MODULE 2

ELECTRICAL PROPERTIES OF MATERIALS

SYLLABUS

Electrical properties of materials: Free-electron concept (Drift velocity, Thermal velocity, Mean collision time, Mean free path, relaxation time). Failure of classical free electron theory. Quantum free electron theory, Assumptions, Fermi factor, density of states (qualitative only), Fermi-Dirac statistics. Expression for electrical conductivity based on quantum free electron theory, Merits of quantum free electron theory.

Conductivity of Semi conducting materials, Concentration of electrons and holes in intrinsic semiconductors, law of mass action. Fermi level in an intrinsic Semiconductor. Hall effect, Hall coefficient.

Temperature dependence of resistivity in metals and superconducting materials. Effect of magnetic field (Meissner effect). Type-I and Type-II superconductors-Temperature dependence of critical field. BCS theory (qualitative). High temperature superconductors. Applications of superconductors- Maglev vehicles.

SYNOPSIS

Students should get the idea on realization of Materials of conductors or semiconductors, how can be used as variety electrical components. This chapter designed to provide basics of Free electron theory, failures of classical theory including Quantum theory assumptions, density of states including Fermi-Dirac characteristics, electron conduction principles in semiconductors, drift-diffusion currents, resistivity, temperature dependency on resistivity, introduction to BCS theory and super conducting phenomena - theory.

QUESTIONS & ANSWERS

1. Give the assumptions of the classical free electron theory.

OR

Bring out the salient features of Drude-Lorentz theory.

(Jan-2009, Feb-2003, Feb-2005, Jan-2004)

Ans. The main assumptions of classical free electron theory are:

1. A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons

2. The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by 3/2 kT, where k is a Boltzmann constant.

   It is related to the kinetic energy,

   $3/2\ kT = ½\ mv_{th}^2$

   Where v_{th} is the thermal velocity same as root mean square velocity.

3. The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant.

4. The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

5. The free electrons are non-interacting and obey pauli's exclusion principle.

6. The motion of free electrons obeys the classical Maxwell-Boltzmann velocity distribution law and laws of kinetic theory of gases.

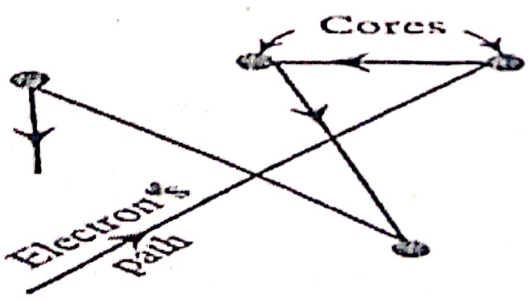
2. Explain the terms: a) Drift velocity b) Relaxation time c) Mean free path d) Mean collision time for free electrons.

(July-2000,July-2008,Feb-2002,Jan-2004,July-2004,Jan-2006)

Ans. Drift velocity (v_d): The average velocity of occupied by the electrons in the steady state in an applied electric field is called drift velocity.

$$\boxed{\text{The drift velocity } V_d = \frac{eE\tau}{m}}$$

Thermal velocity(V_{th}):



The velocity of electrons in random motion due to thermal agitationcalled thermal velocity.

Mean free path (λ):

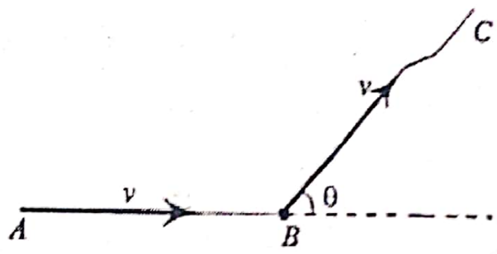
The average distance travelled by the conduction electrons between any two successive collisions with lattice ions.

Relaxation time (τ_r):

From the instant of sudden disappearance of an electric field across a metal, the average velocity of the conduction electrons decays exponentially to zero, and the time required in this process for the average velocity to reduce to (1/e) times its value is known as Relaxation time. The relaxation time τ_r and mean collision time τ are related as,

$$\tau_r = \frac{\tau}{1 - <\cos\theta>}$$

For isotropic scattering or symmetrical scattering $<\cos\theta> = 0$ Then $\tau_r = \tau$



LATTICE SCATTERING

Mean collision time (τ): The average time that elapses between two consecutive collisions of an electron with the lattice points is called mean collision time.

$$\tau = \lambda/v$$

where 'λ' is the mean free path, $v \approx v_{th}$ is velocity same as combined effect of thermal & drift velocities.

3. Derive the expression for drift velocity.

Ans. Consider a conductor subjected to an electric field. In the steady state, conduction electrons move with constant velocity. If 'm' is the mass of the electron, 'v' is the velocity same as the drift velocity 'v_d', 'τ' is the mean collision time, then the resistance force 'F_r' offered to its motion is given by,

$$F_r = \frac{mv_d}{\tau}$$

If 'e' is the charge on an electron, 'E' is the electric field, the driving force on the electron is

$$F = eE$$

In the steady state F = Fr

$$\text{i.e.} \quad \frac{mv_d}{\tau} = eE$$

$$\boxed{\text{The drift velocity } v_d = \frac{eE}{m}\tau}$$

4. Discuss the failure of classical free electron theory.

(Jan-2013, Jan-2009, Jan-2008, July-2008, July-2011)

Ans. Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

(i) Specific heat: The molar specific heat of a gas at constant volume is

$$\boxed{C_V = \frac{3}{2}R}$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was $\boxed{C_V = 10^{-4}RT}$

