SELF DRIVING CARS

Introduction

In 2014 there were 32,675 traffic related fatalities, 2.3 million injuries, and 6.1 million reported collisions. Of these, an estimated 94% are attributed to driver error with 31% involving legally intoxicated drivers, and 10% from distracted drivers.

Autonomous vehicles have the potential to dramatically reduce the contribution of driver error and negligence as the cause of vehicle collisions. They will also provide a means of personal mobility to people who are unable to drive due to physical or visual disability.

The next major milestone in driverless vehicle technology was the first DARPA Grand Challenge in 2004. The objective was for a driverless car to navigate a 150-mile off-road course as quickly as possible. None of the 15 vehicles entered into the event completed the race. In 2005 a similar event was held; this time 5 of 23 teams reached the finish line. Later, in 2007, the DARPA Urban Challenge was held, in which vehicles were required to drive autonomously in a simulated urban setting. Six teams finished the event demonstrating that fully autonomous urban driving is possible.

The availability of on-board computation and wireless communication technology allows cars to exchange information with other cars and with the road infrastructure giving rise to a closely related area of research on connected intelligent vehicles.

Overview of the Decision-Making Hierarchy used in Driverless Cars

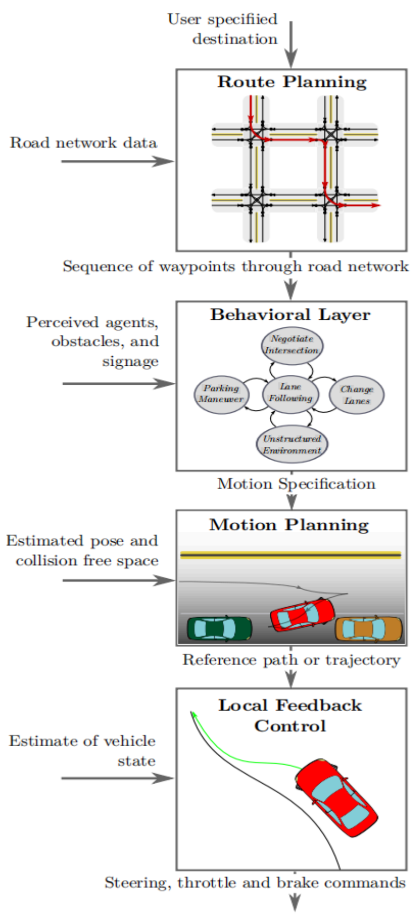
Driverless cars are essentially autonomous decision-making systems that process a stream of observations from on-board sensors such as radars, LIDARs, cameras, GPS/INS units, and odometry. These observations, together with prior knowledge about the road network, rules of the road, vehicle dynamics, and sensor models, are used to automatically select values for controlled variables governing the vehicle's motion.

A. Route Planning

At the highest level, a vehicle's decision-making system must select a route through the road network from its current position to the requested destination. By representing the road network as a directed graph with edge weights corresponding to the cost of traversing a road segment, such a route can be formulated as the problem of finding a minimum-cost path on a road network graph. The graphs representing road networks can however contain millions of edges making classical shortest path algorithms such as Dijkstra or A* impractical.

B. Behavioral Decision Making

After a route plan has been found, the autonomous vehicle must be able to navigate the selected route and interact with other traffic participants according to driving conventions and rules of the road. Given a sequence of road segments specifying the selected route, the behavioral layer is responsible for selecting an appropriate driving behavior at any point of time based on the perceived behavior of other traffic participants, road conditions, and signals from infrastructure.



C. Motion Planning

When the behavioral layer decides on the driving behavior to be performed in the current context, which could be, e.g., cruise-in-lane, change-lane, or turn-right, the selected behavior has to be translated into a path or trajectory that can be tracked by the low-level feedback controller. The resulting path or trajectory must be dynamically feasible for the vehicle, comfortable for the passenger, and avoid collisions with obstacles detected by the on-board sensors.

D. Vehicle Control

In order to execute the reference path or trajectory from the motion planning system a feedback controller is used to select appropriate actuator inputs to carry out the planned motion and correct tracking errors. The tracking errors generated during the execution of a planned motion are due in part to the inaccuracies of the vehicle model.

Modeling for Planning and Control

In this section we will survey the most commonly used models of mobility of car-like vehicles. Such models are widely used in control and motion planning algorithms to approximate a vehicle's behavior in response to control actions in relevant operating conditions.

A. The Kinematic Single-Track Model

In the most basic model of practical use, the car consists of two wheels connected by a rigid link and is restricted to move in a plane. It is assumed that the wheels do not slip at their contact point with the ground, but can rotate freely about their axes of rotation. The front wheel has an added degree of freedom where it is allowed to rotate about an axis normal to the plane of motion. This is to model steering. These two modeling features reflect the experience most passengers have where the car is unable to make lateral displacement without simultaneously moving forward. More formally, the limitation on maneuverability is referred to as a *nonholonomic* constraint. The nonholonomic constraint is expressed as a differential constraint on the motion of the car.

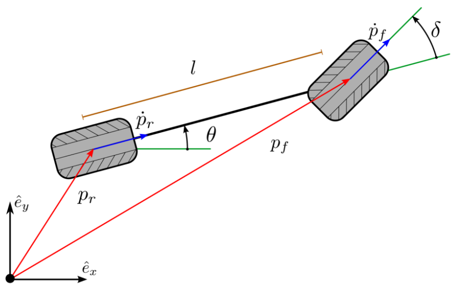


Figure III.1: Kinematics of the single track model. p_r and p_f are the ground contact points of the rear and front tire respectively. θ is the vehicle heading. Time derivatives of p_r and p_f are restricted by the nonholonomic constraint to the direction indicated by the blue arrows. δ is the steering angle of the front wheel.

