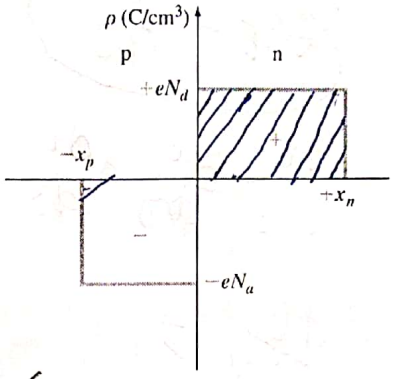

Figure 7.4 | The space charge density in a uniformly doped pn junction assuming the abrupt junction approximation.

7.2.2 Electric Field

An electric field is created in the depletion region by the separation of positive and negative space charge densities. Figure 7.4 shows the volume charge density distribution in the pn junction assuming uniform doping and assuming an abrupt junction approximation. We will assume that the space charge region abruptly ends in the n region at $x = +x_n$ and abruptly ends in the p region at $x = -x_p$ (x_p is a positive quantity).

The electric field is determined from Poisson's equation, which, for a one-dimensional analysis, is

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx} \tag{7.11}$$

where $\phi(x)$ is the electric potential, $E(x)$ is the electric field, $\rho(x)$ is the volume charge density, and ϵ_s is the permittivity of the semiconductor. From Figure 7.4, the charge densities are

$$\rho(x) = -eN_a \qquad -x_p < x < 0 \tag{7.12a}$$

and

$$\rho(x) = eN_d \qquad 0 < x < x_n \tag{7.12b}$$

The electric field in the p region is found by integrating Equation (7.11). We have

$$E = \int \frac{\rho(x)}{\epsilon_s}\,dx = -\int \frac{eN_a}{\epsilon_s}\,dx = \frac{-eN_a}{\epsilon_s}x + C_1 \tag{7.13}$$

where C_1 is a constant of integration. The electric field is assumed to be zero in the neutral p region for $x < -x_p$ since the currents are zero in thermal equilibrium. Since there are no surface charge densities within the pn junction structure, the electric

field is a continuous function. The constant of integration is determined by setting E = 0 at $x = -x_p$. The electric field in the p region is then given by

$$E = \frac{-eN_a}{\epsilon_s}(x + x_p) \qquad -x_p \le x \le 0 \tag{7.14}$$

In the n region, the electric field is determined from

$$E = \int \frac{(eN_d)}{\epsilon_s}\,dx = \frac{eN_d}{\epsilon_s}x + C_2 \tag{7.15}$$

where C_2 is again a constant of integration and is determined by setting E = 0 at $x = x_n$, since the E-field is assumed to be zero in the n region and is a continuous function. Then

$$E = \frac{-eN_d}{\epsilon_s}(x_n - x) \qquad 0 \le x \le x_n \tag{7.16}$$

The electric field is also continuous at the metallurgical junction, or at $x = 0$. Setting Equations (7.14) and (7.16) equal to each other at $x = 0$ gives

$$N_a x_p = N_d x_n \tag{7.17}$$

Equation (7.17) states that the number of negative charges per unit area in the p region is equal to the number of positive charges per unit area in the n region.

Figure 7.5 is a plot of the electric field in the depletion region. The electric field direction is from the n to the p region, or in the negative x direction for this geometry. For the uniformly doped pn junction, the E-field is a linear function of distance through the junction, and the maximum (magnitude) electric field occurs at the metallurgical junction. An electric field exists in the depletion region even when no voltage is applied between the p and n regions.

