2(A)5. Flow of Electric Charges in a Metallic Conductor

In a conductor, free electrons move randomly[4] with a thermal speed of the order of 10^5 to 10^6 m s^{-1} at room temperature. The random motion of an electron in a conductor is shown in Figure 3. In any portion of the conductor, the flow of electrons is so oriented that the average thermal velocity of total number of elecrons in the conductor is zero. As a result of this, the net flow of charge at the *given cross section is zero*. Hence, there is no net flow of current in the conductor.

In short, in the absence of external electric field, the motion of the electrons in the conductor is random such that the *average thermal velocity* of electrons becomes zero *i.e.* $\vec{u} = 0$.

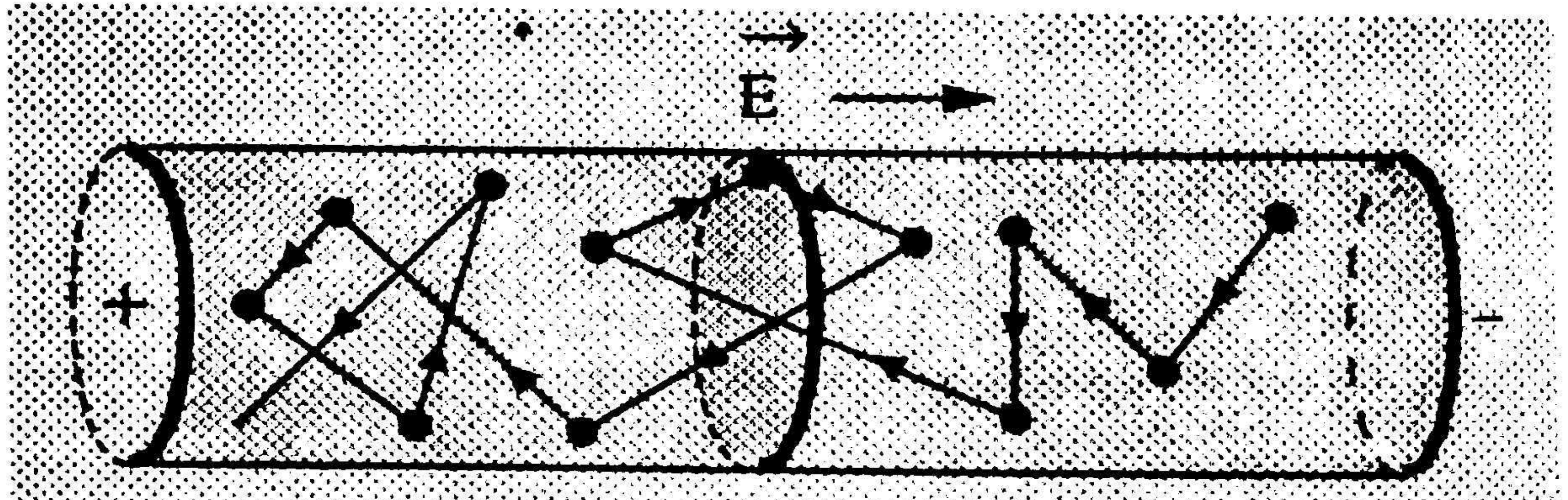


Figure 3

When an electric field is applied across the conductor, the free electrons accelerate in a direction opposite to the direction of the applied field. Due to this acceleration, the electrons gain extra velocity but for a short time because the accelerated electrons collide[5] with other free electrons or the ions in the conductor and during this collision, the extra velocity gained is destroyed. As a net result, the electrons acquire a small velocity called *drift velocity* ($\vec{\upsilon}_d$) in the direction opposite to that of the applied electric field.

Note

When steady current flows through a conductor, the motion of electrons in a conductor has no net acceleration because of steady drift velocity of the electrons.

