

SCALES

There is a wide variation in sizes for engineering objects. Some are very large (eg. Aero planes, rockets, etc) Some are very small (wrist watch, MEMs components) There is a need to reduce or enlarge while drawing the objects on paper. Some objects can be drawn to their actual size. The proportion by which the drawing of an object is enlarged or reduced is called the scale of the drawing.

Definition

A scale is defined as the ratio of the linear dimensions of the object as represented in a drawing to the actual dimensions of the same.

Drawings drawn with the same size as the objects are called full sized drawing. It is not convenient, always, to draw drawings of the object to its actual size. e.g. Buildings, Heavy machines, Bridges, Watches, Electronic devices etc.

Hence scales are used to prepare drawing at:

- Full size
- Reduced size
- Enlarged size

Reducing scales 1:Y (Y>1)	1:2 1:20 1:200 1:2000	1:5 1:50 1:500 1:5000	1:10 1:100 1:1000 1:10000
Enlarging scales X:1 (X>1)	50:1 5:1	20:1 2:1	10:1
Full size scales			1:1

SCALES



DIMENSIONS OF LARGE OBJECTS MUST BE REDUCED TO ACCOMMODATE ON STANDARD SIZE DRAWING SHEET. THIS REDUCTION CREATES A SCALE OF THAT REDUCTION RATIO, WHICH IS GENERALLY A FRACTION..

**SUCH A SCALE IS CALLED REDUCING SCALE
AND
THAT RATIO IS CALLED REPRESENTATIVE FACTOR.**

SIMILARLY IN CASE OF TINY OBJECTS DIMENSIONS MUST BE INCREASED FOR ABOVE PURPOSE. HENCE THIS SCALE IS CALLED ENLARGING SCALE. HERE THE RATIO CALLED REPRESENTATIVE FACTOR IS MORE THAN UNITY.

FOR FULL SIZE SCALE

R.F.=1 OR (1:1)

**MEANS DRAWING
& OBJECT ARE OF
SAME SIZE.**

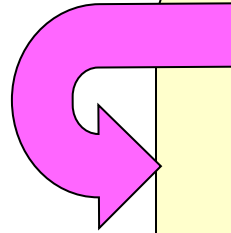
**Other RFs are described
as**

**1:10, 1:100,
1:1000, 1:1,00,000**

USE FOLLOWING FORMULAS FOR THE CALCULATIONS IN THIS TOPIC.

$$\begin{aligned} \text{A} \quad \text{REPRESENTATIVE FACTOR (R.F.)} &= \frac{\text{DIMENSION OF DRAWING}}{\text{DIMENSION OF OBJECT}} \\ &= \frac{\text{LENGTH OF DRAWING}}{\text{ACTUAL LENGTH}} \\ &= \sqrt{\frac{\text{AREA OF DRAWING}}{\text{ACTUAL AREA}}} \\ &= \sqrt[3]{\frac{\text{VOLUME AS PER DRWG.}}{\text{ACTUAL VOLUME}}} \end{aligned}$$

$$\text{B} \quad \text{LENGTH OF SCALE} = \text{R.F.} \times \text{MAX. LENGTH TO BE MEASURED.}$$



BE FRIENDLY WITH THESE UNITS.

1 KILOMETRE = 10 HECTOMETRES

1 HECTOMETRE = 10 DECAMETRES

1 DECAMETRE = 10 METRES

1 METRE = 10 DECIMETRES

1 DECIMETRE = 10 CENTIMETRES

1 CENTIMETRE = 10 MILIMETRES

TYPES OF SCALES:

- | | | |
|----|--------------------|--|
| 1. | PLAIN SCALES | (FOR DIMENSIONS UP TO SINGLE DECIMAL) |
| 2. | DIAGONAL SCALES | (FOR DIMENSIONS UP TO TWO DECIMALS) |
| 3. | VERNIER SCALES | (FOR DIMENSIONS UP TO TWO DECIMALS) |
| 4. | COMPARATIVE SCALES | (FOR COMPARING TWO DIFFERENT UNITS) |
| 5. | SCALE OF CORDS | (FOR MEASURING/CONSTRUCTING ANGLES) |

PLAIN SCALE:- This type of scale represents two units or a unit and its

sub-division

PROBLEM NO.1:- Draw a scale 1 cm = 1m to read decimeters, to measure maximum distance of 6 m. Show on it a distance of 4 m and 6 dm.

CONSTRUCTION:- $\frac{\text{DIMENSION OF DRAWING}}{\text{DIMENSION OF OBJECT}}$

a) Calculate R.F. = $\frac{\text{DIMENSION OF DRAWING}}{\text{DIMENSION OF OBJECT}}$

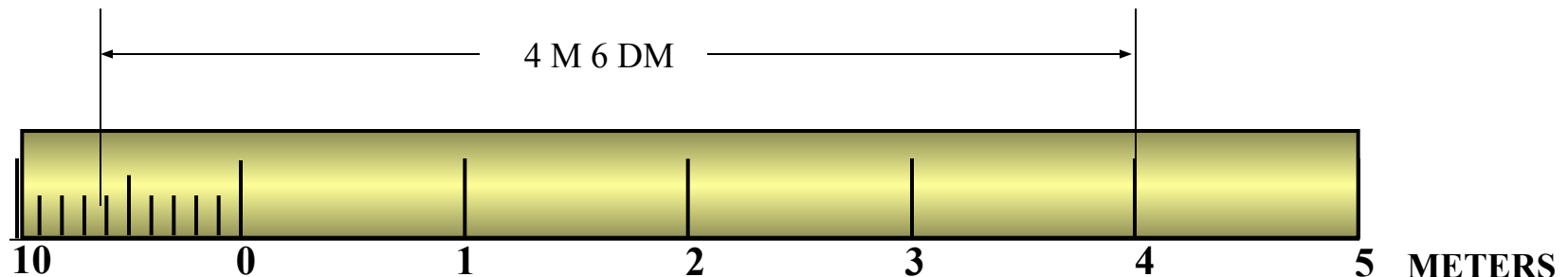
$$\text{R.F.} = 1\text{cm} / 1\text{m} = 1/100$$

$$\begin{aligned} \text{Length of scale} &= \text{R.F.} \times \text{max. distance} \\ &= 1/100 \times 600 \text{ cm} \end{aligned}$$

$$= 6 \text{ cms}$$

- b) Draw a line 6 cm long and divide it in 6 equal parts. Each part will represent larger division unit.
- c) Sub divide the first part which will represent second unit or fraction of first unit.
- d) Place (0) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. Take height of scale 5 to 10 mm for getting a look of scale.
- e) After construction of scale mention it's RF and name of scale as shown.
- f) Show the distance 4 m 6 dm on it as shown.

