - 10 + 2(I_1 + I_2) + 1.5I_1 = 0$$

$$I_1 + 2I_1 + 1.5I_1 + 2I_2 = 10$$

$$4.5I_1 + 2I_2 = 10 \quad \text{--- (2)}$$

$$4.5I_1 + 2I_2 = 10 \quad \text{--- (2)}$$

$$4.5(4I_2) + 2I_2 = 10$$

$$18I_2 + 2I_2 = 10$$

$$20I_2 = 10$$

$$I_2 = \frac{10}{20} = \frac{1}{2} = 0.5 \text{ A}$$

$$I_1 = 2 \text{ A}$$

In loop BOCB

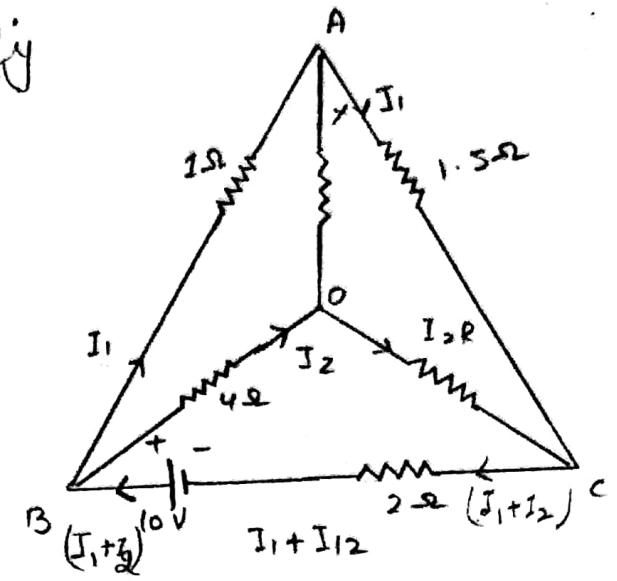
$$-4I_2 - I_2 R - 2(I_1 + I_2) + 10 = 0$$

$$-4(0.5) - 0.5(R) - 2(2.5) + 10 = 0$$

$$-2 - 0.5R - 5 + 10 = 0$$

$$7.5R = 13$$

$$R = \frac{3}{0.5} = \frac{30}{5} = 6 \Omega$$

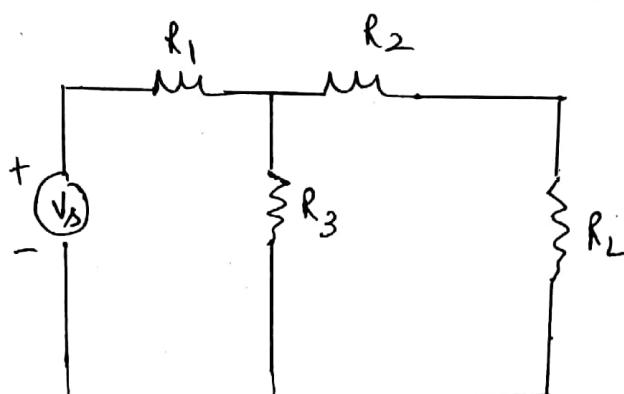


(21)

## Norton's Theorem -

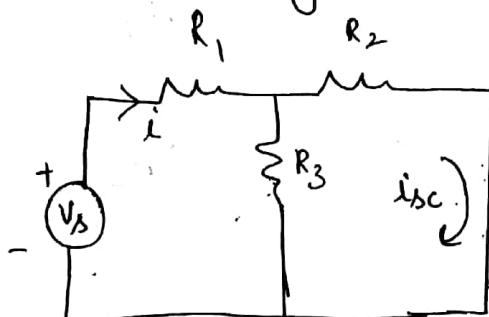
Any two terminal bilateral linear d.c. circuit can be replaced by equivalent circuit consisting of current source in parallel with a resistance.

Explanation:-



To find the current in load resistance  $R_L$

Let replace  $R_2$  by short circuit

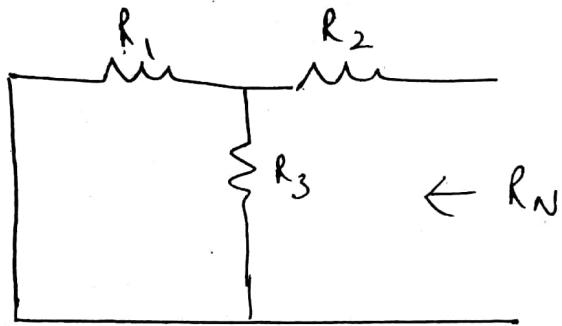


Calculate short circuit  $i_{sc}$

$$i = \frac{V_s}{R_1 + R_2 || R_3} = \frac{V_s}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

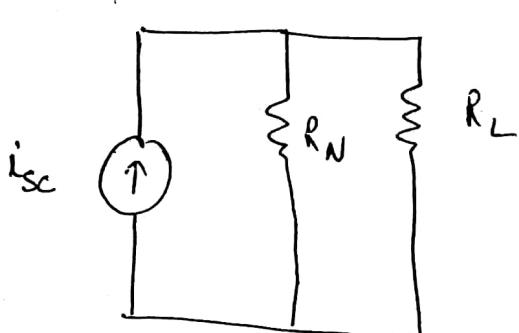
$$i_{sc} = \frac{R_3 \cdot i}{R_2 + R_3} = \frac{R_3}{R_2 + R_3} \cdot \frac{V_s}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

Now calculate equivalent resistance (Norton Resistance)



$$R_N = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Now Norton's equivalent circuit is

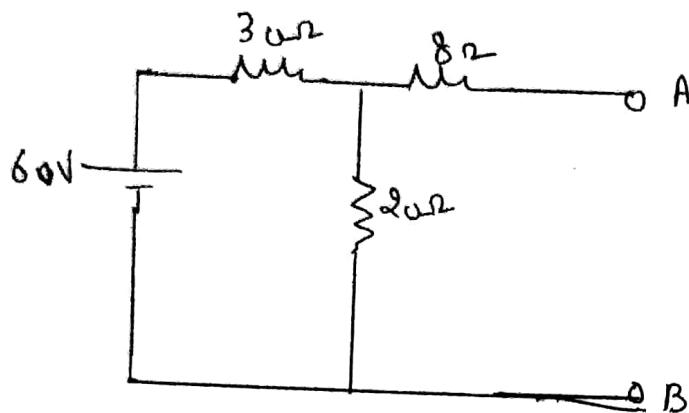


$$I_L = i_{sc} \cdot \frac{R_N}{R_N + R_L}$$

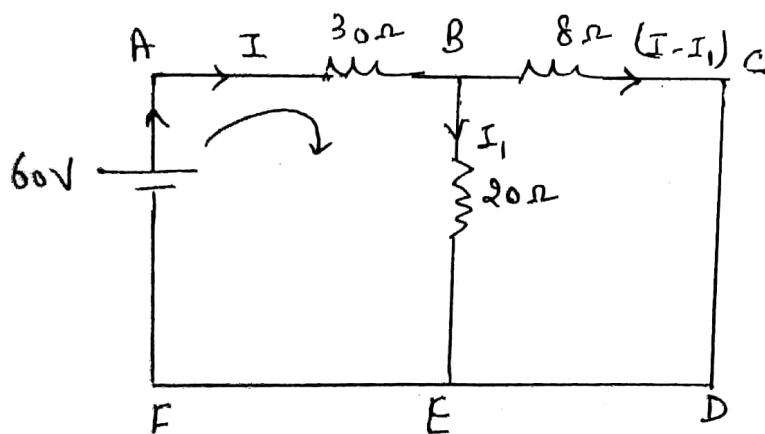
(B)

Examp 1:-

Obtain the Norton Current & equivalent resistance from 'AB'



- Now calculate short circuit current



Now we have to calculate  $(I - I_1) = ?$

Now apply KVL at AB E F A

$$+60 - 30I - 20I_1 = 0$$

$$-30I - 20I_1 = -60$$

$$3I + 2I_1 = 6 \quad \text{--- (1)}$$

apply KVL at B C D E R

$$-8(I - I_1) + 20I_1 = 0$$

$$-8I_1 + 8I_1 + 20I_1 = 0$$

$$-8I_1 + 28I_1 = 0$$

~~408~~ + 201 =

(2)

$$4I - 14I_1 = 0 \quad \text{--- (2)}$$

Solving equation (1) & (2)

$$3I + 2I_1 = 6 \times 7$$

$$4I - 14I_1 = 0$$

$$21I + 14I_1 = 42$$

$$4I - 14I_1 = 0$$

$$25I = 42$$

$$I = \frac{42}{25}$$

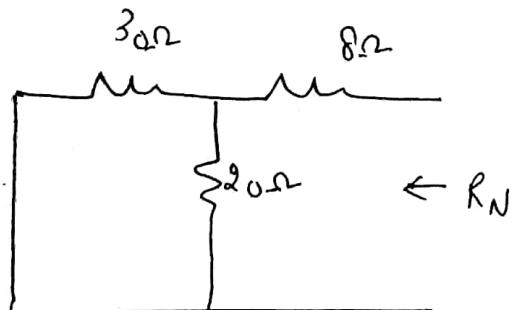
Now put the value of I in (2)

$$4\left(\frac{42}{25}\right) = 14I_1$$

$$I_1 = \frac{4 \times \frac{42}{25}}{25 + 14} = \frac{12}{25}$$

$$\begin{aligned} \text{Now } I_{sc} \text{ is } I - I_1 &= \frac{42}{25} - \frac{12}{25} \\ &= \frac{30}{25} = \frac{6}{5} = 1.2A \end{aligned}$$

Now calculate equivalent resistance

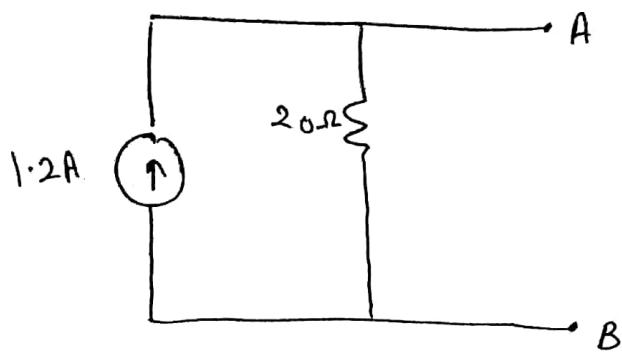


$$\begin{aligned} R_N &= 8 + \frac{30 \times 20}{30 + 20} \\ &= 8 + \frac{60}{5} = 8 + 12 \\ &= 20\Omega \end{aligned}$$

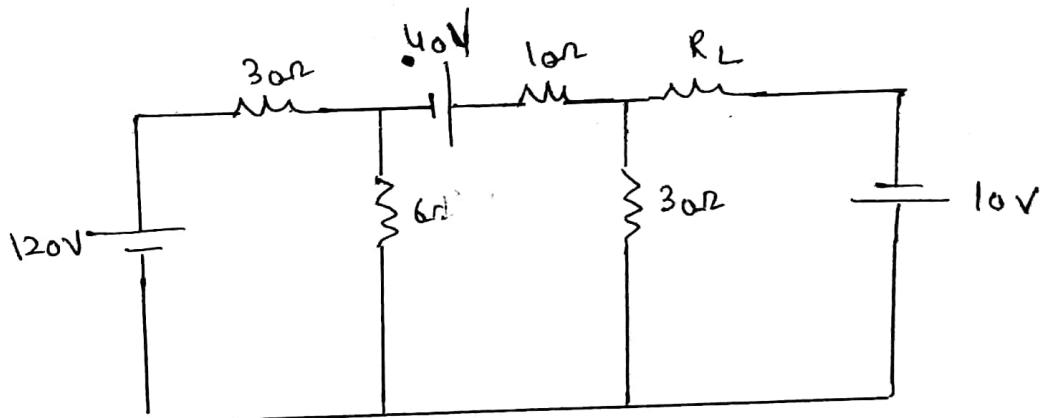
(3)

(16)