The motion of the points p_r and p_f must be collinear with the wheel orientation to satisfy the no-slip assumption. Expressed as an equation, this constraint on the rear wheel is

$$(\dot{p}_r \cdot \hat{e}_y)\cos(\theta) - (\dot{p}_r \cdot \hat{e}_x)\sin(\theta) = 0, \tag{III.1}$$

and for the front wheel:

$$(\dot{p}_f \cdot \hat{e}_y)\cos(\theta + \delta) - (\dot{p}_f \cdot \hat{e}_x)\sin(\theta + \delta) = 0. \tag{III.2}$$

$$\begin{aligned} \dot{x}_f &= v_f \cos(\theta + \delta), & \dot{x}_r &= v_r \cos(\theta), \\ \dot{y}_f &= v_f \sin(\theta + \delta), & \dot{y}_r &= v_r \sin(\theta), \\ \dot{\theta} &= \tfrac{v_f}{l} \sin(\delta), & \dot{\theta} &= \tfrac{v_r}{l} \tan(\delta). \end{aligned}$$

$$\frac{v_r}{v_f} = \cos(\delta). \qquad \delta = \arctan\left(\frac{l\omega}{v_r}\right),$$

The planning and control problems for this model involve selecting the steering angle δ within the mechanical limits of the vehicle $\delta \in [\delta_{min}, \delta_{max}]$, and forward speed v_r within an acceptable range, $v_r \in [v_{min}, v_{max}]$.

$$\dot{\theta} = \omega, \quad \omega \in \left[\frac{v_r}{l}\tan\left(\delta_{min}\right), \frac{v_r}{l}\tan\left(\delta_{max}\right)\right].$$

In this situation, the model is sometimes referred to as the unicycle model since it can be derived by considering the motion of a single wheel. An important variation of this model is the case when v_r is fixed. This is sometimes referred to as the **Dubins car**, after Lester Dubins who derived the minimum time motion between two points with prescribed tangents. Another notable variation is the **Reeds-Shepp car** for which minimum length paths are known when v_r takes a single forward and reverse speed.
The kinematic models are suitable for planning paths at low speeds (e.g. parking maneuvers and urban driving) where inertial effects are small in comparison to the limitations on mobility imposed by the no-slip assumption. A major drawback of this model is that it permits instantaneous steering angle changes which can be problematic if the motion planning module generates solutions with such instantaneous changes.

2. MEASUREMENTS

A measurement provides information about the state of the system. Sensors are those components of the robot that provide measurements

SENSOR CHARACTERISTICS

Passive/Active
Passive sensors measure energy coming to the sensor from the environment (eg. vision)
Active sensors emit energy and measure the reaction of the environment (eg. SODAR sound pulse)

Range
Lower and upper limits of sensor inputs
Set of all possible states that can be measured

Dynamic Range
Defined by spread between lowest and highest inputs, often recorded in decibels (dB)

Important to understand what happens when a sensor receives an input that is out of range e.g. image pixel, LIDAR

Full Scale
Lower and upper limits of sensor output values
Difference between min and max output

Linearity
A linear relationship between input and output is desirable for sensors
Linearity defines how linear this relationship is. Often quoted as 5% non-linearity

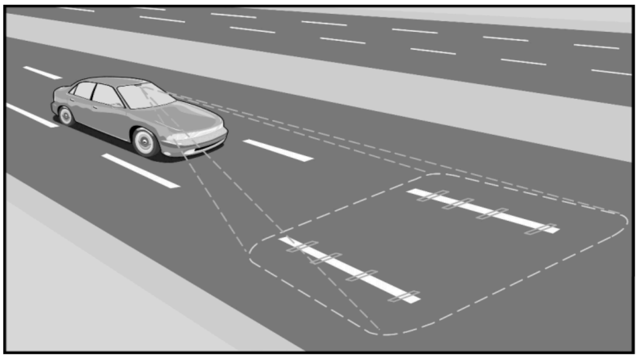
Lateral Dynamics

Lane departures are the number one cause of fatal accidents in the United States, and account for more than 39% of crash-related fatalities. Lane departures are also identified by NHTSA as a major cause of rollover incidents involving sport utility vehicles (SUVs) and light trucks.

Three types of lateral systems have been developed in the automotive industry that address lane departure accidents:

Lane departure warning

system that monitors the vehicle's position with respect to the lane and provides warning if the vehicle is about to leave the lane. An example of a commercial LDW system under development is the AutoVue LDW system by Iteris, Inc.



The AutoVue device is a small, integrated unit consisting of a **camera**, **onboard computer** and **software** that attaches to the windshield, dashboard or overhead. The system is programmed to **recognize the difference between the road and lane markings**. The unit's camera tracks visible lane markings and feeds the information into the unit's computer, which **combines this data with the vehicle's speed**. **Using image recognition software and proprietary algorithms, the computer can predict when a vehicle begins to drift towards an unintended lane change**. When this occurs, the unit automatically emits the commonly known rumble strip sound, **alerting the driver to make a correction**.