The potential in the junction is found by integrating the electric field. In the p region then, we have

$$\phi(x) = -\int E(x)dx = \int \frac{eN_a}{\epsilon_s}(x + x_p)dx \tag{7.18}$$

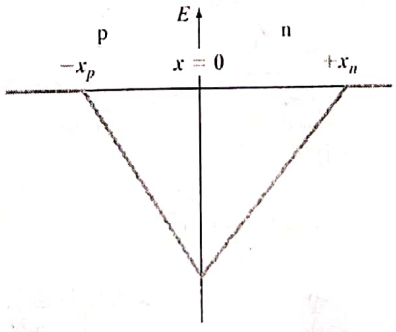

Figure 7.5 | Electric field in the space charge region of a uniformly doped pn junction.

or

$$\phi(x) = \frac{eN_a}{\epsilon_s}\left(\frac{x^2}{2} + x_p \cdot x\right) + C_1' \tag{7.19}$$

where C_1' is again a constant of integration. The potential difference through the pn junction is the important parameter, rather than the absolute potential, so we may arbitrarily set the potential equal to zero at $x = -x_p$. The constant of integration is then found as

$$C_1' = \frac{eN_a}{2\epsilon_s}x_p^2 \tag{7.20}$$

so that the potential in the p region can now be written as

$$\phi(x) = \frac{eN_a}{2\epsilon_s}(x + x_p)^2 \qquad (-x_p \le x \le 0) \tag{7.21}$$

The potential in the n region is determined by integrating the electric field in the n region, or

$$\phi(x) = \int \frac{eN_d}{\epsilon_s}(x_n - x)dx \tag{7.22}$$

Then

$$\phi(x) = \frac{eN_d}{\epsilon_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2' \tag{7.23}$$

where C_2' is another constant of integration. The potential is a continuous function, so setting Equation (7.21) equal to Equation (7.23) at the metallurgical junction, or at $x = 0$, gives

$$C_2' = \frac{eN_a}{2\epsilon_s}x_p^2 \tag{7.24}$$

The potential in the n region can thus be written as

$$\phi(x) = \frac{eN_d}{\epsilon_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + \frac{eN_a}{2\epsilon_s}x_p^2 \qquad (0 \le x \le x_n) \tag{7.25}$$

Figure 7.6 is a plot of the potential through the junction and shows the quadratic dependence on distance. The magnitude of the potential at $x = x_n$ is equal to the built-in potential barrier. Then from Equation (7.25), we have

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right) \tag{7.26}$$

The potential energy of an electron is given by $E = -e\phi$, which means that the electron potential energy also varies as a quadratic function of distance through the space charge region. The quadratic dependence on distance was shown in the energy-band diagram of Figure 7.3, although we did not explicitly know the shape of the curve at that time.

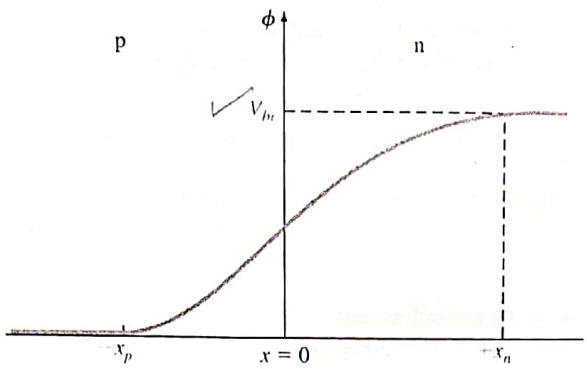

Figure 7.6 | Electric potential through the space charge region of a uniformly doped pn junction.

7.2.3 Space Charge Width

We can determine the distance that the space charge region extends into the p and n regions from the metallurgical junction. This distance is known as the space charge width. From Equation (7.17), we may write, for example,

$$x_p = \frac{N_d x_n}{N_a} \tag{7.27}$$

Then, substituting Equation (7.27) into Equation (7.26) and solving for x_n, we obtain

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2} \tag{7.28}$$

Equation (7.28) gives the space charge width, or the width of the depletion region, x_n extending into the n-type region for the case of zero applied voltage.

Similarly, if we solve for x_n from Equation (7.17) and substitute into Equation (7.26), we find

$$x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2} \tag{7.29}$$

where x_p is the width of the depletion region extending into the p region for the case of zero applied voltage.