2(A)7. Relation between Drift Velocity and Electric Current

Consider a conductor of length l and uniform cross-sectional area A. Let V be the applied potential difference across the ends of the conductor (Figure 4). The magnitude of electric field set up across the conductor is given by

$$E = \frac{V}{l}$$

Let n be the number of free electrons per unit volume of the conductor.

Then, total number of free electrons in the conductor = $n \times$ volume of the conductor

N $= n \times Al.$

If e is the magnitude of charge on each electron then the total charge in the conductor,

$$\boxed{Q = (nAl)e} \quad \text{...(7)}$$

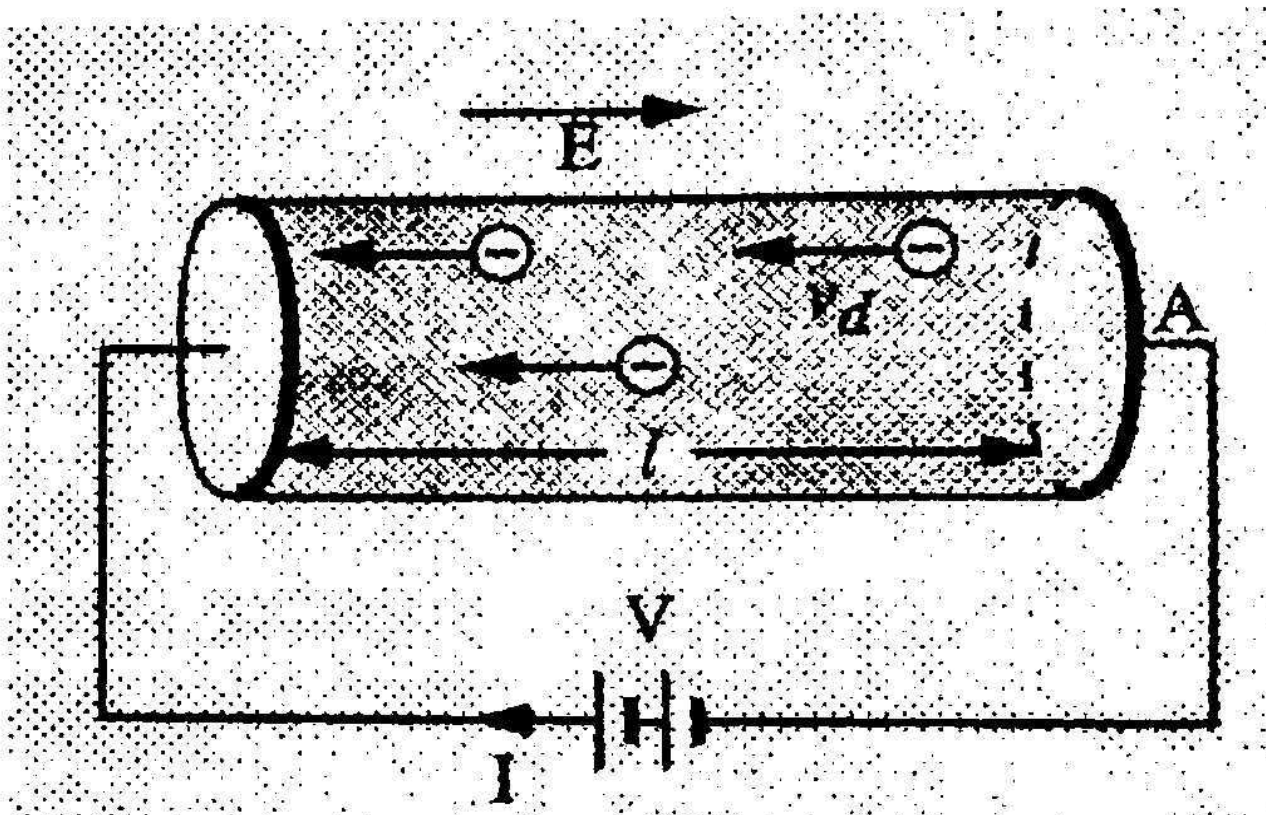


Figure 4

The time taken by the charge to cross the conductor length is given by

$$t = \frac{l}{v_d}$$

where v_d is drift velocity of electrons

According to the definition of electric current,

$$I = \frac{Q}{t} = \frac{nAle}{l / v_d} = neA\, v_d$$

or $$\boxed{I = neA\, v_d} \quad \text{...(8)}$$

or $$I \propto v_d$$ (*as n, e and A are fixed*)