AutoVue is publicized as working effectively both during **day and night**, and in most weather conditions where the lane markings are visible. By simply using the turn signal, a driver indicates to the system that a planned lane departure is intended and the alarm does not sound.
Lane departure warning systems made by Iteris are now in use on trucks manufactured by Mercedes and Freightliner.

Lane keeping systems

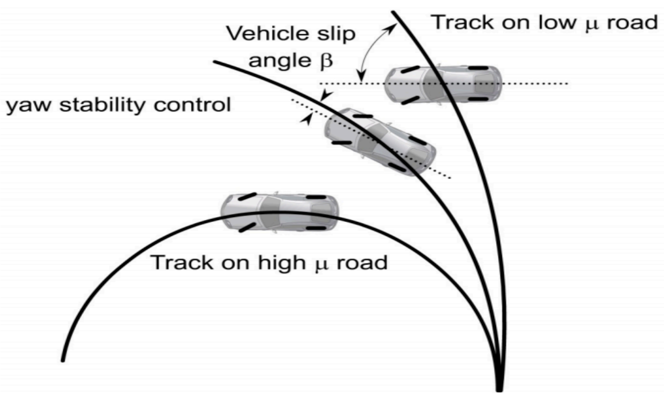
A lane-keeping system automatically controls the steering to keep the vehicle in its lane and also follow the lane as it curves around.
Seeking to strike a balance between system complexity and driver responsibility, the system is targeted at 'monotonous driving' situations. The system operates only on 'straightish' roads (a minimum radius will eventually be specified) and above a minimum defined speed.

Nissan's premise is that drivers feel tired after long hours of continuous expressway driving as a result of having to constantly steer their vehicles slightly to keep them in their lane. The LKS attempts to reduce such fatigue by improving stability on the straight highway road. But the driver must remain engaged in actively steering the vehicle – if he/she does not, the LKS gradually reduces its degree of assistance.

The system uses a single CCD (which is a transistorized light sensor on an integrated circuit. CCDs are sensitive to light and are used as light-detecting components in video and digital cameras, optical scanners, and optical microscopy) camera to recognize the lane demarkation, a steering actuator to steer the front wheels, and an electronic control unit. The camera estimates the road geometry and the host vehicle's position in the lane. Based on this information, along with vehicle velocity and steering wheel angle, the control unit calculates the steering torque needed to keep within the lane.

Yaw(twist or oscillate about a vertical axis) **stability control systems**

Vehicle stability control systems that prevent vehicles from spinning and drifting out have been developed and recently commercialized by several automotive manufacturers.



The lower curve shows the trajectory that the vehicle would follow in response to a steering input from the driver if the road were dry and had a high tire-road friction coefficient. In this case the high friction coefficient is able to provide the lateral force required by the vehicle to negotiate the curved road. If the coefficient of friction were small or if the vehicle speed were too high, then the vehicle would be unable to follow the nominal motion required by the driver – it would instead travel on a trajectory of larger radius (smaller curvature).

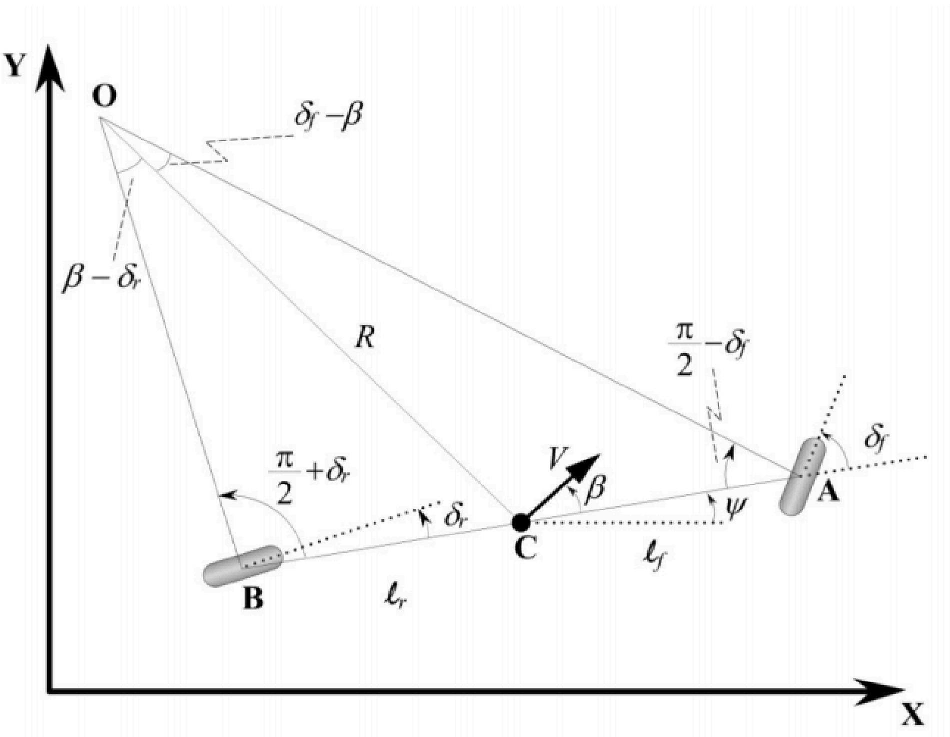
The function of the yaw control system is to restore the yaw velocity of the vehicle as much as possible to the nominal motion expected by the driver. If the friction coefficient is very small, it might not be possible to entirely achieve the nominal yaw rate motion that would be achieved by the driver on a high friction coefficient road surface. In this case, the yaw control system would partially succeed by making the vehicle's yaw rate closer to the expected nominal yaw rate, as shown by the middle curve.

