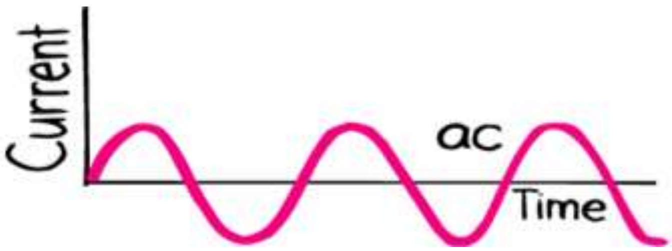
Module 2

A.C. Circuits

The flow of electricity can be done in two ways like AC (alternating current) and DC (direct current). Electricity can be defined as the flow of electrons throughout a conductor such as a wire. The main disparity among AC & DC mainly lies within the direction where the electrons supplies. In direct current, the flow of electrons will be in a single direction & in the alternating current; the flow of electrons will change their directions like going forward & then going backward.

- In Alternating current, movement of electric charge periodically reverses its direction.



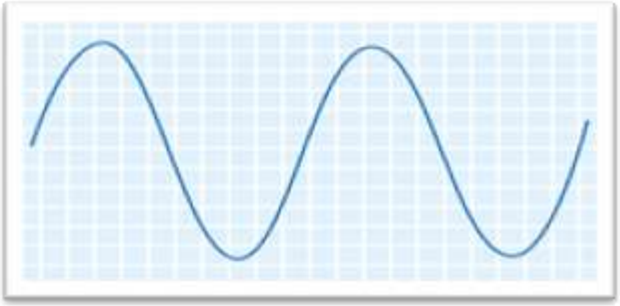
- Whereas in DC, flow of electric charge is only in one direction.

Why we use A.C. in homes

- AC voltage is capable of converting voltage levels just with use of transformers.
  - To transmit AC over a long distance, voltage is stepped up to 400 KV at generating stations and stepped down at a low level , 400/230 V for household and commercial utilization.
  - AC motors are simple in construction, more efficient and robust as compared to DC motors.

Sinusoidal Alternating Quantity:

Alternating quantity that varies according to sin of angle θ.



The instantaneous value of a sine-wave voltage for any angle of rotation is expressed in the formula:

$V = V_m \sin \theta$

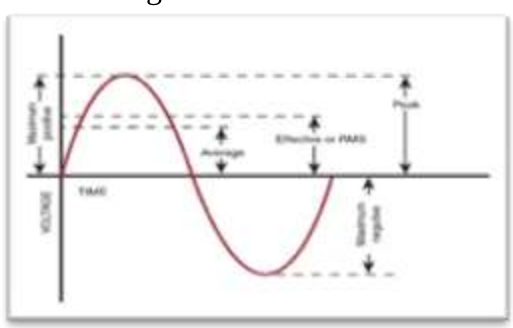
θ is the angle

Vm = the maximum voltage value

V = the instantaneous value of voltage at angle θ

Terms to know

1. Cycle: When an Alternating qty goes through complete set of positive and negative values or goes through 360 electrical degrees.



2. Alternation: One half cycle
3. Time Period: Time taken to complete one cycle by AC.
4. Frequency: No of cycle made per second.
5. Amplitude: Maximum value attained by an alternating quantity in one cycle and also called Peak Value / Max. Value

Peak and RMS Values

The magnitude of alternating quatity can be expressed in three ways:

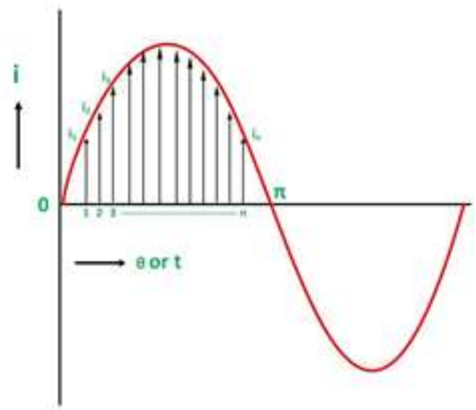
1 Peak Value
2 Average Value
3 Effective or rms value

Peak Value:

The maximum value attained by an alternating quantity during oone cycle is called peak value. This is also called maximum value or amplitude. The peak of an alternating voltage or current is represented by V_m and I_m

Average Value:

The average value of a periodic waveform whether it is a sine wave, square wave or triangular waveform is defined as: “the quotient of the area under the waveform with respect to time”.
In other words, the averaging of all the instantaneous values along time axis with time being one full period, (T).
For symmetrical waves like sinusoidal current or voltage waveform, the positive half cycle will be exactly equal to negative half cycle. Therefore, the average value over a complete cycle will be zero. The work is done by both, positive and negative cycle and hence the average value is determined without considering the signs.So, the only positive half cycle is considered to determine the average value of alternating quantities of sinusoidal waves.



Divide the positive half cycle into (*n*) number of equal parts as shown in the above figure

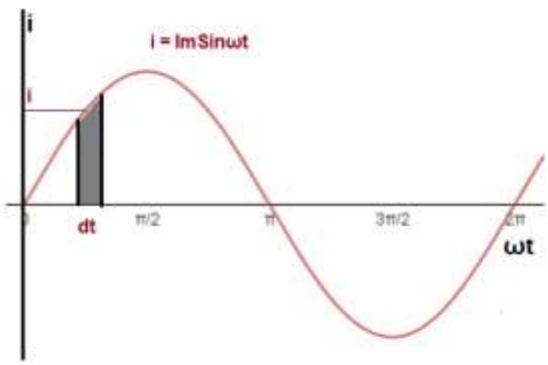
Let i_1, i_2, i_3…….. i_n be the mid ordinates

The Average value of current I_{av} = mean of the mid ordinates

$$I_{av} = \frac{i_1 + i_2 + i_3 + \dots\dots\dots + i_n}{n} = \frac{\text{Area of alternation}}{\text{Base}}$$

Derivation for average value of sinusoidal current:

Let us consider a sinusoidal current i = I_mSinωt as shown in the figure below. We will calculate its average value for one time period ωt = π from the definition of average value of alternating current.



Area of alternation = $\int_0^{\pi} i \, d(wt)$

$= \int_0^{\pi} \text{Im} \, sin \, wt \, d(wt)$

$= I_m \int_0^{\pi} sin \, wt \, d(wt)$

$= I_m[-\cos wt]_0^{\Pi}$

$= I_m[-(-1 - 1)]$

$= 2I_m$

Base = π

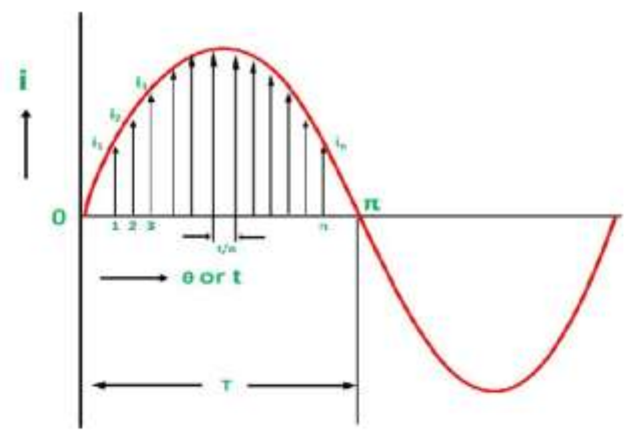
Therefore, Average value of sinusoidal current = $2Im/_{\pi}$

Effective or RMS Value:

"RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, which would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor.
That steady current which, when flows through a resistor of known resistance for a given period of time than as a result the same quantity of heat is produced by the alternating current when flows through the same resistor for the same period of time is called R.M.S or effective value of the alternating current.

In other words, the R.M.S value is defined as the square root of means of squares of instantaneous values.

Let I be the alternating current flowing through a resistor R for time t seconds, which produces the same amount of heat as produced by the direct current (I_{eff}). The base of one alteration is divided into n equal parts so that each interval is of t/n seconds as shown in the figure below.



Let i_1, i_2, i_3,……….in be the mid ordinates

Then the heat produced in

$$\text{First interval} = \frac{i_1^2 Rt}{Jn} \quad \text{calories}$$

$$\text{Second interval} = \frac{i_2^2 Rt}{Jn} \quad \text{calories}$$

$$\text{Third interval} = \frac{i_3^2 Rt}{Jn} \quad \text{calories}$$

$$n^{th} \text{ interval} = \frac{i_n^2 Rt}{Jn} \quad \text{calories}$$

$$\text{Total heat produced} = \frac{Rt}{J}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \dots\dots + i_n^2}{n}\right) \quad \text{calories} \dots\dots\dots (1)$$

Since I_{eff} is considered as the effective value of this current, then the total heat produced by this current will be

$$\frac{I_{eff}^2 Rt}{J} \text{ calories } \dots\dots\dots(2)$$

Now, equating equation (1) and (2) we will get

$$\frac{I_{eff}^2 Rt}{J} = \frac{Rt}{J}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \dots\dots + i_n^2}{n}\right) \text{ or}$$

$$I_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \dots\dots.. + i_n^2}{n}}$$

$$I_{eff} = \sqrt{\text{mean of squares of instantaneous values}}$$

RMS value of an alternating quantity:

An alternating current is given by

$I = I_m \text{ Sin}\theta$

Area of strip $= i^2 d\theta$

Area of squared wave in first half cycle

$$\int_0^\pi i^2 d\theta = \int_0^\pi (I_m \text{ Sin}\theta)^2 d\theta$$

$$= I_m^2 \int_0^\pi \sin^2\theta \cdot d\theta = I_m^2 \int_0^\pi \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2} I_m^2 \int_0^\pi (1 - \cos 2\theta) \cdot d\theta = \frac{1}{2} I_m^2 \left(\theta - \frac{\sin 2\theta}{2}\right)_0^\pi$$