The total depletion or space charge width W is the sum of the two components, or

$$W = x_n + x_p \tag{7.30}$$

Using Equations (7.28) and (7.29), we obtain

$$W = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2} \tag{7.31}$$

The built-in potential barrier can be determined from Equation (7.10), and then the total space charge region width is obtained using Equation (7.31).

EXAMPLE 7.2

Objective: Calculate the space charge width and electric field in a pn junction for zero bias.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

Solution

In Example 7.1, we determined the built-in potential barrier as $V_{bi} = 0.635$ V. From Equation (7.31), the space charge width is

$$W = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 0.951 \times 10^{-4} \text{ cm} = 0.951\ \mu\text{m}$$

Using Equations (7.28) and (7.29), we can find $x_n = 0.8644\ \mu$m, and $x_p = 0.0864\ \mu$m.

The peak electric field at the metallurgical junction, using Equation (7.16) for example, is

$$\text{E}_{\max} = -\frac{eN_d x_n}{\epsilon_s} = -\frac{(1.6 \times 10^{-19})(10^{15})(0.8644 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} = -1.34 \times 10^4 \text{ V/cm}$$

Comment

The peak electric field in the space charge region of a pn junction is quite large. We must keep in mind, however, that there is no mobile charge in this region; hence there will be no drift current. We may also note, from this example, that the width of each space charge region is a reciprocal function of the doping concentration: The depletion region will extend further into the lower-doped region.

EXERCISE PROBLEM

Ex 7.2 A silicon pn junction at $T = 300$ K with zero applied bias has doping concentrations of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine x_n, x_p, W, and $|\text{E}_{\max}|$.

(Ans. $x_n = 4.11 \times 10^{-6}$ cm, $x_p = 4.11 \times 10^{-5}$ cm, $W = 4.52 \times 10^{-5}$ cm, $|\text{E}_{\max}| = 3.18 \times 10^4$ V/cm)

TEST YOUR UNDERSTANDING

TYU 7.1 Calculate V_{bi}, x_n, x_p, W, and $|\text{E}_{\max}|$ for a silicon pn junction at zero bias and $T = 300$ K for doping concentrations of (*a*) $N_a = 2 \times 10^{17}$ cm^{-3}, $N_d = 10^{16}$ cm^{-3} and (*b*) $N_a = 4 \times 10^{15}$ cm^{-3}, $N_d = 3 \times 10^{16}$ cm^{-3}.

[Ans. (*a*) $V_{bi} = 0.772$ V, $x_n = 0.3085\ \mu$m, $x_p = 0.0154\ \mu$m, $W = 0.3240\ \mu$m, $|\text{E}_{\max}| = 4.77 \times 10^4$ V/cm; (*b*) $V_{bi} = 0.699$ V, $x_n = 0.0596\ \mu$m, $x_p = 0.4469\ \mu$m, $W = 0.5064\ \mu$m, $|\text{E}_{\max}| = 2.76 \times 10^4$ V/cm]

TYU 7.2 Repeat Exercise Problem Ex 7.2 for a GaAs pn junction.

(Ans. $V_{bi} = 1.186$ V, $x_n = 0.05590\ \mu$m, $x_p = 0.5590\ \mu$m, $W = 0.6149\ \mu$m, $|\text{E}_{\max}| = 3.86 \times 10^4$ V/cm)

7.3 | REVERSE APPLIED BIAS

If we apply a potential between the p and n regions, we will no longer be in an equilibrium condition—the Fermi energy level will no longer be constant through the system. Figure 7.7 shows the energy-band diagram of the pn junction for the case when a positive voltage is applied to the n region with respect to the p region. As the positive potential is downward, the Fermi level on the n side is below the Fermi level on the p side. The difference between the two is equal to the applied voltage in units of energy.