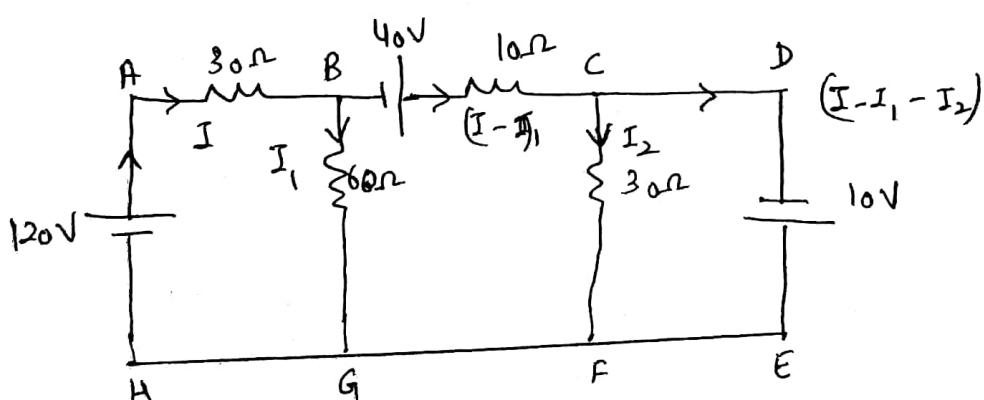
Now equivalent circuit is



(2) Find the Norton's equivalent circuit as seen by  $R_L$



- Replace load resistance by short circuit & calculate  $I_{SC}$



$$\text{Now calculate } I_{SC} = (I - I_1 - I_2)$$

Apply KVL at  $ABGHA$

$$-30I - 6I_1 + 120 = 0$$

$$30I + 60I_1 = 120$$

$$\begin{aligned} 5I + 10I_1 &= 20 \quad \text{--- } (1) \\ I + 2I_1 &= 4 \quad \text{--- } (1) \end{aligned}$$

Apply KVL at BC FB B

$$+40 - 10(I_0 - I_1) - 30I_2 + 60I_1 = 0$$

$$-10I + 10I_1 + 60I_1 - 30I_2 = -40$$

$$-10I + 70I_1 - 30I_2 = -40$$

$$\begin{array}{r} 5I - 8I_1 + 15I_2 = +90 \\ 5I - 8I_1 + 15I_2 = 90 \end{array}$$

$$I - 7I_1 + 3I_2 = 4 \quad (2)$$

Apply KVL at C D E F C

$$+10 + 30I_2 = 0$$

$$30I_2 = -10$$

$$I_2 = -\frac{1}{3}A \quad \text{--- } (3)$$

put value of  $I_2$  in (2)

$$I - 7I_1 - 1 = 4$$

$$I - 7I_1 = 5 \quad \text{--- } (4)$$

$$\begin{array}{r} I + 2I_1 = 4 \\ - - - \\ - 9I_1 = 1 \end{array}$$

$$I_1 = -\frac{1}{9}$$

then  $I$  is

$$\begin{aligned} I + 2I_1 &= 4 \\ I &= 4 + \frac{2}{9} \end{aligned}$$

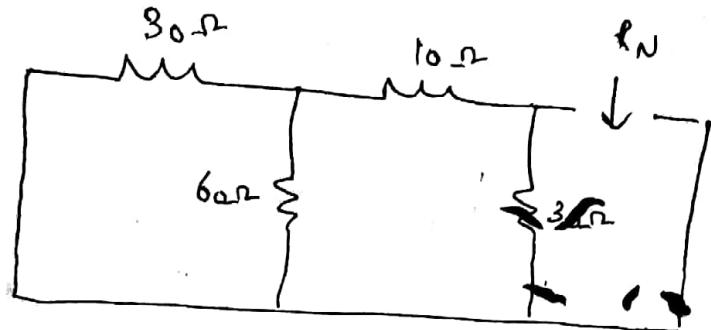
$$I = \frac{38}{9}$$

$$I_{SC} = I - I_1 - I_2$$

$$= \frac{38}{9} + \frac{1}{9} + \frac{1}{3}$$

$$= \frac{38+1+3}{9} = \frac{42}{9} = \frac{14}{3} A$$

Now calculate equivalent resistance  $R_N$

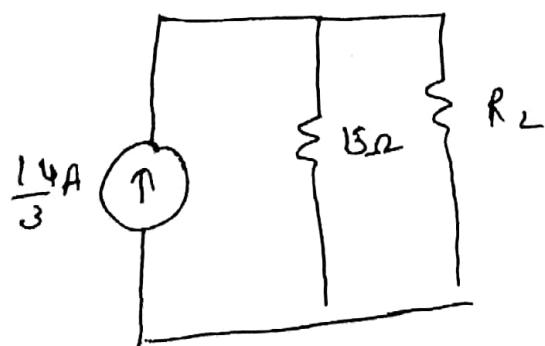


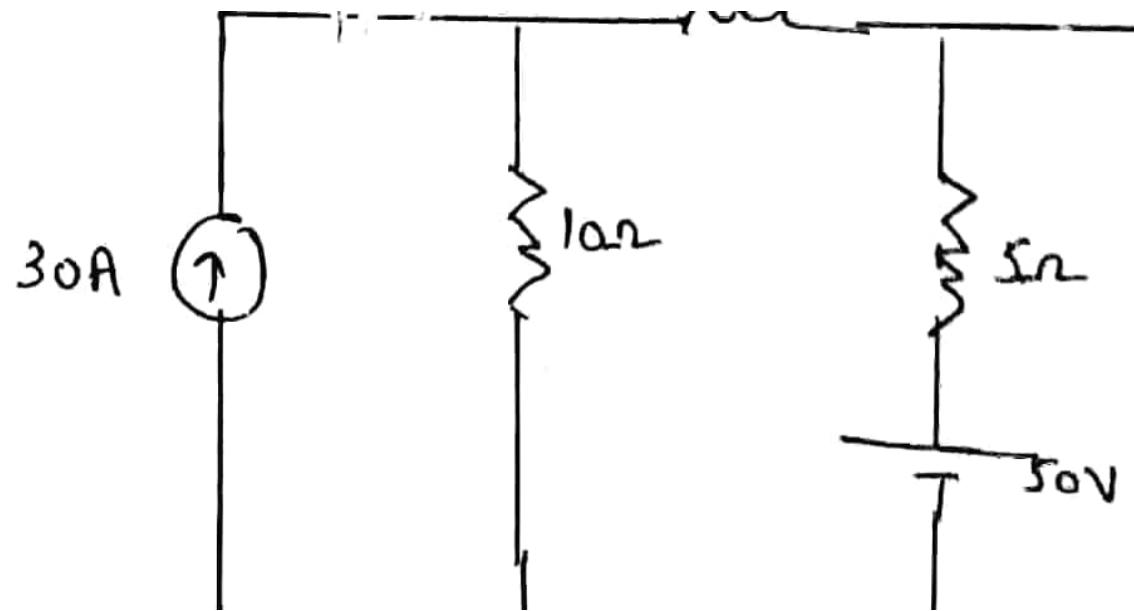
$$R_N = \frac{\frac{30 \times 60}{90} + 10}{\frac{3}{3}}$$

$$= 30 \parallel 30$$

$$= \frac{30 + 30}{60} = 15 \Omega$$

Equivalent circuit is





by kVL at closed loop FCDEF

(5)

$$-5I - 5I_1 - 50 + 300 + 10I = 0$$

(16)

$$-15I - 5I_1 = -250$$

$$3I + I_1 = +50 \quad \text{--- (1)}$$

→ at closed loop CABDC

$$+50 + 5I_1 = 0$$

$$5I_1 = -50$$

$$I_1 = -10 \text{ A}$$

But value of  $I_1$  in eqn (1)

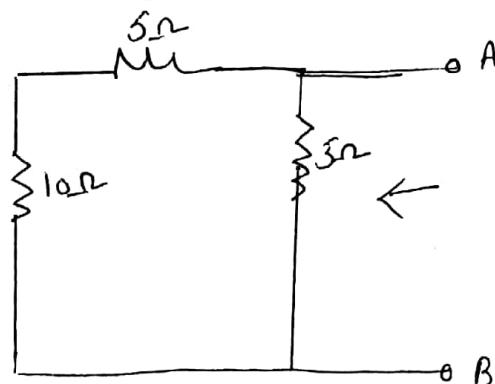
$$3I - 10 = 50$$

$$3I = 60$$

$$I = 20 \text{ A}$$

$$\begin{aligned} \text{Then } I_{AB} (\text{current in branch AB}) &= I - I_1 \\ &= 20 + 10 = 30 \text{ A} \end{aligned}$$

Now calculate equivalent resistance  $R_N$



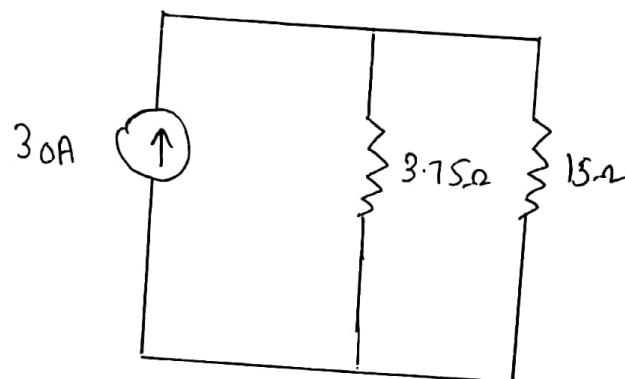
$$R_N = \frac{5 + \cancel{\frac{5 \times 10}{15}}}{\cancel{3}}$$

$$= \frac{5 + \frac{10}{3}}{3} = \frac{25}{9}$$

$$= 5.111 \text{ ohms}$$

$$= \frac{5 \times 15}{20} = \frac{15}{4} = 3.75$$

Now Norton's equivalent circuit

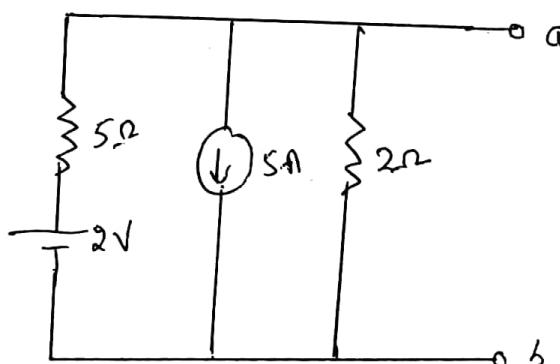


Current in  $15\Omega$

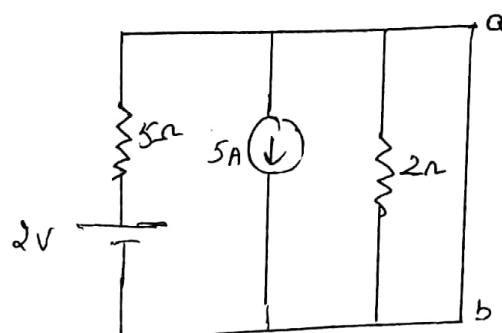
$$I = \frac{3.75}{3.75 + 15} \times 30$$

$$I = \frac{3.75}{18.75} \times 30 = 6A$$

Ques:-4 Find the Norton's equivalent circuit across a-b for network shown in fig

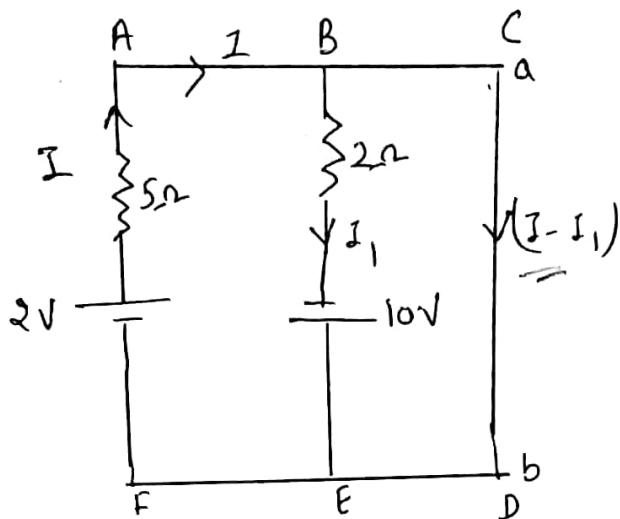


Findably Norton circuit across a-b



at current source into voltage source

(6)



