Norton Analysis

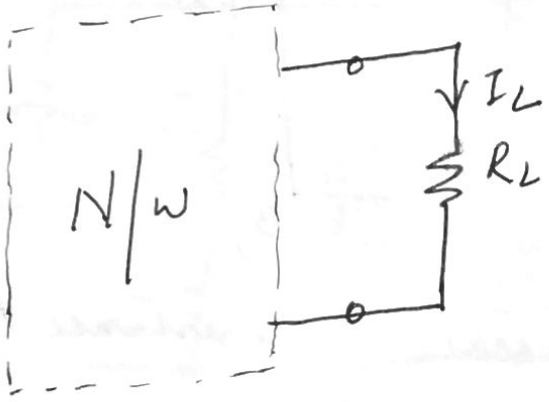
with

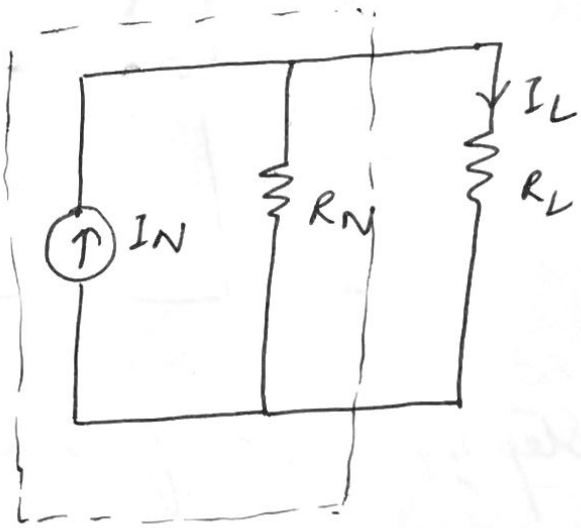
Practice Questions

NORTON'S THEOREM

It states that any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.

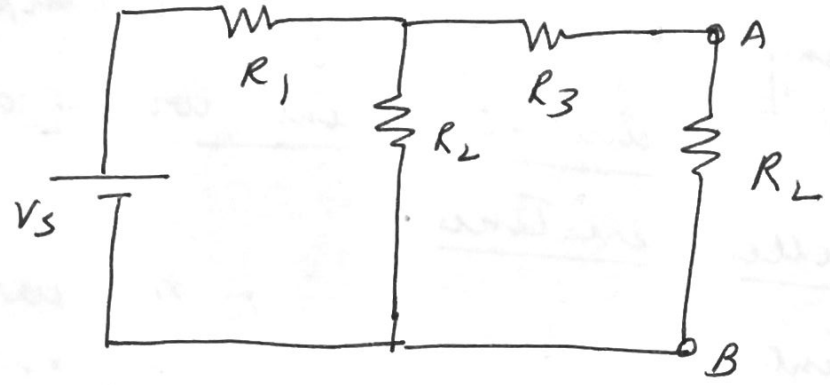
- The constant current is equal to the current which would flow in a short circuit placed across the terminals.
- The parallel resistance is the resistance of the N/w when viewed from these open ckt terminals.





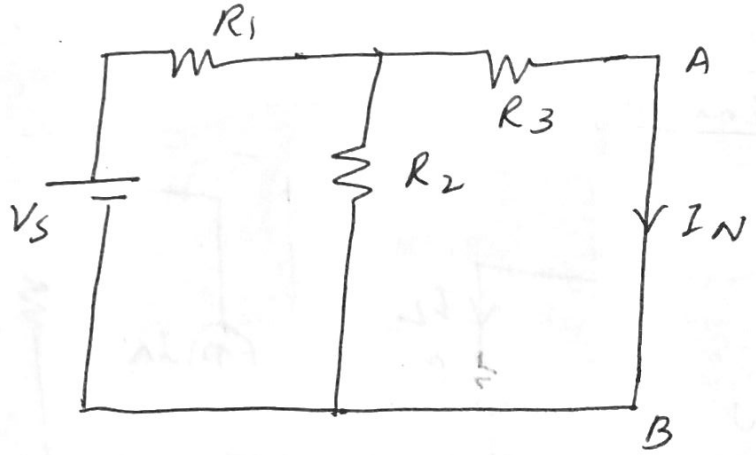
Norton's N/W

EXPLANATION

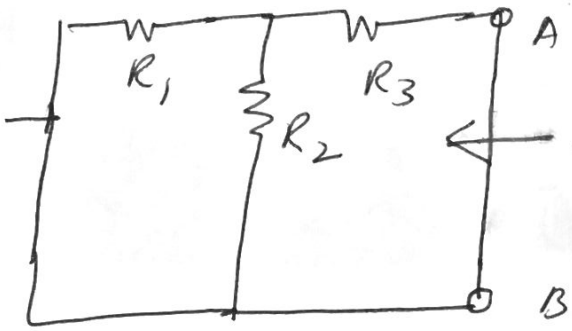


⇒ Find the current through R_L

Step I:- Remove the load resistor R_L from the N/W and calculate short ckt current I_{SC}