Again from equation (8), $$\boxed{v_d = \frac{I}{neA}} \quad \text{...(9)}$$

Current Density and Drift Velocity

Eqn. (8) can be written as $$\frac{I}{A} = nev_d$$

But $$\frac{I}{A} = J \text{ i.e. current density}$$

$\therefore$ $$\boxed{J = nev_d} \quad \text{...(10)}$$

Note The drift velocity is of the order of $10^{-5}\ ms^{-1}$ which is negligible as compared to the ***average electron thermal velocity*** ($10^{5}\ ms^{-1}$) at room temperature.

Mobility (μ)

It is the ratio of the drift velocity (υ_d) of current carrier to the applied electric field (E) i.e.

$$\mu = \frac{\upsilon_d}{E}$$

Using relation (6), we get $$\mu = \frac{\upsilon_d}{E} = \frac{e\tau}{m}$$

For example, in a semiconductor mobility of an electron is more than the mobility of a hole because electron is lighter than the hole.

1.05. ELECTRON MOBILITY

The mobility of free electrons in a conductor is defined as the drift velocity acquired per unit strength of the electric field applied across the conductor. It is denoted by μ.

If v_d is drift velocity attained by free electrons on applying electric field E, then electron mobility is given by

$$\mu = \frac{v_d}{E} \qquad ...(1.11)$$

vd: μE

Now, $$v_d = \frac{e\,E}{m}\tau \qquad \text{(in magnitude)} \qquad ...(1.12)$$

From the equations (1.11) and (1.12), we have

$$\mu = \frac{e\,\tau}{m} \qquad ...(1.13)$$

The equations (1.11) and (1.13) are the expressions for electron mobility.

In the equation (1.09), substituting for $v_d\,(= \mu\,E)$, we have

$$I = n\,A\,\mu\,E\,e \qquad ...(1.14)$$

The equation (1.14) gives the relation between electron mobility and the current through the conductor.

From the equation (1.11), it follows that in SI, the unit of electron mobility is **metre^2 volt^{-1} second^{-1} (m^2 V^{-1} s^{-1})**.

Problem 1.02. A current of 10 A is maintained in a conductor of cross-section 10^{-4} m^2. If the number density of free electrons be 9×10^{28} m^{-3}, calculate the drift velocity of free electrons. Given, charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C.

Sol. Here $I = 10$ A ; $n = 9 \times 10^{28}$ m^{-3} ; $A = 10^{-4}$ m^2

and $\quad e = 1{\cdot}6 \times 10^{-19}$C

Now, $$v_d = \frac{I}{neA} = \frac{10}{9 \times 10^{28} \times 1{\cdot}6 \times 10^{-19} \times 10^{-4}}$$

$$= \mathbf{6{\cdot}94 \times 10^{-6}\ m\ s^{-1}}$$

Q. 1.01. It is found that 10^{20} electrons, each having a charge of $1{\cdot}6 \times 10^{-19}$ C, pass from a point X towards another point Y in 0·1 s. What is the current and its direction ?

Ans. Here, $q = ne = 10^{20} \times 1{\cdot}6 \times 10^{-19} = 16\text{ C}\,; t = 0{\cdot}1\text{ s}$

$$\therefore \qquad I = \frac{q}{t} = \frac{16}{0{\cdot}1} = \mathbf{160\ A}$$

The direction of conventional current is **from the point Y to X.** ['direction of current is opposite to direction of e

Q. 1.02. What is conventional current ?

Ans. The current that flows from positive pole to negative pole of a cell in the external circuit is called conventional current.