(1x)

Now Calculate  $(I - I_1) = ?$

Apply KVL (ABEFA)

$$-2I_1 + 10 + 2 - 5I = 0$$

$$-5I - 2I_1 = -12$$

$$5I + 2I_1 = 12 \quad \text{--- (1)}$$

Apply KVL at BCDEB

$$-10 + 2I_1 = 0$$

$$2I_1 = 10$$

$$I_1 = 5 \text{ A}$$

Put value of  $I_1$  in (1)

$$5I + 10 = 12$$

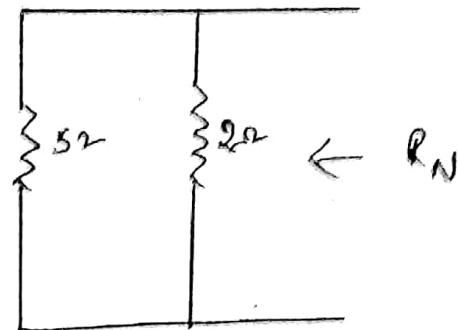
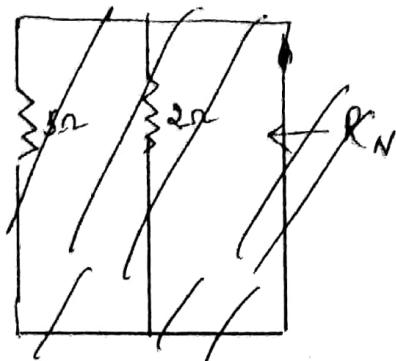
$$5I = 2$$

$$I = \frac{2}{5} = 0.4 \text{ A}$$

Current in branch (ab)

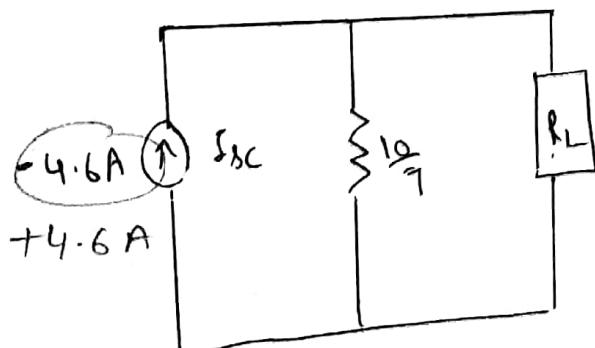
$$I_{bc} = I - I_1 = 0.4 - 5 = -4.6 \text{ A}$$

Now Calculate equivalent resistance ( $r_N$  or  $r_{eq}$ )



$$r_N = \frac{2 \times 5}{7} = \frac{10}{7}$$

Equivalent circuit is

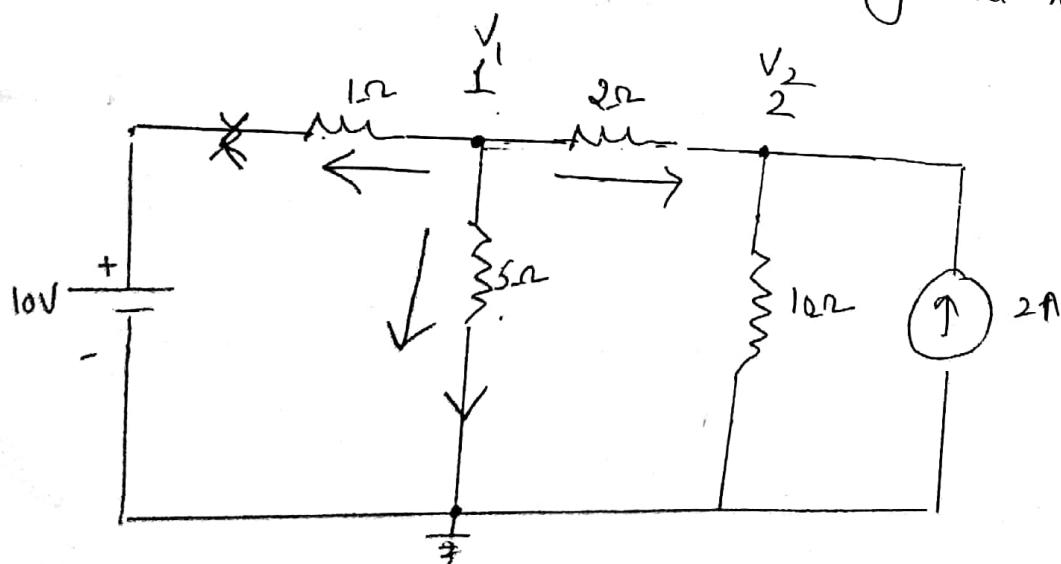


Nodal Analysis:- Nodal analysis provide a general procedure for analysing circuits using node voltage as the circuit variable.

In the node voltage method, we can solve for the unknown voltages in a circuit using KCL.

Steps:-

1. The first step is to select an essential node. (is a node joining three or more elements) Then to select an essential node as a reference. The choice of arbitrary node with the most number of nodes attached is a good choice.
2. Then the node voltage can be defined as voltage at a given node wrt reference node.
3. Finally KCL is applied to all non-reference nodes.



Node '3' is selected as reference node

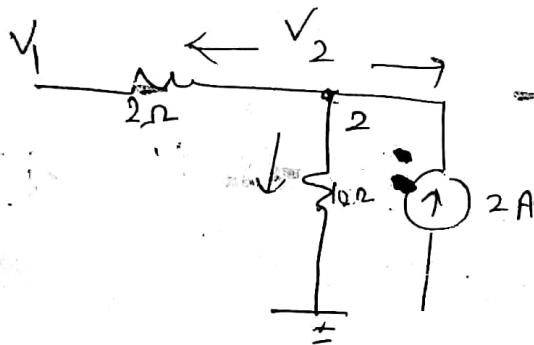
Apply KCL at node '1'

$$\frac{V_1 - 10}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$$

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$$

$$17V_1 - 5V_2 = 100 \quad \text{--- (1)}$$

Apply KCL at node '2'



$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{10} - 2 = 0$$

$$5V_2 - 5V_1 + V_2 - 20 = 0$$

$$-5V_1 + 6V_2 = 20 \quad \text{--- (2)}$$

Solve the eqn '1' & '2'

$$17V_1 - 5V_2 = 100 \times 6$$

$$-5V_1 + 6V_2 = 20 \times 5$$

$$102V_1 - 30V_2 = 600$$

$$-25V_1 + 30V_2 = 100$$

$$77V_1 = 700$$

$$V_1 = \frac{700}{77} = \frac{100}{11} = 9.09V$$

Put value of  $V_1$  in eqn '1'

$$17\left(\frac{100}{11}\right) - 5V_2 = 100$$

$$V_2 = 10.91V$$

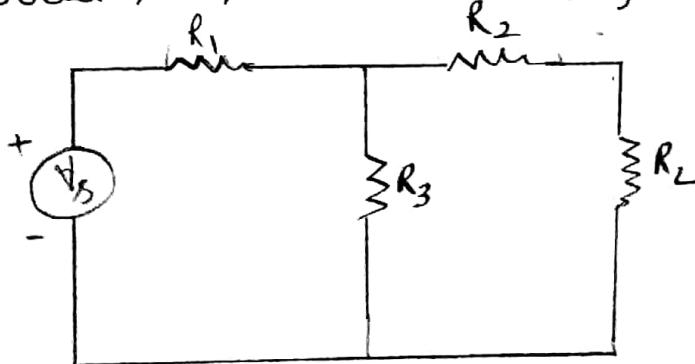
## Thevenin's theorem:-

Statement - Any two terminal bilateral linear d.c. circuit can be replaced by an equivalent circuit consisting of a voltage source + a series resistor.

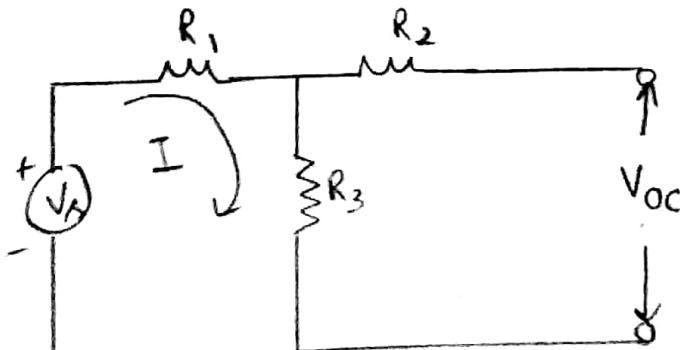
- It is applicable where it is desired to determine current through or voltage across any one element in a network without going through the rigorous method of solving of a network equations

## Explanation -

Simple d.c. circuit as shown in fig. We have to find the current in load resistance ( $R_L$ )

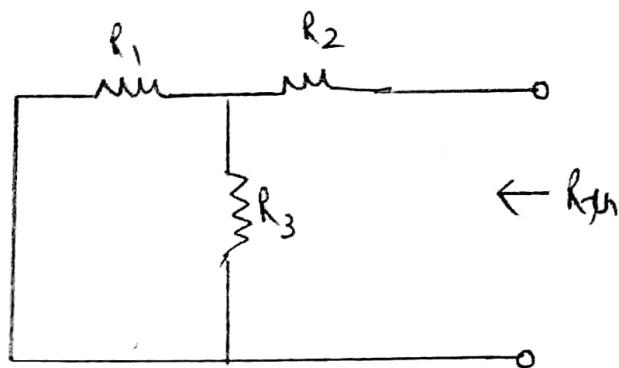


- Remove the load resistance & find open circuit voltage  $V_{OC}$  across the open circuited load terminals

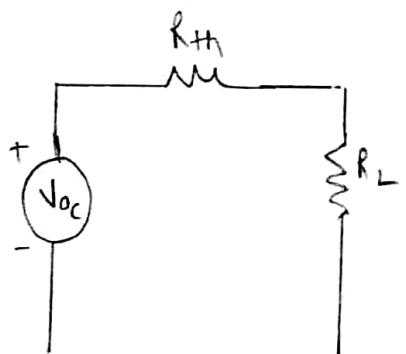


$$V_{OC} = \frac{R_3}{R_1 + R_3} V_s$$

- To find the thevenin resistance (Voltage source is removed by a short circuit)



$$R_{TH} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

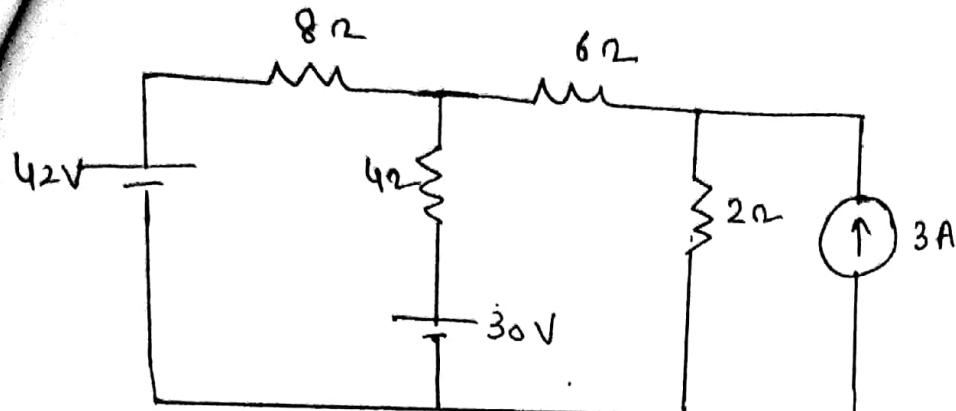


Thevenin's Equivalent circuit

$$I_L = \frac{V_{OC}}{R_{TH} + R_L}$$

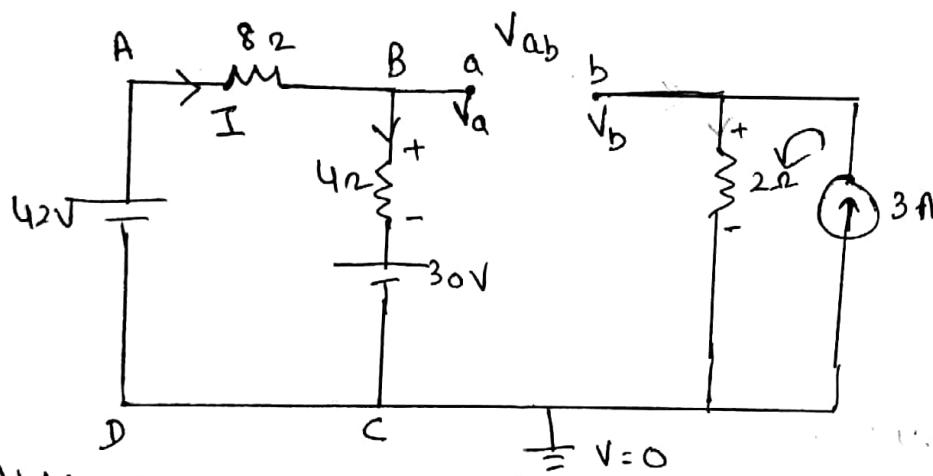
Example 1

(2)