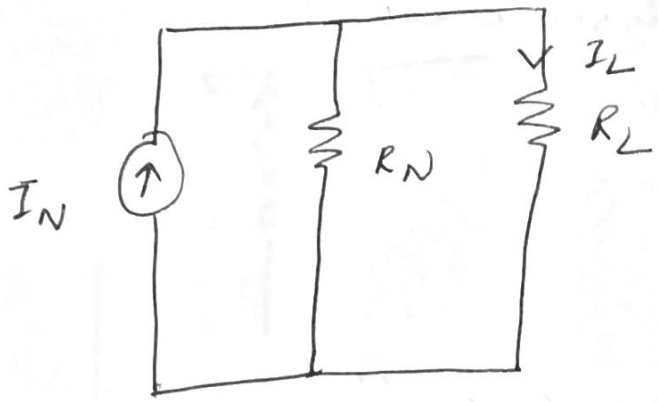
Step II:- For finding parallel resistance R_N, replace voltage source by short ckt



[For RN A-B terminal will be open]

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

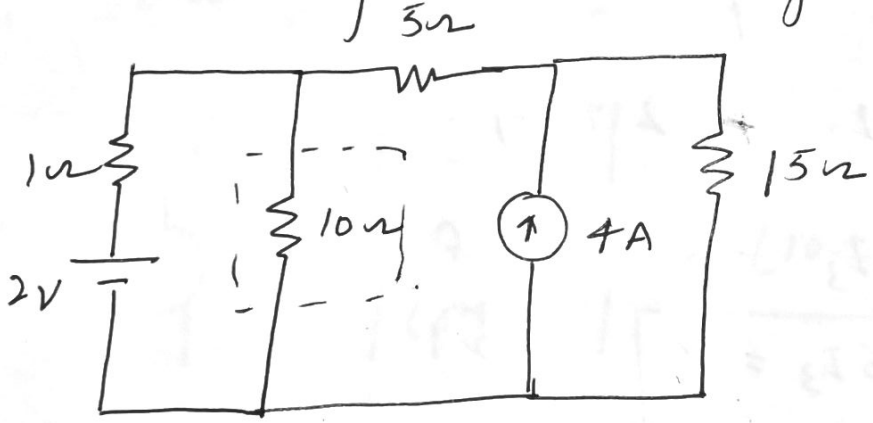
Step 3:- Draw norton's equivalent ckt.



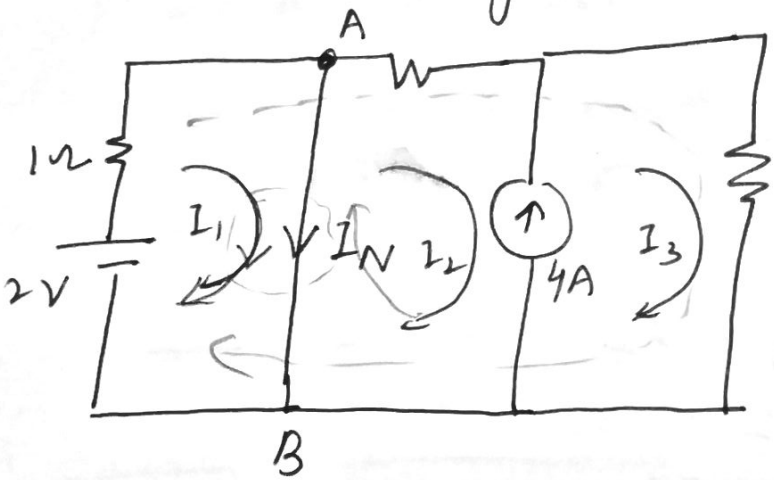
[acc to current divider rule]

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

Numerical 1 :- Find the current through 10Ω resistor using norton's analysis.