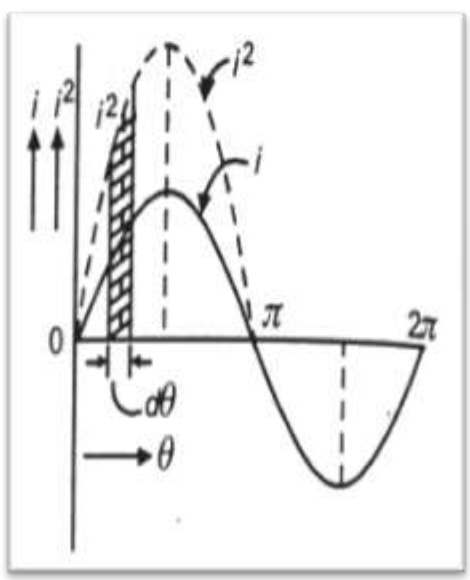
$$= \frac{1}{2} I_m^2 \left\{(\pi - 0) - \frac{\sin 2\pi - \sin\theta}{2}\right\}$$

$$= \frac{1}{2} I_m^2 \{(\pi - 0) - (0 - 0)\}$$

$$= \frac{\pi}{2} I_m^2$$

$$\text{R.M.S. value } I_{rms} = \sqrt{\frac{\text{Area of first half of squared wave}}{\text{Base}}}$$

$$= \sqrt{\frac{\pi I_m^2}{2\pi}} = \sqrt{\frac{I_m^2}{2}} = 0.707 I_m$$



RMS value for pure sinusoidal current is given by $\mathbf{I_{rms} = 0.707\ I_m}$

Form Factor and Peak Factor

Form Factor is defined as the ratio of the root mean square value to the average value of an alternating quantity (current or voltage).

Form Factor = $\frac{Irms}{Iavg}$ = 1.11

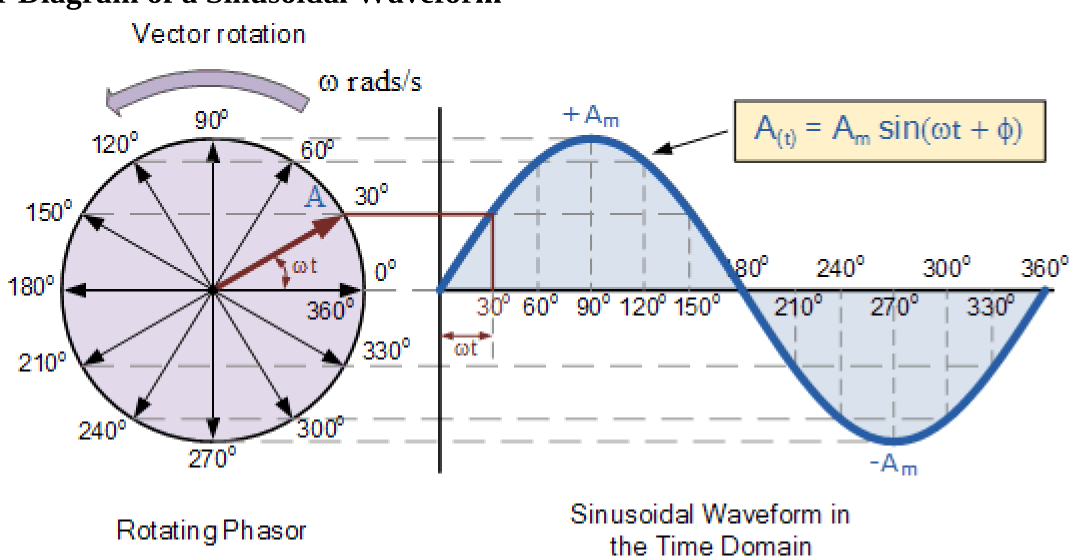
Peak Factor is defined as the ratio of maximum value to the R.M.S value of an alternating quantity. The alternating quantities can be voltage or current.

Peak Factor = $\frac{Imax}{Irms}$ = 1.4142

Phasor Representation of alternating quantity:

An alternating quantity can be represented in the form of wave and equation. The waveform gives the graphical representation whereas equation represents the mathematical expression of the instantaneous value of an alternating quantity. The same alternating quantity can be represented by a line of definite length (representing the maximum value) rotating in counter-clockwise direction at a constant velocity (w rad/sec) Alternating Current (AC) is a type of electric current that reverses its direction periodically in contrast to the Direct Current (DC) which flows in a single direction.

Phasor Diagram of a Sinusoidal Waveform



As the single vector rotates in an anti-clockwise direction, its tip at point A will rotate one complete revolution of 360^o or 2π representing one complete cycle. If the length of its moving tip is transferred at different angular intervals in time to a graph as shown above, a sinusoidal waveform would be drawn starting at the left with zero time. Each position along the horizontal axis indicates the time that has elapsed since zero-time, $t = 0$. When the vector is horizontal the tip of the vector represents the angles at 0^o, 180^o and at 360^o.

Likewise, when the tip of the vector is vertical it represents the positive peak value, (+Am) at 90^o or $\pi/2$ and the negative peak value, (-Am) at 270^o or $3\pi/2$. Then the time axis of the waveform represents the angle either in degrees or radians through which the phasor has moved.

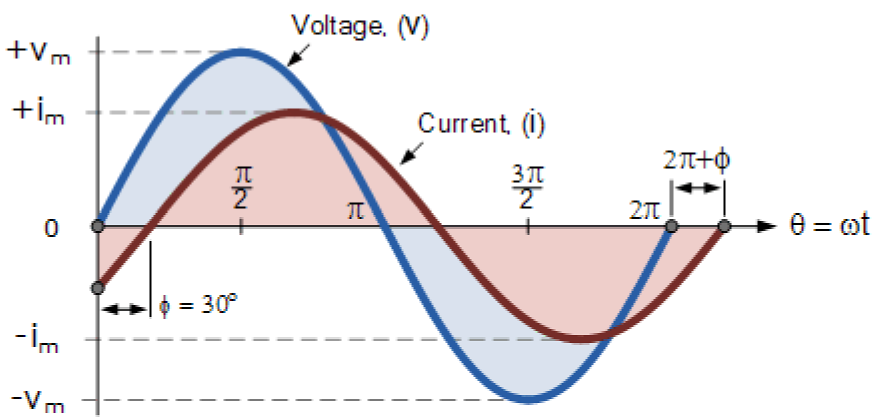
When we are analyzing alternating waveforms, we may need to know the position of the phasor, representing the Alternating Quantity at some particular instant in time especially when we want to compare two different waveforms on the same axis. For example, voltage and current. We have assumed in the waveform above that the waveform starts at time $t = 0$ with a corresponding phase angle in either degrees or radians.

But if a second waveform starts to the left or to the right of this zero-point or we want to represent in phasor notation the relationship between the two waveforms then we will need to take into account this phase difference, Φ of the waveform.

Phase Difference of a Sinusoidal Waveform

The phase of an alternating quantity at an instant is defined as the fractional part of a cycle through which the quantity has advanced from a selected origin.

The two alternating quantities having same frequency, when attain their zero values at different instants, the quantities are said to have a phase difference.



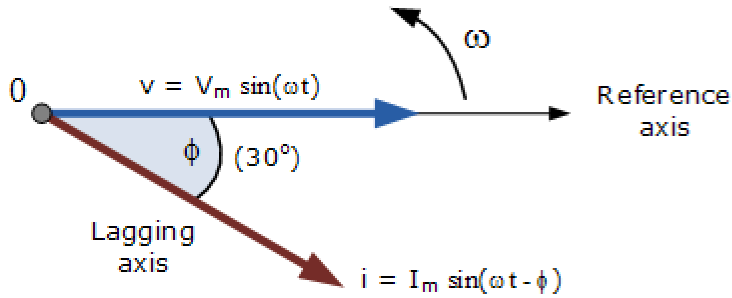
The generalised mathematical expression to define these two sinusoidal quantities will be written as:

$$v_{(t)} = V_m \sin(\omega t)$$

$$i_{(t)} = I_m \sin(\omega t - \phi)$$

The current, i is lagging the voltage, v by angle Φ and in our example above this is 30^o. So, the difference between the two phasors representing the two sinusoidal quantities is angle Φ and the resulting phasor diagram will be.

Phasor Diagram of a Sinusoidal Waveform

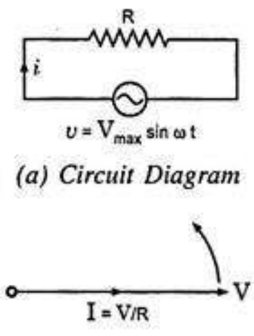


The phasor diagram is drawn corresponding to time zero ($t = 0$) on the horizontal axis. The lengths of the phasors are proportional to the values of the voltage, (V) and the current, (I) at the instant in time that the phasor diagram is drawn. The current phasor lags the voltage phasor by the angle, Φ, as the two phasors rotate in an *anticlockwise* direction, therefore the angle, Φ is also measured in the same anticlockwise direction.

