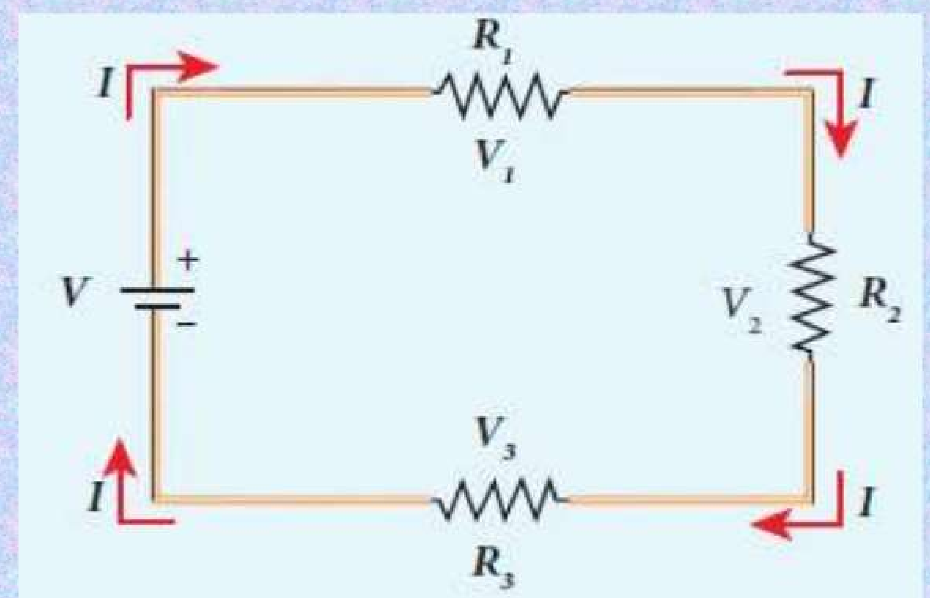
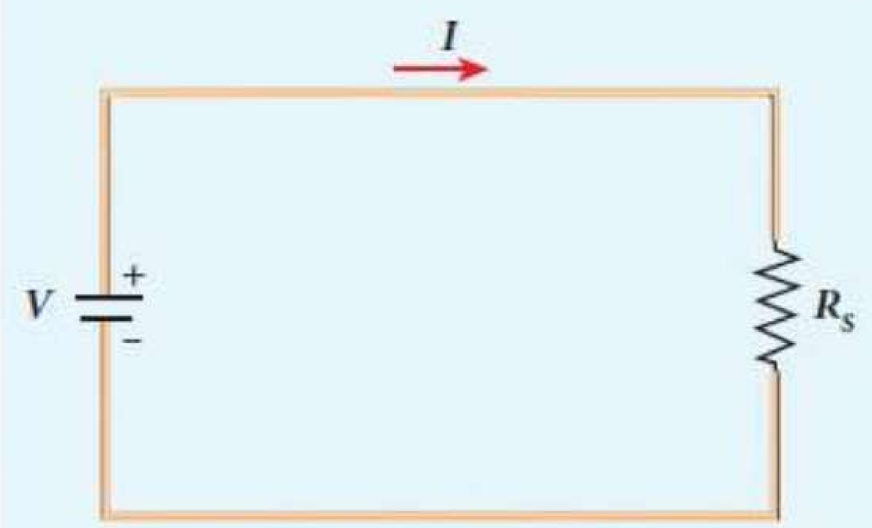
Resistors in series and parallel