Step 1:- Short ckt the given resistor.



Using Mesh analysis in loop 1.

$$1I_1 - 2 = 0$$

$$\boxed{I_1 = 2A}$$

From the ckt it is clear that

$$\boxed{I_N = I_1 - I_2}$$

&

$$\boxed{4 = I_3 - I_2} \quad -(1)$$

Using KVL to supermesh

$$5I_2 + 15I_3 + 2 + I_1 = 0$$

$$5I_2 + 15I_3 - 2 + 2 = 0$$

$$\boxed{5I_2 + 15I_3 = 0} \quad -(2)$$

After solving (1) & (2)

$$I_1 = 2\,A$$

$$I_2 = -3\,A$$

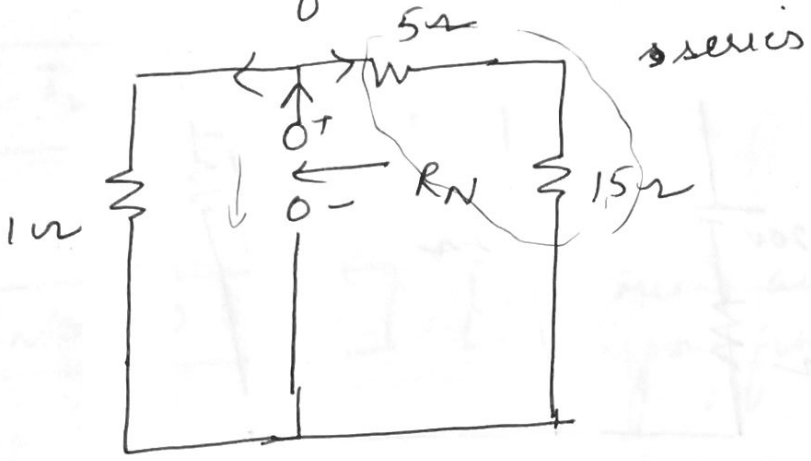
$$I_3 = 1\,A$$

$$\therefore I_N = I_1 - I_2$$

$$= 2 - (-3)$$

$$= 5A$$

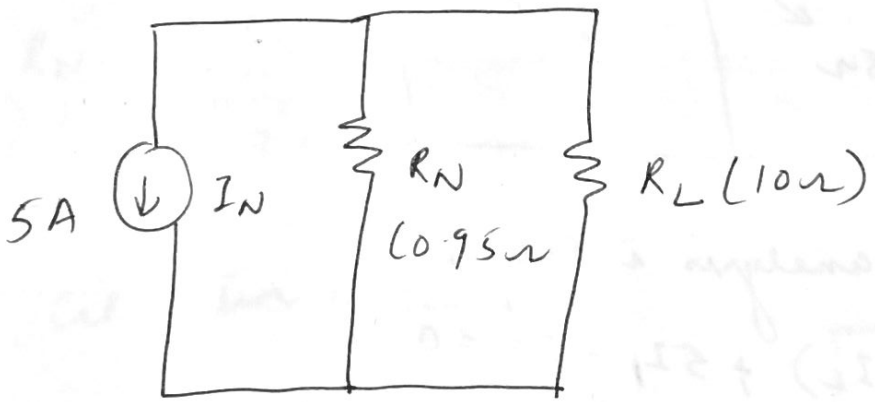
Step II :- Calculation of R_{TH}



$$R_N = 1 \parallel (5+15)$$

$$\boxed{R_N = 0.95\Omega}$$

Step III :- Draw norton equivalent ckt

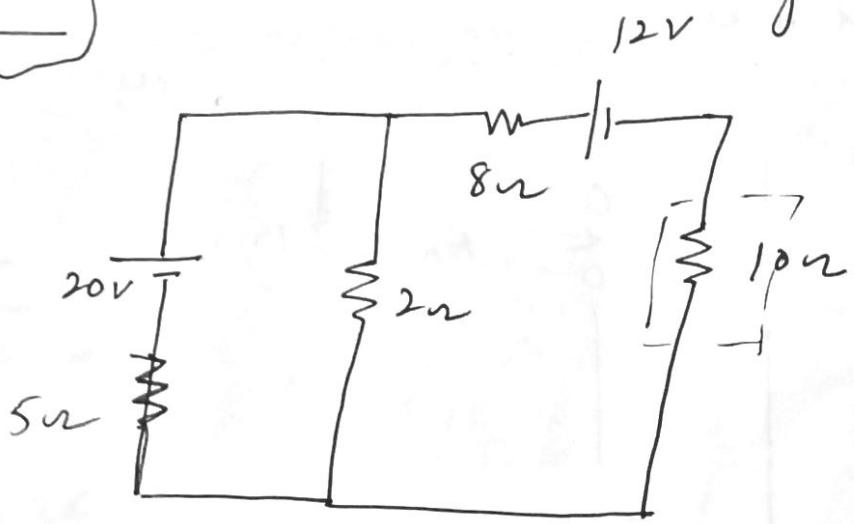