Calculate the current in  $6\Omega$

- Remove the load resistance & calculate open circuit voltage across the open circuited load terminals



$$\frac{V_a - 30}{4} + \frac{V_a - 42}{8} = 0$$

$$2V_a - 60 + V_a - 42 = 0$$

$$3V_a = 102$$

$$V_a = 34V$$

Apply KVL at ABCD (closed loop)

$$42 - 8I - 4I_0 - 30 = 0$$

$$12 = 12I$$

$$I = 1A$$

$$\frac{V_b}{2} = 3$$

$$V_b = 6V$$

Now calculate voltage at terminal 'a'

$$+30 + 4I = V_a$$

$$30 + 4 = V_a$$

$$V_a = 34V$$

Now calculate  $V_b$

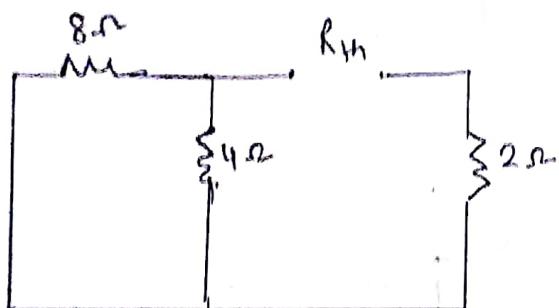
$$0 + 2 \times 3 = V_b$$

$$V_b = 6V$$

Then  $V_{ab} = V_a - V_b = 34 - 6 = 28V$

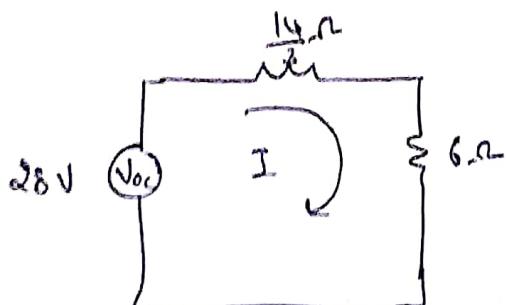
Now calculate  $R_{th}$

All the voltage sources are short circuited & circuit source is open circuited.



$$R_{th} = \frac{8 \times 1}{\frac{12}{3}} + 2$$

$$= \frac{8}{3} + 2 = \frac{8+6}{3} = \frac{14}{3} \text{ ohm}$$

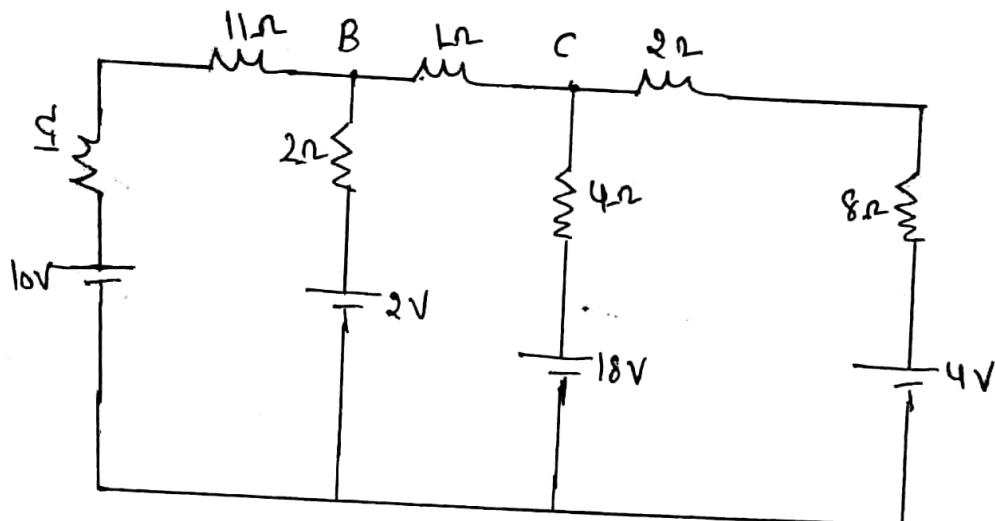


$$\begin{aligned} I &= \frac{28}{6 + \frac{14}{3}} \\ &= \frac{28 \times 3}{32 + 14} = \frac{21}{8} = 2.625A \end{aligned}$$

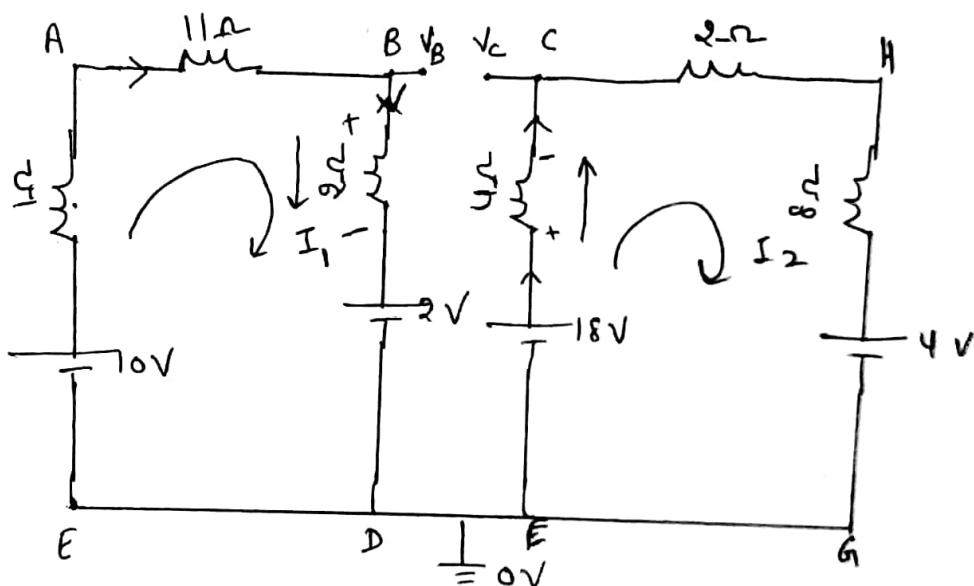
Sample 2

(1)

(3)



Find the Thevenin's equivalent circuit at terminal BC. Determine the current through  $1\Omega$ .



Apply KVL at closed loop ABDEA

$$10 - I_1 - 11I_1 - 2I_1 - 2 = 0$$

$$14I_1 = 8$$

$$I_1 = \frac{8}{14} = \frac{4}{7} \text{ A}$$

Now  $V_B$  is

$$+2 + 2I_1 = V_B \quad V_B = 2 + 2\left(\frac{4}{7}\right) = 2 + \frac{8}{7} = \frac{22}{7} \text{ V}$$

Now Apply KVL at CfGHC

$$+18 - 4I_2 - 2I_2 - 8I_2 - 4 = 0$$

$$14 = 14I_2$$

$$I_2 = 1A$$

$V_C$  is

$$+18 + 4I_2 = V_C$$

$$18 - 4(1) = V_C$$

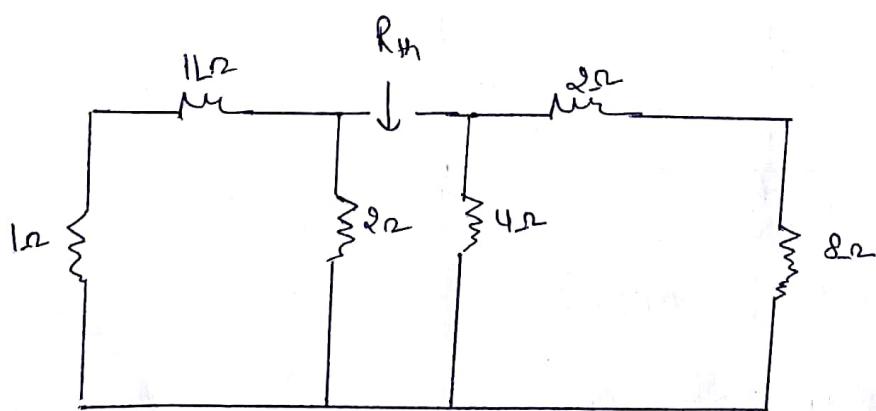
$$V_C = 14V$$

$V_{th}$  voltage at terminal  $V_{BB} = V_B - V_B$

$$= V_B - V_B$$

$$= \frac{14}{7} - \frac{22}{7} = 14 - 3.14 = 10.86V$$

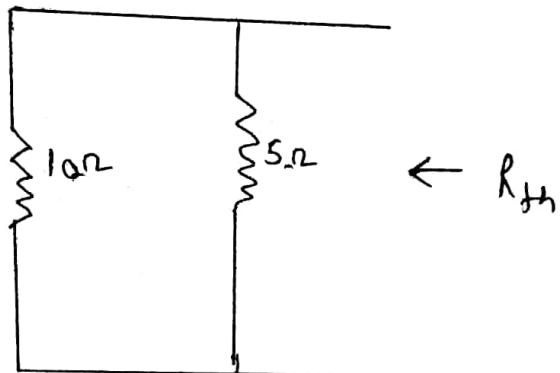
Now calculate  $r_{th}$



$$\left[ 2 \parallel 12 \right] \parallel \left[ 4 \parallel 10\Omega \right] = \left[ \frac{12}{7} \parallel \frac{20}{7} \right]$$

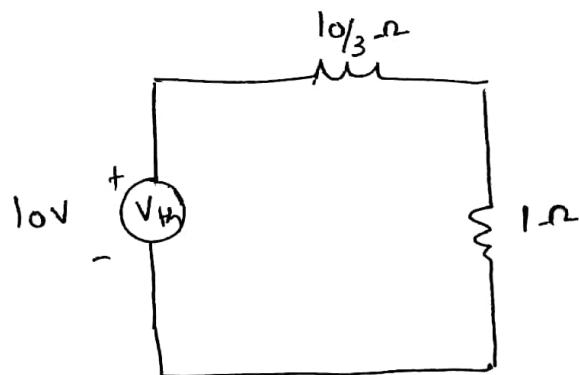
$$\begin{aligned} & \left[ \frac{2+12}{14} \parallel \frac{4 \times 10}{14} \right] = \frac{\frac{12}{7} \times \frac{20}{7}}{\frac{12}{7} + \frac{20}{7}} = \frac{240}{49 \times 32} \\ & \left[ \frac{24}{14} \parallel \frac{40}{14} \right] = 1.0714\Omega \end{aligned}$$

