

Let us consider an energy band which is filled with electrons upto k_1 as shown in Fig. (6.10) (k_1 is less than π/a). As far as the affect of external field is concerned, one is interested in knowing the equivalence to 'free' electrons of the N electrons in the band. The number of free electrons in a band is given as

$$N_{eff} = \sum f_k \quad \dots (6.43)$$

where the summation extends over all the occupied states in the band for a one-dimensional crystal of length L , the number of states in the interval dk is given as,

$$dn = \frac{L}{2\pi} dk \quad \dots (6.44)$$

Because two electrons can occupy each of the states in the shaded region of Fig. 7, the above relation must be multiplied by 2. Hence,

$$N_{eff} = 2 \int dn f_k = \frac{L}{\pi} \int_{-k_1}^{k_1} f_k dk = \frac{2L}{\pi} \int_0^{k_1} f_k dk$$

Putting the value of f_k from eq. (6.42), we get

$$N_{eff} = \frac{2L}{\pi} \frac{m}{\hbar^2} \int_0^{k_1} \frac{d^2 E}{dk^2} dk$$

$$\therefore N_{eff} = \frac{2Lm}{\pi \hbar^2} \left(\frac{dE}{dk} \right)_{k=k_1} \quad \dots (6.45)$$

If the band is completely filled,

$$k = \pm \frac{n\pi}{a}; n = 1, 2, 3, \dots$$

$$\frac{dE}{dk} = 0$$

$$N_{eff} = 0$$

and hence

which means that number of free electrons in a completely filled band is zero.

The effective number of electrons reaches a maximum for a band filled upto $k = k_0$, the inflection point.

In case of insulators there are no effective free electrons. All the bands are completely full in the valence band and conduction band is completely empty and there is a large forbidden energy gap (of the order of 5 – 10 eV) between these two bands and it is impossible to excite any electron across this region Fig 6.11 (a).

All the bands are then either completely filled or empty at any temperature so the external electric field can not produce any current. So, the conductivity of such materials under ordinary conditions is zero and are called insulators. A representative example of an insulator is diamond

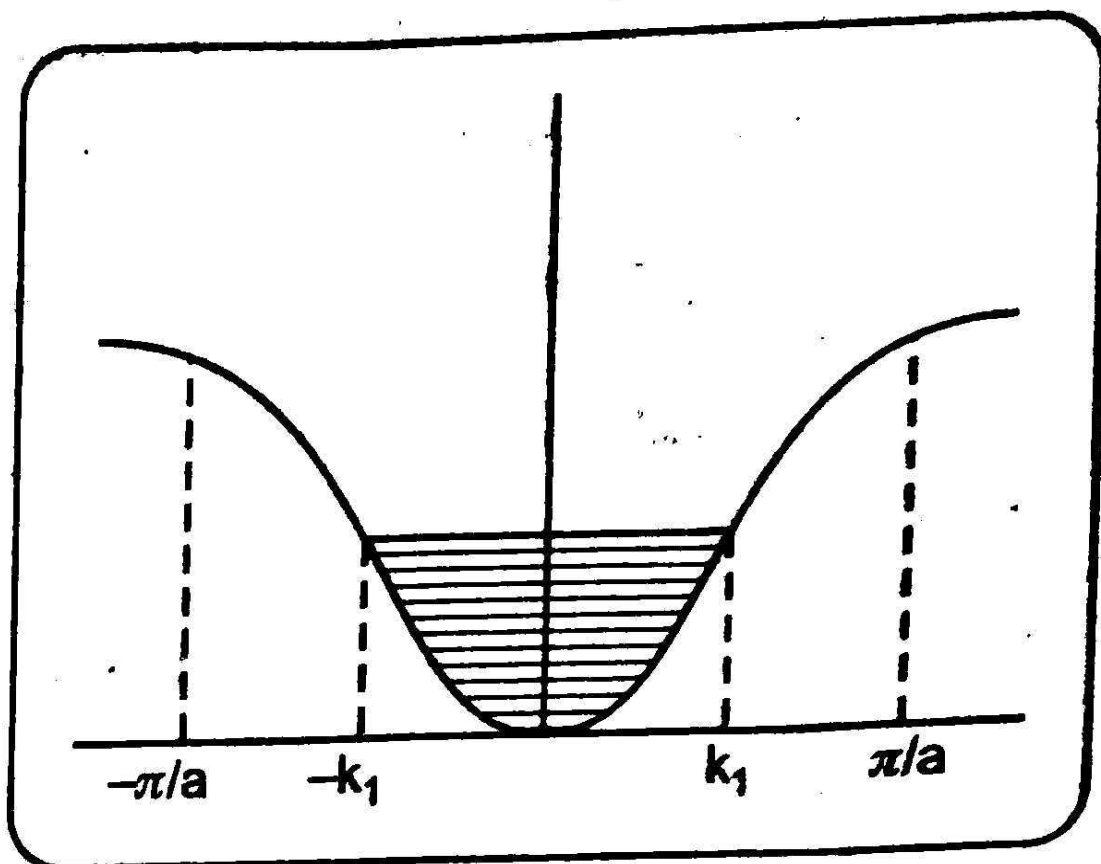


Fig. 6.10 : Energy band filled up to states k_1 at $T = 0$.

