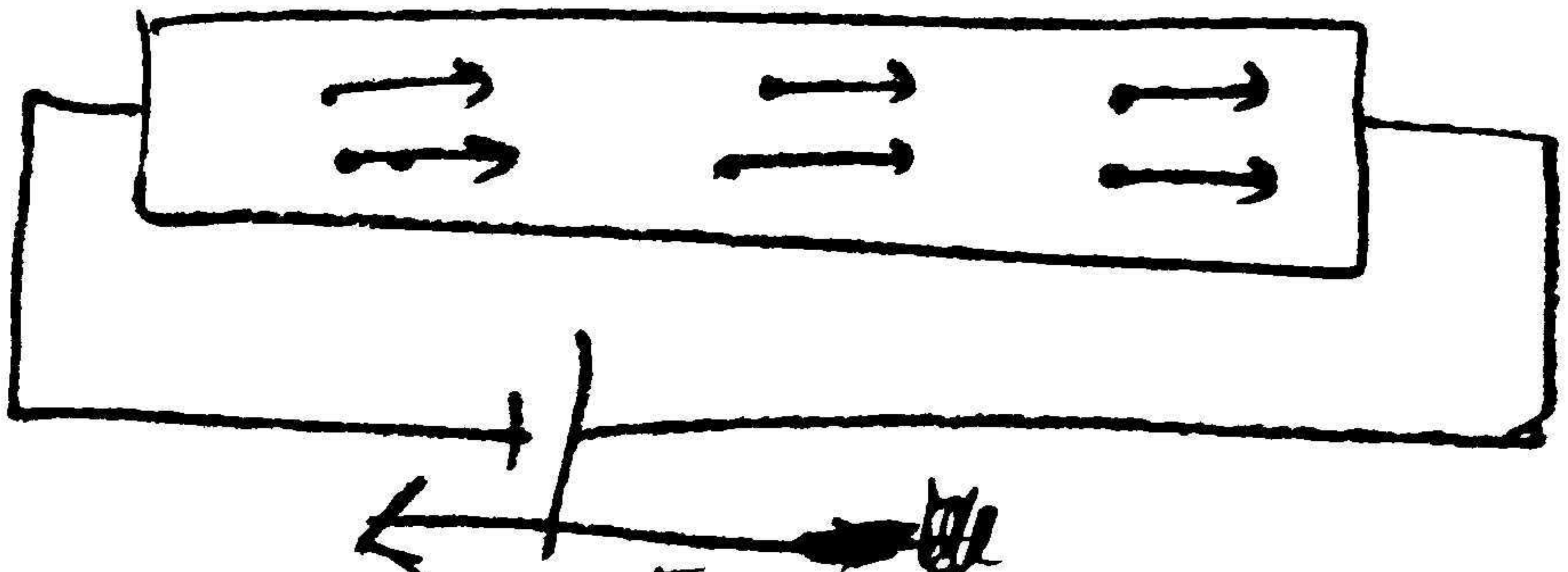
Expression for electrical conductivity & Drift velocity :-

Consider a conductor which is subjected to an E·F of strength E



n = concentration of free $e^{\ominus}s$

m = mass of $e^{\ominus}s$

e = charge of $e^{\ominus}s$

According to Newton's second law of motion the force f acquired by $e^{\ominus}s$ is equal to the force excerted by the field on the $e^{\ominus}s$.

$\therefore$ eqn of motion $ma = -eE$

$$\Rightarrow a = \frac{-eE}{m}$$

$\Rightarrow \int a\,dt = \int -\frac{eE}{m}\,dt$ (Integrate) $\left[a = \frac{dV}{dt}\right]$

elocity of $e^{\ominus}$

$\Rightarrow V = -\frac{eE}{m}t + C$

Integration const.