Now calculate  $r_{th}$



$$r_{th} = \frac{10 + 5}{\frac{10}{3}} = \frac{10}{3} \Omega$$

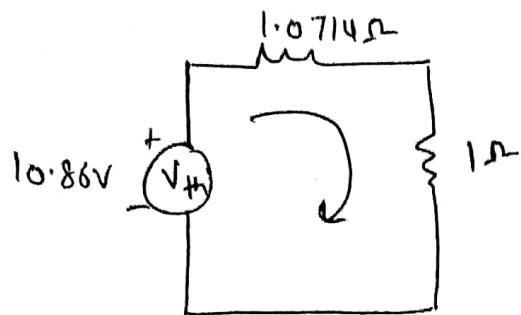
Now the circuit's equivalent circuit is



$$I_{th} = \frac{10}{\frac{10}{3} + 1} = \frac{10}{3.33 + 1} = 2.31 A$$

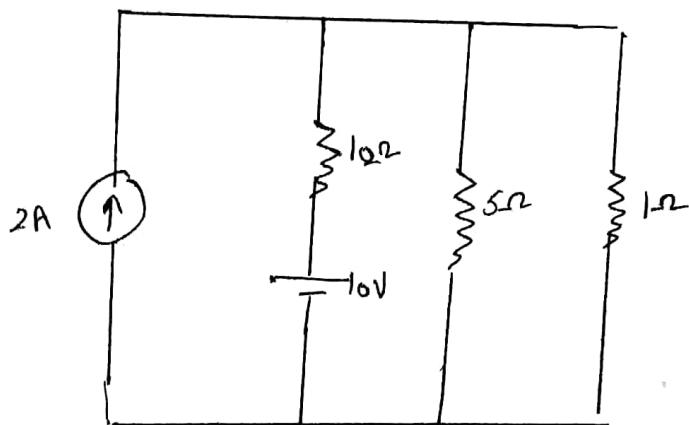
$$\begin{aligned} \text{Now Power loss in } 1\Omega \text{ resistor is} &= I^2 R \\ &= (2.31)^2 \times 1 \\ &= 5.33 W \end{aligned}$$

Now thevin's equivalent circuit is

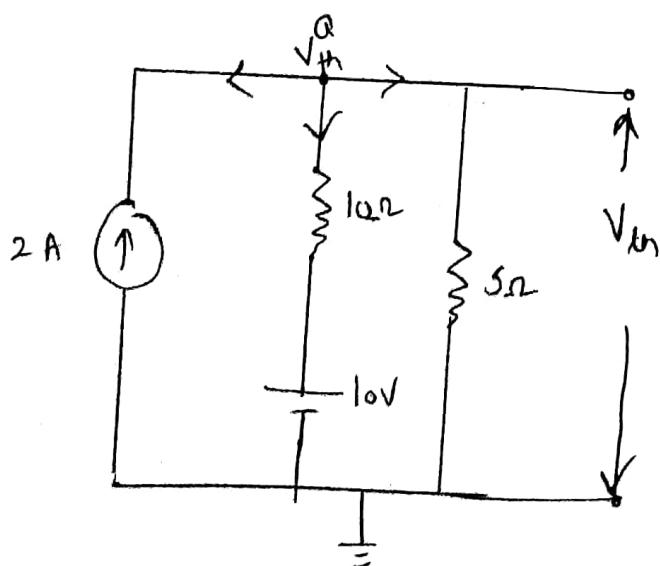


$$I = \frac{10.86}{2.0714} = 5.24 A$$

(3)



Find the power loss in 1 ohm resistor



Apply KCL at node a

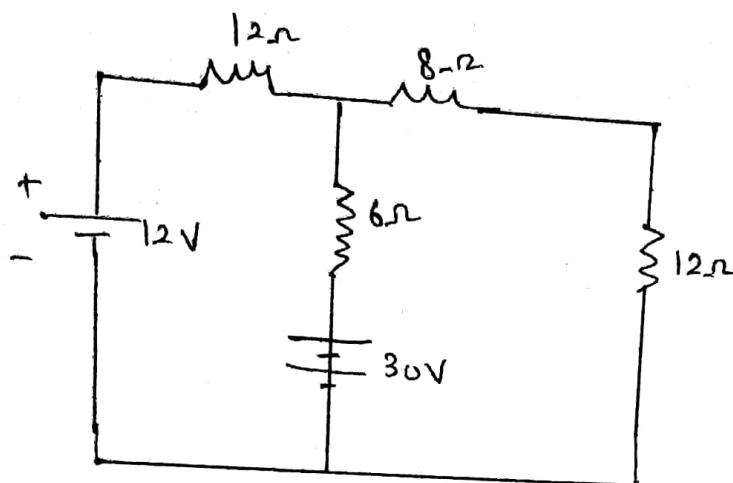
$$\frac{V_{th} - 10}{10} + \frac{V_{th}}{5} = 2$$

$$2V_{th} - 20 + 2V_{th} = 20$$

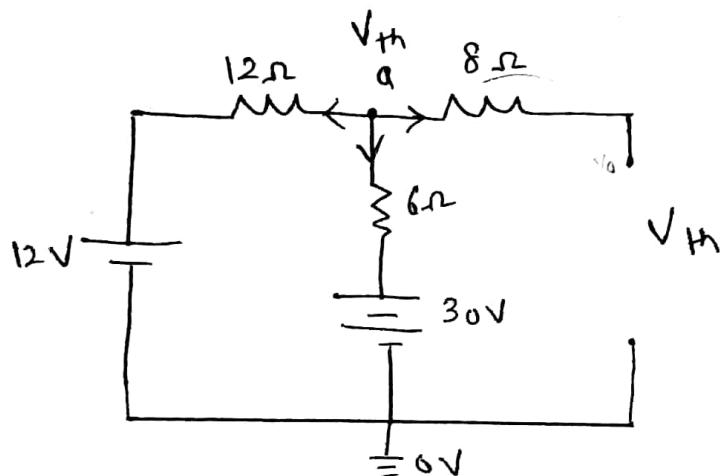
$$3V_{th} = 40$$

$$V_{th} = 10V$$

Determine the current in resistor connected across the terminal A-B



- ① Remove the load resistance & calculate open circuit voltage ( $V_{th}$ )



Apply KCL at node 'a'

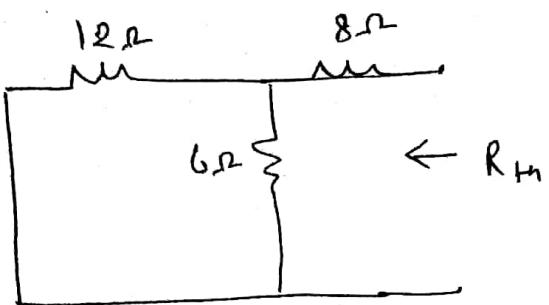
$$\frac{V_{th} - 12}{12} + \frac{V_{th} - 30}{6} = 0$$

$$V_{th} - 12 + 2V_{th} - 60 = 0$$

$$3V_{th} = 72$$

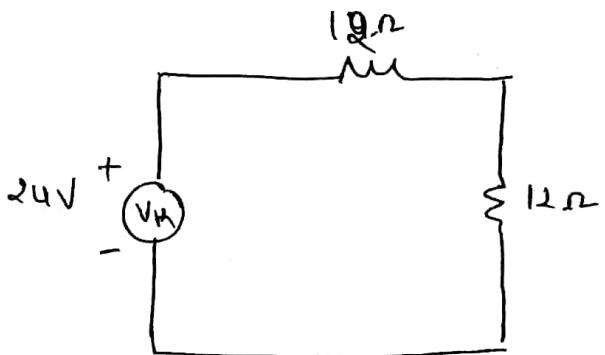
$$V_{th} = 24V$$

Now calculate  $R_{TH}$



$$R_{TH} = \frac{12 \times 8}{12 + 8} = 4\Omega + 8\Omega = 12\Omega$$

Thevenin's Equivalent circuit is



$$I = \frac{24}{12} = \frac{12}{4} = 3A$$

Nodal Analysis:- Nodal analysis provide a general procedure ④

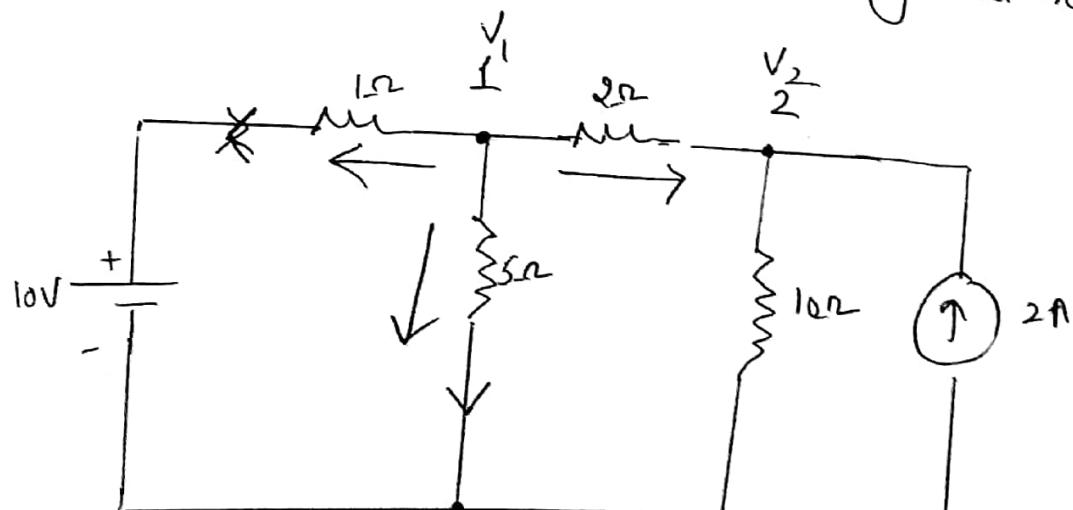
for analysing circuits using node voltage as the circuit variables.

In the node voltage method, we can solve for the unknown

voltages in a circuit using KCL.

Steps:-

1. The first step is to select an arbitrary node joining three or more elements as a reference node. Then to select an arbitrary node as a reference node. The choice is arbitrary, the node with the most branches attached is a good choice.
- Then the node voltage can be defined as voltage at a given node wrt reference node.
- Finally KCL is applied to all non-reference nodes.



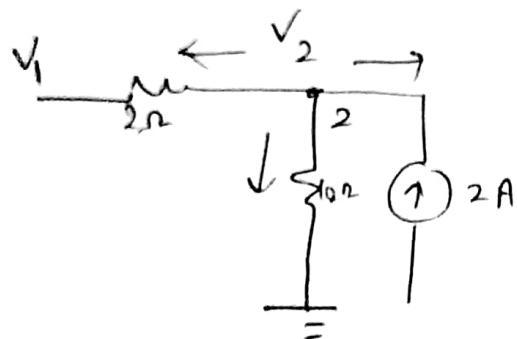
Apply KCL at node '1'

$$\frac{V_1 - 10}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$$

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$$

$$17V_1 - 5V_2 = 100 \quad \text{--- (1)}$$

Apply KCL at node '2'



$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{10} - 2 = 0$$

$$5V_2 - 5V_1 + V_2 - 20 = 0$$

$$-5V_1 + 6V_2 = 20 \quad \text{--- (2)}$$

Solve the eqn '1' & '2'

$$17V_1 - 5V_2 = 100 \times 6$$

$$-5V_1 + 6V_2 = 20 \times 5$$

$$102V_1 - 30V_2 = 600$$

$$-25V_1 + 30V_2 = 100$$

$$\underline{77V_1 = 700}$$

$$V_1 = \frac{700}{77} = \frac{100}{11} = 9.09V$$

Put value of V<sub>1</sub> in eqn '1'

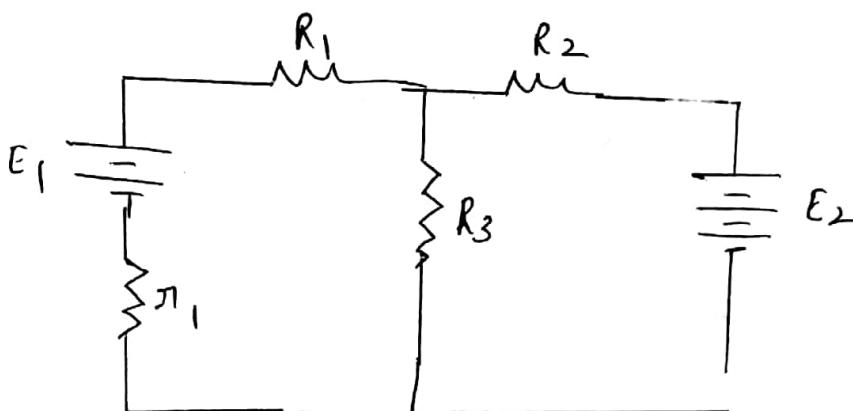
$$17\left(\frac{100}{11}\right) - 5V_2 = 100$$

$$V_2 = 10.91V$$

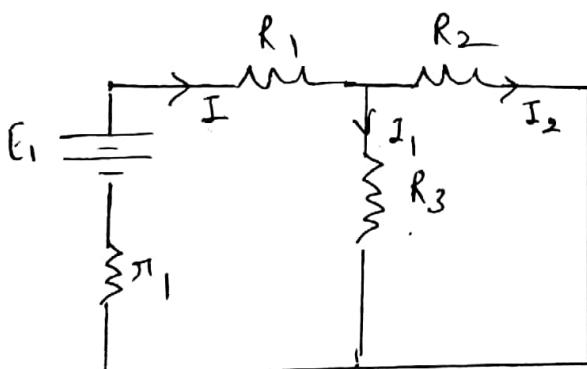
## Superposition Theorem:-

This theorem is applicable to linear bilateral network having two or more than two sources.