Resistors in series

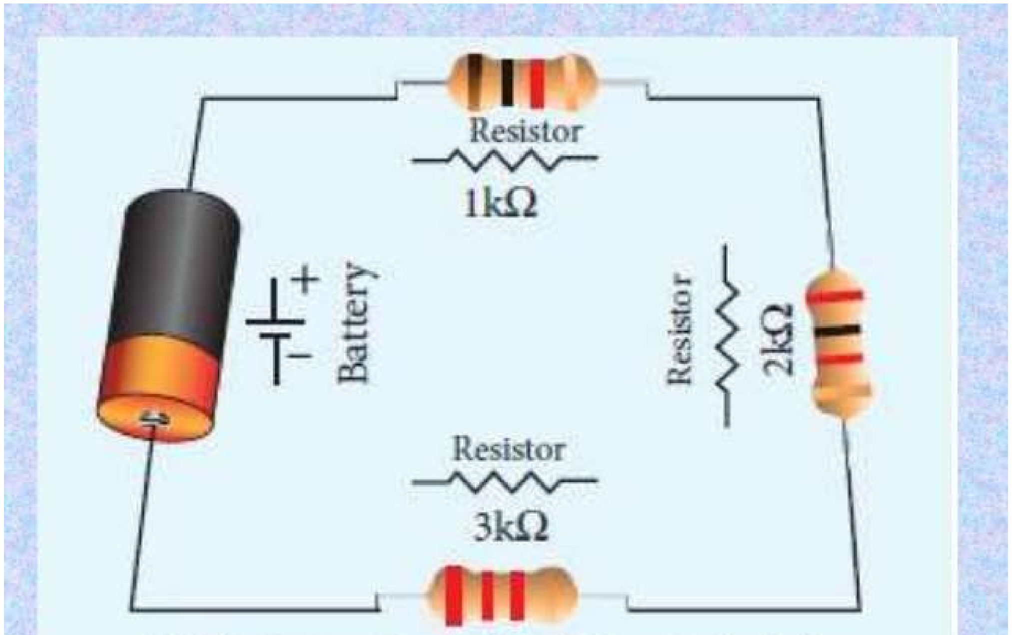
- When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices
- The three resistors R_1, R_2 *and* R_3 *connected in series.*
- The amount of charge passing through resistor R_1 *must also pass through resistors* R_2 and R_3 *since the charges cannot accumulate anywhere in the circuit*



(a) Three resistors in series



(b) Equivalent resistance (R_S) has the same current



(c) Resistors in series (Actual photo)

- Due to this reason, the current I passing through all the three resistors is the same.
- According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different
- Let *V_1, V_2 and V_3 be the potential difference (voltage) across each of the resistors R_1, R_2 and R_3 respectively, then we can write $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$.*
- *But the total voltage V is equal to the sum of voltages across each resistor*

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$V = I\,(R_1 + R_2 + R_3)$$

$$V = I.R_S$$

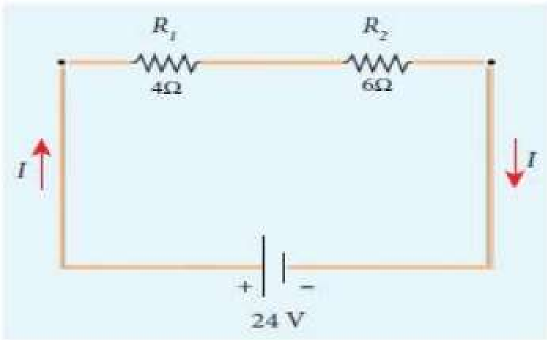
- where R_S *is the equivalent resistance*

$$R_S = R_1 + R_2 + R_3$$

- When several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances

EXAMPLE 2.8

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 Ω and 6 Ω resistors in the circuit.



Solution

Since the resistors are connected in series, the effective resistance in the circuit

$= 4\ \Omega + 6\ \Omega = 10\ \Omega$

$$\text{The Current I in the circuit} = \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,A$$

Voltage across 4Ω resistor

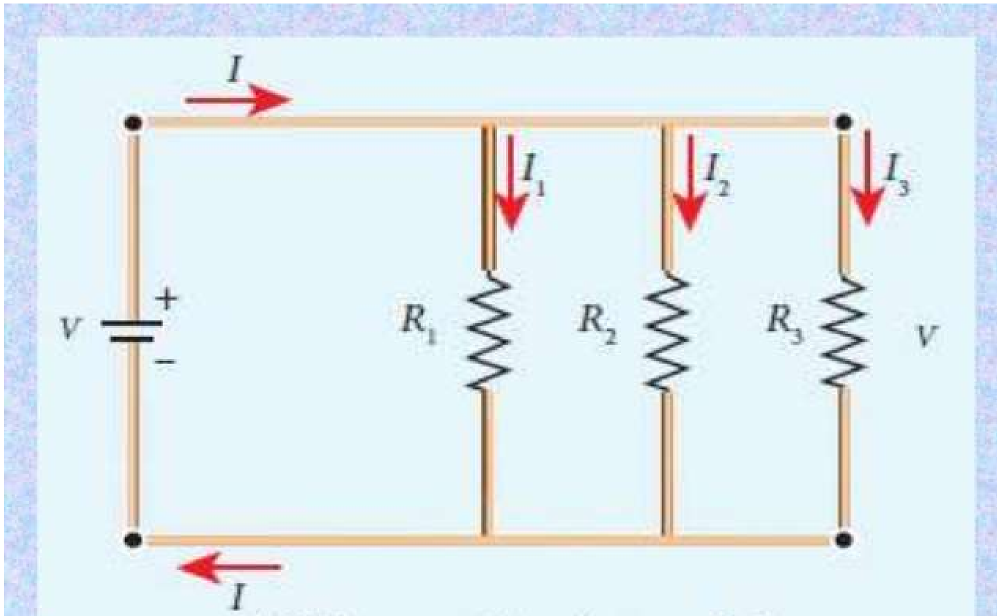
$$V_1 = IR_1 = 2.4\,A \times 4\Omega = 9.6\,V$$