During the absence of the E.F., the average velocity of the $e^{\ominus}$ is zero,

$t = 0, \quad \langle V \rangle = 0$

$$\Rightarrow \boxed{V = -\frac{eEt}{m}}$$

average velocity b/w two successive collisions

$$\bar{V} = \frac{1}{\tau}\int_0^{\tau}\left(-\frac{eEt}{m}\right)dt$$

$$\bar{V} = -\frac{eE}{\tau m}\int_0^{\tau} t\,dt = -\frac{eE}{\tau m}\frac{\tau^2}{2} = \frac{-eE\tau}{2m}$$

$$\bar{V} = -\frac{eE\lambda}{2(mV)}$$

[relaxation time b/w two successive collisions $= \tau$
λ = mean free path of $e^{\ominus}$s.
$\left[\tau = \frac{\lambda}{V}\right]$

$$\bar{V} = \frac{-eE\lambda}{2\left(\frac{3KT}{V}\right)}$$

$\left[\because \frac{1}{2}mV^2 = \frac{3}{2}KT \Rightarrow mV = \frac{3KT}{V}\right]$

$$\bar{V} = \frac{-eE\lambda V}{6KT} \quad \text{—} \quad (1)$$

If n is no. density of $e^{\ominus}$s in conductor then the current density J is :-

$\left[dq = -en A V\,dt, \quad \frac{dq}{dt} = -enAV = -enV \text{ [For unit area of cross sec]}\right]$

$$J = -en\bar{V}$$

$$= -en\left[\frac{-eE\lambda V}{6KT}\right] \quad \text{[from (1)]}$$

$$J = \frac{ne^2E\lambda V}{6KT} \quad \text{—} \quad (2)$$

If q charge is flowing through a conductor cross-section area A in time t,

$$q = \sigma A E t$$

$$\frac{q}{t} = \sigma A E$$

$$\Rightarrow \boxed{I = \sigma A E} \Rightarrow \frac{I}{A} = \sigma E$$ [current density is directly pr-
tional to applied E.F.]

$$\Rightarrow \boxed{J = \sigma E}$$

$$\sigma = \frac{J}{E}$$ [For unit area of cross-sec.]

$$\sigma = \frac{n e^2 \lambda v}{6 k T}$$

$\Rightarrow$ different conductivities of diff. materials are due to different no. of free electrons.

(i) Electrical conductivity

It is defined as the quantity of electricity that flows in unit time per unit area of cross-section of the conductor per unit potential gradient.

According to free electron theory, in a solid the electrons move freely. If E is the applied electric field, then the acceleration of an electron having charge is given by

$$a = \frac{d^2x}{dt^2} = \frac{eE}{m} \qquad \text{... (5.1)}$$

If λ is the mean free path of electrons, then the relaxation time τ between two successive collisions is given by

$$\tau = \frac{\lambda}{\upsilon} \qquad \text{... (5.2)}$$

Integrating eq. 5.1, we get

$$\frac{dx}{dt} = \frac{eE}{m}t + C$$

At

$$t = 0, \frac{dx}{dt} = 0$$

Hence

$$\frac{dx}{dt} = \upsilon = \frac{eE}{m}t$$

So average velocity between two successive collisions

$$\bar{\upsilon} = \frac{1}{\tau}\int_0^\tau \frac{eE}{m}t\,dt = \frac{eE}{\tau m}\int_0^\tau t\,dt$$

$$= \frac{eE}{\tau m}\frac{\tau^2}{2}$$

or

$$\bar{\upsilon} = \frac{e\tau E}{2m}$$

Putting the value of τ from eq. 5.2, we get

$$\bar{\upsilon} = \frac{eE\lambda}{2mv}$$

Since

$$\frac{1}{2}m\upsilon^2 = \frac{3}{2}kT$$

(T is absolute temperature and k is Boltzmann constant)

So

$$m\upsilon = \frac{3kT}{v}$$

So

$$\bar{\upsilon} = \frac{eE\lambda v}{6kT}$$

If n is number density of electrons in the conductor, then the current density i is given by

$$i = ne\bar{\upsilon}$$

$$i = \frac{ne^2E\lambda v}{6kT} \quad \text{... (5.3)}$$

or

If q charge is flowing through a conductor of cross-section area A in time t, then

$$q = \sigma AEt$$

$$\frac{q}{t} = \sigma AE$$

or

$$i = \sigma AE$$

or

$$\sigma = \frac{i}{AE} \quad \text{...(5.4)}$$

or

For unit area of cross-section

$$\sigma = \frac{i}{E}$$

Using eq. (5.3)

$$\sigma = \frac{ne^2\lambda v}{6kTA}$$

This expression shows that different conductivities of different materials are due to different number of free electrons.

(ii) Ohm's law

From eq. (5.4) we have

$$\sigma E = \frac{i}{A}$$

or

$$\sigma E = J$$

or

$$J = \sigma E \quad \text{...(5.5)}$$

This is microscopic form of Ohm's law.