Solved Problem Solid State Physics

Solved Problems

1. *Find the temperature at which there is 1% probability that a state with energy 0.5 eV above Fermi energy.*

(Set-1, Set-3, Set-4–May 2007), (Set-1, Set-2, Set-3–Sept. 2006), (Set-2, Set-3–May 2006), (Set-1, Set-4–June 2005), (Set-1–May 2003)

Sol: Probability, f(E)= 1% = 1/100

$E - E_F = 0.5$ eV

$T = ?$

$$F(E) = \frac{1}{1 + \exp{(E - E_F)}/K_B T}$$

$$K_B = 1.381 \times 10^{-23} \text{ J/K} = 1.381 \times 10^{-23} \times 6.24 \times 10^{18} \text{ eV/K}$$

$$= 8.61744 \times 10^{-5} \text{ eV/K}$$

Substituting the values, we get:

$$\frac{1}{100} = \frac{1}{1 + \exp\left[\dfrac{0.5}{8.61744 \times 10^{-5} T}\right]}$$

$$100 = 1 + \exp\left[\frac{0.5}{8.61744 \times 10^{-5}\, T}\right]$$

$$100 = 1 + \exp\left[\frac{5801.87}{T}\right]$$

$$100 \approx \exp\left[\frac{5801.87}{T}\right]$$

Taking ln on both sides, we get:

$$\text{In } 100 = \frac{5801.87}{T}$$

$$T = \frac{5801.87}{4.605} = 1259.98\ \text{K}.$$

2. *Fermi energy of copper is 7* eV *at room temperature. What is the total number of free electrons/unit volume at the same temperature?*

(Set-2–May 2003)

Sol: Fermi energy, E_F = 7 eV = 7 × 1.602 × 10^{-19} J = 11.214 × 10^{-19} J

$$E_F = \left(\frac{h^2}{8m}\right)\left(\frac{3}{\pi}\right)^{2/3} n^{2/3}$$

$$11.214 \times 10^{-19} = \frac{\left[6.63 \times 10^{-34}\right]^2}{8 \times 9.11 \times 10^{-31}} \times \left[\frac{3 \times 7}{22}\right]^{3/2} n^{2/3}$$

$$11.214 \times 10^{-19} = \frac{43.9569 \times 10^{-68}}{72.88 \times 10^{-31}} \times 0.9326 \times n^{2/3}$$

$$n^{2/3} = \frac{11.214 \times 72.88}{43.9569 \times 0.9326 \times 10^{-18}} = 19.9364 \times 10^{18}$$

$$n = [19.9364 \times 10^{18}]^{3/2} \text{ electrons/m}^3 = 8.9106 \times 10^{28} \text{ electrons/m}^3$$

3. *Find the relaxation time of conduction electrons in a metal of resistivity 1.54 ×* 10^{-8} Ω-m, *if the metal has 5.8 ×* 10^{28} *conduction electrons/m3.*

(Set-3–Sept. 2007), (Set-2–May 2007), (Set-1–May 2006), (Set-4–Sept. 2006), (Set-1–Nov. 2004), (Set-2–May 2004), (Set-2–Nov. 2003), (Set-4–Nov. 2003)

Sol: Given data are:

Resistivity of the metal, $\rho = 1.54 \times 10^{-8}$ Ω–m

Number of conduction electrons, $n = 5.8 \times 10^{28}/\text{m}^3$

Relaxation time, τ = ?

$$\sigma = \frac{ne^2\tau}{m} \quad \text{or} \quad \tau = \frac{\sigma m}{ne^2} = \frac{m}{ne^2\rho}$$

$$= \frac{9.11 \times 10^{-31}}{5.8 \times 10^{28} \times \left[1.602 \times 10^{-19}\right]^2 \times 1.54 \times 10^{-8}}$$

$$= \frac{9.11 \times 10^{-31}}{5.8 \times (1.602)^2 \times 1.54 \times 10^{-18}} = \frac{9.11 \times 10^{-13}}{22.92}$$

$$= \frac{911 \times 10^{-15}}{22.92} = 39.747 \times 10^{-15} \text{ s}$$

4. *For the metal having 6.5×10^{28} conduction electrons/m^3. Find the relaxation time of conduction electrons if the metal has resistivity 1.43×10^{-8}* Ω-m.