Q. 1.10. The potential difference across a given copper wire is increased. What happens to the drift velocity of the charge carriers ? *(C.B.S.E. 1999 S)*

Ans. We know, $v_d = \frac{eE}{m}\tau$

If l is length of the copper wire and V, the potential difference across it, then

$$v_d = \frac{e}{m}\left(\frac{V}{l}\right)\tau$$

Thus, $v_d \propto V$ *i.e.* if potential difference is increased, drift velocity of the electrons will *increase.*

Problem 1.14. A current of 1·8 A flows through a wire of area of cross-section 0·5 mm^2. Find the current density in the wire. If the number density of electrons in the wire is 8·8 × 10^{28} m^{-3}, find the drift velocity of electrons.

Sol. Here, $I = 1 \cdot 8\ \text{A}\ ;\ A = 0 \cdot 5\ \text{mm}^2 = 0 \cdot 5 \times 10^{-6}\ \text{m}^2$

and $n = 8 \cdot 8 \times 10^{28}\ \text{m}^{-3}$

Now, the current density,

$$j = \frac{I}{A} = \frac{1 \cdot 8}{0 \cdot 5 \times 10^{-6}} = \mathbf{3 \cdot 6 \times 10^{6}\ A\ m^{-2}}$$

Also, $j = n\, v_d\, e$

or

$$v_d = \frac{j}{n\,e} = \frac{3 \cdot 6 \times 10^{6}}{8 \cdot 8 \times 10^{28} \times 1 \cdot 6 \times 10^{-19}}$$

$$= \mathbf{2 \cdot 56 \times 10^{-4}\ m\ s^{-1}}$$

1.10. RESISTIVITY

The resistance of a conductor depends upon the following factors :

(i) *It is directly proportional to the length of the conductor i.e.*

$$R \propto l$$

(ii) *It is inversely proportional to the area of cross-section of the conductor i.e.*

$$R \propto \frac{1}{A}$$

Combining the above two factors, we have

$$R \propto \frac{l}{A}$$

or $$R = \rho \frac{l}{A} \qquad \text{...(1.22)}$$

where the constant of proportionality ρ is called ***electrical resistivity*** or ***specific resistance*** of the conductor. Its value depends upon the nature of the material of the conductor and its temperature. The equation (1.22) gives the resistance of a conductor in terms of its length, area of cross-section and resistivity of the material.

If $l = 1$, $A = 1$, then $R = \rho \frac{1}{1}$ or $\rho = R$

Hence, *resistivity or specific resistance of the material of a conductor is the resistance offered by a wire of this material of unit length and unit area of cross-section.*

Unit of resistivity. From the equation (1.22), we have

$$\rho = R \frac{A}{l}$$

Therefore, SI unit of resistivity is **ohm metre (Ω m).**

Problem 1.04. Calculate the resistivity of the material of a wire 1·0 m long, 0·4 mm in diameter and having a resistance of 2·0 ohm. *(P.S.S.C.E. 2009 S, 1994 ; H.S.S.C.E. 2002)*

Sol. Here, $R = 2{\cdot}0\,\Omega ; l = 1{\cdot}0\,\text{m};$

The diameter of wire, $d = 0{\cdot}4\ \text{mm} = 4 \times 10^{-4}\,\text{m}$

Therefore, the area of cross-section of wire,

$= \pi r^2 = \frac{\pi d^2}{4}$

$$A = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi \times (4 \times 10^{-4})^2 = 4\,\pi \times 10^{-8}\,\text{m}^2$$

If ρ is resistivity of the material of the wire, then

$$R = \rho \frac{l}{A}$$

or $$\rho = \frac{RA}{l} = \frac{2{\cdot}0 \times 4\pi \times 10^{-8}}{1{\cdot}0} = \mathbf{2{\cdot}153 \times 10^{-7}\,\Omega\,m}$$

1.11. FACTORS AFFECTING ELECTRICAL RESISTIVITY

The concept of average relaxation time helps in a great deal to understand the nature of electrical resistance. In order to do so, let us derive expression for resistance of a conductor in terms of average relaxation time (τ), electron density (n), etc.

