4. Discuss the failure of classical free electron theory.

(Jan-2013, Jan-2009, Jan-2008, July-2008, July-2011)

Ans. Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

(i) Specific heat: The molar specific heat of a gas at constant volume is

$$\boxed{C_V = \frac{3}{2} R}$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was $\boxed{C_V = 10^{-4} RT}$

Which is for lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

(ii) Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T

i.e. $$\sigma_{exp} \, \alpha \frac{1}{T} \qquad \text{—— (1)}$$

According to the assumptions of classical free electron theory,

$$\frac{3}{2} KT = \frac{1}{2} m v^2_{th}$$

$$V_{th} = \sqrt{\frac{3KT}{m}}$$

i.e. $$V_{th} \, \alpha \sqrt{T}$$

The mean collision time 'τ' is inversely proportional to the thermal velocity

i.e. $$\tau \, \alpha \frac{1}{V_{th}} \quad \text{or} \quad \tau \, \alpha \frac{1}{\sqrt{T}}$$

But $$\sigma = \frac{ne^2\tau}{m}$$

$$\sigma \, \alpha \, \tau$$

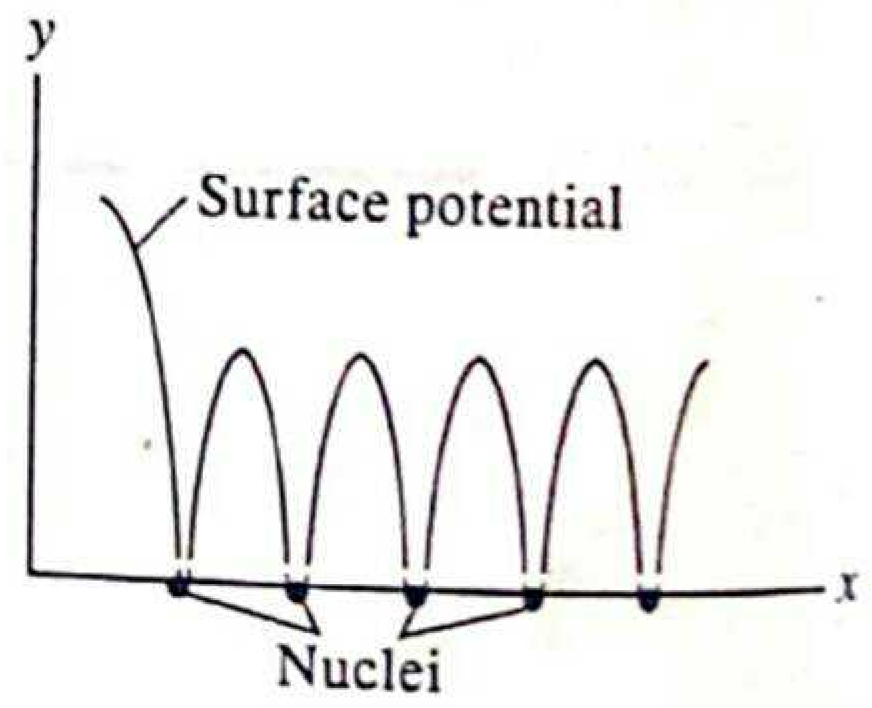
or $$\sigma \, \alpha \frac{1}{\sqrt{T}} \qquad \text{—— (2)}$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

Failures

- Never explain the difference between metal, semiconductor, insulators.
- It tells that F=0 which is not actually true. When an electron passes near an ion , a force acts on it.
- V= constant (by theory) , but actually a periodic change in potential occurs when electron pass from one place to the other.

$$V \propto \frac{1}{r}$$



- It does not explain the concept of specific heat, temperature dependence of electrical conductivity and dependence of electrical conductivity on concentration of electrons.

5. State the assumptions of quantum free electron theory. (July-2003)

Ans. 1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.

2. The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.

3. The electrons travel with a constant potential inside the metal but confined within its boundaries.

4. The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

5. With the increase of temperature the energy levels below E_F are vacated and above E_F are occupied.

6. The distribution electrons among the different energy levels at any temperature is given by Fermi-Dirac distribution function f(E). It is defined as $f(E) = \dfrac{1}{1+\exp\left(\dfrac{E-E_F}{KT}\right)}$.