(Set-4–Sept. 2008), (Set-3–Nov. 2003)

Sol: Number of conduction electrons, $n = 6.5 \times 10^{28}/m^3$

Resistivity of the metal, $\rho = 1.43 \times 10^{-8}$ Ω-m

Relaxation time, τ = ?

$$\sigma = \frac{ne^2\tau}{m} \quad \text{or} \quad \tau = \frac{\sigma m}{ne^2} = \frac{m}{ne^2\rho}$$

$$= \frac{9.1\times10^{-31}}{6.5\times10^{28}\times\left(1.6\times10^{-19}\right)^2\times1.43\times10^{-8}}\ \text{s} = 3.82\times10^{-14}\ \text{s}$$

5. *Calculate the free electron concentration, mobility and drift velocity of electrons in aluminium wire of length of 5* m *and resistance of 0.06* Ω *carrying a current of 15* A, *assuming that each aluminium atom contributes 3 free electrons for conduction.*

Given: [Resistivity for aluminium] = 2.7 × 10 $^{-8}$ Ω–m

[Atomic weight] = 26.98

[Density] = 2.7 × 10^{3} *Kg/m*^{3}

[Avagadro number] = 6.025 × 10 ^{23}

(Set-1, Set-2, Set-4–Sept. 2007), (Set-4–May 2006), (Set-2, Set-3–June 2005)

Sol: Given data are:

Aluminium wire length, L = 5 m

Resistance of wire, $R = 0.06\ \Omega$

Current in wire, $I = 15$ A

Number of conduction electrons of Al atom = 3

Resistivity of aluminium, $\rho = 2.7 \times 10^{-8}\ \Omega$–m

Atomic weight of aluminium, $w = 26.98$

Density of aluminium, $D = 2.7 \times 10^{3}$ Kg/m^3

Avogadro's Number, $N_A = 6.025 \times 10^{26}$ per k-mol

Free electron concentration, n = ?

Mobility of electrons, μ = ?

Drift velocity of electrons, v_d = ?

Number of conduction electrons per m^3, $n = \dfrac{\text{no. of electrons per atom} \times N_A \times D}{\text{atomic weight}}$

$$= \frac{3 \times 6.025 \times 10^{26} \times 2.7 \times 10^{3}}{26.98} = 1.8088 \times 10^{29}/\text{m}^3$$

We know $\rho = \frac{1}{ne\mu}$ or $\mu = \frac{1}{ne\rho}$

mobility, $\mu = \frac{1}{1.8088 \times 10^{29} \times 1.6 \times 10^{-19} \times 2.7 \times 10^{-8}} = 0.00128 \text{ m}^2/\text{VS}$

Drift velocity, $v_d = \left(\frac{eE}{m}\right) \times \tau$

$$E = \frac{V}{L} = \frac{IR}{L}$$

and $\sigma = \frac{ne^2\tau}{m}$ or $\tau = \frac{\sigma m}{ne^2} = \frac{m}{\rho ne^2}$

$$\therefore \; v_d = \frac{e}{m} \times \frac{IR}{L} \times \frac{m}{\rho ne^2} = \frac{IR}{L\rho ne}$$

$$= \frac{15 \times 0.06}{5 \times 2.7 \times 10^{-8} \times 1.8088 \times 10^{29} \times 1.6 \times 10^{-19}}$$

$$= \frac{0.9 \times 10^{-2}}{39.07} = 2.3 \times 10^{-4} \text{ m/s.}$$

6. *Calculate the mobility of the electrons in copper obeying classical laws. Given that the density of copper = 8.92 × 10^3 kg/m^3, Resistivity of copper = 1.73 × 10^{-8}* ohm-m, *atomic weight of copper = 63.5 and Avogadro's number = 6.02 × 10^{26}* per k-mol.

(Set-3–May 2008)

Sol: Density of copper, $D = 8.92 \times 10^3$ kg/m^3

Resistivity of copper, $\rho = 1.73 \times 10^{-8}$ ohm–m

Atomic weight of copper, $W = 63.5$

Avogadro number, $N_A = 6.02 \times 10^{26}$ per K-mol

Mobility μ = ?