Consider a conductor having length l and area of cross-section A. Let n be number of electrons per unit volume in the conductor. If an electric field E is applied across the two ends of the conductor, then drift velocity (in magnitude) of electrons is given by

$$v_d = \frac{eE}{m}\tau$$

The current flowing through the conductor due to drift of electrons is given by

$$I = nAv_d e$$

Substituting for v_d in the above equation, we have

$$I = nA\left(\frac{eE}{m}\tau\right)e$$

or

$$I = \frac{nAe^2E\tau}{m} \qquad ...(1.23)$$

If V is potential difference applied across the two ends of the conductor, then

$$E = \frac{V}{l}$$

Substituting for E in equation (1.23), we have

$$I = \frac{nAe^2V\tau}{ml}$$

or

$$\frac{V}{I} = \frac{ml}{ne^2\tau A}$$

But according to Ohm's law, $\frac{V}{I} = R$, the resistance of the conductor.

$$\therefore \quad R = \frac{m}{ne^2\tau} \cdot \frac{l}{A} \qquad ...(1.24)$$

Comparing the above result with the expression for resistance obtained earlier *i.e.*

$$R = \rho\frac{l}{A},$$

it follows that resistivity of the material of a conductor is given by

$$\rho = \frac{m}{ne^2\tau} \qquad ...(1.25)$$

It follows that resistivity of the material of a conductor depends on the following factors :

1. *It is inversely proportional to the number of free electrons per unit volume (n) of the conductor.* Since the value of n depends upon nature of the material, *the resistivity of a conductor depends upon the nature of the material.*

2. *It is inversely proportional to the average relaxation time (τ) of the free electrons in the conductor.* As we shall study in next section, the value of τ decreases with increase in temperature of the conductor. Since $\rho \propto 1/\tau$, the *resistivity of conductor depends upon its temperature and it increases with increase in temperature of the conductor.*

Resistivity in terms of current density. From the equation (1.17), we have

$$\frac{m}{n e^2 \tau} = \frac{E}{j}$$

Therefore, the equation (1.25) becomes

$$\rho = \frac{E}{j} \qquad ...(1.26)$$

The equation (1.26) gives the relation between current density and resistivity of the material of the conductor.

Resistivity in terms of electron mobility. From the equation (1.19), substituting for j in the equation (1.26), we have

$$\rho = \frac{E}{n e v_d} \qquad ...(1.27)$$

In the equation (1.27), substituting for v_d $(= \mu E)$, we have

$$\rho = \frac{1}{n e \mu} \qquad ...(1.28)$$

The equation (1.28) gives the relation between resistivity of the material of a conductor and the electron mobility.

Note. From the equation (1.27), we have

$$v_d = \frac{E}{n e \rho}$$

Since $E = V/l$, the above equation becomes

$$v_d = \frac{V}{n e l \rho} \qquad ...(1.29)$$

The equation (1.29) can be used to predict the effect of temperature and length of the conductor on drift velocity of electrons.