Voltage across $6\ \Omega$ resistor

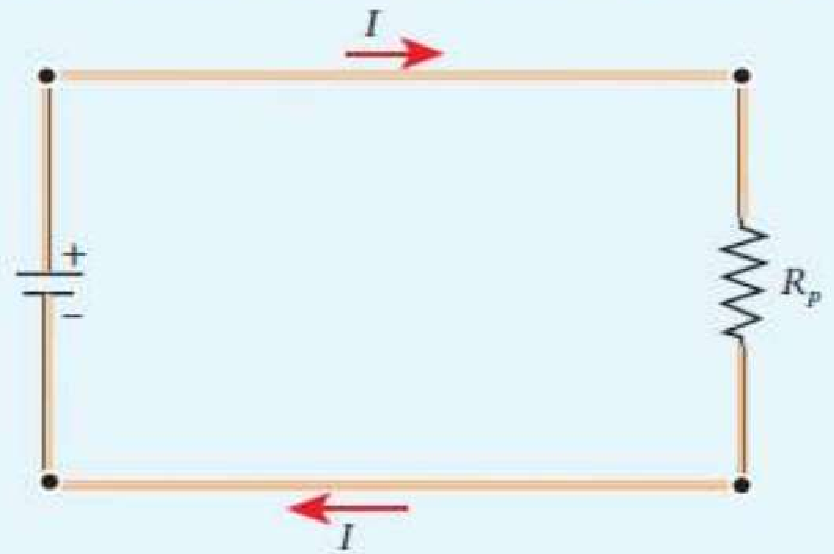
$$V_2 = IR_1 = 2.4\,A \times 6\Omega = 14.4\,V$$

Resistors in parallel

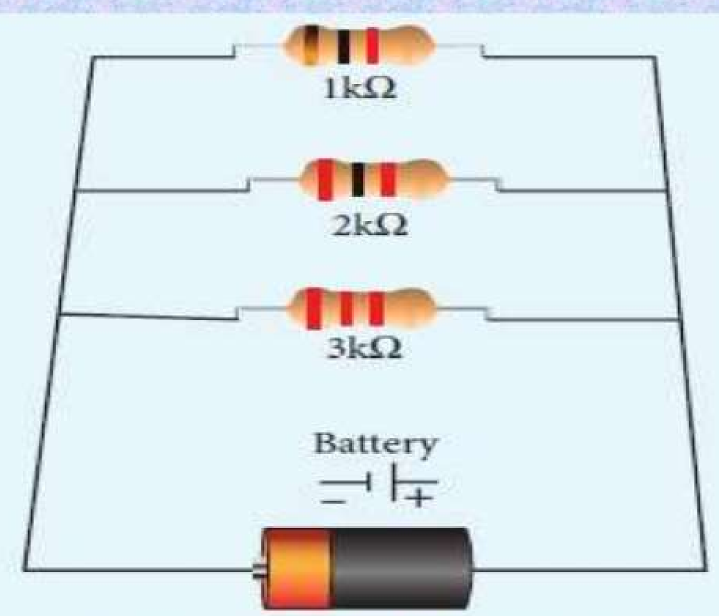
- Resistors are in parallel when they are connected across the same potential difference
- In this case, the total current I that leaves the battery is split into three separate paths.
- Let I_1, I_2 and I_3 be the current through the resistors R_1, R_2 and R_3 respectively.
- Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors.