Analysis of single-phase ac circuits consisting of R, L, C

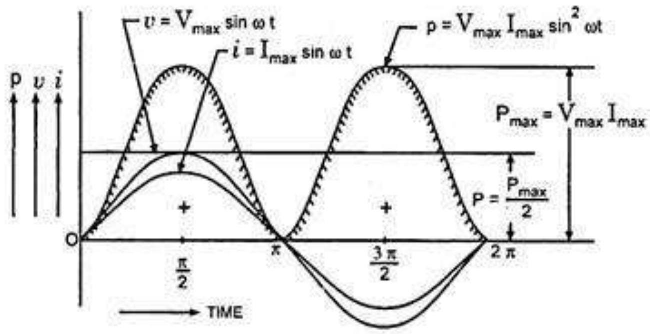
A purely resistive or a non-inductive circuit is a circuit which has inductance so small that at normal frequency its reactance is negligible as compared to its resistance

Consider an ac circuit containing a non-inductive resistance of R ohms connected across a sinusoidal voltage represented by v = V sin wt



(a) Circuit Diagram

(c) Phasor Diagram



(b) Wave Diagram

When the current flowing through a pure resistance changes, no back emf is set up, therefore, applied voltage has to overcome the ohmic drop of iR only

$$i.e. \quad i\,\mathrm{R} = v$$

$$\text{or} \quad i = \frac{v}{\mathrm{R}} = \frac{\mathrm{V_{max}}}{\mathrm{R}} \sin \omega t$$

$$\text{Current will be maximum when } \omega t = \frac{\pi}{2} \text{ or } \sin \omega t = 1$$

$$\therefore \quad \mathrm{I_{max}} = \frac{\mathrm{V_{max}}}{\mathrm{R}}$$

Instantaneous current may be expressed as:

$i = I_{max} \sin \omega t$

From the expressions of instantaneous applied voltage and instantaneous current, it is evident that in a pure resistive circuit, the applied voltage and current are in phase with each other, as shown by wave and phasor diagrams respectively

Power in Purely Resistive Circuit:

The instantaneous power delivered to the circuit in question is the product of the instantaneous values of applied voltage and current

$$i.e.\quad p = v\,i = V_{max} \sin \omega t\; I_{max} \sin \omega t = V_{max}\; I_{max} \sin^2 \omega t$$

$$\text{or}\quad p = \frac{V_{max}\; I_{max}}{2}\,(1 - \cos 2\,\omega\, t) \qquad\qquad \text{Since } \sin^2 \omega\, t = \frac{1 - \cos 2\,\omega\, t}{2}$$

$$= \frac{V_{max}\; I_{max}}{2} - \frac{V_{max}\; I_{max}}{2} \cos 2\,\omega\, t$$

$$\text{Average power, } P = \text{Average of } \frac{V_{max}\; I_{max}}{2} - \textit{average of } \frac{V_{max}\; I_{max}}{2} \cos 2\,\omega\, t$$

Since average of $\frac{V_{max}\; I_{max}}{2} \cos 2\,\omega\, t$ over a complete cycle is zero,

$$P = \frac{V_{max}\; I_{max}}{2} = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = V\,I \text{ watts}$$

<u>AC Circuit containing pure inductance only</u>

An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be had as the inductive coil has always small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance arid is known as a choke coil.

When an alternating voltage is applied to a purely inductive coil, an emf, known as self-induced emf, is induced in the coil which opposes the applied voltage. Since coil has no resistance, at every instant applied voltage has to overcome this self-induced emf only.

Let the applied voltage $v = V_{max} \sin \omega$ t
and self inductance of coil = L henry

Self induced emf in the coil, $e_L = -L \dfrac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self induced emf *i.e.* $v = -e_L$

$$\therefore\quad V_{max} \sin \omega\, t = -\left(-L\frac{di}{dt}\right)$$

$$\text{or}\quad di = \frac{V_{max}}{L} \sin \omega\, t\, dt$$

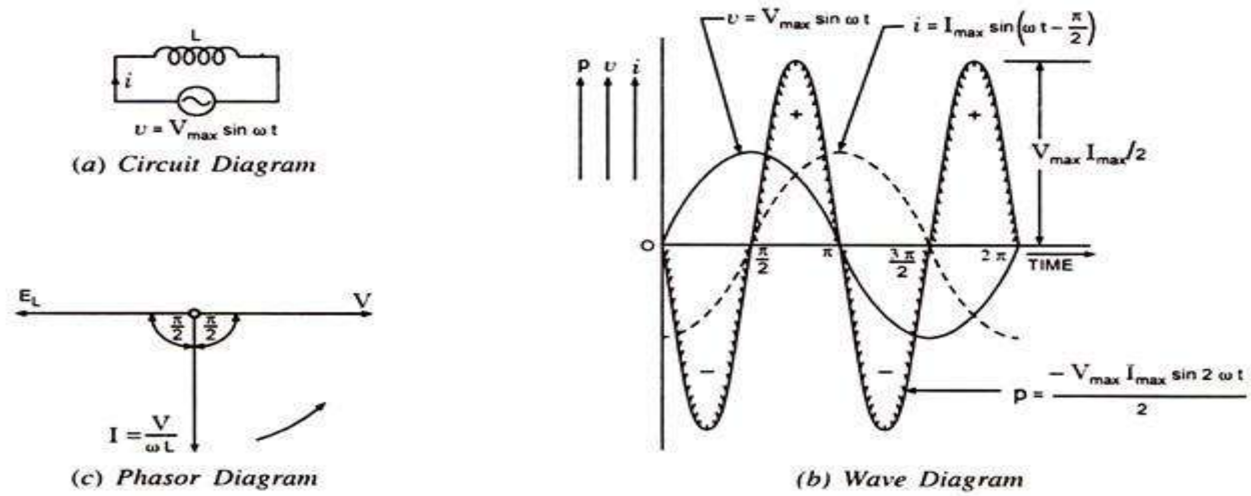
Integrating both sides we get

$$i = \frac{V_{max}}{L} \int \sin \omega\, t\; d\,t = \frac{V_{max}}{\omega L}\,(-\cos \omega\, t) + A$$

where A is a constant of integration, which is found to be zero from initial conditions

$$i.e.\quad i = \frac{-V_{max}}{\omega L} \cos \omega\, t = \frac{V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

From the expressions of instantaneous applied voltage and instantaneous current flowing through a purely inductive coil it is observed that the current lags behind the applied voltage by π/2 as shown by wave diagram and phasor diagram.



(a) Circuit Diagram

(c) Phasor Diagram

(b) Wave Diagram

Inductive Reactance:

ωL in the expression Imax = V_{max}/ωL is known as inductive reactance and is denoted by X_L i.e.,

$X_L = \omega L$

If L is in henry , then X_L will be in ohms.

Power in Purely Inductive Circuit:

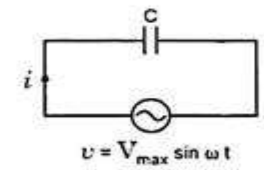
Instantaneous power, $p = v \times i = V_{max} \sin \omega t \; I_{max} \sin (\omega t - \pi/2)$

or $p = - V_{max} I_{max} \sin \omega t \cos \omega t = V_{max} I_{max}/2 \sin 2 \omega t$

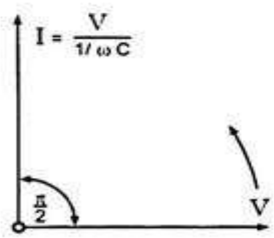
The power measured by wattmeter is the average value of p which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence in a purely inductive circuit power absorbed is zero.

AC circuit containing pure Capacitor only

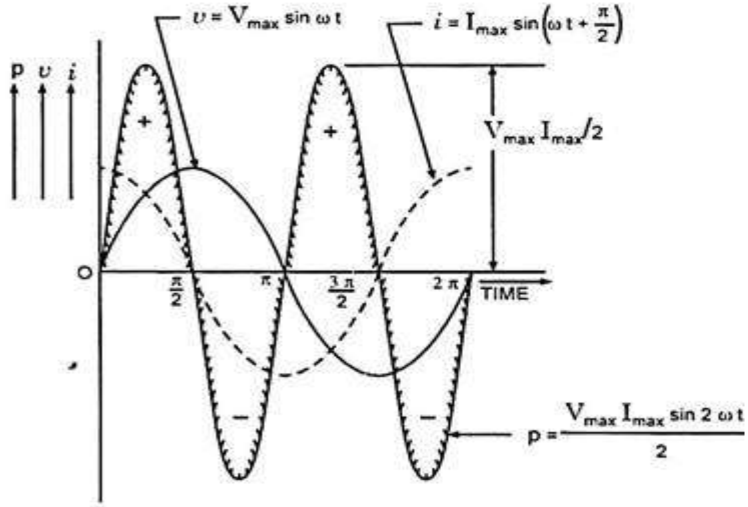
The circuit containing pure capacitor of C farad as shown in fig



(a) Circuit Diagram



(c) Phasor Diagram



(b) Wave Diagram

et an alternating voltage represented by $v = V_{max} \sin \omega t$ be applied across a capacitor of capacitance C farads.

The expression for instantaneous charge is given as:

$q = C\, V_{max} \sin \omega t$

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation:

$$i = \frac{dq}{dt} = [C\, V_{max} \sin \omega t] = \omega C\, V_{max} \cos \omega t = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Current is maximum when $t = 0$

$$\therefore \quad I_{max} = \frac{V_{max}}{1/\omega C}$$

Substituting $\frac{V_{max}}{1/\omega C} = I_{max}$ in the above equation for instantaneous current, we get

$$i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the equations of instantaneous applied voltage and instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by $\pi/2$, as shown in Figs. 4.4 (b) and (c) by wave and phasor diagrams respectively.

Capacitive Reactance:

$1/\omega C$ in the expression $I_{max} = V_{max}/1/\omega C$ is known as capacitive reactance and is denoted by X_C i.e., $XC = 1/\omega C$. If C is in farads and ω is in radians/s, then X_c will be in ohms.

Power in Purely Capacitive Circuit:

$$p = v\,i = V_{max} \sin \omega t.\ I_{max} \sin \left(\omega t + \frac{\pi}{2}\right) = V_{max}\, I_{max} \sin \omega t \cos \omega t$$

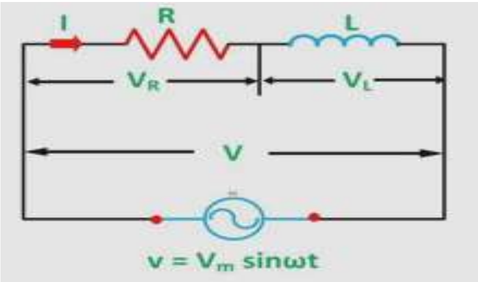
$$= \frac{V_{max}\, I_{max}}{2} \sin 2\omega t$$

Average power, $P = \frac{V_{max}\, I_{max}}{2} \times$ average of $\sin 2\,\omega t$ over a complete cycle $= 0$.