$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

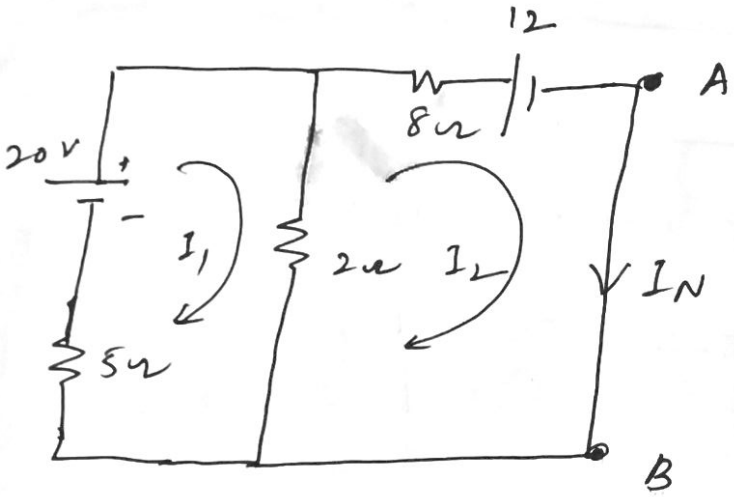
$$= \frac{0.95}{0.95+10} \times 5$$

$$\boxed{I_L = 0.43\ A}$$

Numerical-2 Find the current through 10Ω resi



Solution :- Step 1 :- Calculate I_N



Using Mesh analysis in loop 1.

$$2(I_1 - I_2) + 5I_1 - 20 = 0$$

$$2I_1 - 2I_2 + 5I_1 - 20 = 0$$

$$\boxed{7I_1 - 2I_2 = 20} \quad -(1)$$

$$8I_2 + 12 - 2(I_1 - I_2) = 0$$

$$8I_2 + 12 - 2I_1 + 2I_2 = 0$$

$$\boxed{10I_2 - 2I_1 = -12} \quad -(2)$$

After solving (1) & (2)

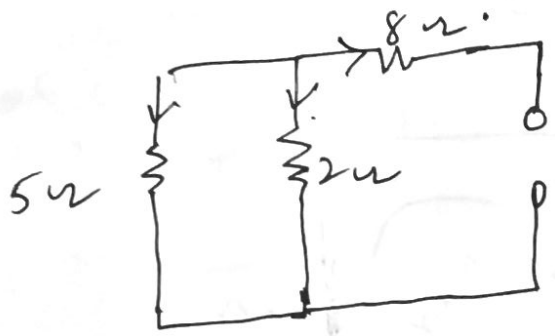
$$\boxed{I_2 = -0.67\ A}$$

Now $I_2 = I_N$

$$\therefore \boxed{I_N = -0.67\ A}$$

[this means actually current is going from B to A]

