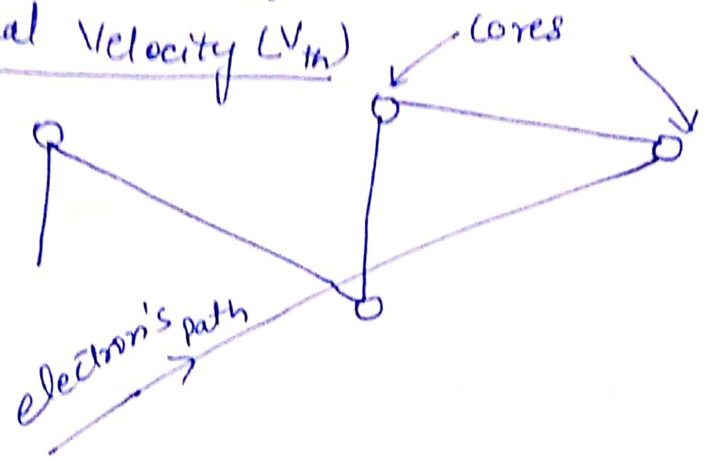
exam

Drift Velocity :- The average velocity of (aquired) occupied by the free electrons in a particular direction during the presence of an electric field.

$$V_d = \frac{eE\tau}{m}$$

Thermal Velocity (V_{th})



The velocity of electrons in random motion due to thermal agitation called thermal velocity.

Mean free path (λ) :- The average distance travelled by the conduction electrons b/w any two successive collisions with lattice ions.

exam

Relaxation time (τ_r) :- The relaxation time is defined as the time taken by a free electron to reach its equilibrium position from its disturbed position, during the presence of an applied field.

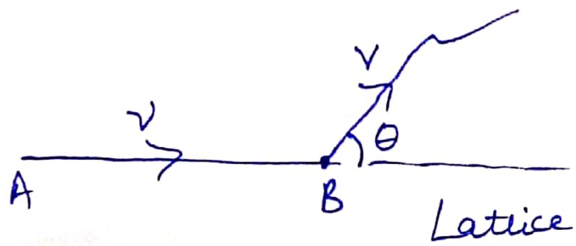
$$\tau = \frac{l}{\langle v \rangle}$$ where l is the distance travelled by the electron.

OR

From the instant of sudden disappearance of an electric field, across a metal, the average velocity of the conduction electrons decays exponentially to zero, and the time required in this process for the average velocity to reduce to $(1/e)$ times its value is known as Relaxation time.

$$\tau_r = \frac{\tau}{1-\langle\cos\theta\rangle}$$, τ is mean collision time

For isotropic scattering or symmetrical scattering $\langle\cos\theta\rangle = 0$ then $\tau_r = \tau$



Lattice scattering

<u>Mean collision time (τ)</u>:- The average time that elapses b/w two consecutive collisions of an $e^{\ominus}$ with the lattice points is called mean collision time.

$$\tau = \frac{\lambda}{V}$$ ← mean free path

<u>Resistivity (ρ)</u>:- Resistivity of the material is the reciprocal of electrical conductivity.

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau_r}$$

gmp exam
<u>Numerical</u>:- Find the relaxation time of conduction electron in a metal of resistivity $1.54\times10^{-8}\,\Omega m$, if the metal has 5.8×10^{28} conduction $e^{\ominus}/m^3$.

$\tau_r = ?$

$\rho = 1.54\times10^{-8}\,\Omega m$

$n = 5.8\times10^{28}\, e^{\ominus}/m^3$

$m = 9.11\times10^{-31}$ kg

$e = 1.602\times10^{-19}$ C

$$\rho = \frac{m}{ne^2\tau_r}$$

$$\Rightarrow \boxed{\tau_r = \frac{m}{ne^2\rho}}$$

(3.)

Q:- Obtain an expression for $\boxed{\text{thermal}}$ conductivity of a metal on the basis of free electron theory.

(OR)

Q:- Give Postulates (Salient features)/(Assumptions) of classical free electron theory (DRUDE - LORENTZ THEORY of Metals) and obtain expression for $\boxed{\text{electrical}}$ conductivity and drift Velocity.