(a) ***Effect of temperature.*** On increasing the temperature of a conductor, the value of resistivity of its material increases. Therefore, from the equation (1.29), it follows that *drift velocity of electrons decreases on increasing the temperature of the conductor.*

(b) ***Effect of length of conductor.*** From the equation (1.29) it follows that *drift velocity of electrons decreases, when length of the conductor is increased.*

1.12. CONDUCTANCE AND CONDUCTIVITY

Conductance. *The reciprocal of resistance of a conductor is called its conductance.*

It is denoted by G. Thus, conductance of a conductor having resistance R is given by

$$G = \frac{1}{R} \qquad ...(1.30)$$

The SI unit of conductance is **ohm^{-1}** (Ω^{-1}), which is also called **mho.** In SI, the unit of conductance is also called **siemen** and is denoted by the symbol **S.**

Conductivity. *The reciprocal of resistivity of the material of a conductor is called its conductivity.* It is denoted by σ. Thus,

$$\sigma = \frac{1}{\rho} \qquad ...(1.31)$$

From the equations (1.26) and (1.31), it follows that

$$\sigma = \frac{j}{E} \qquad ...(1.32)$$

The equation (1.32) gives the relation beween conductivity and current density.

From the equations (1.28) and (1.31), we have

$$\sigma = n\, e\, \mu \qquad ...(1.33)$$

The equation (1.33) gives the relation between conductivity of the material of conductor and the electron mobility.

From the equation (1.31), it follows that unit of conductivity is reciprocal of the unit of resistivity. Therefore, in SI, the unit of conductivity is **ohm^{-1} metre^{-1}** (Ω^{-1} m^{-1}) or **mho metre^{-1}** or **siemen metre^{-1} (S m^{-1})**.

Note. The equation (1.32) can be written as

$$j = \sigma E$$

Since $\vec{j}$ and $\vec{E}$ are parallel vector, we may write

$$\vec{j} = \sigma \vec{E} \qquad ...(1.34)$$

The above relation is treated as an equivalent form of Ohm's law stated by the relation (1.21).

2(A)19. Relation among J, σ and E (microscopic form of Ohm's Law)

We know that current,

$$I = neAv_d \qquad \textit{(Refer eqn. 8)}$$

But drift velocity,

$$v_d = \frac{eE\tau}{m} \qquad \textit{(Refer eqn. 6)}$$

$\therefore$

$$I = neA\left(\frac{eE\tau}{m}\right) = \frac{ne^2AE\tau}{m}$$

or

$$\frac{I}{A} = \frac{ne^2E\tau}{m} \qquad \ldots(i)$$

Now $\frac{I}{A} = J$ and $\frac{m}{ne^2\tau} = \rho$ *(Refer eqn. 16)*

where J is the current density and ρ is the resistivity of the substance.

$\therefore$ eqn. *(i)* becomes $J = \frac{1}{\rho}E$ But $\frac{1}{\rho} = \sigma$