PLAIN SCALE



DECIMETERS

$$\text{R.F.} = 1/100$$

PLANE SCALE SHOWING METERS AND DECIMETERS.

PROBLEM NO.2:- In a map a 36 km distance is shown by a line 45 cm long. Calculate the R.F. and construct a plain scale to read kilometers and hectometers, for max. 12 km. Show a distance of 8.3 km on it.

CONSTRUCTION:-

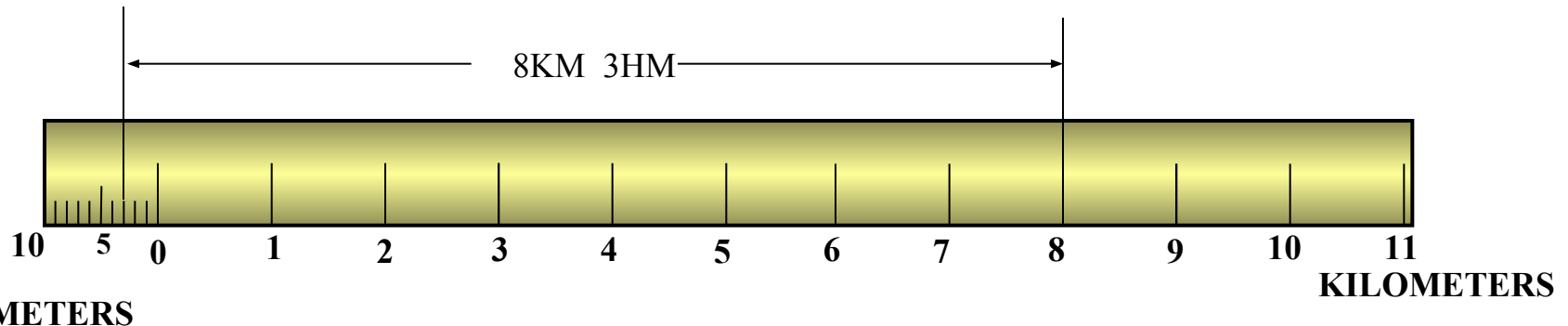
a) Calculate R.F.

$$\text{R.F.} = 45 \text{ cm} / 36 \text{ km} = 45 / 36 \cdot 1000 \cdot 100 = 1 / 80,000$$

$$\begin{aligned} \text{Length of scale} &= \text{R.F.} \times \text{max. distance} \\ &= 1 / 80000 \times 12 \text{ km} \\ &= 15 \text{ cm} \end{aligned}$$

PLAIN SCALE

- b) Draw a line 15 cm long and divide it in 12 equal parts. Each part will represent larger division unit.
- c) Sub divide the first part which will represent second unit or fraction of first unit.
- d) Place (0) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. **Take height of scale 5 to 10 mm for getting a look of scale.**
- e) After construction of scale mention it's RF and name of scale as shown.
- f) Show the distance 8.3 km on it as shown.



$$\text{R.F.} = 1 / 80,000$$

PLANE SCALE SHOWING KILOMETERS AND HECTOMETERS

PROBLEM NO.3:- The distance between two stations is 210 km. A passenger train covers this distance in 7 hours. Construct a plain scale to measure time up to a single minute. RF is 1/200,000 Indicate the distance traveled by train in 29 minutes.

CONSTRUCTION:-

- a) 210 km in 7 hours. Means speed of the train is 30 km per hour (60 minutes)

$$\begin{aligned}\text{Length of scale} &= \text{R.F.} \times \text{max. distance per hour} \\ &= \frac{1}{2,00,000} \times 30\text{km} \\ &= 15 \text{ cm}\end{aligned}$$

PLAIN SCALE

- b) 15 cm length will represent 30 km and 1 hour i.e. 60 minutes.

Draw a line 15 cm long and divide it in 6 equal parts. Each part will represent 5 km and 10 minutes.

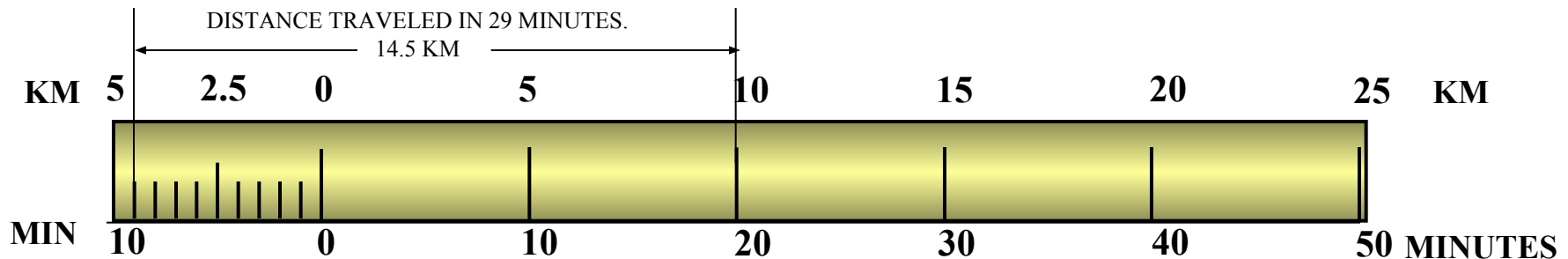
- c) Sub divide the first part in 10 equal parts, which will represent second unit or fraction of first unit. Each smaller part will represent distance traveled in one minute.

- d) Place (0) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. **Take height of scale 5 to 10 mm for getting a proper look of scale.**

- e) Show km on upper side and time in minutes on lower side of the scale as shown.

After construction of scale mention it's RF and name of scale as shown.

- f) Show the distance traveled in 29 minutes, which is 14.5 km, on it as shown.



R.F. = 1/200,000

PLANE SCALE SHOWING KILOMETERS AND MINUTES.

We have seen that the plain scales give only two dimensions, such as a unit and it's subunit or it's fraction.

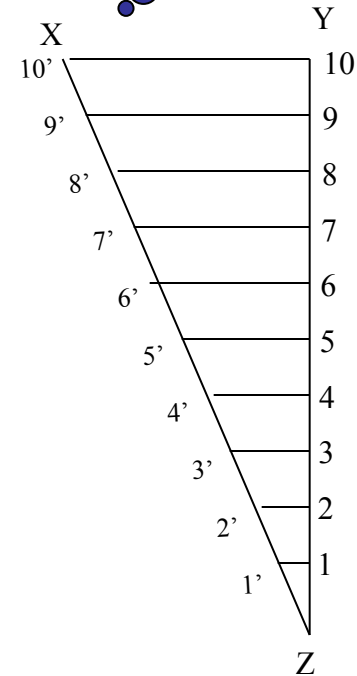
**The diagonal scales give us three successive dimensions
that is a unit, a subunit and a subdivision of a subunit.**

The principle of construction of a diagonal scale is as follows.
Let the XY in figure be a subunit.
From Y draw a perpendicular YZ to a suitable height.
Join XZ. Divide YZ in to 10 equal parts.
Draw parallel lines to XY from all these divisions
and number them as shown.
From geometry we know that similar triangles have
their like sides proportional.