(a) Three resistors in parallel



(b) Equivalent resistance (R_p) has the same current



(c) Resistors in parallel(Actual photo)

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

- Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

- Substituting these values in equation

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_p}$$

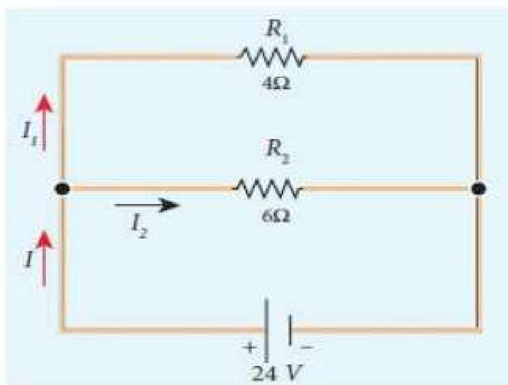
- The value of equivalent resistance in parallel connection will be lesser than each individual resistance.
- House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- Here R_P is the equivalent resistance of the parallel combination of the resistors.
- Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination

EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the current I, I_1 and I_2 in the given circuit.



Solution

Since the resistances are connected in parallel, therefore, the equivalent resistance in the circuit is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_p} = \frac{5}{12}\Omega \qquad \text{or } R_p = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential (voltage) across each resistor is the same.

$$I_1 = \frac{V}{R_1} = \frac{24V}{6\Omega} = 6\,A$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\,A$$

The current I is the total of the currents in the two branches. Then,

$I = I_1 + I_2 = 6\text{ A} + 4\text{ A} = 10\text{ A}$

EXAMPLE 2.11

Calculate the equivalent resistance between A and B in the given circuit.

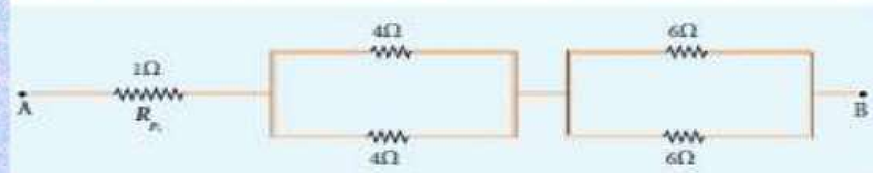
Solution

Parallel connection

Part 1

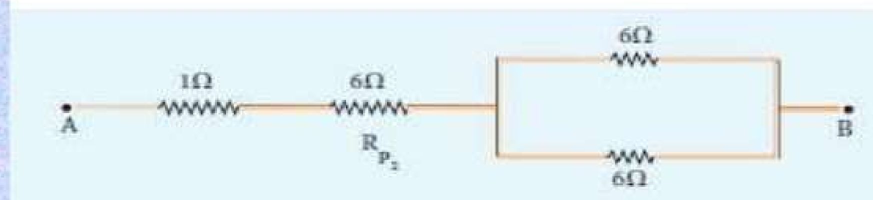
$$\frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \qquad R_{p_1} = 1\Omega$$



Part II

$$\frac{1}{R_{P_2}}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4},\ \frac{1}{R_{P_2}}=\frac{1}{2},\ R_{P_2}=2\Omega$$



Part III

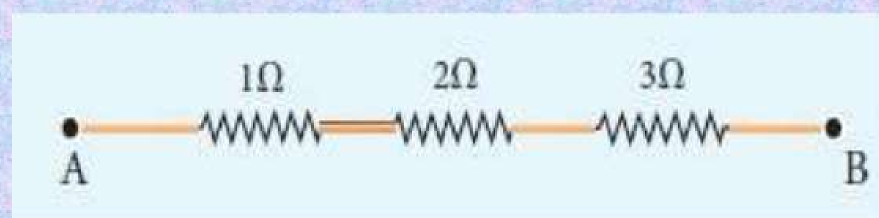
$$\frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{R_{p_3}} = \frac{1}{3}, \qquad R_{p_3} = 3\Omega$$

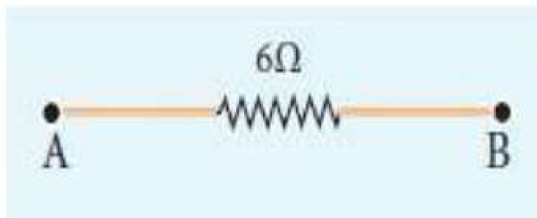
$$R = R_{p_1} + R_{p_2} + R_{p_3}$$

R = 1 + 2 + 3 R = 6 Ω

The circuit became:

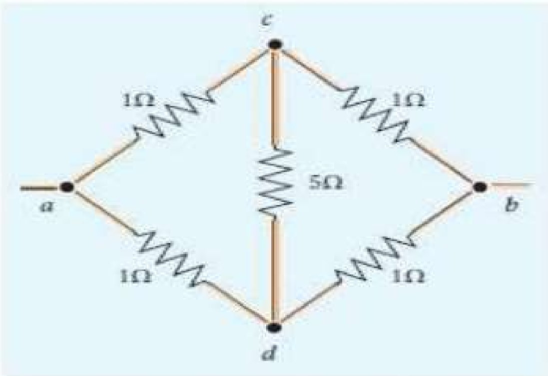


Equivalent resistance between A and B is



EXAMPLE 2.12

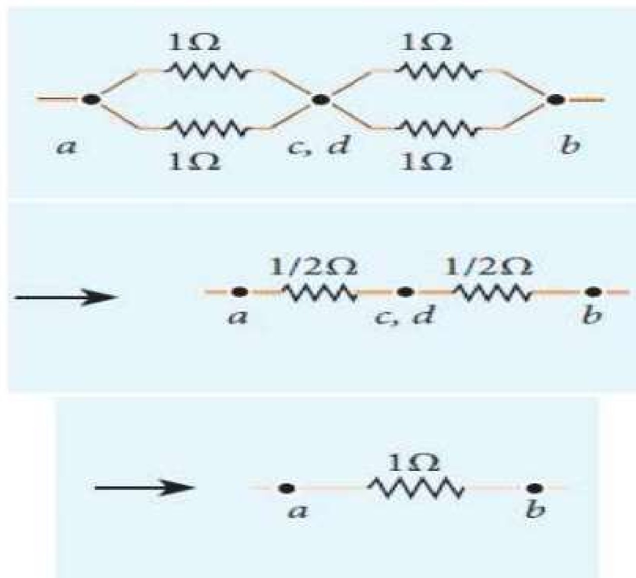
Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.



Solution

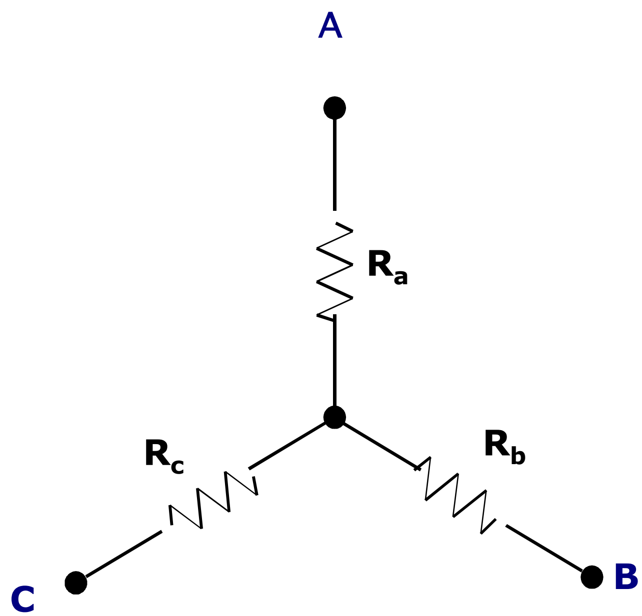
Case (a)

To find the equivalent resistance between the points a and b, we assume that current is entering the junction a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. So the electric potential at the point c and d is the same hence no current flows into $5\ \Omega$ resistance. It implies that the $5\ \Omega$ has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.