Hence power absorbed in a purely capacitive circuit is zero.

RL Series Circuit

A circuit that contains a pure resistance R ohm connected in series with a coil having pure inductance of L (Henry) is known as R L Series Circuit. When an AC supply voltage V is applied the current, I flows in the circuit. I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series circuit is shown below

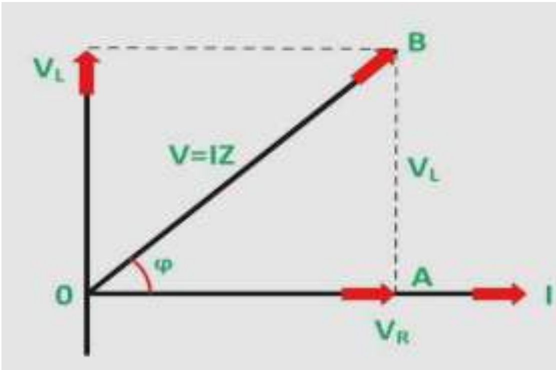


Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

Phasor Diagram of the RL Series Circuit

The phasor diagram of the RL Series circuit is shown below



Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step.

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now,

In right angle triangle OAB

$V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$V = IZ$

Where,

$$Z = \sqrt{R^2 + X_L^2}$$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle

In RL Series Circuit the current lags the voltage by 90-degree angle known as phase angle. It is given by the equation

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation

$v = V_m \, Sinwt$ ……. ………. (1)

The equation of current I is given as

$I = I_m \, Sin(wt-\phi)$ …………….(2)

Then the instantaneous power is given by the equation

$p = vi$ ………………………(3)

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m Sin\omega t) x \, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V \, I \cos\varphi$$

where $\cos\phi$ is called the power factor of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots \ldots \ldots \ldots (4)$$

The power factor is defined as the ratio of resistance to the impedance of an AC Circuit.

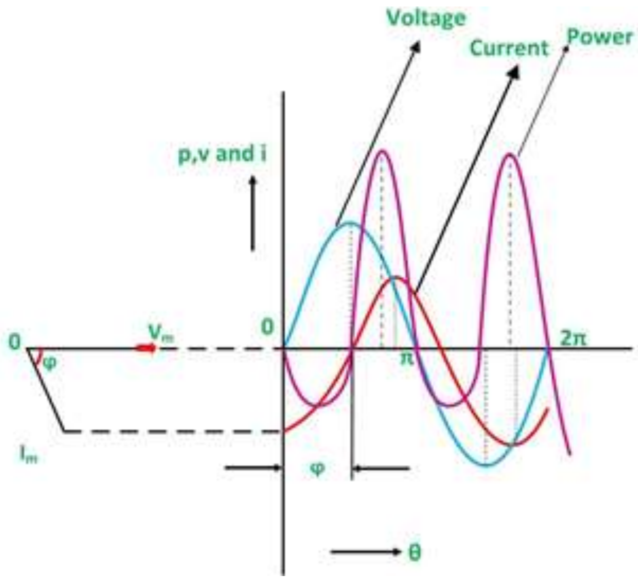
Putting the value of V and $\cos\phi$ from the equation (4) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \ldots \ldots \ldots . . (5)$$

From the equation (5) it can be concluded that the inductor does not consume any power in the circuit.

Waveform and Power Curve of the RL Series Circuit

The waveform and power curve of the RL Series Circuit is shown below



The various points on the power curve are obtained by the product of voltage and current. If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.

Explain the following:

- **a) Real power**
- **b) Reactive power**
- **c) Apparent power**
- **d) Power factor**

a) Real power: The power which is actually consumed or utilized in ac circuit is called true power or active power or real power. It is consumed by the resistive load in the circuit. The unit of real power is watts.

Real power = VI cos ϕ Watts

b) Reactive power: The power which flows back and forth that means it moves in both the directions in the circuit or reacts upon itself, is called Reactive Power. A pure inductor and a pure capacitor do not consume any power since in a half cycle whatever power is received from the source by these components, the same power is returned to the source. This power which returns and flows in both the direction in the circuit, is called Reactive power. This reactive power does not perform any useful work in the circuit. It is measured in a unit called Volt-Amps-Reactive (VAR), rather than watts.

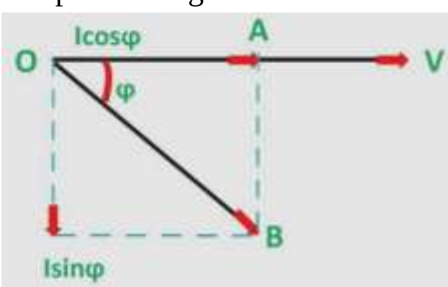
c) Apparent power: The product of root mean square (RMS) value of voltage and current is known as Apparent Power. This power is measured in kVA or MVA.

In a purely resistive circuit, the current is in phase with the applied voltage, whereas in a purely inductive and capacitive circuit the current is 90 degrees out of phase, i.e., if the inductive load is connected in the circuit the current lags voltage by 90 degrees and if the capacitive load is connected the current leads the voltage by 90 degrees.

Hence, from all the above discussion, it is concluded that the current in phase with the voltage produces true or active power, whereas, the current 90 degrees out of phase with the voltage contributes to reactive power in the circuit.
Therefore,

- True power = voltage x current in phase with the voltage
- Reactive power = voltage x current out of phase with the voltage

The phasor diagram for an inductive circuit is shown below:



Taking voltage V as reference, the current I lags behind the voltage V by an angle ϕ. The current I is divided into two components:

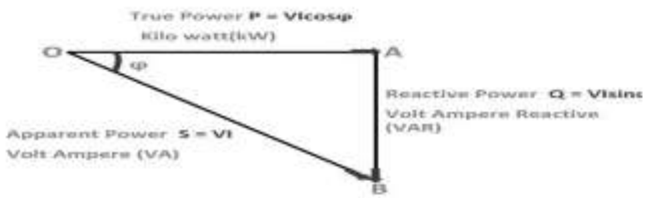
- I Cos ϕ in phase with the voltage V
- I Sin ϕ which is 90 degrees out of phase with the voltage V

Therefore, the following expression shown below gives the active, reactive and apparent power respectively.

- Active power $P = V \times I \cos\phi = V I \cos\phi$
- Reactive power P_r or $Q = V \times I \sin\phi = V I \sin\phi$
- Apparent power P_a or $S = V \times I = VI$

Power Triangle:

Power Triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power. When each component of the current that is the active component (Icosϕ) or the reactive component (Isinϕ) is multiplied by the voltage V, a power triangle is obtained shown in the figure below

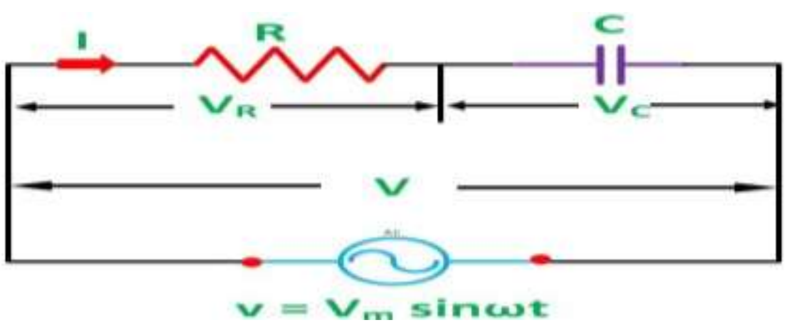


d) Power factor: The power factor in ac circuit is the ratio of the real power that is used to do work and the apparent power that is supplied to the circuit. The power factor can get values in the range from 0 to 1. When all the power is reactive power with no real power (usually inductive load) - the power factor is 0.

Power factor (Cos ϕ) $= \dfrac{Real\ \ power\ \ in\ watts}{Apparent\ \ \ power\ \ in\ Volt - Amp}$

Resistance — Capacitance (R-C) Series Circuit

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit.The RC Series circuit is shown in the figure below:

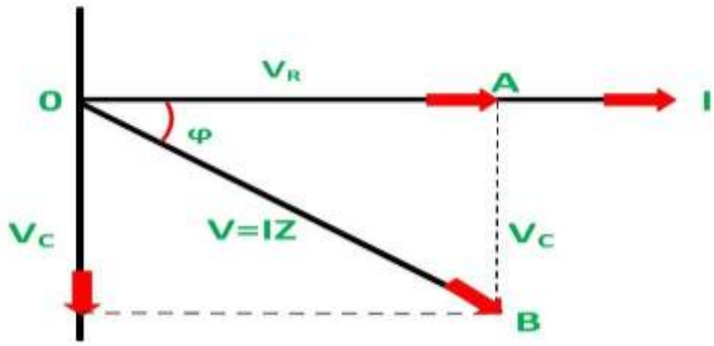


Where,

- V_R – voltage across the resistance R
- V_C – voltage across capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit:

The phasor diagram of the RC series circuit is shown below:



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance VR = IR is taken in phase with the current vector
- Voltage drop in capacitive reactance VC = IXC is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now, $V_R = I_R$ and $V_C = IX_C$, Where $X_C = I/2\pi fC$

In right triangle OAB,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \qquad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Power in RC Series Circuit