Ans:- The $\boxed{\text{main assumptions}}$ of classical free $e^{\ominus}$ theory are:

(1.) A metal is imagined as the structure of 3-D array of ions in b/w which there are free moving valence electrons confined to the body of the material.

(2.) The free $e^{\ominus}$s (electron gas), available in a metal move freely here and there, but are restricted to jump out of the metal due to external forces

(3.) ~~These~~ Such freely moving $e^{\ominus}$s are the cause of conduction in the metal when it is subjected to a potential difference and also called the conduction electrons.

(4.) The free electrons are treated as equivalent to gas molecules & they are assumed to obey the laws of kinetic theory of gases,

$$\frac{3}{2} kT = \frac{1}{2} m v_{th}^2$$

thermal velocity

(5) The electric ~~potential~~ current due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

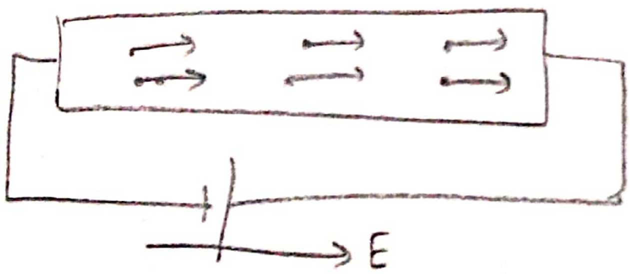
(6) Electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal.

→ The effect of repulsion b/w the e⊖s is considered insignificant.

(7) The free e⊖s are non-interacting & obey Pauli's exclusion principle.

Expression for electrical conductivity & Drift velocity :-

Consider a conductor which is subjected to an E.F of strength E



n = concentration of free e⊖s

m = mass of e⊖s

e = charge of e⊖s

According to Newton's second law of motion the force f acquired by e⊖s is equal to the force exerted by the field on the e⊖s.

∴ eqn of motion $ma = -eE$

$$\Rightarrow a = -\frac{eE}{m}$$

$$\Rightarrow \int a\,dt = \int \frac{-eE}{m}\,dt \quad \text{(Integrate)} \quad \left[a = \frac{dV}{dt}\right]$$

velocity of e^-

$$\Rightarrow V = \frac{-eE}{m}t + C$$

Integration const.

During the absence of the E.F., the average velocity of the $e^{\ominus}$ is zero,

$$t = 0, \quad \langle V \rangle = 0$$

$$\Rightarrow \boxed{V = \frac{-eEt}{m}}$$

average velocity b/w two successive collisions

$$\bar{V} = \frac{1}{\tau}\int_0^\tau \left(\frac{-eE}{m}t\right)dt$$

$$\bar{V} = \frac{-eE}{\tau m}\int_0^\tau t\,dt = \frac{-eE}{\tau m}\frac{\tau^2}{2} = \frac{-eE\tau}{2m}$$

$$\bar{V} = -\frac{eE\lambda}{2(mV)}$$

[relaxation time b/w two successive collisions $= \tau$
λ = mean free path of e^-s.
$\left[\tau = \frac{\lambda}{V}\right]$]

$$\bar{V} = \frac{-eE\lambda}{2\left(\frac{3kT}{V}\right)}$$

$\left[\because \frac{1}{2}mv^2 = \frac{3}{2}kT \Rightarrow mV = \frac{3kT}{V}\right]$

$$\bar{V} = \frac{-eE\lambda V}{6kT} \quad \text{--- (1)}$$

If n is no. density of e^-s in conductor then the current density J is :-

$$J = -en\bar{V}$$

$$= -en\left[\frac{-eE\lambda V}{6kT}\right] \quad \text{[from (1)]}$$

$$J = \frac{ne^2E\lambda V}{6kT} \quad \text{--- (2)}$$

If q charge is flowing through a conductor of cross-section area A in time t,

$$q = \sigma A E t$$

$$\frac{q}{t} = \sigma A E$$

$$\Rightarrow \boxed{I = \sigma A E} \Rightarrow \frac{I}{A} = \sigma E$$ [current density is directly proportional to applied E.F.]

$$\Rightarrow \boxed{J = \sigma E}$$

$$\sigma = \frac{J}{E}$$ [For unit area of cross-sec.]

$$\sigma = \frac{n e^2 \lambda v}{6kT}$$

⇒ different conductivities of diff. materials are due to different no. of free electrons.