Number of free electrons per m^3, $n = \dfrac{\text{No. of free electrons per atom} \times N_A \times D}{\text{Atomic weight}}$

$$n = \frac{1 \times 6.02 \times 10^{26} \times 8.92 \times 10^3}{63.5} \text{ per m}^3$$

$$= 8.456 \times 10^{28} \text{ per m}^3$$

$$\rho = \frac{1}{ne\mu} \qquad \text{where } \mu = \text{mobility}$$

$$\mu = \frac{1}{ne\rho} = \frac{1}{8.456 \times 10^{28} \times 1.6 \times 10^{-9} \times 1.73 \times 10^{-8}}$$

$$= 0.0427 \text{ m}^2/\text{Vs}.$$

7. Calculate the mobility of electrons in copper, considering that each atom contributes one electron for conduction. Resistivity of copper = 1.721 × 10 $^{-8}$ Ω-m, Atomic weight is 63.54, density of copper is 8.95 × 10 ^{3}kg/m^3 and Avogadro number is 6.025 × 10 ^{23}/mole.

Sol: Given data are:

Resistivity of copper, ρ = 1.721 × 10^{-8} Ω-m

Atomic weight of copper, W = 63.54

Density of copper, D = 8.95 × 10^3 kg/m^3

Avogadro's number, N_A = 6.025 × 10^{26} per K-mol

Number of free electrons per atom = 1

Mobility of conduction electrons of copper, μ = ?

Number of conduction electrons per m^3, $n = \dfrac{\text{no. of electrons per atom} \times N_A \times D}{\text{At.weight}}$

$$n = \frac{1 \times 6.025 \times 10^{26} \times 8.95 \times 10^{3}}{63.54} / \text{m}^3$$

$$= 8.487 \times 10^{28} / \text{m}^3$$

we know that $\rho = \dfrac{1}{ne\mu}$

or $\mu = \dfrac{1}{ne\rho} = \dfrac{1}{8.487 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.721 \times 10^{-8}}$

$$\frac{1}{23.37 \times 10} = 0.0428 \text{ m}^2/\text{Vs.}$$

8. *Find the relaxation time of conduction electrons in a metal contains 6.5 × 10* ^{28} *conduction electrons per*m^3*. The resistivity of the metal is 1.50 ×* 10^{-8} Ω–m.

Sol: Given data are:

Number of conduction electrons, $n = 6.5 \times 10^{28}/\text{m}^3$

Resistivity of the metal, $\rho = 1.50 \times 10^{-8}$ ohm–m

Relaxation time, τ = ?

we know that $\sigma = \dfrac{ne^2\tau}{m}$

$$\tau = \frac{\sigma m}{ne^2} = \frac{m}{ne^2 \rho}$$

$$= \frac{9.11 \times 10^{-31}}{6.5 \times 10^{28} \times \left[1.602 \times 10^{-19}\right]^2 \times 1.50 \times 10^{-8}}$$

$$= \frac{9.11 \times 10^{-31}}{25.022 \times 10^{-18}} = 0.364 \times 10^{-13} \text{ s}$$

$$= 3.64 \times 10^{-14} \text{s}$$

9. *A uniform silver wire has a resistivity of 1.54 × 10*$^{-8}$ Ω–m *at a temperature 300* K. *For an electric field along the wire of 1* V/cm. *Calculate:*

1. *the drift velocity*
2. *the mobility and relaxation time of electrons assuming that there are 5.8 × 10* ^{28} *conduction electrons per m*^{3}*of the material and*
3. *calculate the thermal velocity of conduction electrons.*

Sol: Given data are:

Resistivity of silver wire, $\rho = 1.54 \times 10^{-8}$ Ω–m

Electric field, E = 1 V/cm = 10^2 V/m

Number of electrons per unit volume, n = 5.8×10^{28} / m^3

Relaxation time, τ = ?

Drift velocity, v_d = ?

Mobility of conduction electrons, n = ?

$$\sigma = \frac{ne^2\tau}{m} \quad \text{or} \quad \tau = \frac{\sigma m}{ne^2} = \frac{m}{ne^2\rho}$$

Relaxation time, $\tau = \dfrac{9.11 \times 10^{-31}}{5.8 \times 10^{28} \times [1.602 \times 10^{-19}]^2 \times 1.54 \times 10^{-8}}$

$$= \frac{9.11 \times 10^{-31}}{22.93 \times 10^{-18}} = 3.97 \times 10^{-14}\,\text{s}$$

Drift velocity, $v_d = \left(\dfrac{eE}{m}\right) \times \tau$

$$= \left[\frac{1.602 \times 10^{-19} \times 10^2}{9.11 \times 10^{-31}}\right] \times 3.97 \times 10^{-14} = 0.7\,\text{m/s}$$