Consider two similar triangles XYZ and 7' 7Z,
we have $7Z / YZ = 7'7 / XY$ (each part being one unit)
Means $7'7 = 7 / 10 \times XY = 0.7 XY$

\therefore
Similarly
 $1' - 1 = 0.1 XY$
 $2' - 2 = 0.2 XY$
Thus, it is very clear that, the sides of small triangles,
which are parallel to divided lines, become progressively
shorter in length by 0.1 XY.

DIAGONAL SCALE



**The solved examples ON NEXT PAGES will
make the principles of diagonal scales clear.**

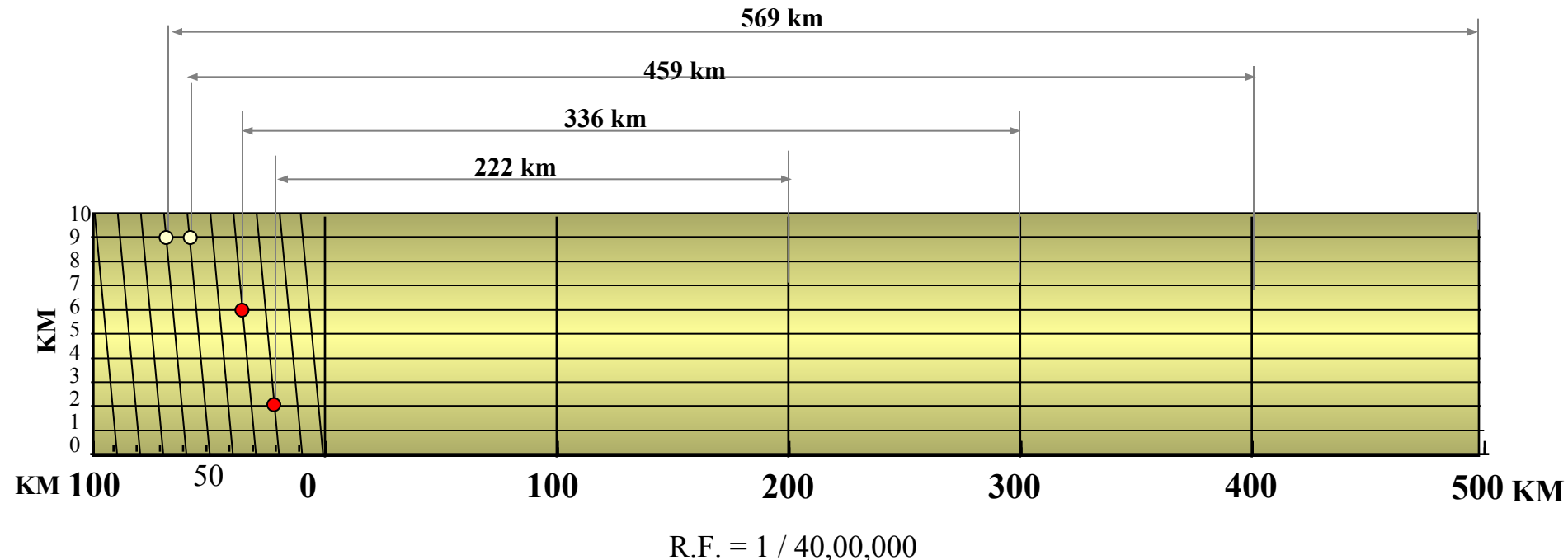


PROBLEM NO. 4 : The distance between Delhi and Agra is 200 km.
In a railway map it is represented by a line 5 cm long. Find it's R.F.
Draw a diagonal scale to show single km. And maximum 600 km.
Indicate on it following distances. 1) 222 km 2) 336 km 3) 459 km 4) 569 km

DIAGONAL SCALE

SOLUTION STEPS: $RF = 5 \text{ cm} / 200 \text{ km} = 1 / 40,00,000$
Length of scale = $1 / 40,00,000 \times 600 \times 10^5 = 15 \text{ cm}$

Draw a line 15 cm long. It will represent 600 km. Divide it in six equal parts. (each will represent 100 km.)
Divide first division in ten equal parts. Each will represent 10 km. **Draw** a line upward from left end and mark 10 parts on it of any distance. **Name** those parts 0 to 10 as shown. Join 9th sub-division of horizontal scale with 10th division of the vertical divisions. **Then** draw parallel lines to this line from remaining sub divisions and complete diagonal scale.



DIAGONAL SCALE SHOWING KILOMETERS.

PROBLEM NO.5: A rectangular plot of land measuring 1.28 hectares is represented on a map by a similar rectangle of 8 sq. cm. Calculate RF of the scale. Draw a diagonal scale to read single meter. Show a distance of 438 m on it.

SOLUTION :

1 hectare = 10,000 sq. meters

1.28 hectares = $1.28 \times 10,000$ sq. meters

$$= 1.28 \times 10^4 \times 10^4 \text{ sq. cm}$$

8 sq. cm area on map represents

$$= 1.28 \times 10^4 \times 10^4 \text{ sq. cm on land}$$

1 cm sq. on map represents

$$= 1.28 \times 10^4 \times 10^4 / 8 \text{ sq cm on land}$$

1 cm on map represent

$$\sqrt{1.28 \times 10^4 \times 10^4 / 8} \text{ cm}$$

$$= 4,000 \text{ cm}$$

1 cm on drawing represent 4,000 cm,

Means RF = $1 / 4000$

Assuming length of scale 15 cm, it will represent 600 m.

DIAGONAL SCALE

Draw a line 15 cm long.

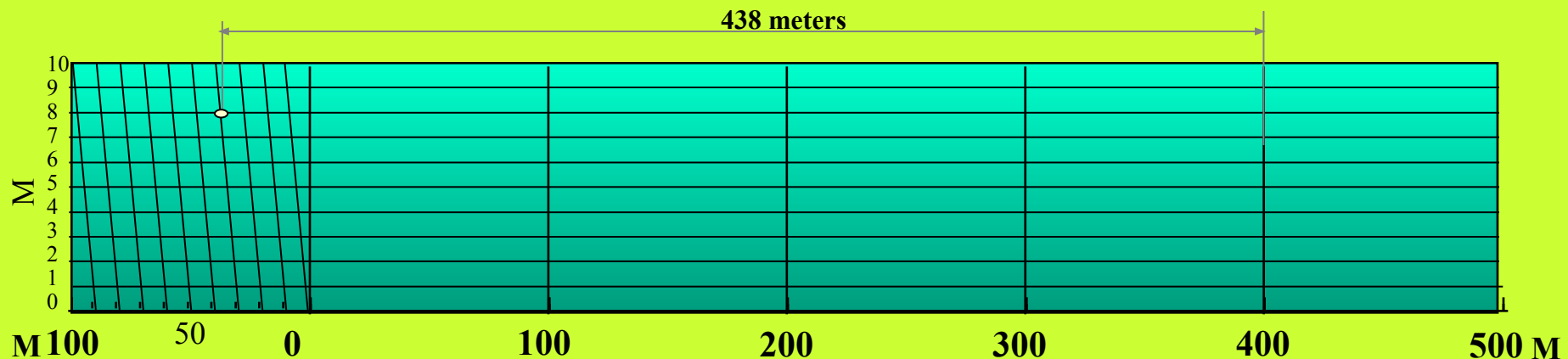
It will represent 600 m . Divide it in six equal parts. (each will represent 100 m.)