7. Electrons are treated as wave-like particles.

2(A)5. Flow of Electric Charges in a Metallic Conductor

In a conductor, free electrons move randomly[4] with a thermal speed of the order of 10^5 to 10^6 m s^{-1} at room temperature. The random motion of an electron in a conductor is shown in Figure 3. In any portion of the conductor, the flow of electrons is so oriented that the average thermal velocity of total number of elecrons in the conductor is zero. As a result of this, the net flow of charge at the *given cross section is zero*. Hence, there is no net flow of current in the conductor.

In short, in the absence of external electric field, the motion of the electrons in the conductor is random such that the *average thermal velocity* of electrons becomes zero *i.e.* $\vec{u} = 0$.

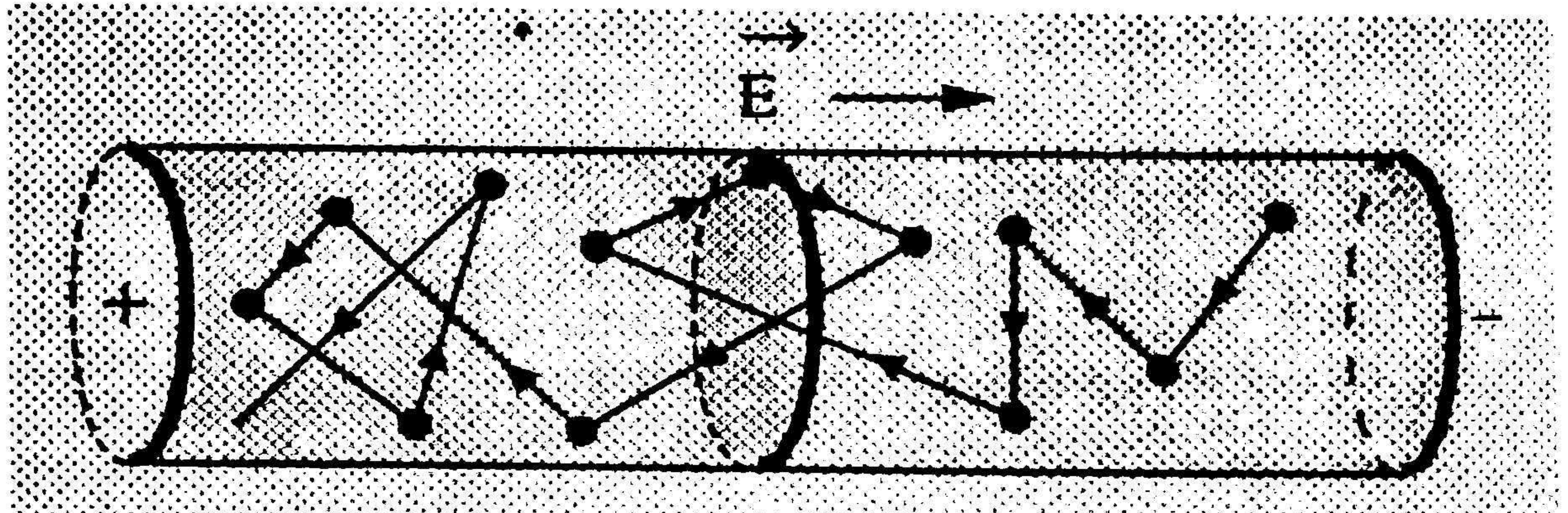


Figure 3

When an electric field is applied across the conductor, the free electrons accelerate in a direction opposite to the direction of the applied field. Due to this acceleration, the electrons gain extra velocity but for a short time because the accelerated electrons collide[5] with other free electrons or the ions in the conductor and during this collision, the extra velocity gained is destroyed. As a net result, the electrons acquire a small velocity called *drift velocity* ($\vec{\upsilon}_d$) in the direction opposite to that of the applied electric field.

Note

When steady current flows through a conductor, the motion of electrons in a conductor has no net acceleration because of steady drift velocity of the electrons.

2(A)7. Relation between Drift Velocity and Electric Current

Consider a conductor of length l and uniform cross-sectional area A. Let V be the applied potential difference across the ends of the conductor (Figure 4). The magnitude of electric field set up across the conductor is given by

$$E = \frac{V}{l}$$

Let n be the number of free electrons per unit volume of the conductor.