The total potential barrier, indicated by V_{total}, has increased. The applied potential is the reverse-biased condition. The total potential barrier is now given by

$$V_{total} = |\phi_{Fn}| + |\phi_{Fp}| + V_R \tag{7.32}$$

where V_R is the magnitude of the applied reverse-biased voltage. Equation (7.32) can be rewritten as

$$V_{total} = V_{bi} + V_R \tag{7.33}$$

where V_{bi} is the same built-in potential barrier we had defined in thermal equilibrium.

7.3.1 Space Charge Width and Electric Field

Figure 7.8 shows a pn junction with an applied reverse-biased voltage V_R. Also indicated in the figure are the electric field in the space charge region and the electric field E_{app}, induced by the applied voltage. The electric fields in the neutral p and n regions are essentially zero, or at least very small, which means that the magnitude of the electric field in the space charge region must increase above the thermal-equilibrium value due to the applied voltage. The electric field originates on positive charge and terminates on negative charge; this means that the number of positive and negative

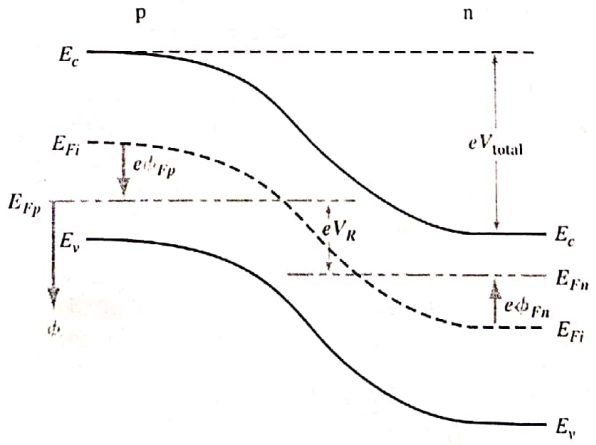

Figure 7.7 | Energy-band diagram of a pn junction under reverse bias.

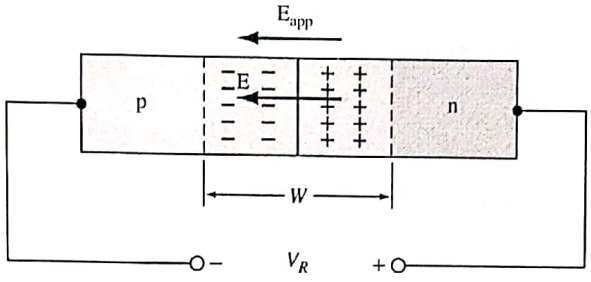

Figure 7.8 | A pn junction, with an applied reverse-biased voltage, showing the directions of the electric field induced by V_R and the space charge electric field.

charges must increase if the electric field increases. For given impurity doping concentrations, the number of positive and negative charges in the depletion region can be increased only if the space charge width W increases. The space charge width W increases, therefore, with an increasing reverse-biased voltage V_R. We are assuming that the electric field in the bulk n and p regions is zero. This assumption will become clearer in the next chapter when we discuss the current–voltage characteristics.

In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier. The total space charge width can be written from Equation (7.31) as

$$W = \left\{\frac{2\epsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2} \tag{7.34}$$

showing that the total space charge width increases as we apply a reverse-biased voltage. By substituting the total potential barrier V_{total} into Equations (7.28) and (7.29), the space charge widths in the n and p regions, respectively, can be found as a function of applied reverse-biased voltage.

EXAMPLE 7.3

Objective: Calculate the width of the space charge region in a pn junction when a reverse-biased voltage is applied.

Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Assume that $n_i = 1.5 \times 10^{10}$ cm^{-3} and $V_R = 5$ V.

■ Solution

The built-in potential barrier was calculated in Example 7.1 for this case and is $V_{bi} = 0.635$ V. The space charge width is determined from Equation (7.34). We have

$$W = \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

so that

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83\ \mu\text{m}$$