Divide first division in ten equal parts. Each will represent 10 m.

Draw a line upward from left end and mark 10 parts on it of any distance.

Name those parts 0 to 10 as shown. Join 9th sub-division of horizontal scale with 10th division of the vertical divisions.

Then draw parallel lines to this line from remaining sub divisions and complete diagonal scale.



$$\text{R.F.} = 1 / 4000$$

DIAGONAL SCALE SHOWING METERS.

PROBLEM NO.6: Draw a diagonal scale of R.F. 1: 2.5, showing centimeters and millimeters and long enough to measure up to 20 centimeters. Show the distance 13.4 cm on it.

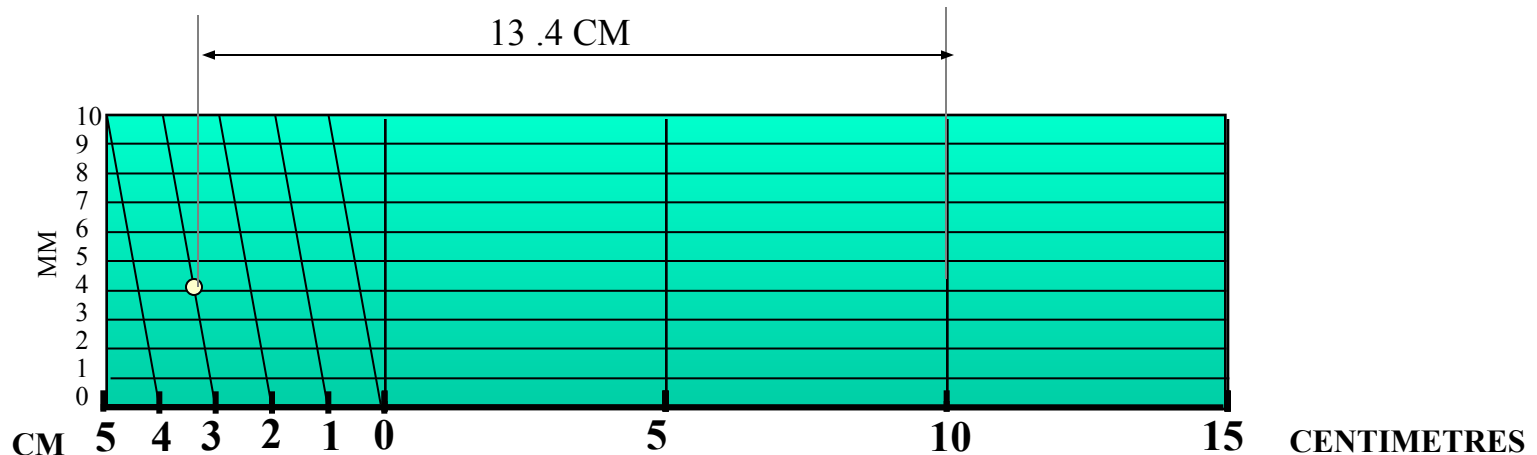
SOLUTION STEPS:

R.F. = 1 / 2.5

Length of scale = $1 / 2.5 \times 20$ cm.
= 8 cm.

1. Draw a line 8 cm long and divide it into 4 equal parts.
(Each part will represent a length of 5 cm.)
2. Divide the first part into 5 equal divisions.
(Each will show 1 cm.)
3. At the left hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.
4. Complete the scale as explained in previous problems.
Show the distance 13.4 cm on it.

DIAGONAL SCALE



R.F. = 1 / 2.5

DIAGONAL SCALE SHOWING CENTIMETERS.

COMPARATIVE SCALES:

These are the Scales having same R.F. but graduated to read different units.

These scales may be Plain scales or Diagonal scales and may be constructed separately or one above the other.

EXAMPLE NO. 7 :

A distance of 40 miles is represented by a line 8 cm long. Construct a plain scale to read 80 miles. Also construct a comparative scale to read kilometers upto 120 km (1 m = 1.609 km)

SOLUTION STEPS:

Scale of Miles:

40 miles are represented = 8 cm

: 80 miles = 16 cm

R.F. = $8 / 40 \times 1609 \times 1000 \times 100$

= $1 / 8,04,500$

CONSTRUCTION:

Take a line 16 cm long and divide it into 8 parts. Each will represent 10 miles. Subdivide the first part and each sub-division will measure single mile.

Scale of Km:

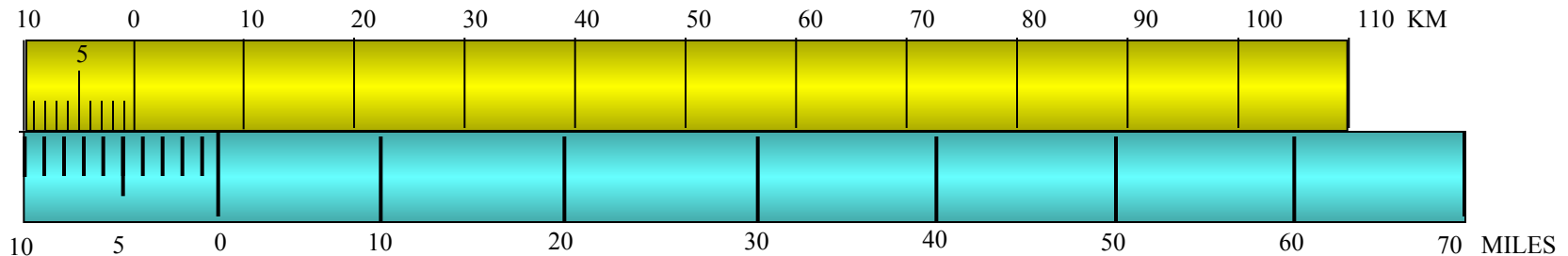
Length of scale

= $1 / 8,04,500 \times 120 \times 1000 \times 100$

= 14.90 cm

CONSTRUCTION:

On the top line of the scale of miles cut off a distance of 14.90 cm and divide it into 12 equal parts. Each part will represent 10 km. Subdivide the first part into 10 equal parts. Each subdivision will show single km.