Step III :- Calculate R_N



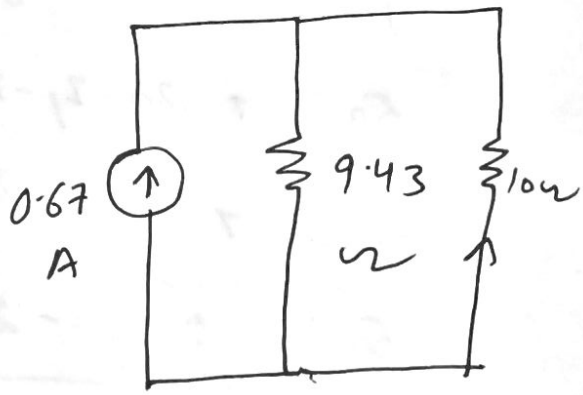
$$R_N = (5 \| 2) + 8$$

$$= 9.43$$

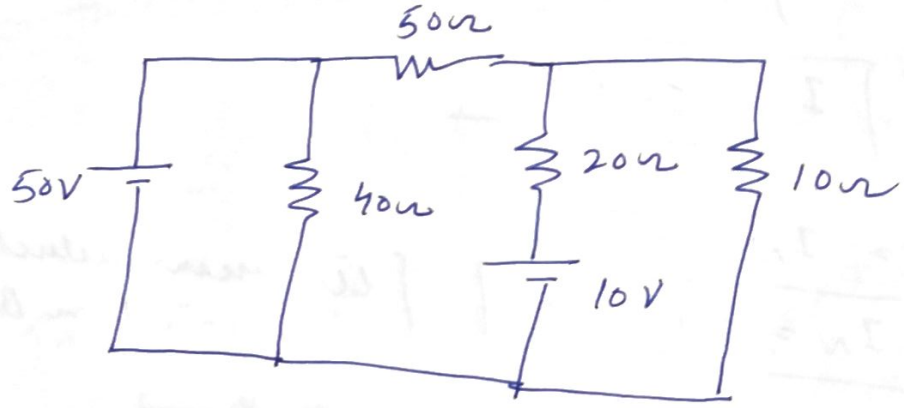
Step IV :- Calculation of I_L

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10}$$

$$= 0.33\ A\ (\uparrow)$$

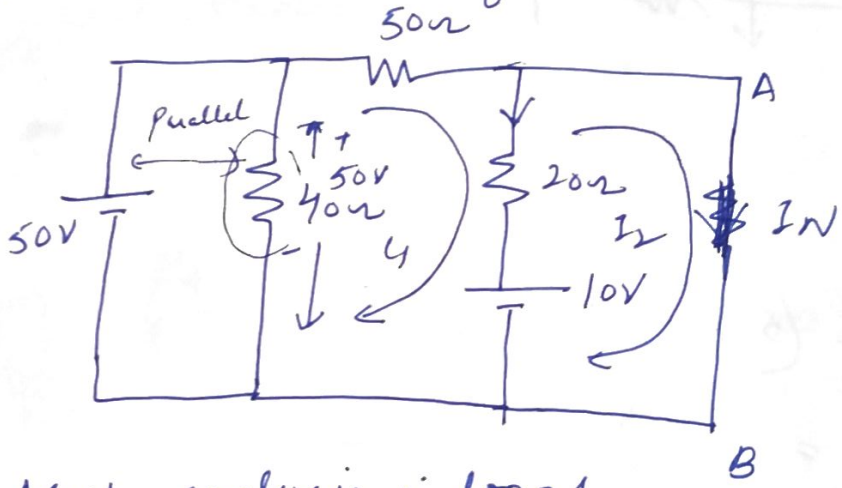


Numerical-3 Find the current in the 10Ω resistor



Solution :-

Step 1 :- Calculation of I_N



Using Mesh analysis in loop 1.

$$50I_1 + 20(I_1 - I_2) + 10 - 50 = 0$$

$$50I_1 + 20I_1 - 20I_2 - 40 = 0$$

$$50I_1 + 20I_1 - 20I_2 = 40$$

$$\boxed{70I_1 - 20I_2 = 40} \quad -(1)$$

Applying Mesh analysis in loop 2.

$$-10 - 20(I_1 - I_2) = 0$$

$$\boxed{-20I_1 + 20I_2 = 10} \quad -(2)$$

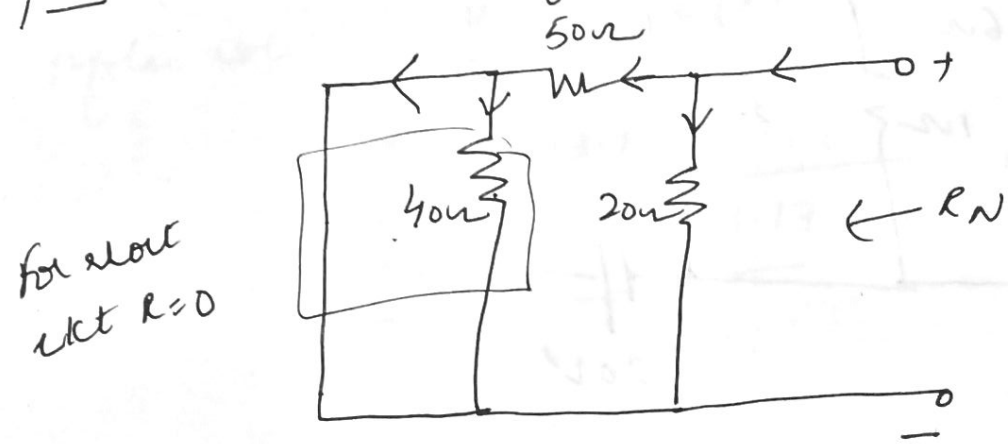
After solving = n (1) & (2)