Mobility, $\mu = \dfrac{v_d}{E} = \dfrac{0.7}{10^2} = 0.7 \times 10^{-2}\ \text{m}^2/\text{Vs}$

$$\frac{3}{2} K_B T = \frac{1}{2} m v_{th}^2$$

so, thermal velocity, $V_{th} = \sqrt{\dfrac{3K_BT}{m}} = \sqrt{\dfrac{3 \times 1.381 \times 10^{-23} \times 300}{9.11 \times 10^{-31}}}$

$$= 1.17 \times 10^5\,\text{m/s}$$

10. The Fermi energy of silver is 5.5 eV, *and the relaxation time of electrons is 3.97 × 10* $^{-14}$ s. *Calculate the Fermi velocity and the mean free path for the electrons in silver.*

Sol: The given data are:

The Fermi energy of silver, E_F = 5.5 eV = 5.5 × 1.602 × 10–19 J

The relaxation time of electrons in silver, $\tau = 3.97 \times 10^{-14}$ S

Fermi velocity, V_F = ?

Mean free path, λ = ?

We know that $\frac{1}{2}mv_F^2 = E_F$

or $V_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 5.5 \times 1.602 \times 10^{-19}}{9.11 \times 10^{-31}}}$

1.39×10^6 m/s

The mean free path, $\lambda = V_F\tau$

$= 1.39 \times 10^6 \times 3.97 \times 10^{-14}$

$= 5.52 \times 10^{-8}$ m

11. Calculate the Fermi energy in eV *for silver at 0* K, *given that the density of silver is 10500* kg/m^3, *its atomic weight is 107.9 and it has one conduction electron per atom.*

Sol: The given data are:

Density of silver, D = 10500 kg/m^3

Atomic weight of silver, M = 107.9

Number of free electrons per atom = 1

$$\text{Electronic concentration, } n = \frac{\text{number of free electrons per atom} \times N_A \times D}{M}$$

$$= \frac{1 \times 6.025 \times 10^{26} \times 10500}{107.9} = 5.863 \times 10^{28} \text{ per m}^3$$

$$\text{Fermi energy, } E_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\right)^{2/3} n^{2/3}$$

$$= \frac{\left[6.63 \times 10^{-34}\right]^2}{8 \times 9.11 \times 10^{-31}} \times \left(\frac{3 \times 7}{22}\right)^{2/3} \times \left(5.863 \times 10^{28}\right)^{2/3}$$

$$= 5.85 \times 10^{-38} \times 1.5091 \times 10^{19} = 8.83 \times 10^{-19}\,\text{J}$$

12. *Find the drift velocity of free electrons in a copper wire of cross sectional area 10* mm^2. *When the wire carries a current of 100 A. Assume that each copper atom contributes one electron to the electron gas.* [*Density of copper = 8.92 × 10* ^{3} kg/m3, *Atomic weight of copper = 63.5 and Avogadro's number = 6.02 × 10* ^{26} per K-mol]

Sol: Area of cross section of wire, A = 10 mm^2

= 10 × 10^{-6} m^2

Current through the wire, I = 100 amperes

Number of free electrons per atom = 1

Density of copper, $D = 8.92 \times 10^3$ kg/m^3

Atomic weight of copper, $W = 63.5$

Avogadro's number, $N_A = 6.02 \times 10^{26}$ per K-mol

Drift velocity of free electron, $v_d = ?$

Current density, $J = \frac{I}{A} = \frac{100}{10 \times 10^{-6}} = 10^7 \text{ Amp/m}^2$

But $J = nev_d$ where n = free electron concentration

No. of free electrons per m^3, $n = \frac{\text{No. of electrons per atom} \times N_A \times D}{\text{Atomic weight}}$

$$n = \frac{1 \times 6.02 \times 10^{26} \times 8.92 \times 10^3}{63.5} \text{ per m}^3$$

$$v_d = \frac{J}{ne} = \frac{10^7 \times 63.5}{6.02 \times 10^{26} \times 8.92 \times 10^3 \times 1.6 \times 10^{-19}}$$

$$= 0.7391 \times 10^{-3} \text{ m/s}$$

1. *Find the resistivity of an intrinsic semiconductor with intrinsic concentration of 2.5×10^{19}* per m^3. *The mobilities of electrons and holes are 0.40* m^2/ V-s *and 0.20* m^2/ V-s.