Expression for thermal conductivity

Consider a metallic rod having two ends namely H & C ~~with the temp of the Hot end H~~

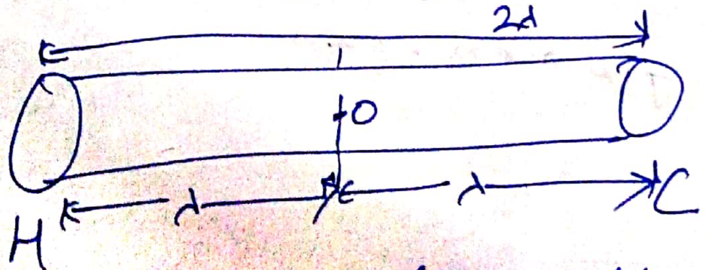
H – Hot end, C – Cold end

End H is at higher temp. than C

⇒ In collision, the e^-s near H loose their K.E while the e^-s near C gain the energy.

amount of heat Q passing through a cross-section of the rod per unit area per sec.

$$Q = \frac{1}{3} n v \lambda \frac{dE}{dt} \quad -(1)$$



λ = mean free path
v = velocity of e^-s

From kinetic theory of gases

$$E = \frac{3}{2}kT$$

$$\frac{dE}{dt} = \frac{3}{2}k\frac{dT}{dt}$$

$$\therefore \quad Q = \frac{1}{2}nv\lambda k\frac{dT}{dt} \qquad [\text{From } ①]$$

$$K\frac{dT}{dt} = \frac{1}{2}nv\lambda k\frac{dT}{dt} \qquad \left[\because Q = K\frac{dT}{dt}\right]$$

$$\boxed{K = \frac{1}{2}nv\lambda k}$$

Wiedemann-Franz relation :-

Good electrical conductors are also good thermal conductors. &

⇒ The ratio of thermal conductivity to the electrical conductivity is constant at any temperature (but not too low temperature) is constant for all metals.

$$\frac{K}{\sigma} = \text{constant}$$

$$\Rightarrow \frac{\frac{1}{2}nv\lambda k}{\frac{ne^2\lambda v}{6kT}} = \frac{1}{2}\frac{nv\lambda k \, 6kT}{ne^2\lambda v} = 3\left(\frac{k}{e}\right)^2 T$$

$$\Rightarrow \boxed{\frac{K}{\sigma} \propto T}$$

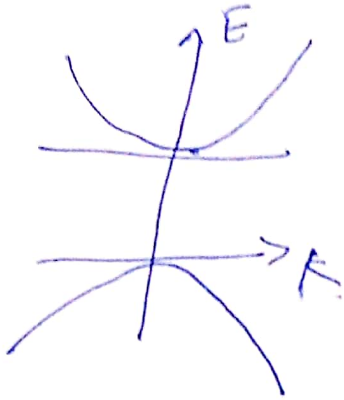
Density of States :- its Total no. of available electron states per unit Energy range at E.

Parabolic approximation

$$E = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$= E_V - \frac{\hbar^2 k^2}{2m_V}$$

$$\left[\because E_{KE} = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}\right]$$



Effective mass

$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$

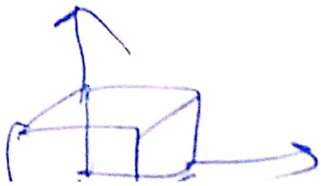
mass exhibited by $e^{\ominus}$ when inside a semiconductor