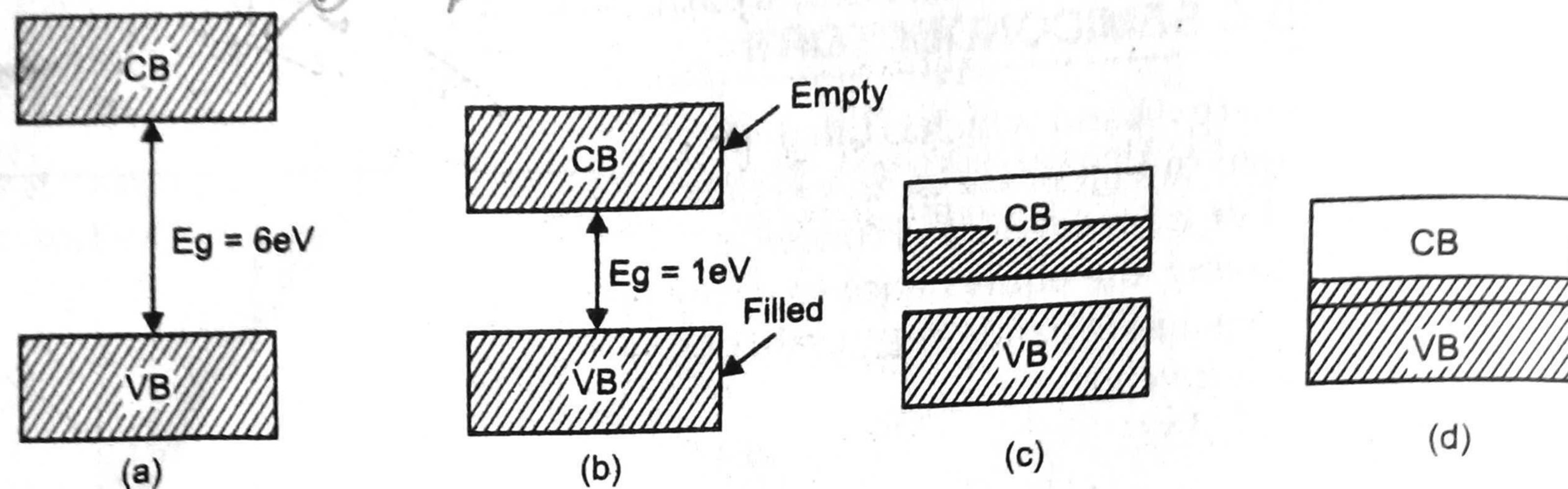


Fig. 6.11 : Energy band model for insulator, semiconductor and metal.

where the covalent bond splits $2s$ and $2p$ levels into two bands separated by an energy gap of $\Delta E_g = 7 \text{ eV}$; with the valence electrons filling the lower band.

When the forbidden gap is small, say of the order of 1 eV or less Fig 6.11 (b), so that electrons could be excited thermally from states near the top of the filled band to states near the bottom of the next empty band across the band gap. Hence a limited number of electrons are available for conduction in the conduction band which is almost empty, on applying an electric field. The state near top of filled band also contributes to the flow of electric current through the mechanism of hole conduction. A material of this type is called semiconductor. At low temperature (0K) the valence band is completely filled and the conduction band is completely empty. So, a semiconductor virtually behaves as an insulator at low temperature. Even at room temperature some electrons (about one electron for 10^{14} atoms) cross the conduction and impart little conductivity to the semiconductor. As the temperature is increased, more and more electrons cross over to the conduction band and the conductivity increases. Examples of semiconductors are Germanium (band gap 0.78 eV) and silicon (band gap 1.21 eV).

The energy band structure in solids have two possibilities. (i) A solid is a conductor if either its conduction band is not completely filled and the valence band may be completely filled and there is extremely small energy gap between them as shown in Fig. 6.11 (c) e.g. Li, Na, K etc. (ii) The valence band is completely filled and the empty conduction band overlap with valence band Fig. 6.11 (d) (eg; Ba, Cd, Zn etc.) so the energy gap is zero. The electron in the valence band are free to move inside the crystal lattice. The electrons under the influence of small applied field acquire additional energy and move to higher energy state. These mobile electrons constitute current.