Three types of stability control systems have been proposed and developed for yaw control:

Differential Braking systems which utilize the ABS brake system on the vehicle to apply differential braking between the right and left wheels to control yaw moment.
Steer-by-Wire systems which modify the driver's steering angle input and add a correction steering angle to the wheels
Active Torque Distribution systems which utilize active differentials and all wheel drive technology to independently control the drive torque distributed to each wheel and thus provide active control of both traction and yaw moment.

KINEMATIC MODEL OF LATERAL VEHICLE MOTION

Under certain assumptions described below, a kinematic model for the lateral motion of a vehicle can be developed. Such a model provides a mathematical description of the vehicle motion without considering the forces that affect the motion. The equations of motion are based purely on geometric relationships governing the system.



The wheelbase of the vehicle is L=lf + lr
Delta-r and delta-f are steering angles of rear and front wheel respectively.
Psi describes the orientation/heading of the vehicle.
The velocity at the c.g. of the vehicle is denoted by V and makes an angle beta with the longitudinal axis of the vehicle. The angle beta is called the slip angle of the vehicle.
The point O is the instantaneous rolling center for the vehicle.

The major assumption used in the development of the kinematic model is that **the velocity vectors at points A and B are in the direction of the orientation of the front and rear wheels respectively**.

This is equivalent to assuming that the "slip angles" at both wheels are zero. This is a reasonable assumption for low speed motion of the vehicle (for example, for speeds less than 5 m/s). At low speeds, the lateral force generated by the tires is small. In order to drive on any circular road of radius, the total lateral force from both tires is **mv^2/R**

Apply the sine rule to triangle OCA.

$$\frac{\sin(\delta_f - \beta)}{\ell_f} = \frac{\sin\left(\frac{\pi}{2} - \delta_f\right)}{R}$$

$$\frac{\sin(\delta_f)\cos(\beta) - \sin(\beta)\cos(\delta_f)}{} = \frac{\cos(\delta_f)}{R}$$

$$\tan(\delta_f)\cos(\beta) - \sin(\beta) = \frac{\ell_f}{R} \qquad \text{-------------------------- 1.}$$

Apply the sine rule to triangle OCB.

$$\frac{\sin(\beta - \delta_r)}{\ell_r} = \frac{\sin\left(\frac{\pi}{2} + \delta_r\right)}{R}$$

$$\frac{\cos(\delta_r)\sin(\beta) - \cos(\beta)\sin(\delta_r)}{\ell_r} = \frac{\cos(\delta_r)}{R}$$

$$\sin(\beta) - \tan(\delta_r)\cos(\beta) = \frac{\ell_r}{R} \qquad \text{------------------------------ 2.}$$

Add eqn 1 and 2:

$$\{\tan(\delta_f) - \tan(\delta_r)\}\cos(\beta) = \frac{\ell_f + \ell_r}{R}$$

angular velocity of the vehicle is $\frac{V}{R}$, it follows that

$$\dot{\psi} = \frac{V}{R}$$

$$\dot{\psi} = \frac{V\cos(\beta)}{\ell_f + \ell_r}\left(\tan(\delta_f) - \tan(\delta_r)\right)$$

The overall equations of motion are therefore given by

$$\dot{X} = V\cos(\psi + \beta)$$

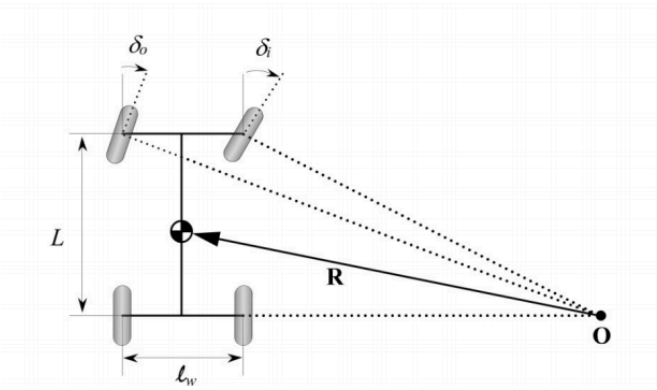
$$\dot{Y} = V\sin(\psi + \beta)$$

$$\dot{\psi} = \frac{V\cos(\beta)}{\ell_f + \ell_r}\left(\tan(\delta_f) - \tan(\delta_r)\right)$$

In this model there are three inputs: V, delta-r and delta-f. The velocity is an external variable and can be assumed to be a time varying function or can be obtained from a longitudinal vehicle model.

$$\beta = \tan^{-1}\left(\frac{\ell_f \tan\delta_r + \ell_r \tan\delta_f}{\ell_f + \ell_r}\right)$$

Here it is appropriate to include a note on the “bicycle” model assumption. Both the left and right front wheels were represented by one front wheel in the bicycle model. It should be noted that the left and right steering angles in general will be approximately equal, but not exactly so. This is because the radius of the path each of these wheels travels is different.