$\downarrow m^*$
$\uparrow$ mobility of $e^{\ominus}$s

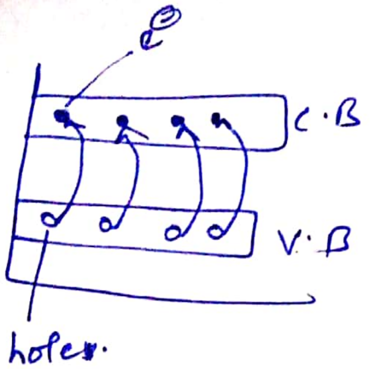
Bloch's Theorem :-

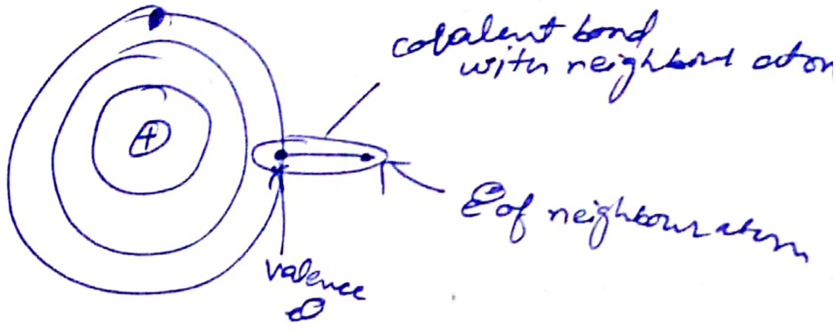
$$\psi_k = \underbrace{u_k(r)}_{\text{Periodic Bloch cell function}} \underbrace{e^{i\vec{k}\cdot\vec{r}}}_{\text{Plane wave}}$$

also



$e^{\ominus}$ hole pair





↑ temp
covalent bond break.
$e^{\ominus}$s are free to move.
→ called conduction $e^{\ominus}$s.

Effective mass of $e^{\ominus}$:

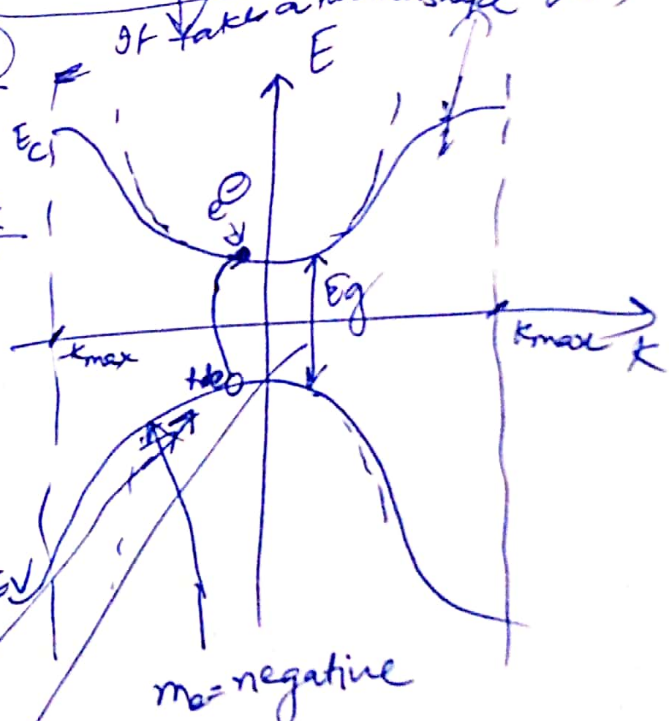
when we plot E with k
if take a parabola shape allowed energy E(k)

$$E = \frac{p^2}{2m^*} = \frac{\hbar^2 k^2}{2m^*}$$

$$\Rightarrow \frac{dE}{dk} = \frac{\hbar^2 2k}{2m^*} = \frac{\hbar^2 k}{m^*}$$

$$\Rightarrow \frac{d^2E}{dk^2} = \frac{\hbar^2}{m^*}$$

$$\Rightarrow \boxed{m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}}$$



$e^{\ominus}$ jump from valence band to C.B & leaves behind a hole

energy required to jump one $e^{\ominus}$ to ~~break a bond~~ ~~& convert~~ jump from valence band to reach conduction band.

The mass of an electron in a periodic potential of a crystal is different from the free electron mass & is usually referred to as the effective mass.