Sol: Given data are:

Intrinsic concentration (n_i) = $2.5 \times 10^{19}/\text{m}^3$

Mobility of electrons (μ_n) = 0.40 m^2/V-s

The mobility of holes (μ_p) = 0.20 m^2/V-s

The conductivity of an intrinsic semiconductor (σ_i) = $n_i e[\mu_n + \mu_p]$

$$\text{The resistivity } (\rho_i) = \frac{1}{\sigma_i} = \frac{1}{n_i e\left[\mu_n + \mu_p\right]}$$

$$= \frac{1}{2.5 \times 10^{19} \times 1.6 \times 10^{-19}\left[0.40 + 0.20\right]}$$

$$= \frac{1}{2.5 \times 1.6 \times 0.6} = 0.4166\ \Omega\text{-m.}$$

2. *Calculate the number of donor atoms per* m^3 *of n-type material having resistivity of 0.25* Ω-m, *the mobility of electrons is 0.3* m^2/V-s.

Sol: We know:

$$\frac{1}{\sigma} = \rho = \frac{1}{ne\mu_n}$$

[Since n = number of free electron per m^3 ≈ number of donor atoms in n-type]

So $n = \frac{1}{\rho\, e\, \mu_n} = \frac{1}{0.25 \times 1.6 \times 10^{-19} \times 0.3} = 8.333 \times 10^{19}$ per m^3

3. *At 300* K, *find the diffusion coefficient of electrons in silicon. Given the mobility of electrons* (μ_n) *is 0.21*m^2/V-s.

Sol: From Einstein's equation, we know:

$$\frac{D_n}{\mu_n} = \frac{K_B T}{e}$$

$$D_n = \mu_n \frac{K_B T}{e} = \frac{0.21 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 54.34 \times 10^{-4}\ m^2/s$$

4. *The Hall coefficient* (R_H) *of a semiconductor is 3.22 × 10*$^{-4}$ $m^3\ C^{-1}$. *Its resistivity is 8.50 × 10*$^{-3}$ Ω-m.*Calculate the mobility and carrier concentration of the carriers.*

Sol: Since R_H is positive, so the given semiconductor is p-type.

So $R_H = \frac{1}{pe}$ where $p =$ hole concentration

(or) $p = \frac{1}{R_H e} = \frac{1}{3.22 \times 10^{-4} \times 1.6 \times 10^{-19}} = 19.4 \times 10^{21}\ m^{-3}$

Mobility of holes μ_p is:

$$\mu_p = \frac{\sigma_p}{pe} = \sigma_p R_H = \frac{R_H}{\rho} \text{ ; where } \rho = \text{resistivity} = 8.50 \times 10^{-3}\ \Omega\text{-m}$$

$$\mu_p = \frac{3.22 \times 10^{-4}}{8.50 \times 10^{-3}} = 0.0378\ \text{m}^2/\text{V-s}$$

5. *Mobilities of electrons and holes in a sample of intrinsic germanium at 300* K *are 0.36* m^2/V-s *and 0.17* m^2/ V-s, *respectively. If the resistivity of the specimen is 2.12* Ω-m, *compute the intrinsic concentration.*

Sol: Mobility of electrons (μ_e) = 0.36 m^2/ V-s

Mobility of holes (μ_h) = 0.17 m^2/ V-s

Resistivity ρ_i = 2.12 Ω-m

Energy gap (E_g) = ?

$$\sigma_i = \frac{1}{\rho_i} = n_i e\,(\mu_e + \mu_h)$$

$$\frac{1}{2.12} = n_i \times 1.6 \times 10^{-19}\left[0.36 + 0.17\right]$$

$$n_i = \frac{10^{19}}{2.12 \times 1.6 \times 0.53} = 556.25 \times 10^{16}/\text{m}^3$$

6. *The following data are given for intrinsic germanium at 300* K $n_i = 2.4 \times 10^{19}$/m^3; $\mu_e = 0.39$ m^2/V-

s; μ_h =*0.19* m²/V-s. *Calculate the resistivity of the sample.*

(Set-1–Sept. 2007), (Set-2–Sept. 2006), (Set-1–May 2003)