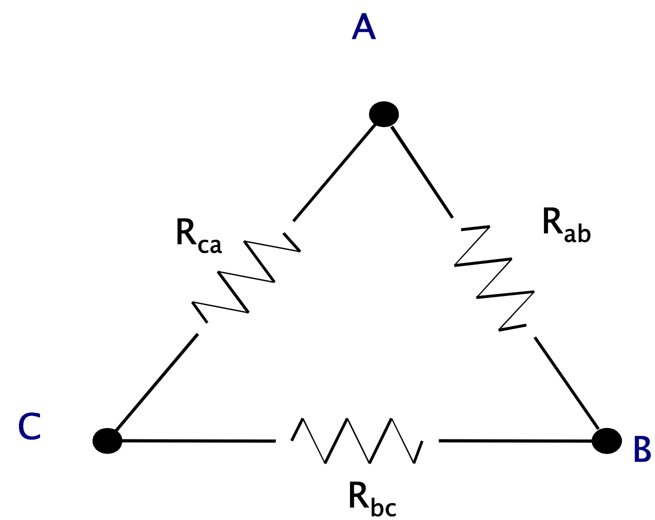


The equivalent resistance of the circuit between a and b is $R_{eq} = 1\Omega$

Star & Delta Connections



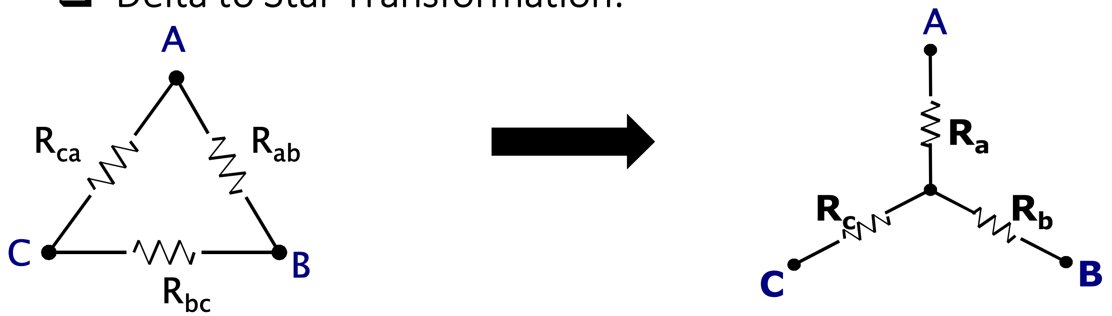
Star Connection



Delta Connection

Star-Delta Transformation

- Delta to Star Transformation:



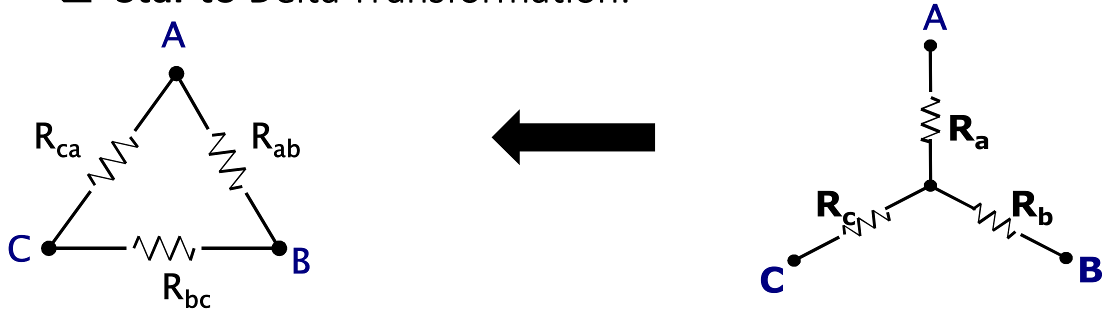
$$R_a = \frac{R_{ab}\, R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_b = \frac{R_{bc}\, R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{ca}\, R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

Star-Delta Transformation contd…

❑ **Star to** Delta Transformation:



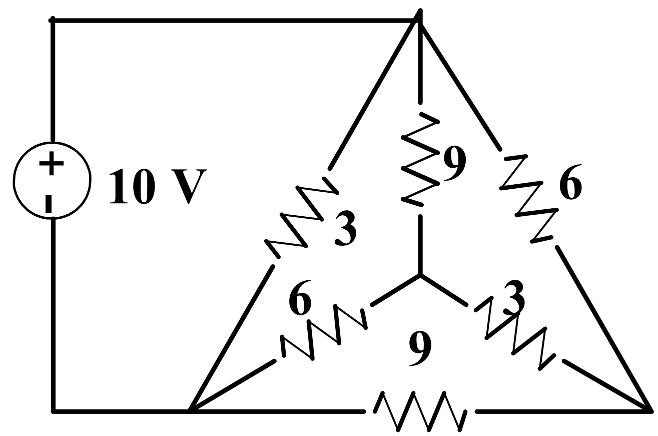
$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c}$$

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}$$

$$R_{ca} = R_c + R_a + \frac{R_c R_a}{R_b}$$

Example 1

For the circuit shown, determine the power supplied to the resistive network (all resistor values are in Ω's)



$P_{supplied}$ =47.06 W