Let ℓ_w be the track width of the vehicle and δ_o and δ_i be the outer and inner steering angles respectively. Let the wheelbase $L = \ell_f + \ell_r$ be small compared to the radius R. If the slip angle β is small, then Eq. (2.12) can be approximated by

$$\dot{\psi} = \frac{V\cos(\beta)}{\ell_f + \ell_r}\left(\tan(\delta_f) - \tan(\delta_r)\right)$$

$$\frac{\dot{\psi}}{V} \approx \frac{1}{R} = \frac{\delta}{L}$$

OR

$$\delta = \frac{L}{R}$$

Since the radius at the inner and outer wheels are different, we have

$$\delta_o = \frac{L}{R + \frac{\ell_w}{2}} \qquad \delta_i = \frac{L}{R - \frac{\ell_w}{2}} \qquad ($$

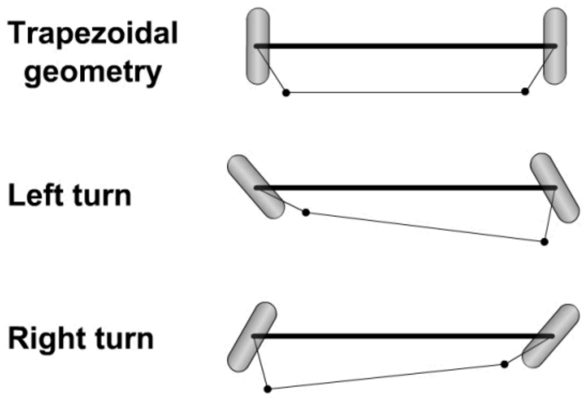
The average front wheel steering angle is approximately given by

$$\delta = \frac{\delta_o + \delta_i}{2} \cong \frac{L}{R}$$

The difference between δ_o and δ_i is

$$\delta_i - \delta_o = \frac{L}{R^2}\ell_w = \delta^2 \frac{\ell_w}{\mathrm{L}}$$

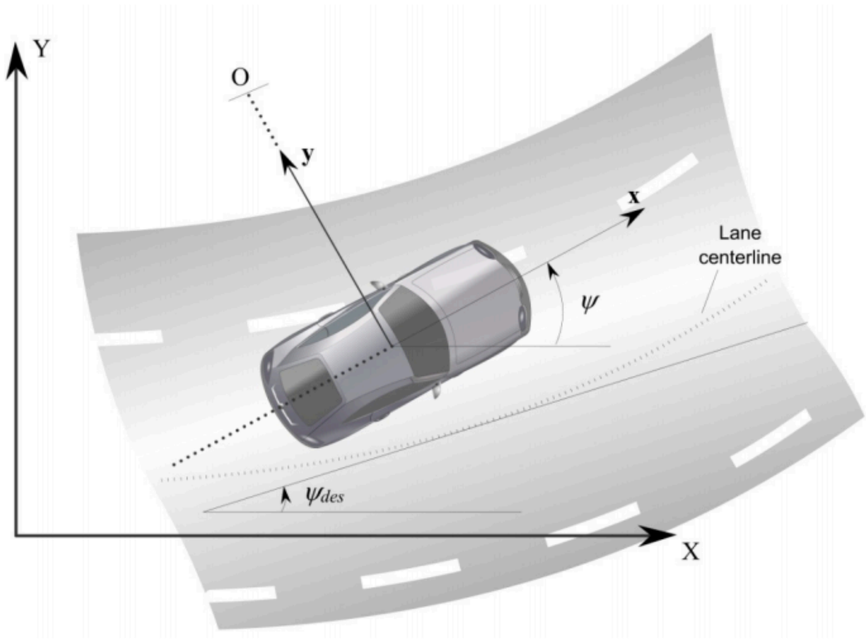
Thus the difference in the steering angles of the two front wheels is proportional to the square of the average steering angle. Such a differential steer can be obtained from a trapezoidal tie rod arrangement.



BICYCLE MODEL OF LATERAL VEHICLE DYNAMICS

At higher vehicle speeds, the assumption that the velocity at each wheel is in the direction of the wheel can no longer be made. In this case, instead of a kinematic model, a dynamic model for lateral vehicle motion must be developed.

A "bicycle" model of the vehicle with two degrees of freedom is considered. The two degrees of freedom are represented by the vehicle lateral position y and the vehicle yaw angle psi . The vehicle lateral position is measured along the lateral axis of the vehicle to the point O which is the center of rotation of the vehicle. The vehicle yaw angle is measured with respect to the global axis. The longitudinal velocity of the vehicle at the c.g. is denoted by V_x.



$$ma_y = F_{yf} + F_{yr}$$

where $a_y = \left(\dfrac{d^2y}{dt^2}\right)_{inertial}$ is the inertial acceleration of the vehicle at the c.g. in the direction of the y axis and F_{yf} and F_{yr} are the lateral tire forces of the front and rear wheels respectively. Two terms contribute to a_y: the acceleration $\ddot{y}$ which is due to motion along the y axis and the centripetal acceleration $V_x\dot{\psi}$. Hence

$$a_y = \ddot{y} + V_x\dot{\psi}$$

$$m(\ddot{y} + \dot{\psi}V_x) = F_{yf} + F_{yr}$$

Moment balance about the z axis yields the equation for the yaw dynamics as

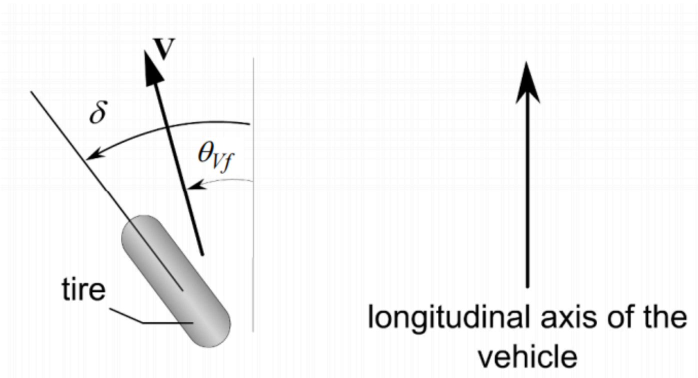
$$I_z\ddot{\psi} = \ell_f F_{yf} - \ell_r F_{yr} \tag{2.22}$$