This theorem states that current flowing through any section is the algebraic sum of all currents which should flow in that section if each source of eng would be considered separately and other sources are replaced by their internal resistance.



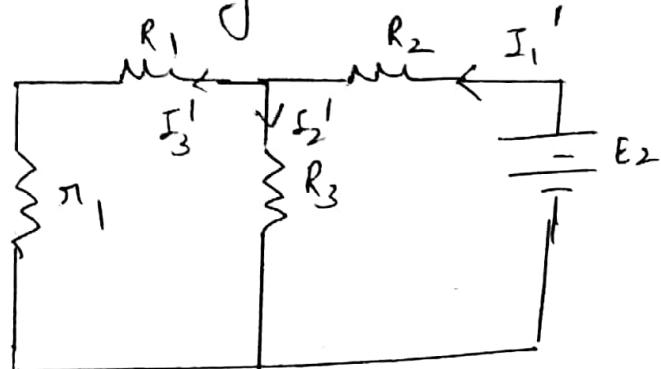
First we activate source of eng  $E_1$  &  $E_2$  is deactivated replaced by short circuit



Determine the current in  
in section ( $I, I_1 + I_2$ )

Then current in each branch is

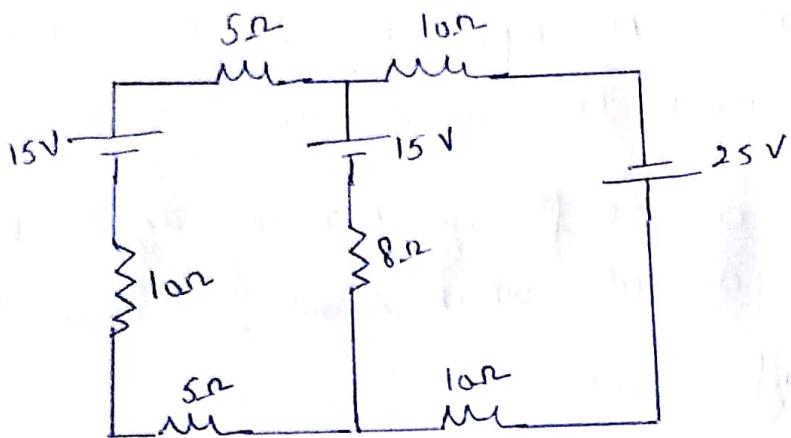
Now  $E_2$  is activated &  $E_1$  is replaced by short circuit



$$I_1 = I - I_3' \quad \text{and} \quad I_3 = I_1 + I_2'$$

$$I_2 = I_2 - I_1'$$

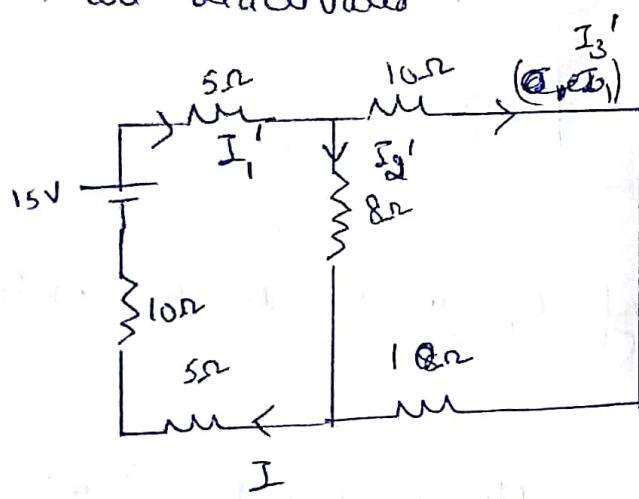
Ques:-1 Find the current in different branches of the Network



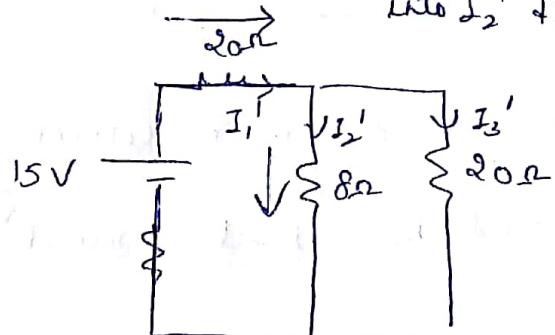
Solution:-

First we activate 15V (battery) + all the voltage sources are deactivated

Currents are deactivated



Total Current  $I_1'$  is splitted into  $I_2'$  +  $I_3'$



$$\begin{aligned} \text{Total resistance is } &= (20||8) + 20 \\ &= \frac{8 \times 20}{28} + 20 \\ &= 5.714 + 20 \\ &= 25.714 \Omega \end{aligned}$$

$$\begin{aligned} I_3' &= \frac{8}{28} \times 0.583 \\ &= 0.168 A \end{aligned}$$

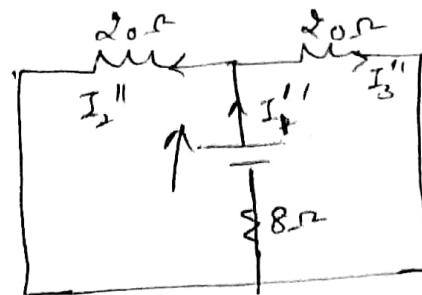
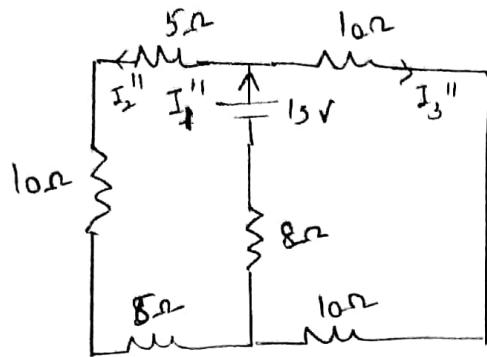
$$\begin{aligned} \text{Total Current} &= I_1' \\ I_1' &= \frac{15}{25.714} = 0.583 A \end{aligned}$$

$$\begin{aligned} I_2' &= \frac{20}{28} \times 0.583 \\ &= 0.4166 A \end{aligned}$$

(2)

Now activate the 15V source of current  $I_1''$  in branch '2' + all voltage  
are deactivated

(26)



$$\begin{aligned}\text{Total resistance} &= (20 \parallel 20) + 8 \\ &= \frac{10}{20 + 20} + 8 \\ &= 18 \Omega\end{aligned}$$

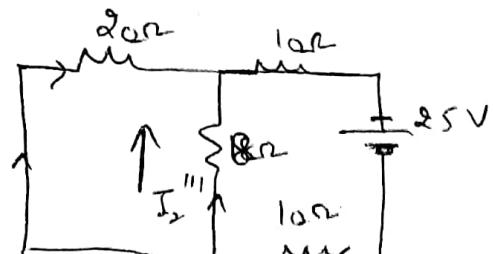
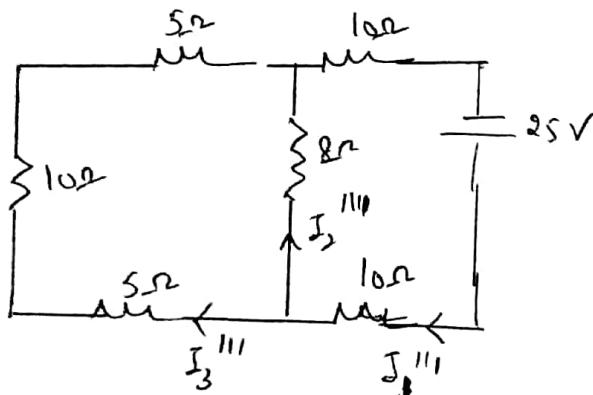
Current supply

$$I_1'' = \frac{15}{18} = 0.833 A$$

$$I_2'' = \frac{20}{40} \times 0.833 = 0.4167 A$$

$$I_3'' = \frac{20}{40} \times 0.833 = 0.4167 A$$

Now activate 25V voltage source



$$\begin{aligned}\text{Total resistance} &= \frac{20 \times 10}{20 + 10} + 20 \\ &= 5.714 + 20 \\ &= 25.714\end{aligned}$$

$$I_1''' = \frac{25}{25.714} = 0.9722 A$$

$$I_2''' = \frac{20}{28} \times 0.9722 \\ = 0.6944 A$$

$$I_3''' = \frac{8}{28} \times 0.9722 \\ = 0.2777 A$$

Now current in each branch is

$$I_1 = I_1' - I_2'' + I_3''' \\ = 0.444 A$$

~~$I_2 = I_1' - I_2'' + I_3'''$~~ 

$$= -I_2' + I_1'' + I_2''' \\ = 1.111 A$$

$$I_3 = I_3' + I_3'' + I_3''' \\ = 1.555 A$$

Sc

10

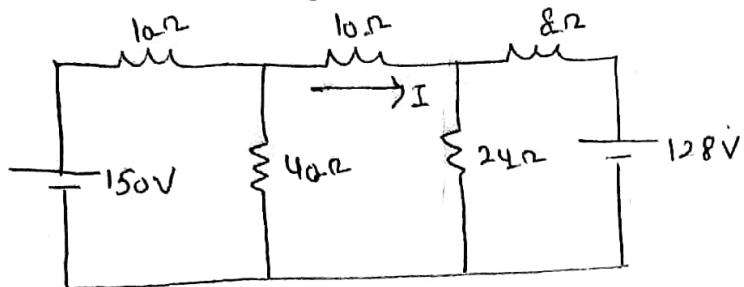
-2

7

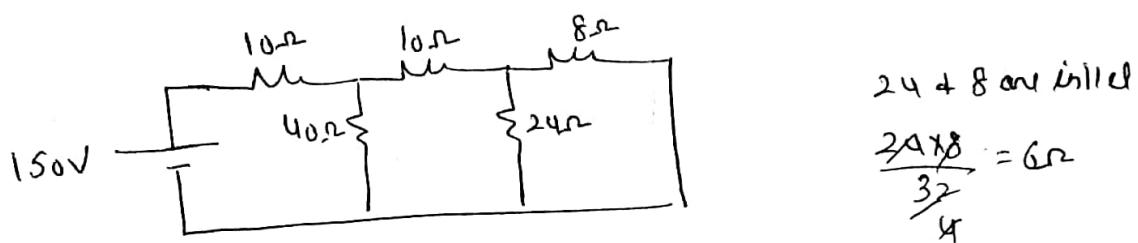
(3)

Ques. 2 Find I using superposition theorem

(27)