Sol: $\rho_i = \dfrac{1}{n_i e(\mu_e + \mu_h)}$

$n_i = 2.4 \times 10^{19}/\text{m}^3$; $\mu_e = 0.39\ \text{m}^2/\text{V-s}$ $\mu_h = 0.19\ \text{m}^2/\text{V-s}$

$$\rho_i = \frac{1}{2.4\times10^{19}\times1.6\times10^{-19}\times(0.39+0.19)} = \frac{1}{2.4\times1.6\times0.58} = 0.449\ \Omega\text{-m}$$

7. The electron and hole mobilites in a silicon sample are 0.135 and 0.048 m²/V-s, *respectively. Determine the conductivity of intrinsic Si at 300* K *if the intrinsic carrier concentration is 1.5 ×* *10^{16}* atoms/m³. *The sample is doped with 10^{23} phosphorous* atoms/m³. *Determine the hole concentration and conductivity.*

(Set-3–May 2004), (Set-4–May 2003)

Sol: Mobility of electrons (μ_e) = 0.135 m²/V-s

Mobility of holes (μ_h) = 0.048 m²/V-s

Mobility of holes (μ_h) = 0.048 m²/ V-s

Instrinsic carrier concentration (n_i) = $1.5 \times 10^{16}/m^3$

Conductivity (σ) = $n_i e\,(\mu_e + \mu_h) = 1.5 \times 10^{16} \times 1.6 \times 10^{-19}\,[0.135 + 0.048]$

$= 1.5 \times 1.6 \times 0.183 \times 10-3 = 0.439 \times 10^{-3}/\Omega\text{-m}.$

Doping concentration, $N_D = 10^{23}$ phosphorous atoms/m^3

hole concentration, p = ?

conductivity (σ_n) = ?

$$p = \frac{n_i^2}{N_D} = \frac{\left(1.5 \times 10^{16}\right)^2}{10^{23}} = 2.25 \times 10^9/\text{m}^3$$

$$\sigma_n = N_D e \mu_e = 10^{23} \times 1.6 \times 10^{-19} \times 0.135 = 2.16 \times 10^3/\Omega\text{-m}.$$

8. *The R_H of a specimen is 3.66×10^{-4}* m³/c. *Its resistivity is 8.93×10^{-3}* Ω-m. *Find* μ and n.

(Set-1–May 2004), (Set-2–May 2003)

Sol: Since R_H is positive, the given specimen is p-type material, $R_H = \frac{1}{pe}$

$$\left.\begin{array}{l}\text{Carrier concentration} \\ [\text{hole concentration}]\end{array}\right\} \quad (p) = \frac{1}{R_H e} = \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}} = 1.7 \times 10^{22}\,\text{m}^{-3}$$

$$\text{Mobility } (\mu) = \sigma_H R_H = \frac{R_H}{\rho_H} = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}} = 4.099 \times 10^{-2}\,\text{m}^2/\text{V-s}$$

9. *Find the conductivity of intrinsic silicon at 300* K. *It is given that n_i at 300* K *in silicon is 1.5* × *10^{16}*/m^3 *and the mobilities of electrons and holes in silicon are 0.13* m^2/V-s *and 0.05* m^2/V-s, *respectively*

(Set-2–May 2003)

Sol: Intrinsic concentration (n_i) = 1.5 × 10^{16}/m^3

Mobility of electrons (μ_e) = 0.13 m^2/V-m

Mobility of holes (μ_h) = 0.05 m^2/V-m

Conductivity (σ) = $n_i e$ ($\mu_e + \mu_h$) = 1.5 × 10^{16} × 1.6 × 10^{-19} (0.13 + 0.05)/Ω-m

$= 4.32 \times 10^{-4}/\Omega\text{-m}$

10. A pure silicon material has an intrinsic concentration of $1.5 \times 10^{16}/m^3$ at 300 K. If it is doped with donor impurity atoms at the rate of 1 in 10^8 atoms of silicon, then calculate its conductivity. Assume that all the impurity atoms are ionized. Given that the atomic weight of silicon is 28.09, density = 2.33×10^3 kg/m^3 electron and hole mobilities are 0.14 m^2/V-s and 0.05 m^2/V-s, respectively.