If the alternating voltage applied across the circuit is given by the equation

$$v = V_m \text{Sin}\omega t \text{ ....}(1)$$

Then,

$$i = I_m \sin(\omega t + \varphi) \dots \dots \dots . (2)$$

Therefore, the instantaneous power is given by p = vi
Putting the value of v and i from the equation (1) and (2) in p = vi

$$P = (V_m \text{Sin}\omega t) x\ I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the **power factor** of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \dots \dots \dots \dots . (3)$$

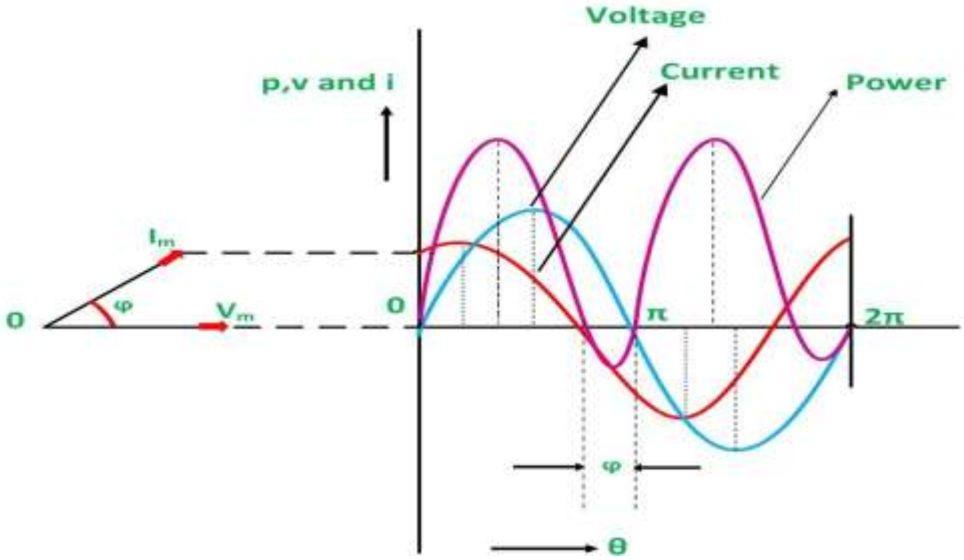
Putting the value of V and cosϕ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots..(4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

Waveform and Power Curve of the RC Series Circuit

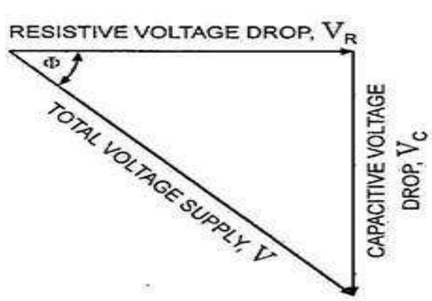
The waveform and power curve of the RC circuit is shown below:



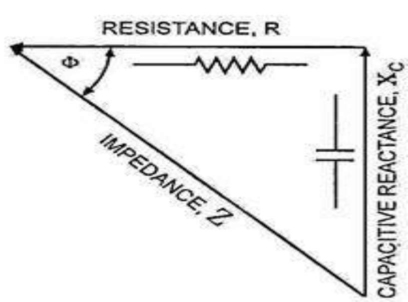
The various points on the power curve are obtained from the product of the instantaneous value of voltage and current.

The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.

<u>**Impedance Triangle**</u>



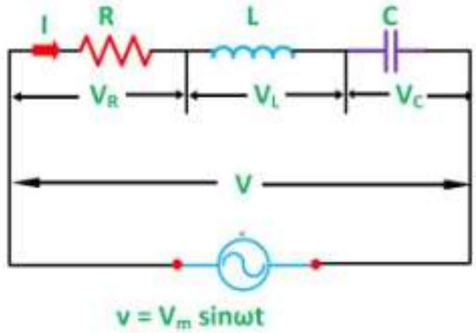
(a) Voltage Triangle



(b) Impedance Triangle

RLC Series Circuit

The **RLC Series Circuit** is defined as when a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other. As all the three elements are connected in series so, the current flowing in each element of the circuit will be same as the total current I flowing in the circuit.
The **RLC Circuit** is shown below-



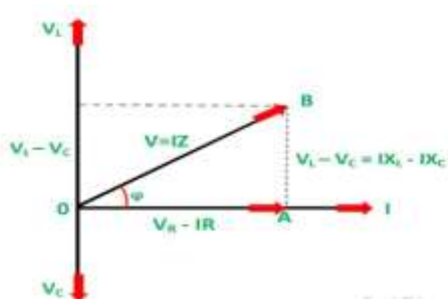
In the RLC Series Circuit, $X_L = 2\pi fL$ and $X_C = 1/2\pi fC$

When the AC voltage is applied through the RLC Series Circuit the resulting current I flows through the circuit, and thus the voltage across each element will be

- V_R = IR that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across the capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC Series Circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Steps to draw the Phasor Diagram of the RLC Series Circuit

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90 degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

It is the total opposition offered to the flow of current by an RLC Circuit and is known as Impedance of the circuit.

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

The product of voltage and current is defined as power.

$$P = VI\cos\varphi = I^2R$$

Where cosϕ is the power factor of the circuit and is expressed as

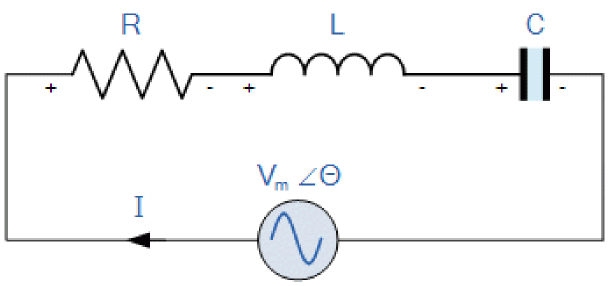
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as a RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of power factor is unity.

RLC Series Resonance

Resonance occurs in a series circuit when the supply frequency causes the voltages across L and C to be equal and opposite in phase.



In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L = X_C$. The point at which this occurs is called the Resonant Frequency point, (f_r) of the circuit, and as we are analyzing a series RLC circuit this resonance frequency produces a Series Resonance.

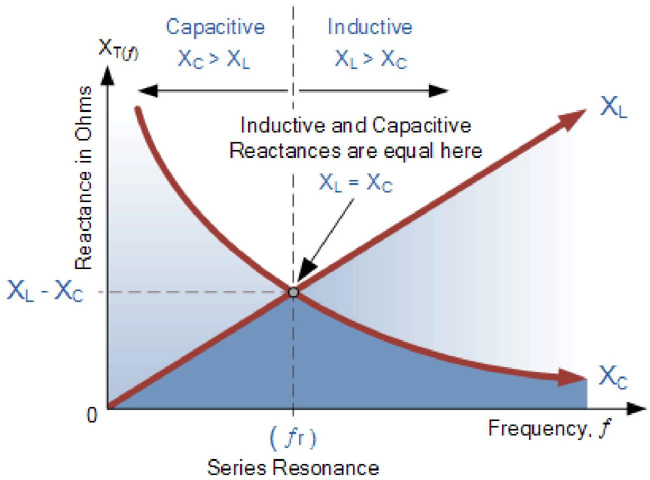
Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels.

Various terms in RLC series circuit:

- Inductive reactance: $X_L = 2\pi f L = \omega L$
- Capacitive reactance: $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive
- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

Series Resonance Frequency

Electrical resonance occurs in an AC circuit when the two reactances (inductive reactance and capacitive reactance) which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is where the two reactance curves cross each other.



where: f_r is in Hertz, L is in Henries and C is in Farads.

In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

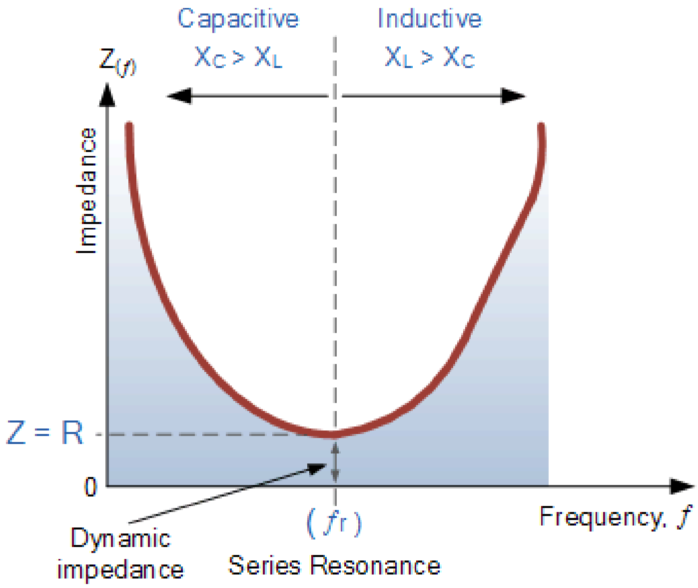
$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore\ f_r = \frac{1}{2\pi\sqrt{LC}}\ (\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\ (\mathrm{rads})$$

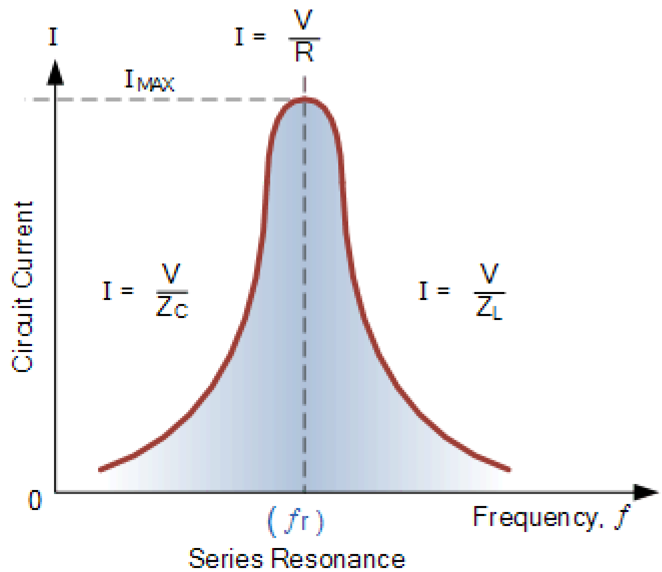
Impedance in a Series Resonance Circuit

At resonance, the two reactance cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R.So, the total impedance of the series circuit becomes just the value of the resistance and therefore: Z = R.Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit. The circuit impedance at resonance is called the “dynamic impedance” of the circuit and depending upon the frequency, X_C (typically at high frequencies) or X_L (typically at low frequencies) will dominate either side of resonance as shown below.