The next step is to model the lateral tire forces that act on the vehicle. Experimental results show that the lateral tire force of a tire is proportional to the "slip-angle" for small slip-angles. The slip angle of a tire is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel

$$\alpha_f = \delta - \theta_{Vf} \tag{2.23}$$

where θ_{vf} is the angle that the velocity vector makes with the longitudinal axis of the vehicle and δ is the front wheel steering angle. The rear slip angle is similarly given by

$$\alpha_r = -\theta_{Vr} \tag{2.24}$$



$$F_{yf} = 2C_{\alpha f}(\delta - \theta_{Vf}) \tag{2.25}$$

where the proportionality constant $C_{\alpha f}$ is called the cornering stiffness of each front tire, δ is the front wheel steering angle and θ_{Vf} is the front tire velocity angle. The factor 2 accounts for the fact that there are two front wheels.

Similarly the lateral tire for the rear wheels can be written as

$$F_{yr} = 2C_{\alpha r}(-\theta_{Vr}) \tag{2.26}$$

where $C_{\alpha r}$ is the cornering stiffness of each rear tire and θ_{Vr} is the rear tire velocity angle.

The following relations can be used to calculate θ_{Vf} and θ_{Vr}:

Using small angle approximations and using the notation $V_y = \dot{y}$,

$$\theta_{Vf} = \frac{\dot{y} + \ell_f \dot{\psi}}{V_x}$$

$$\theta_{Vr} = \frac{\dot{y} - \ell_r \dot{\psi}}{V_x}$$

After all substitutions, the space state representation of the model is :

$$\frac{d}{dt}\begin{Bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & 0 & -V_x-\dfrac{2C_{\alpha f}\ell_f-2C_{\alpha r}\ell_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{2\ell_f C_{\alpha f}-2\ell_r C_{\alpha r}}{I_z V_x} & 0 & -\dfrac{2\ell_f{}^2 C_{\alpha f}+2\ell_r{}^2 C_{\alpha r}}{I_z V_x} \end{bmatrix}$$

$$+\begin{Bmatrix} 0 \\ \dfrac{2C_{\alpha f}}{m} \\ 0 \\ \dfrac{2\ell_f C_{\alpha f}}{I_z} \end{Bmatrix}\delta$$

LONGITUDINAL VEHICLE DYNAMICS

Common systems involving longitudinal control available on today's passenger cars include **cruise control, anti-lock brake systems and traction control systems**.

The two major elements of the longitudinal vehicle model are the **vehicle dynamics** and the **powertrain dynamics**. The vehicle dynamics are influenced by **longitudinal tire forces, aerodynamic drag forces, rolling resistance forces and gravitational forces**. The longitudinal powertrain system of the vehicle consists of the **internal combustion engine, the torque converter, the transmission and the wheels.**

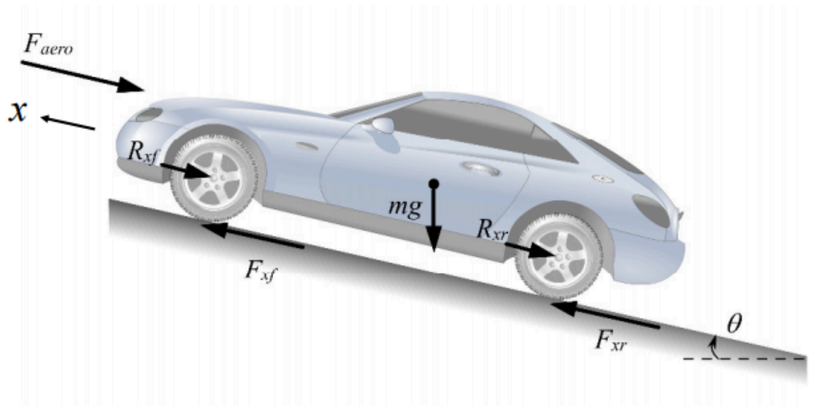


Figure 4-1. Longitudinal forces acting on a vehicle moving on an inclined road

A force balance along the vehicle longitudinal axis yields

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin(\theta)$$

where

F_{xf} is the longitudinal tire force at the front tires

F_{xr} is the longitudinal tire force at the rear tires

F_{aero} is the equivalent longitudinal aerodynamic drag force

R_{xf} is the force due to rolling resistance at the front tires

R_{xr} is the force due to rolling resistance at the rear tires

m is the mass of the vehicle

g is the acceleration due to gravity

θ is the angle of inclination of the road on which the vehicle is traveling

Aerodynamic drag force

The equivalent aerodynamic drag force on a vehicle can be represented as :

$$F_{aero} = \frac{1}{2}\rho C_d A_F (V_x + V_{wind})^2$$

where ρ is the mass density of air, C_d is the aerodynamic drag coefficient, A_F is the frontal area of the vehicle, which is the projected area of the vehicle in the direction of travel, $V_x = \dot{x}$ is the longitudinal vehicle velocity, V_{wind} is the wind velocity (positive for a headwind and negative for a tailwind).