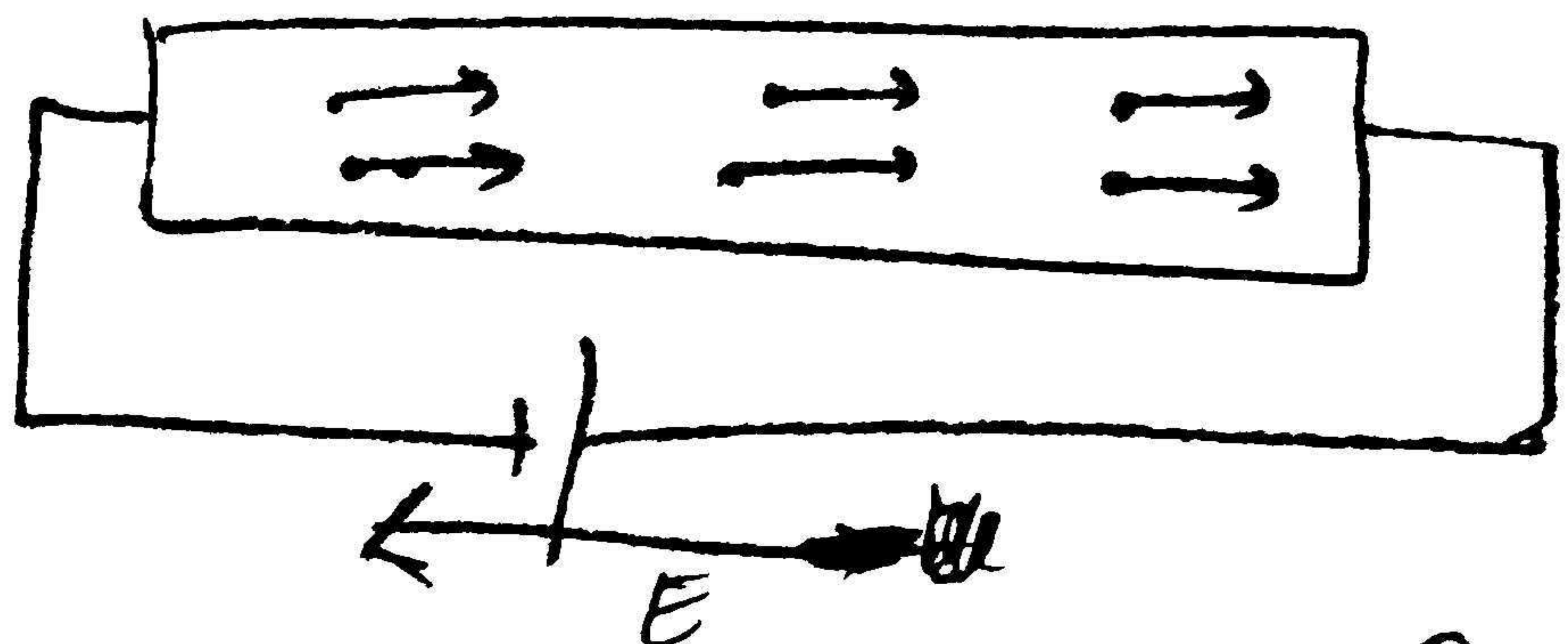
where σ is the conductivity of the substance.

$$\therefore \quad \boxed{J = \sigma E} \qquad \ldots(24)$$

which is the *microscopic form of Ohm's law.*

Expression for electrical conductivity & Drift velocity :-

Consider a conductor which is subjected to an E·F of strength E



n = concentration of free $e^{\ominus}$s

m = mass of $e^{\ominus}$s

e = charge of $e^{\ominus}$s

According to Newton's second law of motion the force f acquired by $e^{\ominus}$s is equal to the force excerted by the field on the $e^{\ominus}$s.

$\therefore$ eqn of motion $ma = -eE$

$$\Rightarrow a = \frac{-eE}{m}$$

$$\Rightarrow \int a\, dt = \int \frac{-eE}{m} dt$$ (Integrate) $\left[a = \frac{dv}{dt}\right]$

locity
of $e^{\ominus}$

$$\Rightarrow v = -\frac{eE}{m}t + C$$

Integration const.

During the absence of the E.F., the average velocity of the $e^{\ominus}$ is zero,

$$t = 0, \quad \langle v \rangle = 0$$

$$\Rightarrow \boxed{v = -\frac{eEt}{m}}$$

average velocity b/w two successive collisions

$$\bar{v} = \frac{1}{\tau}\int_0^\tau \left(-\frac{eEt}{m}\right) dt$$

$$\bar{v} = -\frac{eE}{\tau m}\int_0^\tau t\, dt = -\frac{eE}{\tau m}\frac{\tau^2}{2} = \frac{-eE\tau}{2m}$$

$$\bar{v} = -\frac{eE\lambda}{2(mv)}$$

[relaxation time b/w two successive collisions $= \tau$
λ = mean free path of $e^{\ominus}$s.
$\left[\tau = \frac{\lambda}{v}\right]$

$$\bar{v} = \frac{-eE\lambda}{2\left(\frac{3kT}{v}\right)}$$

$\left[\because \frac{1}{2}mv^2 = \frac{3}{2}kT \Rightarrow mv = \frac{3kT}{v}\right]$

$$\bar{v} = \frac{-eE\lambda v}{6kT} \quad \text{— (1)}$$

If n is no. density of $e^{\ominus}$s in conductor [redacted] then the current density J [redacted] is :-