Series Circuit Current at Resonance

Since the current flowing through a series resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its minimum value, (=R). Therefore, the circuit current at this frequency will be at its maximum value of V/R as shown below.



The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite. The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition, they cancel each other out.

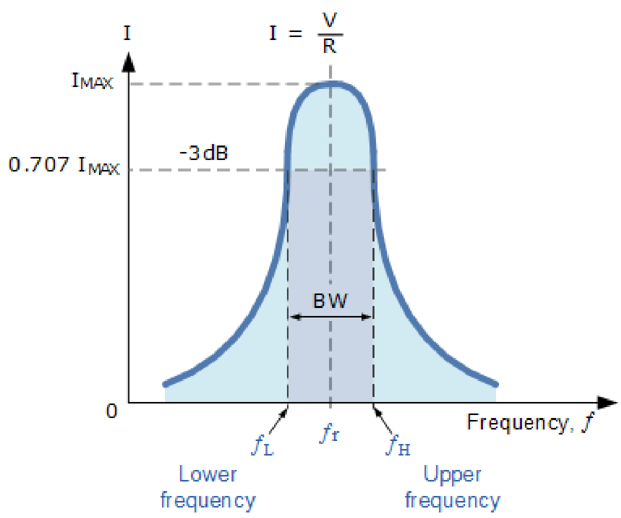
As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an Acceptor Circuit because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency.

Bandwidth of a Series Resonance Circuit

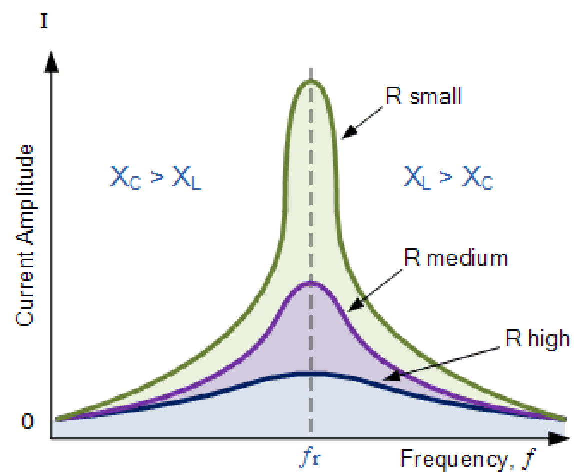
If the series RLC circuit is driven by a variable frequency at a constant voltage, then the magnitude of the current, I is proportional to the impedance, Z, therefore at resonance the power absorbed by the circuit must be at its maximum value as $P = I^2Z$.

If we now reduce or increase the frequency until the average power absorbed by the resistor in the series resonance circuit is half that of its maximum value at resonance, we produce two frequency points called the half-power points which are -3dB down from maximum, taking 0dB as the maximum current reference.

The -3dB points give us a current value that is 70.7% of its maximum resonant value which is defined as: $0.5(I^2 R) = (0.707 \times I)^2 R$. Then the point corresponding to the lower frequency at half the power is called the "lower cut-off frequency", labelled f_L with the point corresponding to the upper frequency at half power being called the "upper cut-off frequency", labelled f_H. The distance between these two points, i.e. $(f_H - f_L)$ is called the Bandwidth, (BW) and is the range of frequencies over which at least half of the maximum power and current is provided as shown.



The frequency response of the circuit's current magnitude above, relates to the "sharpness" of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the Quality factor, Q of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, $Q = f_r / BW$.

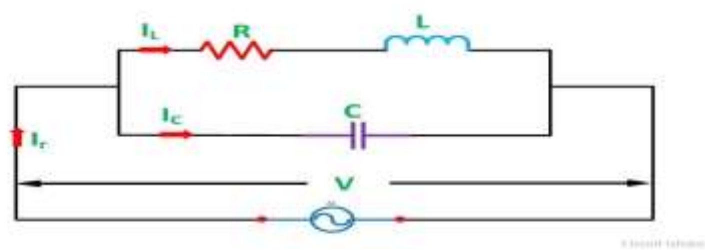


Quality factor of series resonance circuit is given by

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r C R} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

RLC Parallel Circuit and resonance

Parallel Resonance means when the circuit current is in phase with the applied voltage of an AC circuit containing an Inductor and a Capacitor connected together in parallel. Let us understand the Parallel Resonance with the help of a circuit diagram shown below.

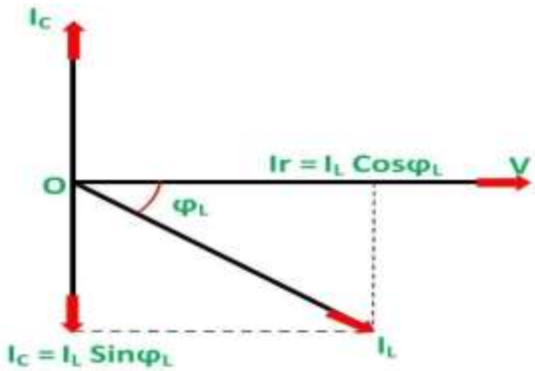


Consider an Inductor of L Henry having some resistance of R ohms connected in parallel with a capacitor of capacitance C farads. A supply voltage of V volts is connected across these elements. The circuit current Ir will only be in phase with the supply voltage when the following condition given below in the equation is satisfied.

$$I_C = I_L \sin\varphi_L$$

Phasor Diagram

The phasor diagram of the given circuit is shown below



At the Resonance condition, the circuit draws the minimum current as under this (resonance) condition the reactive component of current is suppressed.

Frequency at Resonance Condition in Parallel resonance Circuit

The value of inductive reactance $X_L = 2\pi fL$ and capacitive reactance $X_C = 1/2\pi fC$ can be changed by changing the supply frequency. As the frequency increases the value of X_L and consequently the value of Z_L increases. As a result, there is a decrease in the magnitude of current I_2 and this I_2 current lags behind the voltage V.

On the other hand, the value of capacitive reactance decreases and consequently the value of I_C increases.

At some frequency, f_r called resonance frequency.

$$I_C = I_L \, Sin\varphi_L$$

Where,

$$I_L = \frac{V}{Z_L}$$

$$Sin\varphi_L = \frac{X_L}{Z_L} \quad \text{and} \quad I_C = \frac{V}{X_C}$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L} \quad \text{or} \quad X_L X_C = Z_L^2 \quad \text{or}$$

$$\frac{\omega L}{\omega C} = Z_L^2 = (R^2 + X_L^2) \quad \text{or}$$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2 \quad \text{or}$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2} \quad \text{or}$$

$$f_r = \frac{1}{2\pi L} = \sqrt{\frac{L}{C} - R^2} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is very small as compared to L, then resonant frequency will be

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Effect of Parallel Resonance

At parallel resonance line current $I_r = I_L \cos\phi$ or

$$\frac{V}{Z_r} = \frac{V}{Z_l} \times \frac{R}{Z_l} \quad \text{or} \quad \frac{1}{Z_r} = \frac{R}{Z_L^2} \quad \text{or}$$

$$\frac{1}{Z_r} = \frac{R}{L/C} = \frac{CR}{L} \quad \left(\text{as } Z_L^2 = \frac{L}{C}\right)$$

Therefore, the circuit impedance will be given as

$$Z_r = \frac{L}{CR}$$

The following **conclusions** are made from the above overall discussion about the **Parallel Resonance.**

- The circuit impedance is purely resistive because there is no frequency term present in it. If the value of Inductance, Capacitance and Resistance is in Henry, Farads and Ohm than the value of circuit impedance Z_r will be in Ohms.
- The value of Zr will be very high because the ratio L/C is very large at parallel resonance.
- The value of circuit current, Ir = V/Zr is very small because the value of Zr is very high.
- The current flowing through the capacitor and the coil is much greater than the line current because the impedance of each branch is quite lower than that of circuit impedance Zr.

Three-phase balanced circuits, voltage and current relations in star and delta connections.

Poly-phase system: An ac system having a group of (two or more than two) equal voltages of same frequency arranged to have equal phase difference between them is called a poly-phase system.

Phase difference = $\frac{360\ electrical\ \ degree}{Number\ \ of\ phases}$

Advantage of three –phase system over single phase system.

1. Power delivered is constant. In single phase circuit the power delivered is pulsating and objectionable for many applications.
2. For a given frame size a poly-phase machine gives a higher output than a single phase machine.
3. Poly-phase induction motors are self starting and are more efficient. Single phase motor has no starting torque and requires an auxiliary means for starting.
4. Comparing with single phase motor, three phase induction motor has higher power factor and efficiency.
5. Three phase motors are very robust, relatively cheap, and generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single phase motors.

Generation of 3 Phase E.M.Fs in a 3 Phase Circuit

In a 3 phase system, there are three equal voltages or EMFs of the same frequency having a phase difference of 120 degrees. These voltages can be produced by a three-phase AC generator having three identical windings displaced apart from each other by 120 degrees electrical. When these windings are kept stationary, and the magnetic field is rotated as shown in the figure A below or when the windings are kept stationary, and the magnetic field is rotated as shown below in figure B, an emf is induced in each winding. The magnitude and frequency of these EMFs are same but are displaced apart from one another by an angle of 120 degrees.