R.F. = $1 / 804500$

COMPARATIVE SCALE SHOWING MILES AND KILOMETERS

COMPARATIVE SCALE:

EXAMPLE NO. 8 :

A motor car is running at a speed of 60 kph.
On a scale of $RF = 1 / 4,00,000$ show the distance traveled by car in 47 minutes.

SOLUTION STEPS:

Scale of km.

$$\begin{aligned} \text{length of scale} &= RF \times 60 \text{ km} \\ &= 1 / 4,00,000 \times 60 \times 10^5 \\ &= 15 \text{ cm.} \end{aligned}$$

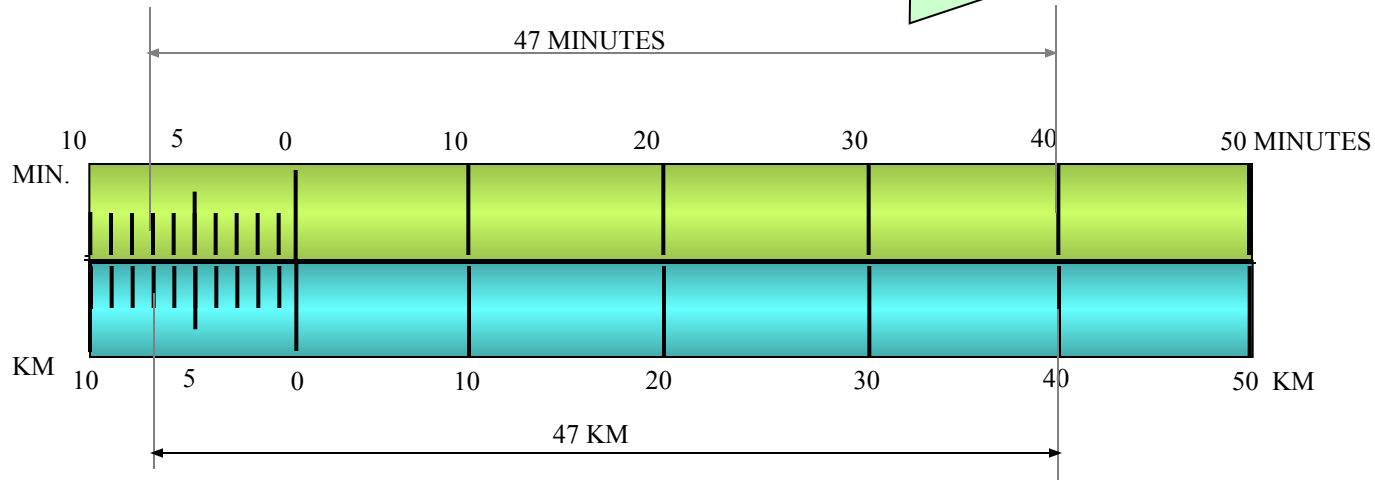
CONSTRUCTION:

Draw a line 15 cm long and divide it in 6 equal parts.
(each part will represent 10 km.)

Subdivide 1st part in 10 equal subdivisions.
(each will represent 1 km.)

Time Scale:

Same 15 cm line will represent 60 minutes.
Construct the scale similar to distance scale.
It will show minimum 1 minute & max. 60min.



$$R.F. = 1 / 4,00,000$$

COMPARATIVE SCALE SHOWING MINUTES AND KILOMETERS

EXAMPLE NO. 9 :

A car is traveling at a speed of 60 km per hour. A 4 cm long line represents the distance traveled by the car in two hours. Construct a suitable comparative scale up to 10 hours. The scale should be able to read the distance traveled in one minute. Show the time required to cover 476 km and also distance in 4 hours and 24 minutes.

SOLUTION:

4 cm line represents distance in two hours , means for 10 hours scale, 20 cm long line is required, as length of scale.This length of scale will also represent 600 kms. (as it is a distance traveled in 10 hours)

CONSTRUCTION:

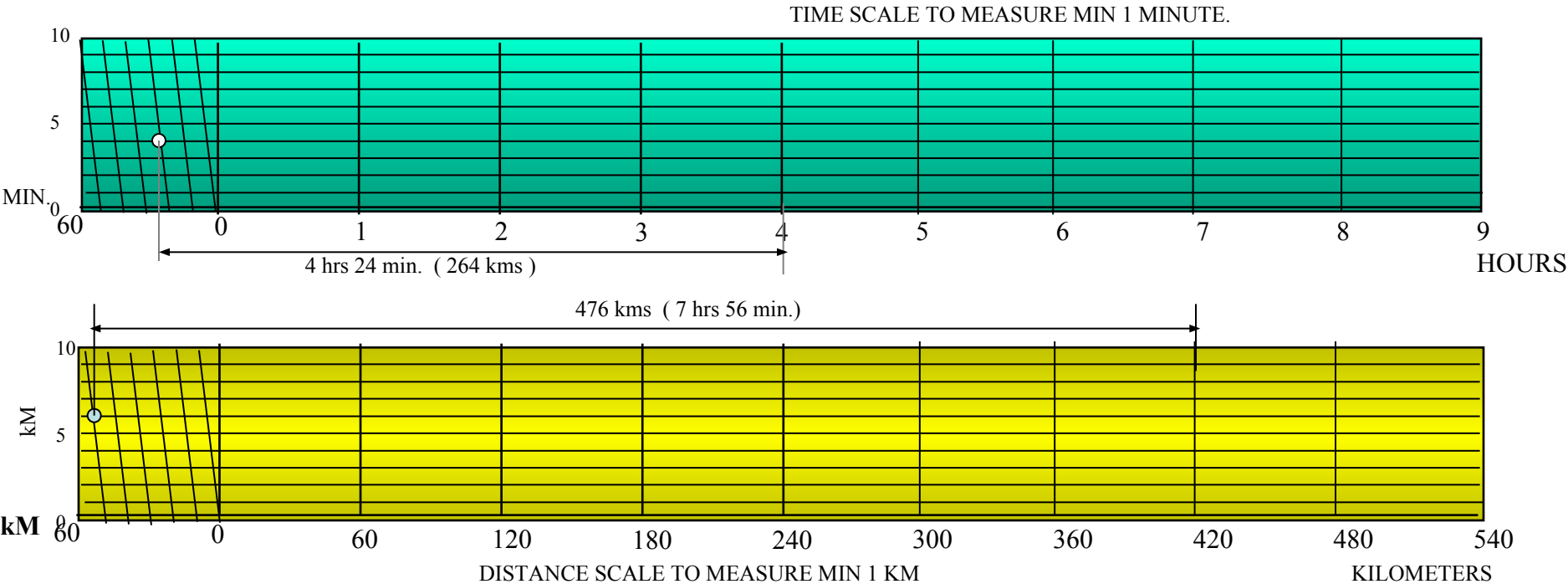
Distance Scale (km)

Draw a line 20 cm long. Divide it in TEN equal parts.(Each will show 60 km)
Sub-divide 1st part in SIX subdivisions.(Each will represent 10 km)
At the left hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.
And complete the diagonal scale to read minimum ONE km.

