interval.

Density of States (3D-case) :- Mathematically, density of states is represented by D(E) where

$$D(E) = \frac{dN}{dE}$$

where dN represents the number of energy states lying between the energy interval from E to E+dE. Since each energy level can accomodate two electrons (with opposite spin in accordance with Pauli Exclusion Principle), the actual density of states will be twice the above value, ie.

$$\boxed{D(E) = 2\frac{dN}{dE}} \quad -(i)$$

Now the energy of a free electron inside a metal (3D) is known to be

$$E = \frac{h^2}{8mL^2}(n_x^2+n_y^2+n_z^2)$$

The above expression can be rewritten as

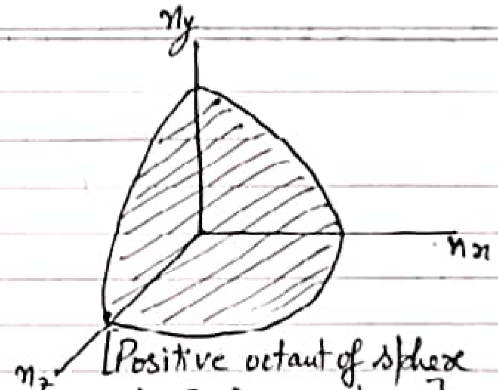
$$n_x^2+n_y^2+n_z^2 = \left(\sqrt{\frac{8mL^2E}{h^2}}\right)^2$$

The above equation represents the equation of a sphere of radius given by

$$\sqrt{\frac{8mL^2E}{h^2}}$$

in a 3-dimensional n-space with ⊥ axis n_x, n_y and n_z. [Note that eqn. of a sphere in 3-D actual space with ⊥ axes x, y, z is $x^2+y^2+z^2 = r^2$ where r is the radius of the sphere].

Note that every point in the n-space will have a fixed value of (n_x, n_y, n_z) and hence the different points in the n-space will represent different energy states of the electron inside the metal. Furthermore, ∵ the quantum numbers n_x, n_y, n_z can only have positive integral values, so the points corresponding to the allowed energy states for the electrons inside the metal can exist only in the positive octant of the sphere in n-space, i.e the number of states of energy less than E must be equal to the volume of the positive octant of the sphere in n-space.



[Positive octant of sphere in 3-D n-space].

ie $N = \frac{1}{8} \times \frac{4}{3}\pi R^3$

ie $N = \frac{1}{8} \times \frac{4}{3}\pi \left(\sqrt{\frac{8mL^2E}{h^2}}\right)^3$

ie $N = \frac{\pi}{6}\left(\frac{8mL^2E}{h^2}\right)^{3/2}$

ie $N = \frac{\pi}{6} L^3 \left(\frac{8mE}{h^2}\right)^{3/2}$

i.e. $\boxed{N = \frac{\pi}{6} V \left(\frac{8m}{h^2}\right)^{3/2} E^{3/2}}$ ($\because V = L^3$ for cubical metal lattice) —(ii)

→ Formula for No. of states with energy less than or equal to E

In terms of $\hbar$ we can rewrite the above expression as

$$N = \frac{V}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2} \quad \left(\because \hbar = h/2\pi\right)$$

Now, by definition, the density of states

$$D(E) = 2\frac{dN}{dE} = 2 \times \frac{V}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \times \frac{3}{2} E^{1/2}$$

i.e $\boxed{D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} = 4\pi V\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}}$ —(iii)

→ Formula for Density of states for electrons in a 3-D metal lattice.