(v) At low temperature (but > 20 K),

$$\rho \propto T^5$$

(vi) The resistivity varies in presence of magnetic field. The effect is known as Magneto resistance.

(vii) For most metals $\rho \propto \frac{1}{P}$, where P is pressure.

(viii) According to Matthiessen's rule, for metals containing small amounts of impurity,

$$\rho = \rho_0 + \rho\,(T)$$

where ρ_0 is constant and increase with impurity content and $\rho(T)$ is temperature dependent part of resistivity.

(ix) Above Debye temperature, the ratio of thermal (K) to electrical (σ) conductivity is proportional to T.

$$\frac{K}{\sigma} \propto T \text{ (Wiedemann-Franz law)}$$

where the constant of proportionality is nearly same for all the metals.

(x) At 0 K, most of metals show the phenomenon of super conductivity.

On the basis of free electron theory following properties of solids have been explained.

(i) Electrical conductivity

It is defined as the quantity of electricity that flows in unit time per unit area of cross-section of the conductor per unit potential gradient.

According to free electron theory, in a solid the electrons move freely. If E is the applied electric field, then the acceleration of an electron having charge is given by

$$a = \frac{d^2x}{dt^2} = \frac{eE}{m} \qquad \text{... (5.1)}$$

If λ is the mean free path of electrons, then the relaxation time τ between two successive collisions is given by

$$\tau = \frac{\lambda}{\upsilon} \qquad \text{... (5.2)}$$

Integrating eq. 5.1, we get

$$\frac{dx}{dt} = \frac{eE}{m}t + C$$

At $\quad t = 0, \frac{dx}{dt} = 0$

Hence $\quad \frac{dx}{dt} = \upsilon = \frac{eE}{m}t$

So average velocity between two successive collisions

$$\bar{\upsilon} = \frac{1}{\tau}\int_0^\tau \frac{eE}{m}t\,dt = \frac{eE}{\tau m}\int_0^\tau t\,dt$$

$$= \frac{eE}{\tau m}\frac{\tau^2}{2}$$

or $\quad \bar{\upsilon} = \cancel{\frac{eE\tau}{2m}} \quad \frac{eE\tau}{2m}$

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Putting the value of τ from eq. 5.2, we get

$$\bar{\upsilon} = \frac{eE\lambda}{2mv}$$

Since $$\frac{1}{2}m\upsilon^2 = \frac{3}{2}kT$$ (T is absolute temperature and k is Boltzmann constant)

So $$m\upsilon = \frac{3kT}{v}$$

So $$\bar{\upsilon} = \frac{eE\lambda v}{6kT}$$

If n is number density of electrons in the conductor, then the current density i is given by

$$i = ne\bar{v}$$

or $$i = \frac{ne^2E\lambda v}{6kT} \quad \text{... (5.3)}$$

If q charge is flowing through a conductor of cross-section area A in time t, then

$$q = \sigma AEt$$

or $$\frac{q}{t} = \sigma AE$$

or $$i = \sigma AE$$

or $$\sigma = \frac{i}{AE} \quad \text{...(5.4)}$$

For unit area of cross-section $$\sigma = \frac{i}{E}$$

Using eq. (5.3)

$$\sigma = \frac{ne^2\lambda v}{6kTA}$$

This expression shows that different conductivities of different materials are due to different number of free electrons.

(ii) Ohm's law

From eq. (5.4) we have $$\sigma E = \frac{i}{A}$$

or $$\sigma E = J$$

or $$J = \sigma E \quad \text{...(5.5)}$$

This is microscopic form of Ohm's law.

(iii) Thermal conductivity

We know that if there is no temperature difference between two points in a specimen i.e. $T_1 = T_2$, there is no transfer of energy. So to discuss the thermal conductivity of metals, we suppose that a temperature gradient exists across the specimen instead of voltage gradient, hence the transport of thermal energy takes place due to this gradient.

If we consider the specimen in the form of a metallic rod having two ends namely A and B and the end A is at higher temperature than B, then the conduction of heat from A to B takes place by electrons. In collision, the electrons near A loose their kinetic energy while the electrons near B gain the energy.

The amount of heat Q passing through a cross-section of the rod per unit area per second is given by :

$$Q = \frac{1}{3}nv\lambda\frac{dE}{dt} \quad \text{... (5.6)}$$

where λ = mean free path

v = velocity of electrons

n = number density of free electrons

PU 2011 obtain an expression for thermal conductivity of a metal. on the basis of free e⁻ theory PU 2012

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$\frac{1}{3} n v \lambda \frac{3}{2} k \frac{dT}{dx}$

From kinetic theory of gases

$$E = \frac{3}{2}kT$$

$$\therefore \quad \frac{dE}{dt} = \frac{3}{2}k\frac{dT}{dt}$$

Put above value in 5.6

Hence $$Q = \frac{1}{2}nv\lambda k\frac{dT}{dt}$$

or $$K\frac{dT}{dt} = \frac{1}{2}nv\lambda k\frac{dT}{dt} \qquad \left[\because Q = K\frac{dT}{dt}\right]$$

or $$K = \frac{1}{2}nv\lambda k \qquad \text{... (5.7)}$$

This value of K is verified experimentally and the free electrons theory is found to be successful to explain thermal conductivity.

(iv) Wiedemann-Franz relation PU 2012

Wiedemann and Franz in 1853 discovered that all good electrical conductors are also good thermal conductors and the ratio of thermal conductivity to the electrical conductivity at any temperature (but not too low temperature) is constant for all metals.

i.e., $$\frac{K}{\sigma} = \text{constant}$$

Using eqs. (5.7) and (5.4), we get

$$\frac{K}{\sigma} = \frac{\frac{1}{2}nv\lambda k.6kT}{ne^2\lambda v}$$

$$= 3\left(\frac{k}{e}\right)^2 T$$

or $$\frac{K}{\sigma} \propto T.$$

$K = \frac{1}{2} n v \lambda k$
and
$\sigma = \frac{n e^2 \lambda v}{6kT}$

This is Wiedemann-Franz relation.

(v) ...