Time scale:

Draw a line 20 cm long. Divide it in TEN equal parts.(Each will show 1 hour) Sub-divide 1st part in SIX subdivisions.(Each will represent 10 minutes) At the left hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.
And complete the diagonal scale to read minimum ONE minute.

COMPARATIVE SCALE:

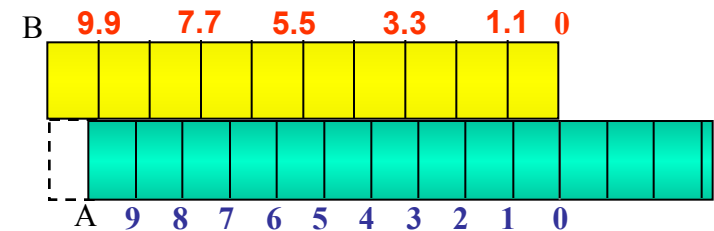


Vernier Scales:

These scales, like diagonal scales, are used to read to a very small unit with great accuracy. It consists of two parts – a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions.

As it would be difficult to sub-divide the minor divisions in ordinary way, it is done with the help of the vernier. The graduations on vernier are derived from those on the primary scale.

Figure to the right shows a part of a plain scale in which length A-O represents 10 cm. If we divide A-O into ten equal parts, each will be of 1 cm. Now it would not be easy to divide each of these parts into ten equal divisions to get measurements in millimeters.



Now if we take a length BO equal to $10 + 1 = 11$ such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent $11 / 10 = 1.1$ cm.

The difference between one part of AO and one division of BO will be equal $1.1 - 1.0 = 0.1$ cm or 1 mm.

This difference is called Least Count of the scale.

Minimum this distance can be measured by this scale.

The upper scale BO is the vernier. The combination of plain scale and the vernier is vernier scale.

Example 10:

Draw a vernier scale of RF = 1 / 25 to read centimeters upto 4 meters on it, and show lengths 2.39 m and 0.91 m

Vernier Scale

SOLUTION:

Length of scale = RF X max. Distance
 $= 1 / 25 \times 4 \times 100$
 $= 16 \text{ cm}$

CONSTRUCTION: (Main scale)

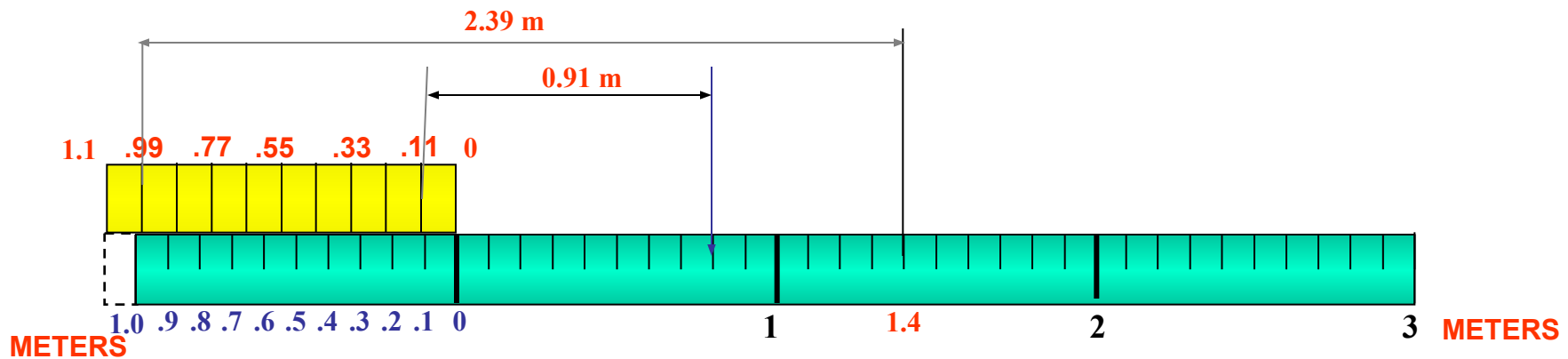
Draw a line 16 cm long.
 Divide it in 4 equal parts.
 (each will represent meter)
 Sub-divide each part in 10 equal parts.
 (each will represent decimeter)
 Name those properly.

CONSTRUCTION: (vernier)

Take 11 parts of Dm length and divide it in 10 equal parts.
 Each will show 0.11 m or 1.1 dm or 11 cm and construct a rectangle
 Covering these parts of vernier.

TO MEASURE GIVEN LENGTHS:

- (1) For 2.39 m : Subtract 0.99 from 2.39 i.e. $2.39 - .99 = 1.4 \text{ m}$
 The distance between 0.99 (left of Zero) and 1.4 (right of Zero) is 2.39 m
- (2) For 0.91 m : Subtract 0.11 from 0.91 i.e. $0.91 - 0.11 = 0.80 \text{ m}$
 The distance between 0.11 and 0.80 (both left side of Zero) is 0.91 m



Example 11: A map of size 500cm X 50cm wide represents an area of 6250 sq.Kms. Construct a vernier scale to measure kilometers, hectometers and decameters and long enough to measure upto 7 km. Indicate on it a) 5.33 km b) 59 decameters.

Vernier Scale

SOLUTION:

$$\begin{aligned} \text{RF} &= \sqrt{\frac{\text{AREA OF DRAWING}}{\text{ACTUAL AREA}}} \\ &= \sqrt{\frac{500 \times 50 \text{ cm sq.}}{6250 \text{ km sq.}}} \\ &= 2 / 10^5 \end{aligned}$$

Length of

$$\begin{aligned} \text{scale} &= \text{RF} \times \text{max. Distance} \\ &= 2 / 10^5 \times 7 \text{ kms} \\ &= 14 \text{ cm} \end{aligned}$$

CONSTRUCTION: (*Main scale*)

Draw a line 14 cm long.
Divide it in 7 equal parts.
(each will represent km)
Sub-divide each part in 10 equal parts.
(each will represent hectometer)
Name those properly.