Which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

(ii) Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T

i.e. $$\sigma_{exp} \; \alpha \frac{1}{T} \qquad \text{—— (1)}$$

According to the assumptions of classical free electron theory,

$$\frac{3}{2}KT = \frac{1}{2}mv^2_{th}$$

$$V_{th} = \sqrt{\frac{3KT}{m}}$$

i.e. $$V_{th} \; \alpha \sqrt{T}$$

The mean collision time 'τ' is inversely proportional to the thermal velocity

i.e. $$\tau \; \alpha \frac{1}{V_{th}} \quad \text{or} \quad \tau \; \alpha \frac{1}{\sqrt{T}}$$

But $$\sigma = \frac{ne^2\tau}{m}$$

$$\sigma \; \alpha \; \tau$$

or $$\sigma \; \alpha \frac{1}{\sqrt{T}} \qquad \text{—— (2)}$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

5. State the assumptions of quantum free electron theory. **(July-2003)**

Ans. 1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.

2. The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.

3. The electrons travel with a constant potential inside the metal but confined within its boundaries.

4. The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

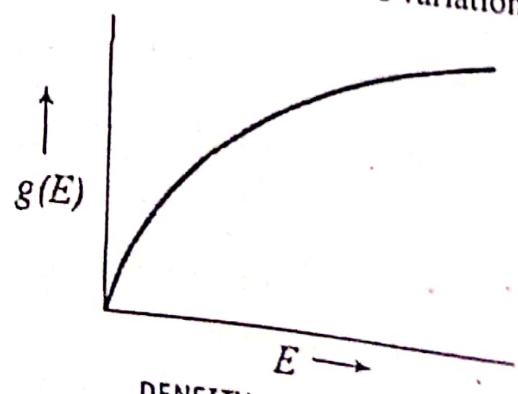
5. With the increase of temperature the energy levels below E_F are vacated and above E_F are occupied.

6. The distribution electrons among the different energy levels at any temperature is given by Fermi-Dirac distribution function f(E). It is defined as $f(E) = \dfrac{1}{1+\exp\left(\dfrac{E-E_F}{KT}\right)}$.

7. Electrons are treated as wave-like particles.

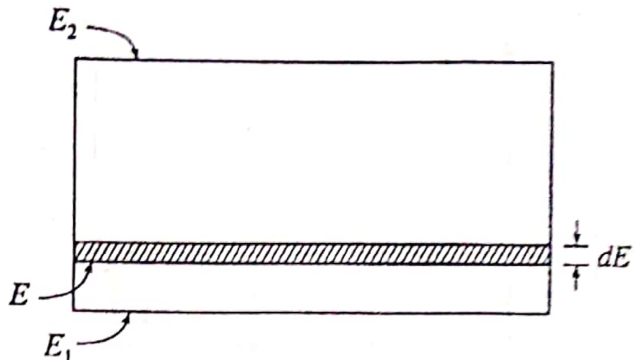
6. Explain density of states. **(Jan-2007,Jan-2006,July-2004,Jan-2003)**

Ans. There are large numbers of allowed energy levels for electrons in solid materials. A group of energy levels close to each other is called as energy band. Each energy band is spread over a few electron-volt energy ranges. In 1mm³ volume of the material, there will be a more than a thousand billion permitted energy levels in an energy range of few electron-volts. Because of this, the energy values appear to be virtually continuous over a band spread. To represent it technically it is stated as density of energy levels. The dependence of density of energy levels on the energy is denoted by g(E). The graph shows variation of g(E) versus E.



DENSITY OF STATES

It is called density of states function. It is the number of allowed energy levels per unit energy interval in the band associated with material of unit volume. In an energy band as E changes g(E) also changes.



ENERGY INTERVAL IN AN ENERGY BAND

Consider an energy band spread in an energy interval between E_1 and E_2. Below E_1 and above E_2 there are energy gaps. g(E) represents the density of states at E. As dE is small, it is assumed that g(E) is constant between E and E+dE. The density of states in range E and (E+dE) is denoted by g(E)dE.

i.e. $$g(E)dE = \left[\frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3}\right] E^{\frac{1}{2}}\, dE$$

It is clear g(E) is proportional to $\sqrt{E}$ in the interval dE.

7. Explain fermi energy and fermi factor. Discuss the variation of fermi factor with temperature and energy. (Jan-2007,July-2007,Jan-2010,Jan-2011,June-2012)

Ans. Fermi energy: In a metal having N atoms, there are N allowed energy levels in each band. In the energy band the energy levels are separated by energy differences. It is characteristic of the material. According to Pauli's exclusion principle, each energy level can accommodate a maximum of two electrons, one with spin up and the other with spin down. The filling up of energy levels occurs from the lowest level. The next pair of electrons occupies the next energy level and so on till all the electrons in the metal are accommodated. Still number of allowed energy levels, are left vacant. This is the picture when there is no external energy supply for the electrons. The energy of the highest occupied level at absolute zero temperature (0K) is called the Fermi energy and the energy level is called Fermi level. It is denoted by 'E_f'.

Fermi factor: The electrons in the energy levels for below Fermi level cannot absorb the energy above absolute zero temperature. At ordinary temperature because there are no vacant energy levels above Fermi level into which electrons could get into after absorbing the thermal energy. Though the excitations are random, the distributions of electrons in various energy levels will be systematically governed by a statistical function at the steady state.

The probability f(E) that a given energy state with energy E is occupied at a steady temperature

T is given by, $f(E) = \dfrac{1}{e^{\frac{E-E_F}{kT}}+1}$ F(E) is called the fermi factor.