$I_1 = 1\ A$

$I_2 = 1.5\ A$

$$\boxed{I_N = I_2 = 1.5\ A}$$

Step II :- Calculation of R_N

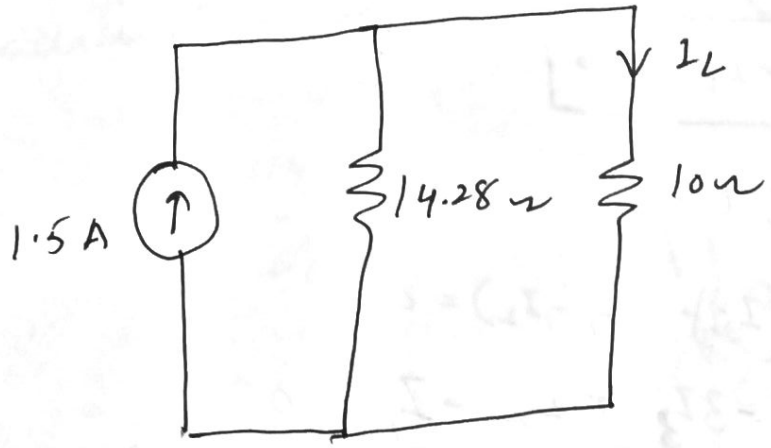


$R_N = 40 \| 0 = 0$

$\therefore = 50 \| 20$

$$\boxed{R_N = 14.28\ \Omega}$$

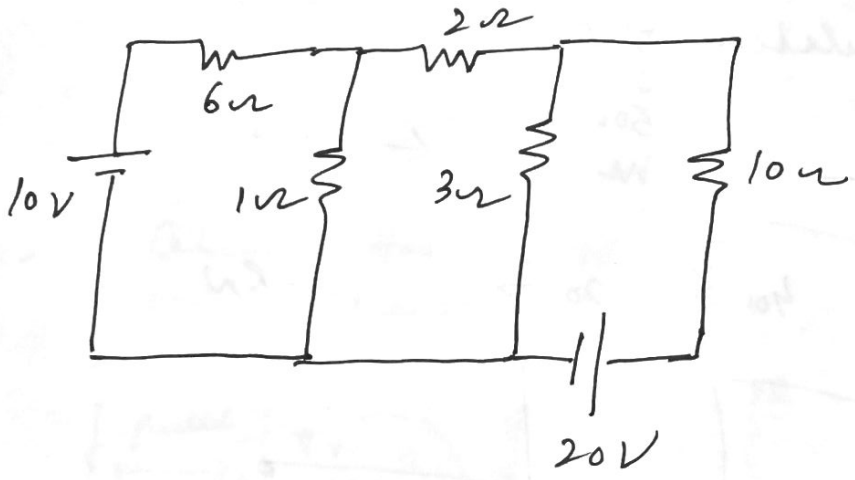
Step III :- Calculation of I_L



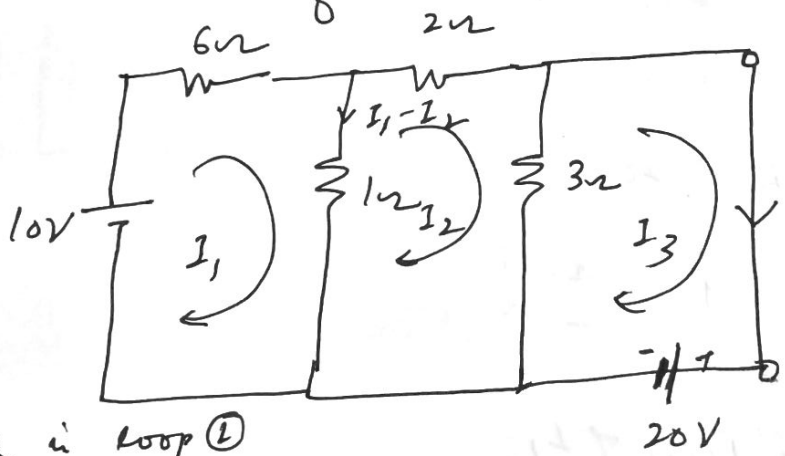
$$I_L = 1.5 \times \frac{14.28}{14.28 + 10}$$