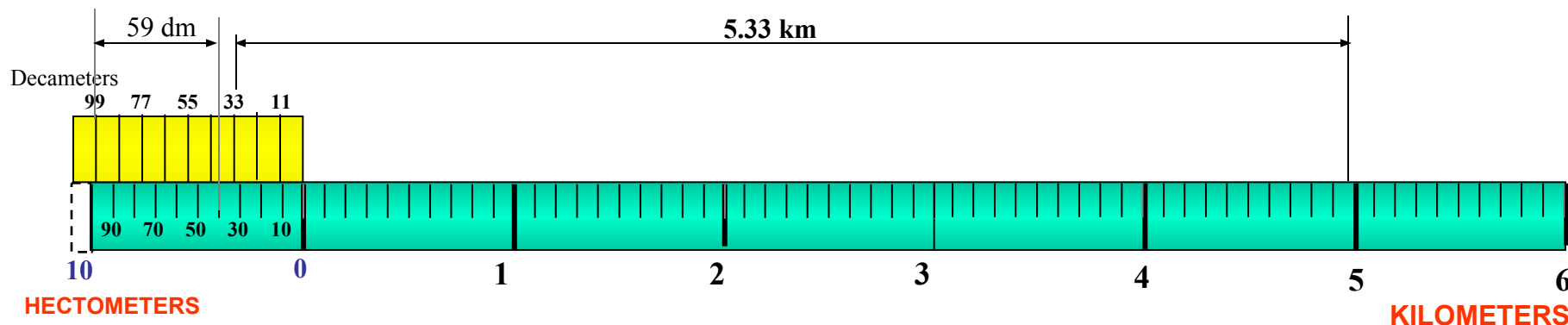
CONSTRUCTION: (*vernier*)

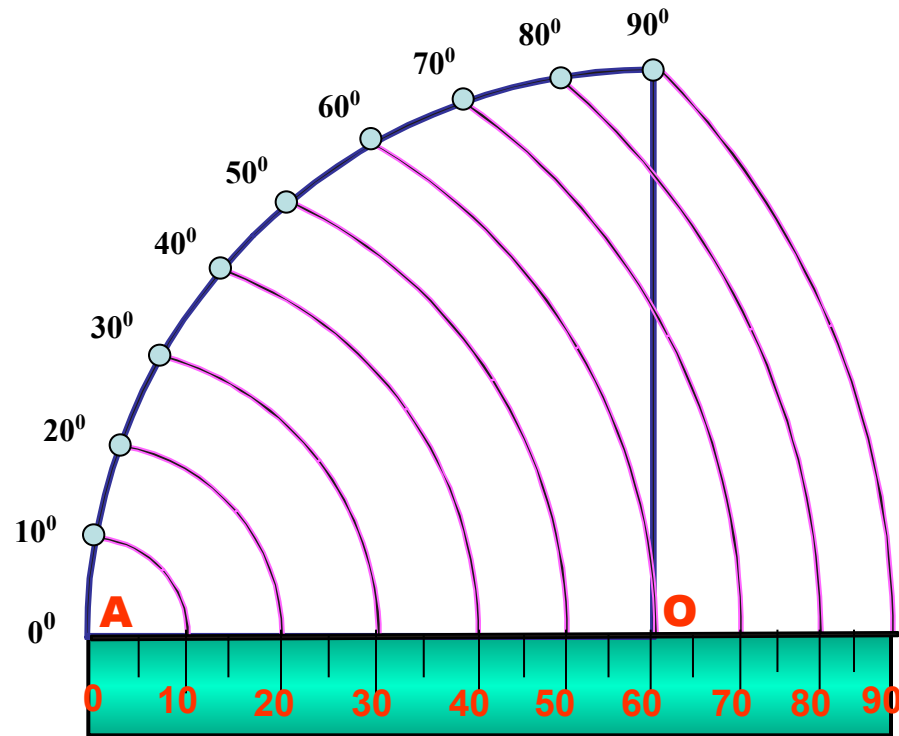
Take 11 parts of hectometer part length and divide it in 10 equal parts.
Each will show 1.1 hm m or 11 dm and
Covering in a rectangle complete scale.

TO MEASURE GIVEN LENGTHS:

a) For 5.33 km :
Subtract 0.33 from 5.33
i.e. $5.33 - 0.33 = 5.00$
The distance between 33 dm
(left of Zero) and
5.00 (right of Zero) is 5.33 k m

(b) For 59 dm :
Subtract 0.99 from 0.59
i.e. $0.59 - 0.99 = - 0.4 \text{ km}$
(- ve sign means left of Zero)
The distance between 99 dm and
- .4 km is 59 dm
(both left side of Zero)





SCALE OF CORDS

CONSTRUCTION:

1. DRAW SECTOR OF A CIRCLE OF 90° WITH ' OA ' RADIUS.
(' OA ' ANY CONVINIENT DISTANCE)
2. DIVIDE THIS ANGLE IN NINE EQUAL PARTS OF 10° EACH.
3. NAME AS SHOWN FROM END ' A ' UPWARDS.
4. FROM ' A ' AS CENTER, WITH CORDS OF EACH ANGLE AS RADIUS DRAW ARCS DOWNWARDS UP TO ' AO ' LINE OR IT'S EXTENSION AND FORM A SCALE WITH PROPER LABELING AS SHOWN.

AS CORD LENGTHS ARE USED TO MEASURE & CONSTRUCT
DIFERENT ANGLES IT IS CALLED SCALE OF CORDS.

PROBLEM 12: Construct any triangle and measure it's angles by using scale of cords.



CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then construct a triangle of given sides. (You are supposed to measure angles x, y and z)

To measure angle at x:

Take O-A distance in compass from cords scale and mark it on lower side of triangle as shown from corner **x**. **Name** O & A as shown. **Then** O as center, O-A radius draw an arc upto upper adjacent side. **Name** the point B.

Take A-B cord in compass and place on scale of cords from Zero.

It will give value of angle at x

To measure angle at y:

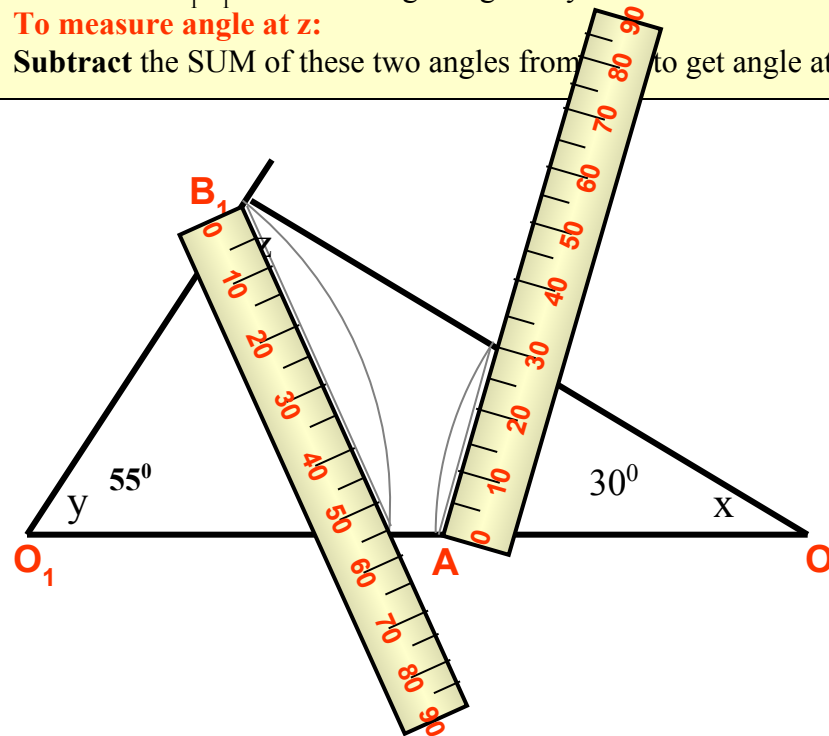
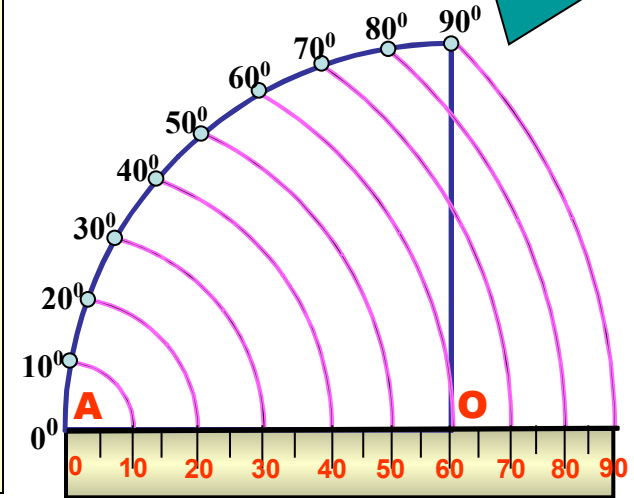
Repeat same process from O_1 . Draw arc with radius O_1A_1 .

Place Cord A_1B_1 on scale and get angle at y.

To measure angle at z:

Subtract the SUM of these two angles from 180 to get angle at z.

SCALE OF CORDS



$$\text{Angle at } z = 180 - (55 + 30) = 95^\circ$$

PROBLEM 12: Construct 25° and 115° angles with a horizontal line, by using scale of cords.

CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then Draw a horizontal line. Mark point O on it.

To construct 25° angle at O.

Take O-A distance in compass from cords scale and mark it on on the line drawn, from O

Name O & A as shown. **Then** O as center, O-A radius draw an arc upward..

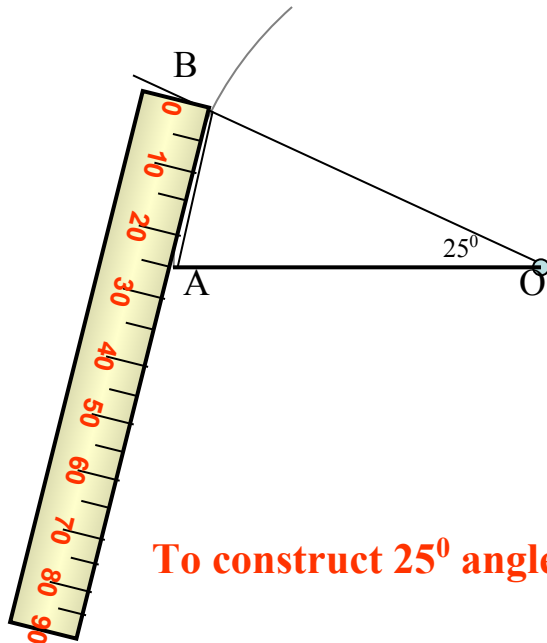
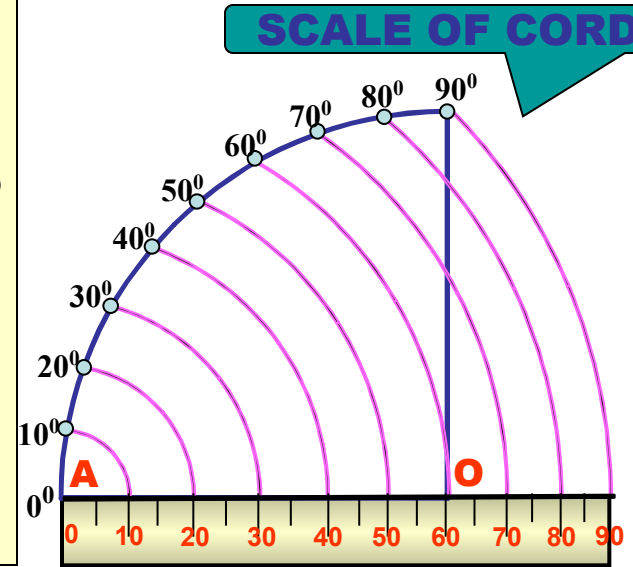
Take cord length of 25° angle from scale of cords in compass and from A cut the arc at point B. Join B with O. The angle AOB is thus 25°

To construct 115° angle at O.

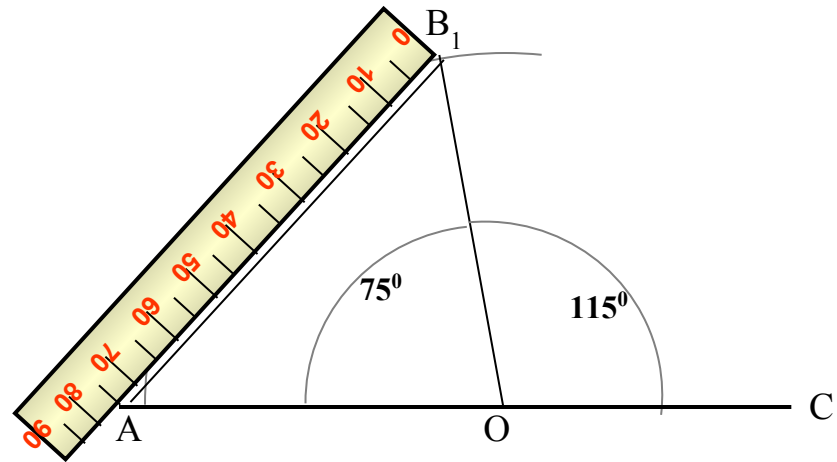
This scale can measure or construct angles upto 90° only directly.

Hence Subtract 115° from 180° . We get 75° angle, which can be constructed with this scale.

Extend previous arc of OA radius and taking cord length of 75° in compass cut this arc at B_1 with A as center. Join B_1 with O. Now angle AOB_1 is 75° and angle COB_1 is 115° .



To construct 25° angle at O.



To construct 115° angle at O.