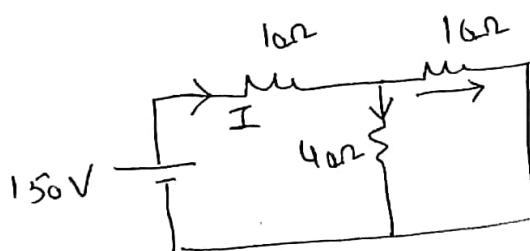
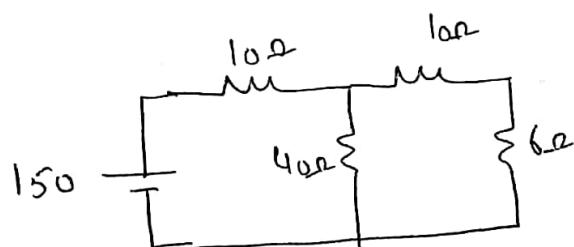


Solution First we activate 150V battery source & deactivate 128V



24 + 8 are in parallel

$$\frac{24 \times 8}{32} = 6\Omega$$



$$\text{Total equivalent resistance} = \frac{40 \times 16}{56} + 10 \\ = 21.49\Omega$$

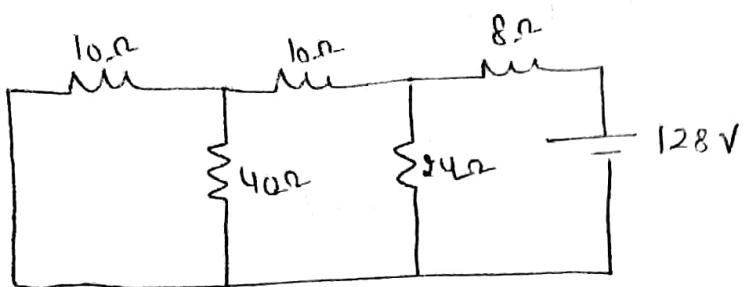
Current in 10Ω  
resistor =

$$= 6.97 \times \frac{40}{56}$$

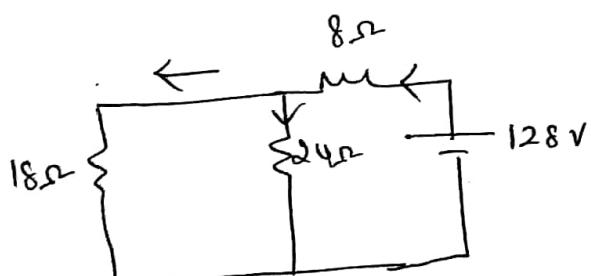
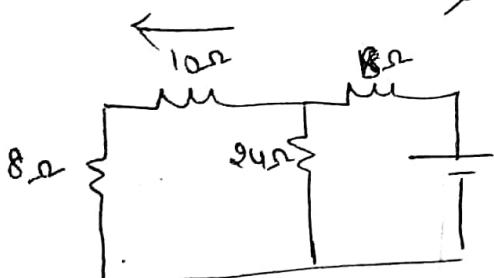
$$\text{Total current supplied (I)} = \frac{150}{21.49} \\ = 6.97$$

$$= 4.98A \\ = 5A$$

Now activated 128V, & deactivated the 150V battery source



$$40 + 10 \text{ are in } 11\text{d} \text{ ie. } \frac{40+10}{50} = 8\Omega$$



$$\begin{aligned}\text{Total resistance} &= \frac{18 \times 24}{42} + 8 \\ &= \frac{72}{7} + 8 \\ &= 18.286\Omega\end{aligned}$$

$$\text{Total current supplied} = \frac{128}{18.286} = 6.99\text{A}$$

$$\begin{aligned}\text{Current in } 10\Omega \text{ resistor} &= \frac{24}{42} \times 6.99 = 3.99\text{A} \\ &= 4\text{A}\end{aligned}$$

$$\text{Total current in } 10\Omega = 5 - 4 = 1\text{A}$$

(12)

## Delta - Star and Star - Delta Transformation:-

Three resistances connected nose-to-tail as shown in fig are said to be delta or mesh connected (or  $\Delta$ )

Three resistances connected together at a common point O are said to be star- (or  $Y$ ) connected.

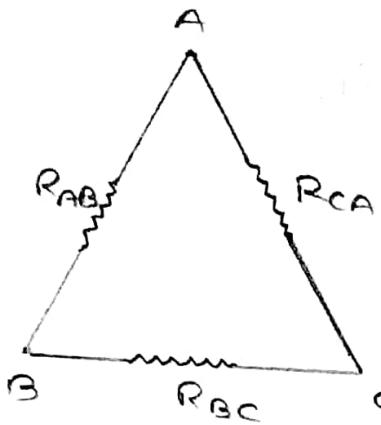


fig (a)

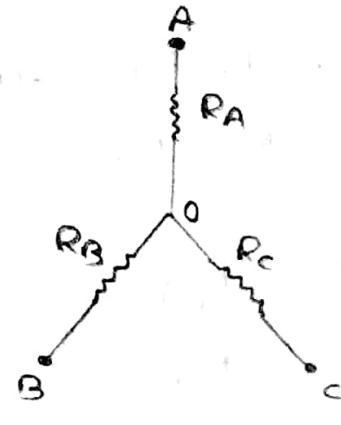


fig (b)

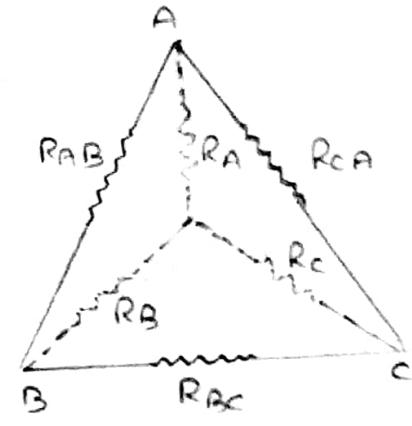


fig (c)

## Delta - Star transformation:-

Consider a circuit as shown in fig where three resistances  $R_{AB}$ ,  $R_{BC}$  and  $R_{CA}$  are connected in delta. If this circuit is converted to star connections let the resistances be  $R_A$ ,  $R_B$  and  $R_C$ .

For two circuits to be equivalent, the resistance measured between any two of the terminals A, B and C must be same in the two cases.

$$R_{AB} \text{ for star connection} = R_{AB} \text{ for delta connection} \quad (i)$$

$$R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$$

$$R_A + R_B = \frac{R_{AB} \cdot (R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \quad (i)$$

$$\text{Similarly } R_B + R_C = \frac{R_{BC} (R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \quad (ii)$$

(23)

$$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad \text{--- (iii)}$$

Subtract (ii) - (i)

$$R_C - R_A = \frac{R_{BC}(R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} - \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C - R_A = \frac{R_{CA}R_{AB} + R_{BC}R_{CA} - R_{AB}R_{BC} - R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C - R_A = \frac{R_{CA}(R_{BC} - R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad \text{--- (iv)}$$

Add (iii) + (iv)

$$2R_C = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} + \frac{R_{CA}(R_{BC} - R_{AB})}{R_{AB} + R_{BC} + R_{CA}}$$

$$2R_C = \frac{R_{CA}R_{AB} + R_{CA}R_{BC} + R_{CA}R_{BC} - R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{CA}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

Similarly

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

(24)

(13)

## Delta Transformation

We know that

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{--- (i)}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{--- (ii)}$$

$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{--- (iii)}$$

Divide (i) by (ii)

$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}}$$

$$\boxed{R_{CA} = \frac{R_A}{R_B} \times R_{BC}} \quad \text{--- (iv)} \quad R_{BC} = \frac{R_B}{R_A} \times R_{CA}$$

Divide (i) by (iii)

$$\frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}}$$

$$\boxed{R_{AB} = \frac{R_A}{R_C} \times R_{BC}} \quad \text{--- (v)}$$

put these values in (i)

$$R_A = \frac{\frac{R_A}{R_C} \times R_{BC} \times \frac{R_A}{R_B} \times R_{BC}}{\frac{R_A}{R_C} \times R_{BC} + R_{BC} + \frac{R_A}{R_B} \times R_{BC}}$$

$$\cancel{R_A} = \frac{\frac{R_{BC}}{R_B} \frac{R_A^2}{R_C}}{\frac{R_A}{R_C} + 1 + \frac{R_A}{R_B}}$$

(25)

$$I = \frac{\frac{R_{BC} R_A}{R_B R_C}}{\frac{R_A R_B + R_B R_C + R_A R_C}{R_B R_C}}$$

$$I = \frac{\frac{R_{BC} R_A}{R_B R_C}}{R_A R_B + R_B R_C + R_A R_C}$$

$$R_A R_{BC} = R_A R_B + R_B R_C + R_A R_C$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \quad \text{(i)}$$

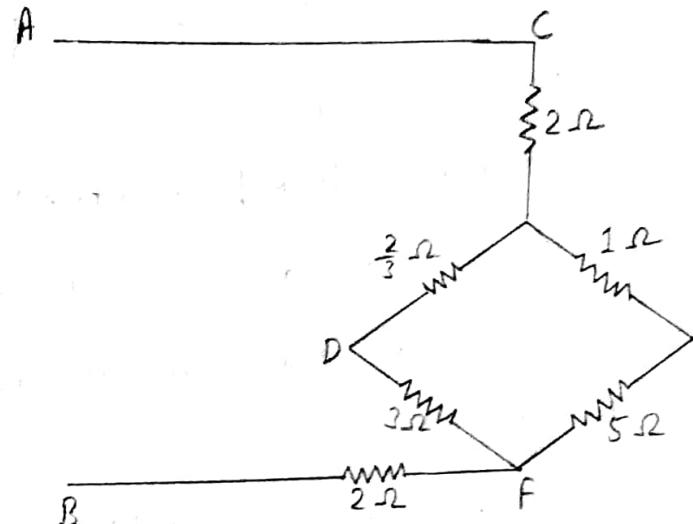
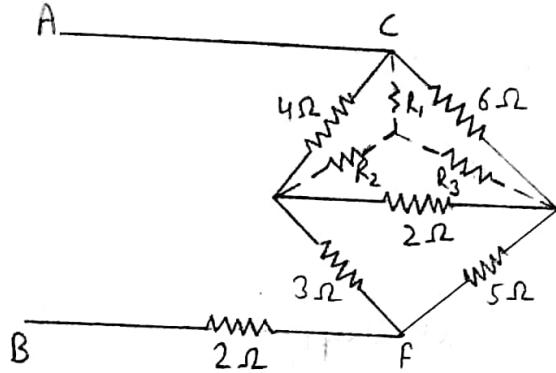
Similarly

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{CA} = R_A + R_C + \frac{R_C R_A}{R_B}$$

### Numericals on Star-Delta

1- Find the resistance between AB of the circuit shown in the figure using Star-Delta Transformation



(a) Solution:-

$$R_1 = \frac{4 \times 6}{4+6+2} = \frac{24}{12} = 2 \Omega$$

$$R_2 = \frac{4 \times 2}{4+6+2} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3} \Omega$$

$$R_3 = \frac{2 \times 6}{4+6+2} = \frac{12}{12} = 1 \Omega$$

Resistance across CF or AB

$$R_{AB} = 2 + \left[ \left( \frac{2}{3} + 3 \right) || (5 + 1) \right] + 2$$

$$= 2 + \left[ \frac{11}{3} || 6 \right] + 2$$

$$= 2 + \frac{\frac{11}{3} \times 6^2}{\frac{11}{3} + 6} + 2$$

$$= 2 + \frac{22 \times 3}{29} + 2$$

$$= 4 + \frac{66}{29} = \frac{116 + 66}{29} = \frac{182}{29} = 6.276 \Omega$$

(27)

## Transient Response:-

The short-lived electrical phenomenon that occurs in a system due to sudden change in voltage, current or load is called transient response.

In general, transient took place whenever:

- 1) Load is connected or disconnected to/from the supply.
- 2) There is a sudden change in the applied voltage from one finite value to the other.
- 3) The circuits get short-circuited

Infact, transients are not caused due to any part of the applied voltage rather than are associated with the changes that occur in the stored energy in inductors & capacitors.

Since, no energy is stored in the resistors, no transient occurs in pure resistive circuit.