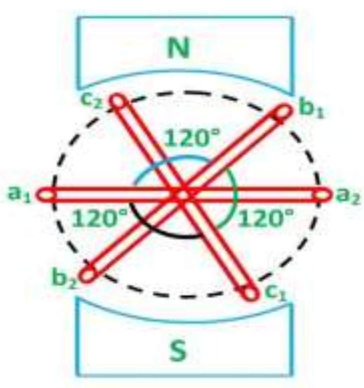


Figure A

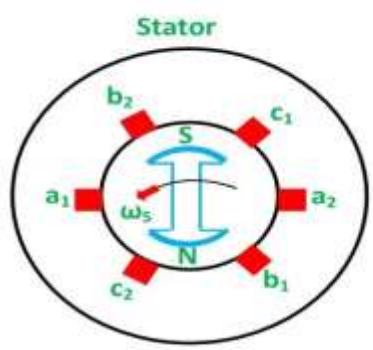


Figure B

Consider three identical coils a_1a_2, b_1b_2 and c_1c_2 as shown in the above figure. In this figure a_1, b_1 and c_1 are the starting terminals, whereas a_2, b_2 and c_2 are the finish terminals of the three coils. The phase difference of 120 degrees has to be maintained between the starts terminals a_1, b_1 and c_1.

Now, let the three coils mounted on the same axis, and they are rotated by either keeping coil stationary and moving the magnetic field or vice versa in an anticlockwise direction at (ω) radians per seconds. Three EMFs are induced in the three coils respectively.

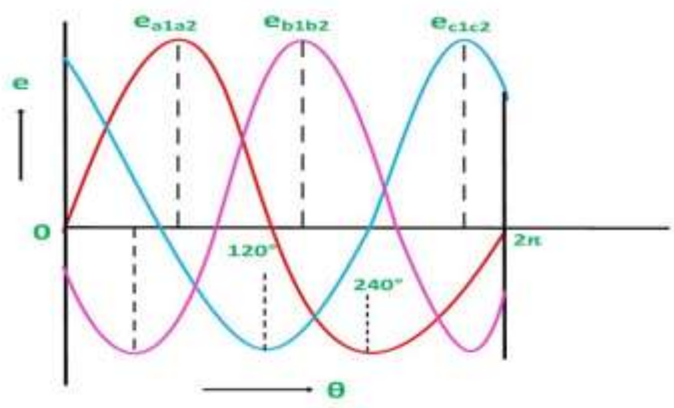


Figure C

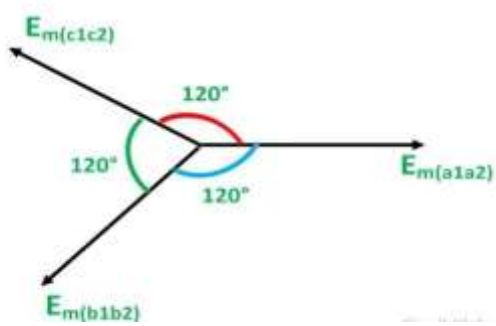
Considering the figure C, the analysis about their magnitudes and directions are given as follows. The emf induced in the coil a_1a_2 is zero and is increasing in the positive direction as shown by the waveform in the above figure C represented as e_{a1a2}.

The coil b_1b_2 is 120 degrees electrically behind the coil a_1a_2. The emf induced in this coil is negative and is becoming maximum negative as shown by the wave e_{b1b2}.

Similarly, the coil c_1c_2 is 120 degrees electrically behind the coil b_1b_2, or we can also say that the coil c_1c_2 is 240 degrees behind the coil a_1a_2. The emf induced in the coil is positive and is decreasing as shown in the figure C represented by the waveform e_{c1c2}.

Phasor Diagram

The EMFs induced in the three coils in a 3 phase circuits are of the same magnitude and frequency and are displaced by an angle of 120 degrees from each other as shown below in the phasor diagram.



These EMFs of a 3 phase circuits can be expressed in the form of the various equations given below.

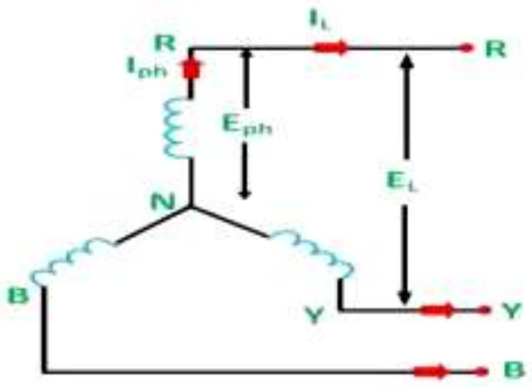
$$e_{a1a2} = E_m \sin \omega t$$

$$e_{b1b2} = E_m \sin(\omega t - 2\pi/3) = E_m \sin(\omega t - 120°)$$

$$e_{c1c2} = E_m \sin(\omega t - 4\pi/3) = E_m \sin(\omega t - 240°)$$

Star or Y connections

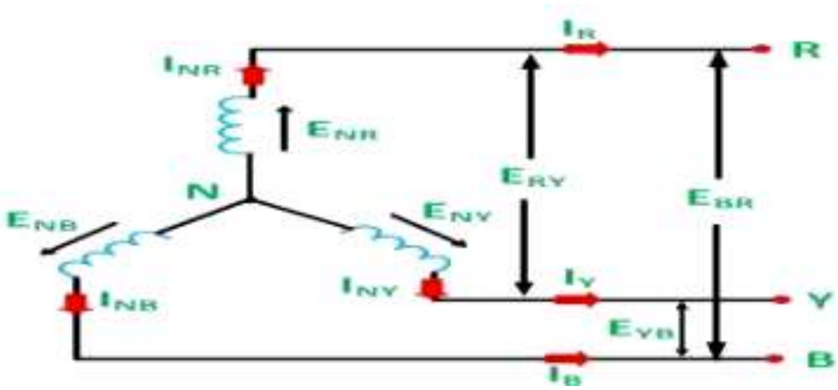
The star connection is shown in the diagram below.



Considering the above figure, the finish terminals of the three windings are connected to form a star or neutral point. The three conductors named as R, Y and B run from the remaining three free terminals as shown in the above figure.

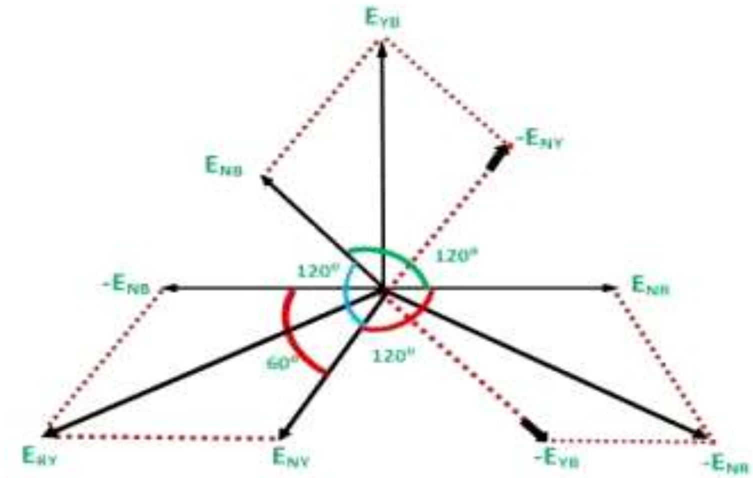
The current flowing through each phase is called Phase current I_{ph}, and the current flowing through each line conductor is called Line Current I_L. Similarly, the voltage across each phase is called Phase Voltage E_{ph}, and the voltage across two line conductors is known as the Line Voltage E_L.

Relation between Phase Voltage and Line Voltage in Star Connection The Star connection is shown in the figure below.



As the system is balanced, a balanced system means that in all the three phases, i.e., R, Y and B, the equal amount of current flows through them. Therefore, the three voltages E_{NR}, E_{NY} and E_{NB} are equal in magnitude but displaced from one another by 120 degrees electrical.

The Phasor Diagram of Star Connection is shown below.



The arrowheads on the emfs and current indicate direction and not their actual direction at any instant.
Now,
$E_{NR}=E_{NY}=E_{NB}=E_{ph}$ (in magnitude)
There are two phase voltages between any two lines.
Tracing the loop NRYN

$$\overline{E_{NR}} + \overline{E_{RY}} - \overline{E_{NY}} = 0 \quad \text{or}$$

$$\overline{E_{RY}} = \overline{E_{NY}} - \overline{E_{NR}} \quad \text{(vector difference)}$$

To find the vector sum of E_{NY} and $-E_{NR}$, we have to reverse the vector E_{NR} and add it with E_{NY} as shown in the phasor diagram above.
Therefore,

$$E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2E_{NY}E_{NR}\cos 60^0} \quad \text{or}$$

$$E_L = \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph}E_{ph} \times 0.5} \quad \text{or}$$

$$E_L = \sqrt{3E_{ph}^2} = \sqrt{3}\, E_{ph} \quad \text{(in magnitude)}$$

Hence, in Star Connections Line voltage is root 3 times of phase voltage.

$$\text{Line voltage} = \sqrt{3} \times \text{Phase voltage}$$

Relation between Phase Current and Line Current in Star Connection

The same current flows through phase winding as well as in the line conductor as it are connected in series with the phase winding.

$$I_R = I_{NR}$$

$$I_Y = I_{NY} \quad \text{and}$$

$$I_B = I_{NB}$$

Where the phase current will be

$$I_{NR} = I_{NY} = I_{NB} = I_{ph}$$

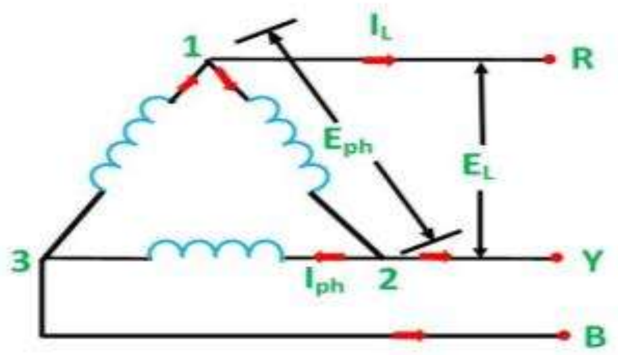
The line current will be

$$I_R = I_Y = I_B = I_L$$

Hence, in a 3 Phase system of Star Connections, the Line Current is equal to Phase Current.