Then, total number of free electrons in the conductor = $n \times$ volume of the conductor

$$N \quad = n \times Al.$$

If e is the magnitude of charge on each electron then the total charge in the conductor,

$$\boxed{Q = (nAl)e} \qquad \ldots(7)$$

The time taken by the charge to cross the conductor length is given by

$$t = \frac{l}{v_d}$$

where v_d is drift velocity of electrons

According to the definition of electric current,

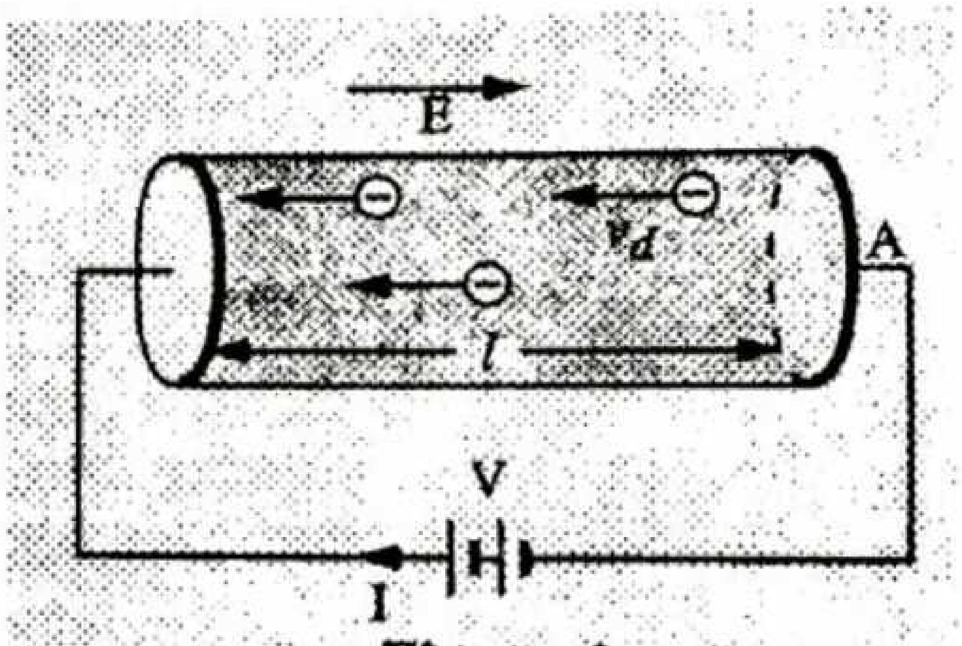


Figure 4

$$I = \frac{Q}{t} = \frac{nAle}{l / v_d} = neA\, v_d$$

or

$$\boxed{I = neA\, v_d} \qquad \ldots(8)$$

or

$$I \propto v_d \qquad \textit{(as n, e and A are fixed)}$$

Again from equation (8), $$\boxed{v_d = \frac{I}{neA}} \qquad \ldots(9)$$

Current Density and Drift Velocity

Eqn. (8) can be written as $$\frac{I}{A} = nev_d$$

But $$\frac{I}{A} = J \text{ i.e. current density}$$

$\therefore$ $$\boxed{J = nev_d} \qquad \ldots(10)$$

1.05. ELECTRON MOBILITY

The mobility of free electrons in a conductor is defined as the drift velocity acquired per unit strength of the electric field applied across the conductor. It is denoted by μ.

If v_d is drift velocity attained by free electrons on applying electric field E, then electron mobility is given by

$$\mu = \frac{v_d}{E} \qquad \text{...(1.11)}$$

$v_d : \mu E$

Now,
$$v_d = \frac{eE}{m}\tau \qquad \text{(in magnitude)} \qquad \text{...(1.12)}$$

From the equations (1.11) and (1.12), we have

$$\mu = \frac{e\tau}{m} \qquad \text{...(1.13)}$$

The equations (1.11) and (1.13) are the expressions for electron mobility.

In the equation (1.09), substituting for v_d $(= \mu E)$, we have

$$I = n A \mu E e \qquad \text{...(1.14)}$$

The equation (1.14) gives the relation between electron mobility and the current through the conductor.

From the equation (1.11), it follows that in SI, the unit of electron mobility is **metre^2 volt^{-1} second^{-1} (m^2 V^{-1} s^{-1}).**

Mobility (μ)

It is the ratio of the drift velocity (υ_d) of current carrier to the applied electric field (E) i.e.

$$\mu = \frac{\upsilon_d}{E}$$

Using relation (6), we get $$\mu = \frac{\upsilon_d}{E} = \frac{e\tau}{m}$$

For example, in a semiconductor mobility of an electron is more than the mobility of a hole because electron is lighter than the hole.