$$= 0.88\ A$$

Numerical-4 :- Find the current the 10Ω resistor



Step-I :- Calculation of I_N



Applying KVL in loop ①

$$6I_1 + I_1 - I_2 - 10 = 0$$

$$\boxed{7I_1 - I_2 = 10} \quad \text{①}$$

Applying KVL in loop ②

$$2I_2 + 3(I_2 - I_3) + (I_1 - I_2) = 0$$

$$5I_2 - 3I_3 + I_2 - I_1 = 0$$

$$\boxed{6I_2 - I_1 - 3I_3 = 0} \quad \text{②}$$

Applying KVL in Mesh (3)

(6)

$$20 - 3(I_2 - I_3) = 0$$

$$-3I_2 + 3I_3 = -20$$

$$\boxed{3I_2 - 3I_3 = 20} \quad (3)$$

After solving eq (1), (2) & (3)

$$I_3 = -13.17\ A$$

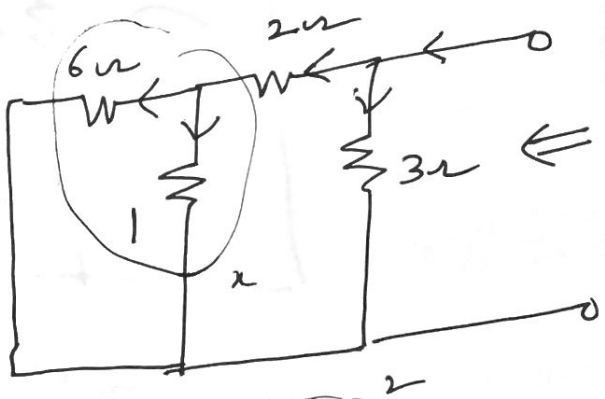
$$\boxed{I_N = I_3 = -13.17\ A}$$

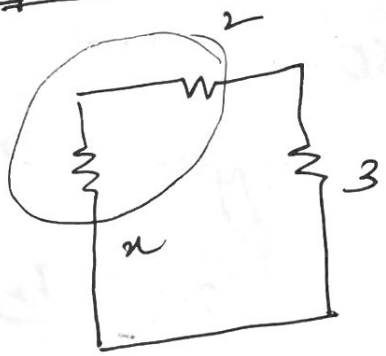
(-ve means current is going downward to upward)

Step II :- Calculation of R_N

$$R_N = [6 \| 1 + 2] \| 3$$

$$= 1.46\ \Omega$$

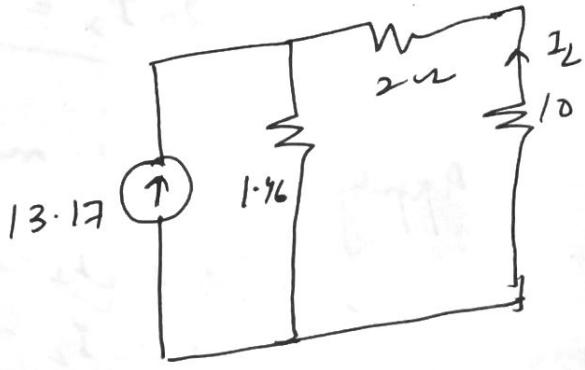




Step III :- Calculation of I_L

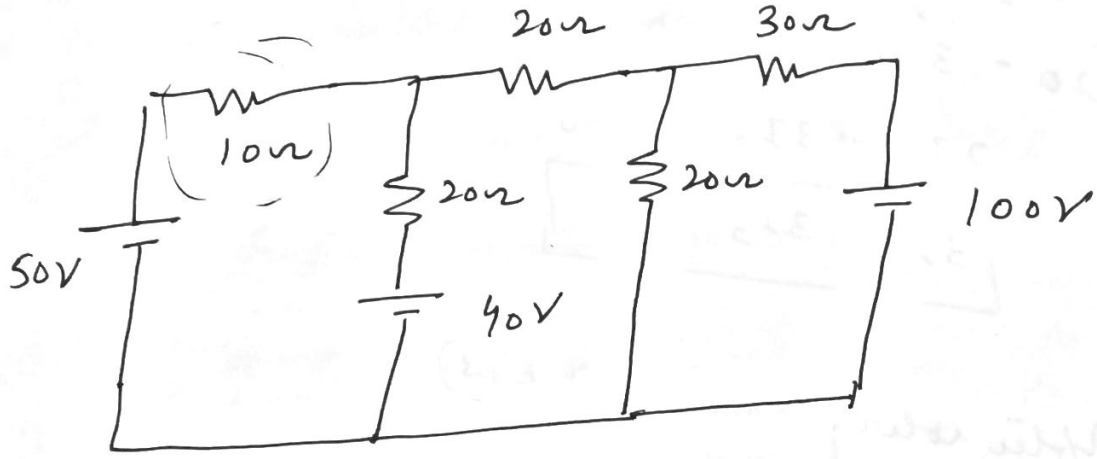
$$I_L = \frac{13.17 \times 1.46}{1.46 + 10}$$