In Inductive circuit, energy is stored in the form of electromagnetic field.

In Capacitive circuit, energy is stored in the form of electrostatic field.

When the circuit is associated with single energy storing element i.e. R-L or R-C circuit, the transient which occurs are called single energy transients.

When the circuit is associated with both energy storing elements i.e. R-L-C circuits, the transient which occurs are called double energy transients.

## Transient Response of R-L series circuit:-

Consider a circuit containing resistance  $R$  (ohms) & inductance  $L$  (Henry) with a switch as shown in the fig. The switch has been kept open for a long time, & is closed at  $t=0$ .

Applying Kirchhoff's Voltage Law to the circuit after switching.

$$V = Ri + L \frac{di}{dt}$$

$$L \frac{di}{dt} = V - Ri$$

$$\frac{di}{dt} = \frac{V}{L} - \frac{R}{L} i$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \quad \text{--- (i)}$$

We know that

$$( ) \quad \frac{dy}{dx} + Py = Q \quad (\text{is a linear differential equation}) \quad \text{--- (1)}$$

$P, Q$  are the functions of  $x$  only

$$I.F = e^{\int P dx}$$

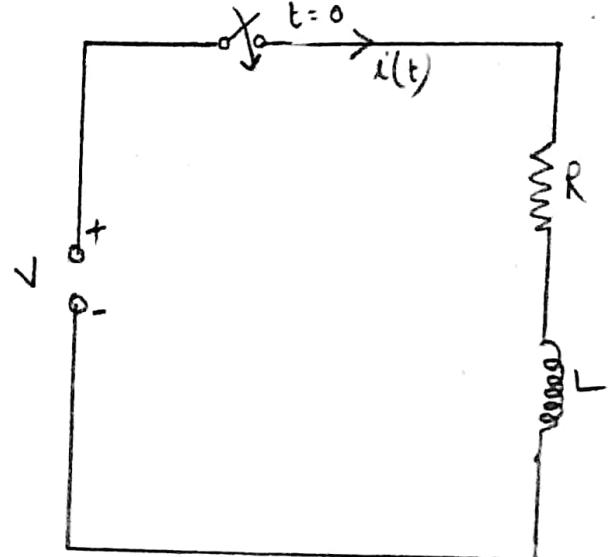
Soln of this equation is

$$\boxed{y \times I.F = \int Q \times I.F dx + C}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \quad (\text{compare it with 1})$$

$$P = \frac{R}{L}, \quad Q = \frac{V}{L}$$

$$I.F = e^{\int P dx} = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}} = e^{\frac{Rt}{L}} \quad (29)$$



R-L series circuit

Solution is

$$i e^{\frac{Rt}{L}} = \int \frac{V}{L} e^{\frac{Rt}{L}} dt + C$$

$$i e^{\frac{Rt}{L}} = \frac{V}{R} \frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} + C$$

$$i e^{\frac{Rt}{L}} = \frac{V}{R} e^{\frac{Rt}{L}} + C$$

$$i = \frac{V}{R} + C e^{-\frac{Rt}{L}} \quad \text{(ii)}$$

where  $C$  is a constant & it must satisfy the initial condition on inductance current

$$i(0^+) = i(0^-) = 0$$

Substituting in equation (ii)

$$0 = \frac{V}{R} + C$$

$$C = -\frac{V}{R}$$

Hence

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{Rt}{L}}$$

$$\boxed{i = \frac{V}{R} \left( 1 - e^{-\frac{Rt}{L}} \right)} \quad \text{(iii)}$$

In above expression, the first term

$\frac{V}{R}$  → corresponds to steady state condition for response ( $i_f$ )

$-\frac{V}{R} e^{-\frac{Rt}{L}}$  corresponds to transient state (natural response)

which approaches to zero as  $t \rightarrow \infty$

(30)

The ratio  $\frac{L}{R}$  is called time constant & denoted by  $\tau$ .

The reciprocal of the time constant (i.e.  $\frac{1}{\tau}$ ) is called damping coefficient of the circuit.

The voltage across the resistor is

$$\begin{aligned} V_R(t) &= i(t) \cdot R \\ &= \frac{V}{R} \left( 1 - e^{-\frac{R}{L}t} \right) \\ &= V \left( 1 - e^{-\frac{R}{L}t} \right) \\ &= V \left( 1 - e^{-\frac{t}{\tau}} \right) \quad \text{---(iv)} \end{aligned}$$

The voltage across inductance is

$$\begin{aligned} V &= V_R + V_L \\ V_L &= V - V_R \\ &= V - V \left( 1 - e^{-\frac{t}{\tau}} \right) \\ &= V e^{-\frac{t}{\tau}} \end{aligned}$$

Now  $i = \frac{V}{R} \left( 1 - e^{-\frac{R}{L}t} \right)$

$$V_R = V e^{-\frac{t}{\tau}}$$

$$V_L = V e^{-\frac{t}{\tau}}$$

At  $t=0$

$$\rightarrow i = 0$$

At  $t = \frac{L}{R} = \tau$

$$i = \frac{V}{R} \left( 1 - e^0 \right)$$

$$= \frac{V}{R} (1 - 0.37)$$

$$\rightarrow i = 0.63 \frac{V}{R}$$

At  $t = \infty$   
 $\rightarrow i = \frac{V}{R}$

$$V_R = 0$$

$$V_R = V (1 - e^0)$$

$$V_R = V (1 - 0.37)$$

$$V_R = 0.63V$$

$$V_R = V$$

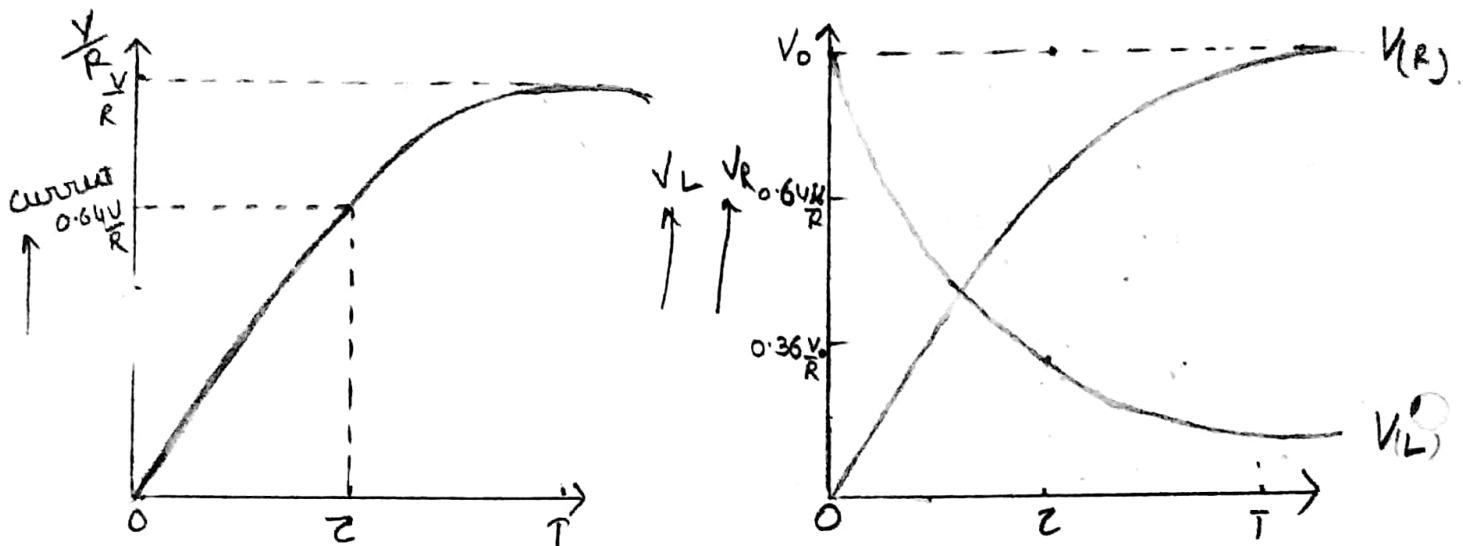
$$V_L = V$$

$$V_L = V e^{-1}$$

$$V_L = V (0.37)$$

$$V_L = 0$$

Now according to these values, the graph b/w current and time is shown in fig.



### Transient Response of R-C series circuit:-

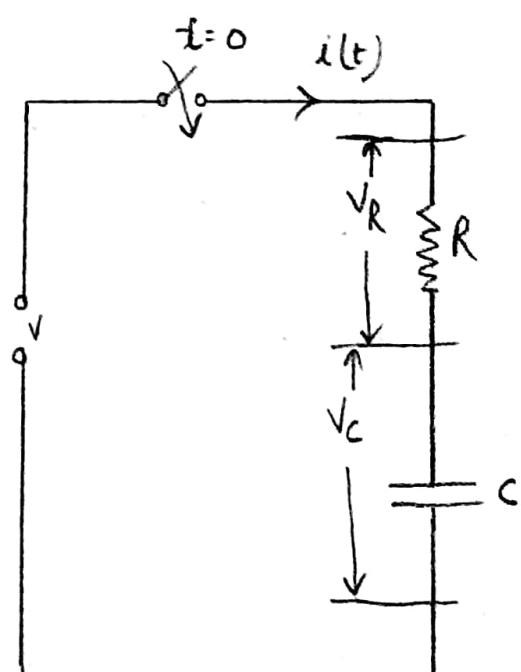
Consider a circuit containing resistance ( $R \text{ ohm}$ ) & capacitor  $C$  joined with a switch as shown in fig. The switch is closed at time  $t=0$ .

Applying Kirchhoff's voltage law to the circuit after switching

$$V = V_R + V_C$$

$$V = iR + \frac{1}{C} \int_{-\infty}^t i(t) dt$$

$$V = iR + \frac{1}{C} \int_{-\infty}^t i(t) dt + V_C(0^+)$$



R-C series circuit.

Assuming capacitor initially uncharged

(32)

$$V = R i(t) + \frac{1}{C} \int_0^t i(t) dt$$

Differentiating

$$R \frac{di}{dt} + \frac{1}{C} i = 0 \quad \text{--- (i)}$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

$$P = \frac{1}{RC}, \quad Q = 0$$

$$\bullet I.F. = e^{\int P dt} = e^{\int \frac{1}{RC} dt} = e^{\frac{t}{RC}}$$

Solution

$$i \times I.F. = \int Q I.F. dt + C$$

$$i e^{\frac{t}{RC}} = \int 0 \times e^{\frac{t}{RC}} dt + C$$

$$i e^{\frac{t}{RC}} = C$$

$$i(t) = C e^{-\frac{t}{RC}} \quad (C \text{ is constant})$$

As ~~at~~  $t=0^+ = i(0^+) = \frac{V}{R}$  since capacitor behaves as a short circuited at  $t=0^+$

$$\frac{V}{R} = C$$

$$\boxed{i(t) = \frac{V}{R} e^{-\frac{t}{RC}}} \quad \text{--- (ii)}$$

$\tau = RC$  is time constant

Voltage across the resistor

$$V_R(t) = R i(t) = V e^{-\frac{t}{T}}$$

Voltage across the capacitor

$$V = V_R + V_C$$

$$V_C = V - V_R = V - V e^{-\frac{t}{T}}$$

$$V_C = V \left(1 - e^{-\frac{t}{T}}\right)$$

$$\downarrow \\ i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$t = 0$

$$\downarrow \\ i = \frac{V}{R}$$

$t = T = \frac{1}{RC}$

$$\downarrow \\ i = \frac{V}{R} e^0$$

$t = \infty$

$$V_R = V e^{-\frac{t}{T}}$$

$$V_R = V$$

$$V_R = V C^{-1}$$

$$V_R = 0.37 \frac{V}{R}$$

$$V_R = 0$$

$$V_C = V \left(1 - e^{-\frac{t}{T}}\right)$$

$$V_C = 0$$

$$V_C = V \left(1 - e^0\right)$$

$$V_C = V \left(1 - 0.37\right) \\ = 0.63 V$$

$$V_C = V$$

Using these values, Now graph between current & time.