Atmospheric conditions affect air density ρ and hence can significantly affect aerodynamic drag. The commonly used standard set of conditions to which all aerodynamic test data are referred to are a temperature of 15 ^oC and a barometric pressure of 101.32 kPa (Wong, 2001). The corresponding mass density of air ρ may be taken as 1.225 kg/m^3.

Longitudinal tire force

The longitudinal tire forces F_{xf} and F_{xr} are friction forces from the ground that act on the tires. Experimental results have established that the longitudinal tire force generated by each tire depends on

a) the slip ratio,
b) the normal load on the tire and
c) the friction coefficient of the tire-road interface.

The vertical force on a tire is called the **tire normal load**. The normal load on a tire
a) comes from a portion of the weight of the vehicle
b) is influenced by fore-aft location of the c.g., vehicle longitudinal acceleration, aerodynamic drag forces and grade of the road.

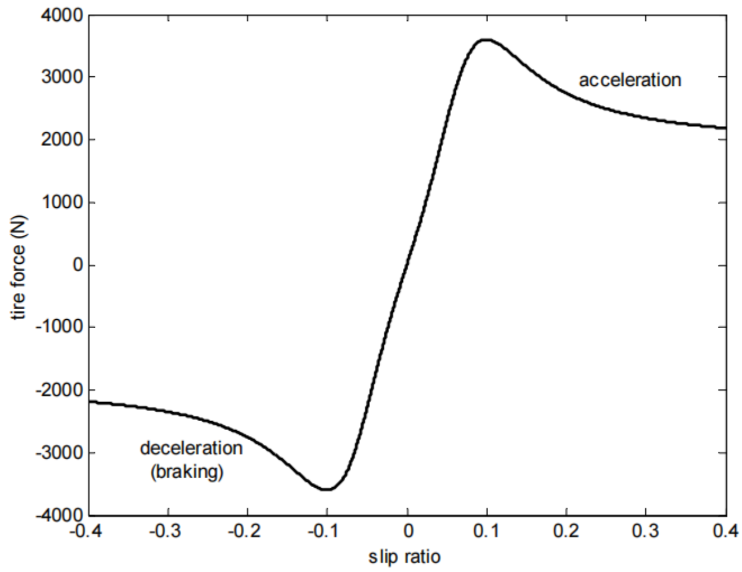
Slip Ratio

The difference between the actual longitudinal velocity at the axle of the wheel V_x and the equivalent rotational velocity $r_{eff}w_w$ of the tire is called longitudinal slip. In other words, longitudinal slip is equal to $r_{eff}w_w$ - V_x. Longitudinal slip ratio is defined as :

$$\sigma_x = \frac{r_{eff}\omega_w - V_x}{V_x} \text{ during braking}$$

$$\sigma_x = \frac{r_{eff}\omega_w - V_x}{r_{eff}\omega_w} \text{ during acceleration}$$

If the friction coefficient of the tire-road interface is assumed to be 1 and the normal force is assumed to be a constant, the typical variation of longitudinal tire force as a function of the slip ratio is shown :



As can be seen from the figure, in the case where longitudinal slip ratio is small (typically less than 0.1 on dry surface), as it is during normal driving, the longitudinal tire force is found to be proportional to the slip ratio. The tire force in this small-slip region can then be modeled as

$$F_{xf} = C_{\sigma f}\sigma_{xf}$$

$$F_{xr} = C_{\sigma r}\sigma_{xr}$$

where $C_{\sigma f}$ and $C_{\sigma r}$ are called the longitudinal tire stiffness parameters of the front and rear tires respectively.

Rolling resistance

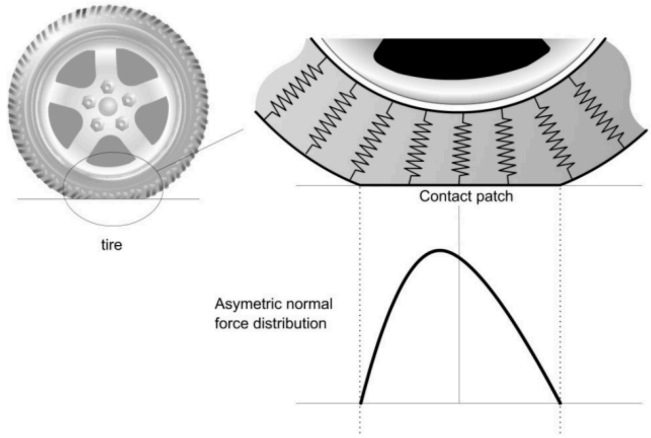
As the tire rotates, both the tire and the road are subject to **deformation in the contact patch**. The road is of course much stiffer and so its deformation can be neglected. But the **tire is elastic** and new material from the tire continuously enters the contact patch as the tire rotates. **Due to the normal load, this material is deflected vertically as it goes through the contact patch and then springs back to its original shape after it leaves the contact patch**. Due to the internal damping of the tire material, the energy spent in deforming the tire material is not completely recovered when the material returns to its original shape. **This loss of energy can be represented by a force on the tires called the rolling resistance that acts to oppose the motion of the vehicle.**

The loss of energy in tire deformation also results in a **non-symmetric distribution of the normal tire load over the contact patch**. When the tires are static (not rotating), then the distribution of the normal load *Fz* in the contact patch is symmetric with respect to the center of the contact patch. However, when the tires are rotating, the normal load distribution is non symmetric

Typically, the rolling resistance is modeled as being roughly proportional to the normal force on each set of tires i.e.