Sol: Given data are:

Intrinsic concentration (n_i) = $1.5 \times 10^{16}/\text{m}^3$

Atomic weight of silicon (A) = 28.09

Density of silicon (D) = 2.33×10^3 kg/m^3

Electron mobility (μ_e) = 0.14 m^2/V-s

Hole mobility (μ_h) = 0.05 m^2/V-s

$$\text{No. of silicon atoms per unit volume } (N) = \frac{N_A \times D}{A}$$

$$= \frac{6.025 \times 10^{26} \times 2.33 \times 10^3}{28.09} = 5 \times 10^{28}/\text{m}^3$$

Since the doping concentration is 1 in 10^8 silicon atoms

∴ Electron concentration (n) = $\frac{N}{10^8} = \frac{5\times10^{28}}{10^8} = 5\times10^{20}/\text{m}^3$

From law of mass action, hole concentration $p = \frac{n_i^2}{n} = \frac{1.5\times10^{162}}{5\times10^{20}}$

$= 4.5 \times 10^{11}/\text{m}^3$

∴ Conductivity (σ) = e[$n\mu_e + p\mu_h$] = $1.6 \times 10^{-19}[5 \times 10^{20} \times 0.14 + 4.5 \times 10^{11} \times 0.05]$

$= 1.6 \times 10^{-19}$ [70,000 × 10^{15} + 0.0000225 × 10^{15}] = 1.6×10^{-19} × 70,000.0000225 × 10^{15}

= 11.2/Ω–m

11. Pure germanium at 300 K *has a density of charge carries 2.5 × 10^{19}*/m^3. *A specimen of pure germanium is doped with donor impurity atoms at the rate of one impurity atom for every 10^6 atoms of germanium. Assuming that all the impurity atoms are ionized, find the resistivity of the doped germanium if the electron and hole mobilities are 0.36 m^2*/V-s *and 0.18 m^2*/V-s, *respectively and the number of germanium atoms/unit volume is 4.2 × 10^{28}* atoms/m^3.

Sol: Given data are:

Density of charge carriers (n_i) = $2.5 \times 10^{19}/\text{m}^3$

Mobility of electrons (μ_e) = 0.36 m^2/ V-s

Mobility of holes (μ_h) = 0.18 m^2/ V-s

Since doping concentration is 1 in 10^6

$$\text{Hence, impurity atoms per m}^3 = \frac{\text{No. of Germanium atoms in m}^3}{10^6}$$

$$= \frac{4.2 \times 10^{28}}{10^6} = 4.2 \times 10^{22}/\text{m}^3$$

As all the impurity atoms are ionized,

So, the number of free electrons per m^3 = n = 4.2 × 10^{22}/m^3

The hole concentration p is obtained from law of mass action as:

$np = n_i^2$

$$p = \frac{n_i^2}{n} = \frac{\left(2.5 \times 10^{19}\right)^2}{4.2 \times 10^{22}} = 1.488 \times 10^{16}/\text{m}^3$$

$$\text{Resistivity } (\rho_i) = \frac{1}{\sigma_i} = \frac{1}{ne\mu_e + pe\mu_h} = \frac{1}{e\left[n\mu_e + p\mu_h\right]}$$

$$= \frac{1}{1.6 \times 10^{-19}\left[4.2 \times 10^{22} \times 0.36 + 1.488 \times 10^{16} \times 0.18\right]}$$

$$= \frac{1}{1.6 \times 10^{-19}\left[1512 \times 10^{19} + 0.0002678 \times 10^{19}\right]}$$

$$= \frac{1}{1.6 \times 1512.0002678} = 4.13 \times 10^{-4}/\Omega\text{-m}$$

12. An intrinsic Ge at room temperature with a carrier concentration of 2.4 × *10*^{9} m^{-3} *is doped with one Sb atom in 10*^{6} *Ge atoms. What would be the concentration of holes if the Ge atom concentration is 4* × *10*^{28} m^{-3}?

Sol: Carrier concentration in Ge at room temperature, $(n + p) = 2.4 \times 10^{9}$ m^{-3}

Doping concentration of Sb atoms = 1 in 10^{6} Ge atoms

Concentration of Ge atoms, $N = 4 \times 10^{28}$ m^{-3}

Since Sb atoms are pentavalent atoms, their ionization contributes free electrons and positive ions in the material, but holes will not be affected.

So, hole concentration, $p = \frac{1}{2} \times$ carrier concentration

$$= \frac{1}{2} \times 2.4 \times 10^{9}\ \text{m}^{-3} = 1.2 \times 10^{9}\ \text{m}^{-3}.$$

13. Calculate the density of donor atoms to produce an n-type material with 0.2 Ω-m *resistivity and 0.35* m^{2}V^{-1}*electron mobility.*

Sol: Resistivity of the material, ρ = 0.2 Ω-m

Mobility of electrons, $\mu_n = 0.35$ m^2 V^{-1}

Density of donor atoms, n = ?