<u>Delta or Mesh connection</u>

In **Delta (Δ) or Mesh connection**, the finished terminal of one winding is connected to start terminal of the other phase and so on which gives a closed circuit. The three line conductors are run from the three junctions of the mesh called Line Conductors. The connection in Delta form is shown in the figure below.

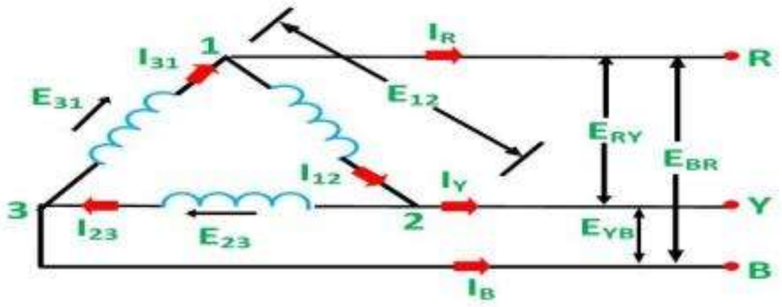


The current flowing through each phase is called **Phase Current (Iph)**, and the current flowing through each line conductor is called **Line Current (I_L).**

The voltage across each phase is called **Phase Voltage (E_{ph})**, and the voltage across two line conductors is called **Line Voltage (E_L).**

Relation between Phase Voltage and Line Voltage in Delta Connection

To understand the relationship between the phase voltage and line voltage in the Delta consider the figure A shown below.



It is clear from the figure that the voltage across terminals 1 and 2 is the same as across the terminals R and Y. Therefore,

$E_{12}= E_{RY}$

Similarly,

$E_{23}= E_{YB}$ and $E_{31} = E_{BR}$

Where, the phase voltages are

$E_{12}= E_{23}= E_{31} = E_{ph}$

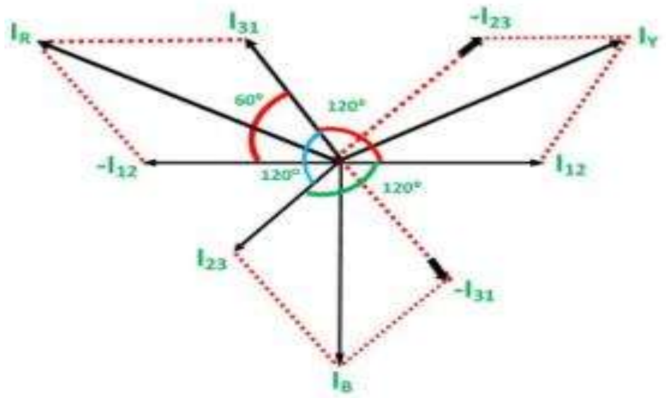
The line voltages are

$E_{RY} = E_{YB} = E_{BR}= E_L$

Hence, in Delta Connection Line Voltage is equal to Phase Voltage.

Relation between Phase Current and Line Current in Delta Connection

As in the balanced system the three phase current I_{12}, I_{23} and I_{31} are equal in magnitude but are displaced from one another by 120 degrees electrical.The **phasor diagram** is shown below.



Hence, $I_{12} = I_{23} = I_{31} = I_{ph}$

If we look at figure, it is seen that the current is divided at every junction 1, 2 and 3.

Applying Kirchhoff's Law at junction 1

The Incoming currents are equal to outgoing currents.

$$\overline{I_{31}} = \overline{I_R} + \overline{I_{12}}$$

And their vector difference will be given as

$$\overline{I_R} = \overline{I_{31}} - \overline{I_{12}}$$

The vector I_{12} is reversed and is added in the vector I_{31} to get the vector sum of I_{31} and $-I_{12}$ as shown above in the phasor diagram. Therefore,

$$I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2I_{31}I_{12}\cos 60^\circ} \quad \text{or}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph} \times 0.5}$$

As we know, $I_R = I_L$, therefore,

$$I_L = \sqrt{3I_{ph}^2} = \sqrt{3}I_{ph}$$

Line current = $\sqrt{3}$ * Phase Current

Hence, in Delta connection line current is root three times of phase current.

Short questions with answers
Module 2

Q1. Differentiate between rms value and average value.
Ans. "RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, which would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor.
The average value of a periodic waveform whether it is a sine wave, square wave or triangular waveform is defined as: "the quotient of the area under the waveform with respect to time". In other words, the averaging of all the instantaneous values along time axis with time being one full period, (T).

Q2. What do you mean by resonance? Explain.
Ans. Resonance in an AC circuit refers to that state of the circuit in which the inductive reactance of the circuit is equal to its capacitive reactance
At resonance, the series impedance of the two elements is at a minimum and the parallel impedance is a maximum.

Q3. Discuss the significance of phasor diagram in electrical engineering.
Ans. Phasor Diagram is a graphical way of representing the magnitude and directional relationship between two or more alternating quantities. Sinusoidal waveforms of the same frequency can have a Phase Difference between themselves which represents the angular difference of the two sinusoidal waveforms. Every phasor in the diagram will have the same angular velocity because they represent sine waves of identical frequency. The length of the each phasor arm is directly related to the amplitude of the wave it represents, and the angle between the phasors is the same as the angle of phase difference between the sine waves.

Q4. Write the mathematical expression for 50Hz sinusoidal voltage supplied for domestic purposes at 230V.
Ans. $V = V_m \, Sinwt$

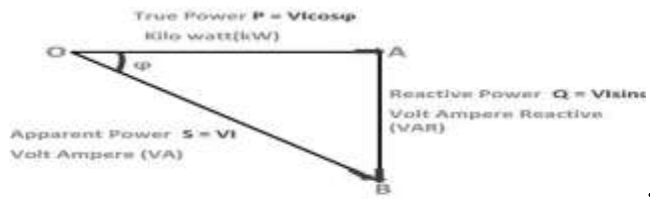
$V_{rms} = 230 \text{ V}$

$V_m = \sqrt{2} * V_{rms} = \sqrt{2} * 230 = 325.22\text{V}$

$w = 2\pi f = 2\pi 50 = 100\pi$

$V = V_m \, Sinwt = 325.22 \, Sin(100\, \pi t)$

Q5. Draw the power triangle and define various types of power.
Ans. Power Triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power. When each component of the current that is the active component ($I\cos\phi$) or the reactive component ($I\sin\phi$) is multiplied by the voltage V, a power triangle is obtained shown in the figure below



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True power: The power which is actually consumed or utilized in an ac circuit.
Reactive power: The power which flow back and forth or reacts upon itself.
Apparent power: The product of the rms voltage and the rms current is called apparent power

Q6. How will you differentiate between AC and DC circuit.

Basis	**Alternating current**	**Direct current**
Definition	The direction of the current reverse periodically.	The direction of the current remain same.
Causes of flow of electrons	Rotating a coil in a uniform magnetic field or rotating a uniform magnetic field within a stationary coil	Constant magnetic field across the wire
Frequency	50 or 60 Hertz	Independent of frequency
Direction of flow of electrons.	Bidirectional	Unidirectional
Power Factor	Lies between 0 and 1	Always 1
Polarity	It has polarity (+, -)	Do not have polarity
Obtained From	Alternators	Generators, battery, solar cell, etc.
Type of load	Their load is resistive, inductive or capacitive.	Their load is usually resistive in nature.
Graphical Representation	It is represented by irregular waves like triangular wave, square wave, square tooth wave, sine wave.	It is represented by the straight line.
Transmission	Can be transmitted over long distance with some losses.	It can be transmitted over very long distance with negligible losses.
Convertible	Easily convert into direct current	Easily convert into alternating current
Harazdous	Dangerous	Very dangerous
Application	Factories, Industries and for the domestic purposes.	Electroplating, Electrolysis, Electronic Equipment etc.

Q7. Define Form Factor and Peak Factor.

Ans. The ratio of the root mean square value to the average value of an alternating quantity (current or voltage) is called Form Factor.

Peak Factor is defined as the ratio of maximum value to the R.M.S value of an alternating quantity. The alternating quantities can be voltage or current.

Q8. Describe the advantage of three –phase system over single phase system.

Ans.

1. Power delivered is constant. In single phase circuit the power delivered is pulsating and objectionable for many applications.
2. For a given frame size a polyphase machine gives a higher output than a single phase machine.
3. Polyphase induction motors are self starting and are more efficient. Single phase motor has no starting torque and requires an auxiliary means for starting.
4. Comparing with single phase motor, three phase induction motor has higher power factor and efficiency.
5. Three phase motors are very robust, relatively cheap, and generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single phase motors.
6. For transmitting the same amount of power at the same voltage, a three phase transmission line requires less conductor material than a single phase line. The three phase transmission system is so cheaper.
7. For a given amount of power transmitted through a system, the three phase system requires conductors with a smaller cross-sectional area.
8. This means a saving of copper and thus the original installation costs are less. Polyphase motors have uniform torque whereas most of the single phase motors have pulsating torque.
9. Parallel operation of three-phase generators is simpler than that of single phase generator.
10. Polyphase system can set up rotating magnetic field in stationary windings.