$$= 1.68\ A$$

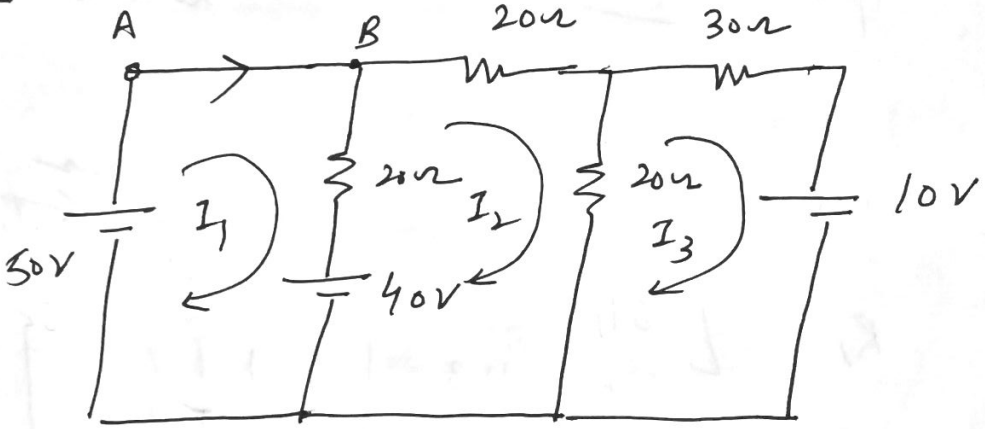


Numerical

(5) Find the current through the 10Ω resistor



Solution;-



Step I calculation of I_N

Applying KVL to mesh 1

$$20(I_1 - I_2) + 40 - 50 = 0$$

$$\boxed{2I_1 - 2I_2 = 1} \quad -(1)$$

Applying KVL to mesh (2)

$$2I_2 + 20(I_2 - I_3) - 40 - 20(I_1 - I_2) = 0$$

$$2I_2 + 2I_2 - 2I_3 - 4 - 2I_1 + 2I_2 = 0$$

$$\boxed{6I_2 - 2I_1 - 2I_3 = 4} \quad (2)$$

Applying KVL to Mesh 3

$$+20(I_2 - I_3) + 30I_3 + 100 = 0$$

$$-20I_2 + 20I_3 + 30I_3 = -100$$

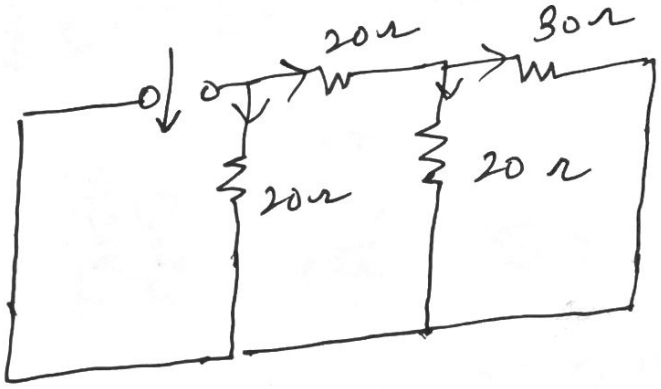
$$\boxed{50I_3 - 20I_2 = 100} \quad -(3)$$

After solving eq (1), (2) & (3)

$$I_1 = 0.81\ A$$

$$\boxed{I_N = I_1 = 0.81\ A}$$

Step II :- Calculation of R_N



$$R_N = \left[\left(20 \| 30\right) + 20\right] \| 20$$

$$= 12.3\ \Omega$$

Step III :- Calculation of I_L

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10}$$

$$= 0.45\ A$$