Electrical conductivity, $\sigma_e = ne\mu_e$

or $$n = \frac{\sigma_e}{e\,\mu_e} = \frac{1}{\rho_e e \mu_e} = \frac{1}{0.2 \times 1.602 \times 10^{-19} \times 0.35} = 8.92 \times 10^{19} \text{ electron/m}^3$$

14. If resistivity of an intrinsic semiconductor is 5 Ω-m *at* 300 K *and* 2.5 Ω-m *at* 320 K, *what would be its energy gap?*

Sol: Resistivity at 300 K, ρ_1 = 5 Ω-m

Resistivtiy at 320 K, ρ_2 = 2.5 Ω-m

Energy gap of intrinsic semiconductor, E_g = ?

For intrinsic semiconductor,

We know $\sigma = A \exp\left(\frac{-E_g}{2K_B T}\right)$ where A is constant

$$\rho = \frac{1}{\sigma} = \frac{1}{A \exp\left(\frac{-E_g}{2K_B T}\right)}$$

$$\rho_1 \propto \exp\left(\frac{E_g}{2K_B T_1}\right) \quad \text{and} \quad \rho_2 \propto \exp\left(\frac{E_g}{2K_B T_2}\right)$$

$$\frac{\rho_1}{\rho_2} = \frac{\exp\left(\frac{E_g}{2K_B T_1}\right)}{\exp\left(\frac{E_g}{2K_B T_2}\right)} = \exp\left[\frac{E_g}{2K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

$$l_n\left(\frac{\rho_1}{\rho_2}\right) = \frac{E_g}{2K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$E_g = \frac{2K_B \ln\left(\frac{\rho_1}{\rho_2}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \frac{2 \times 1.38 \times 10^{-23} \ln\left(\frac{5}{2.5}\right)}{\left(\frac{1}{300} - \frac{1}{320}\right)} = \frac{1.913 \times 10^{-23}}{2.0833 \times 10^{-4}} = 0.918 \times 10^{-19}\ \text{J}$$

$$= \frac{0.918 \times 10^{-19}}{1.602 \times 10^{-19}}\ \text{eV} = 0.573\ \text{eV}$$

15. *Find the diffusion coefficient of electrons in Silicon at 300* K *if* μ_e *is 0.19* m²/V-s.

(Set-2–Sept. 2007), (Set-3–May 2007), (Set-4–June 2003), (Set-2–May 2004)

Sol: Probability of electrons, μ_e = 0.19 m^2/V-s

Temperature of specimen, T = 300 K

Diffusion coefficient of electrons, D_n = ?

$D_n = \frac{\mu_e K_B T}{e}$, where K_B = Boltzmann constant

$= 1.38 \times 10^{-23}$ J/K

and e = charge on electrons = 1.6×10^{-19} C

$$D_n = \frac{0.19 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 4.92 \times 10^{-3} \, m^2/sec$$

16. The resistivity of an intrinsic semiconductor is 4.5 Ω-m *at 20°* C *and 2.0* Ω-m *at 32°* C. *What is the energy band gap?*

(Set-4–May 2004)

Sol: ρ_1 = 4.5 Ω-m

ρ_2 = 2.0 Ω-m

T_1 = 20° C = 293 K

T_2 = 32° C = 305 K

Energy band gap, E_g = ?

We know:

Resistivity, $\rho = A \exp\left[\frac{E_g}{2k_B T}\right]$

where A = constant

k_B = Boltzmann constant

= 1.38×10^{-23} J/k

$$\frac{\rho_1}{\rho_2} = \frac{A \exp\left[\frac{E_g}{2k_B T_1}\right]}{A \exp\left[\frac{E_g}{2k_B T_2}\right]} = \exp\left[\frac{E_g}{2k_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

Taking logarithm on both sides, we get:

$$\ln\left(\frac{\rho_1}{\rho_2}\right) = \frac{E_g}{2k_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$E_g = \frac{2k_B}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \ln\left(\frac{\rho_1}{\rho_2}\right)$$

$$= \frac{2 \times 1.38 \times 10^{-23}}{\frac{1}{293} - \frac{1}{305}} \times \ln\left(\frac{4.5}{2.0}\right) \text{J}$$

$$= 1.6669 \times 10^{-19}\ \text{J} = 1.04\ \text{eV}$$