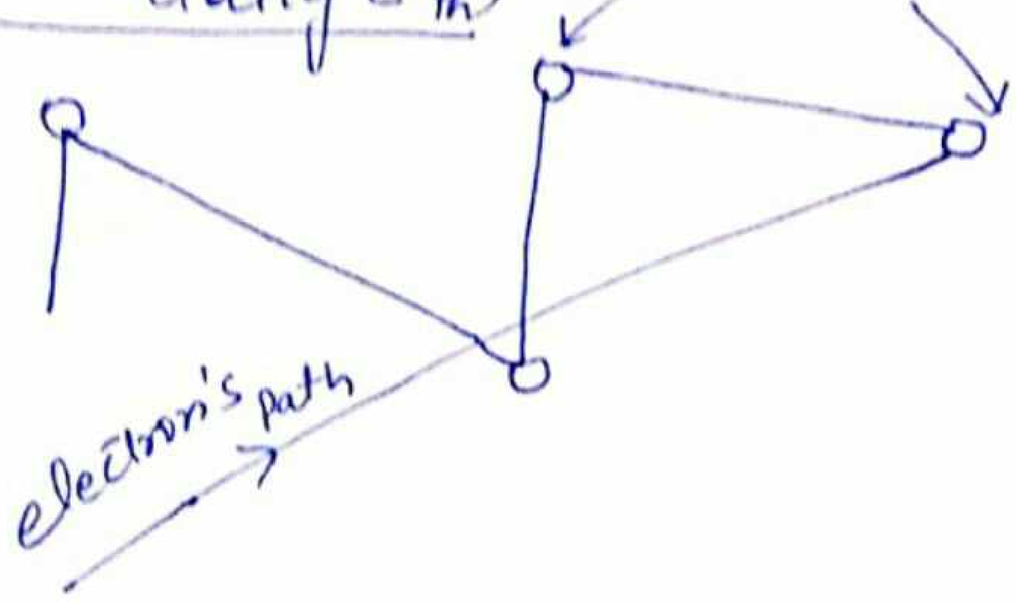
Mean collision time (τ) :- The average time that elapses b/w two consecutive collisions of an e^- with the lattice points is called mean collision time.

$$\tau = \frac{\lambda}{v}$$ ← mean free path

Resistivity (ρ) :- Resistivity of the material is the reciprocal of electrical conductivity.

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau_r}$$

Thermal Velocity (V_{th})



The velocity of electrons in random motion due to thermal agitation called thermal velocity.

Mean free path (λ):– The average distance travelled by the conduction electrons b/w any two successive collisions with lattice ions.

exam

Relaxation time (τ_r):– The relaxation time is defined as the time taken by a free electron to reach its equilibrium position from its disturbed position, during the presence of an applied field.

$$\tau = \frac{l}{\langle v \rangle}$$ where l is the distance travelled by the electron.

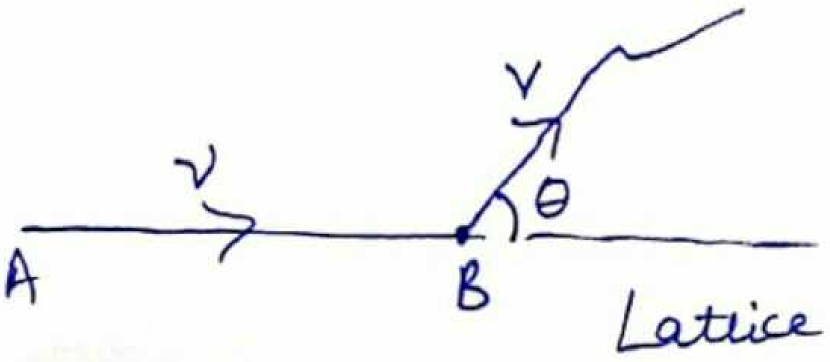
OR

From the instant of sudden disappearance of an electric field, across a metal, the average velocity of the conduction electrons decays exponentially to zero, and the time required in this process for

the average velocity to reduce to $(1/e)$ times its value is known as Relaxation time.

$$\tau_r = \frac{\tau}{1 - \langle \cos\theta \rangle}$$, τ is mean collision time

For isotropic scattering or symmetrical scattering $\langle \cos\theta \rangle = 0$ then $\tau_r = \tau$



Latice scattering