[$dq = -enAv\,dt$
$\frac{dq}{dt} = -enAv$
$= -env$ [for unit area of cross sec]]

[redacted]

$$J = -en\bar{v}$$

$$= -en\left[\frac{-eE\lambda v}{6kT}\right] \quad [\text{from (1)}]$$

$$J = \frac{ne^2E\lambda v}{6kT} \quad \text{— (2)}$$

If q charge is flowing through a conductor cross-section area A in time t,

$$q = \sigma A E t$$

$$\frac{q}{t} = \sigma A E$$

$$\Rightarrow \boxed{I = \sigma A E} \Rightarrow \frac{I}{A} = \sigma E$$

[current density is directly pr-
tional to applied E.F.]

$$\Rightarrow \boxed{J = \sigma E}$$

$$\sigma = \frac{J}{E}$$

[For unit area of cross-sec.]

$$\sigma = \frac{n e^2 \lambda v}{6 k T}$$

⇒ different conductivities of diff. materials are due to different no. of free electrons.

(i) Electrical conductivity

It is defined as the quantity of electricity that flows in unit time per unit area of cross-section of the conductor per unit potential gradient.

According to free electron theory, in a solid the electrons move freely. If E is the applied electric field, then the acceleration of an electron having charge is given by

$$a = \frac{d^2x}{dt^2} = \frac{eE}{m} \qquad \text{... (5.1)}$$

If λ is the mean free path of electrons, then the relaxation time τ between two successive collisions is given by

$$\tau = \frac{\lambda}{\upsilon} \qquad \text{... (5.2)}$$

Integrating eq. 5.1, we get

$$\frac{dx}{dt} = \frac{eE}{m}t + C$$

At $t = 0, \frac{dx}{dt} = 0$

Hence $\frac{dx}{dt} = \upsilon = \frac{eE}{m}t$

So average velocity between two successive collisions

$$\bar{\upsilon} = \frac{1}{\tau}\int_0^\tau \frac{eE}{m}t\,dt = \frac{eE}{\tau m}\int_0^\tau t\,dt$$

$$= \frac{eE}{\tau m}\frac{\tau^2}{2}$$

or $\bar{\upsilon} = \frac{eE\tau}{2m} \quad \frac{eE\tau}{2m}$

Putting the value of τ from eq. 5.2, we get

$$\bar{\upsilon} = \frac{eE\lambda}{2m\upsilon}$$

Since

$$\frac{1}{2}m\upsilon^2 = \frac{3}{2}kT$$

(T is absolute temperature and k is Boltzmann constant)

So

$$m\upsilon = \frac{3kT}{\upsilon}$$

So

$$\bar{\upsilon} = \frac{eE\lambda\upsilon}{6kT}$$

If n is number density of electrons in the conductor, then the current density i is given by

$$i = ne\bar{\upsilon}$$

or

$$i = \frac{ne^2E\lambda\upsilon}{6kT} \quad \text{... (5.3)}$$

If q charge is flowing through a conductor of cross-section area A in time t, then

$$q = \sigma AEt$$

$$\frac{q}{t} = \sigma AE$$

or

$$i = \sigma AE$$

or

$$\sigma = \frac{i}{AE} \quad \text{...(5.4)}$$

or

For unit area of cross-section

$$\sigma = \frac{i}{E}$$

Using eq. (5.3)

$$\sigma = \frac{ne^2\lambda\upsilon}{6kTA}$$

This expression shows that different conductivities of different materials are due to different number of free electrons.

(ii) Ohm's law

From eq. (5.4) we have

$$\sigma E = \frac{i}{A}$$

or

$$\sigma E = J$$

or

$$J = \sigma E \quad \text{...(5.5)}$$

This is microscopic form of Ohm's law.

... conductivity