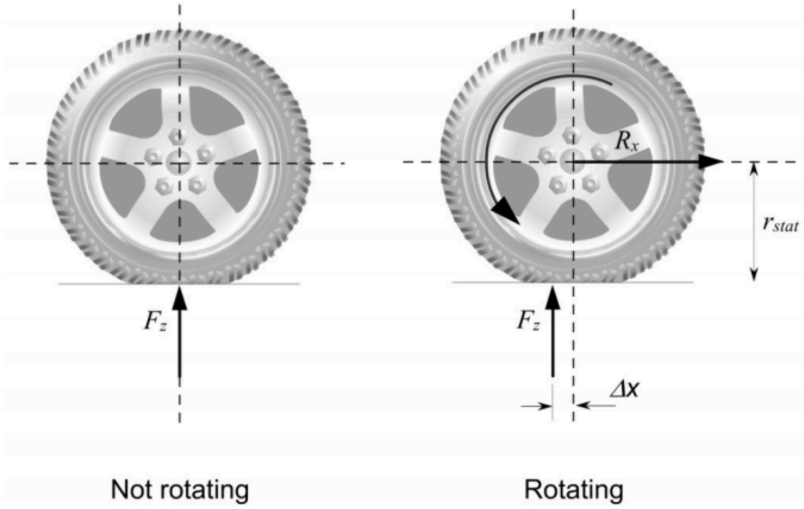
$$R_{xf} + R_{xr} = f(F_{zf} + F_{zr})$$

where *f* is the rolling resistance coefficient



The moment $F_z(\Delta x)$ due to the offset normal load is balanced by the moment due to the rolling resistance force $R_x r_{stat}$, where r_{stat} is the statically loaded radius of the tire. Hence

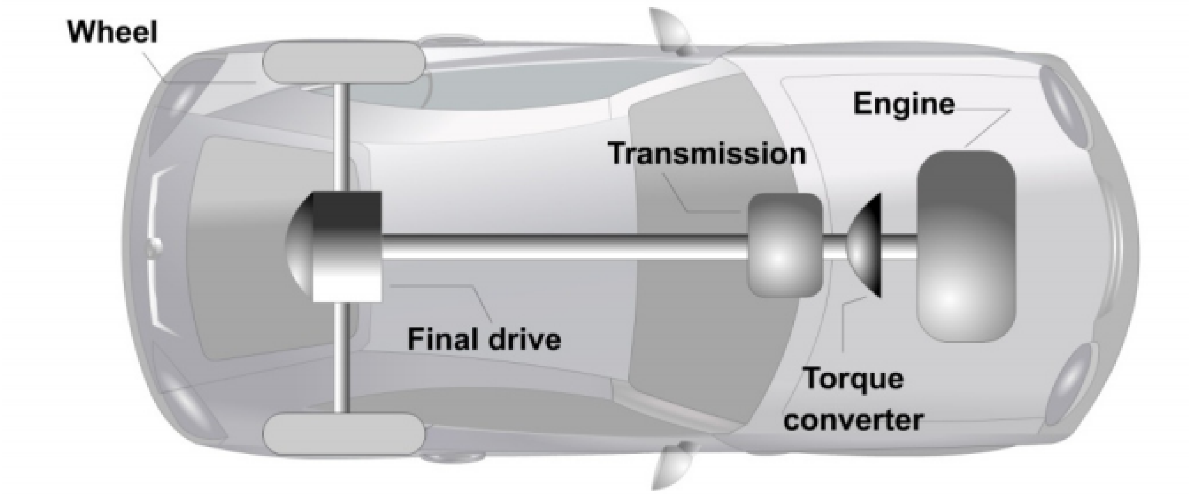
$$R_x = \frac{F_z(\Delta x)}{r_{stat}}$$

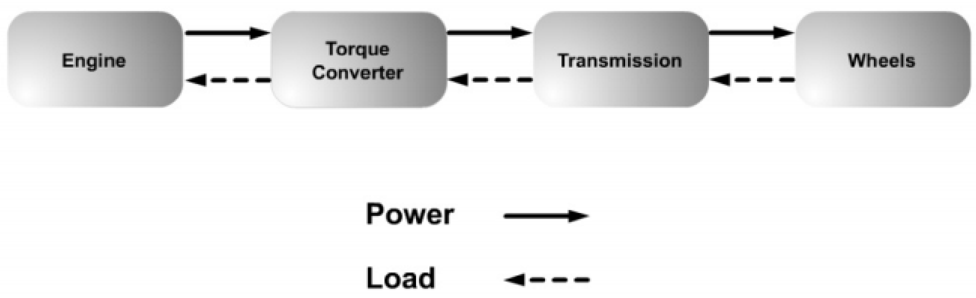


DRIVELINE DYNAMICS

$$m\ddot{x} = F_{xf} + F_{xr} - R_{xf} - R_{xr} - F_{aero} - mg\sin(\theta)$$

where F_{xf} and F_{xr} are the longitudinal tire forces. The longitudinal tire forces on the driving wheels are the primary forces that help the vehicle move forward. These forces depend on the difference between the rotational wheel velocity $r_{eff}w_w$ and the vehicle longitudinal velocity x . The wheel rotational velocity Z_w is highly influenced by the driveline dynamics of the vehicle.





Torque converter

The torque converter is a type of fluid coupling that connects the engine to the transmission. If the engine is turning slowly, such as when the car is idling at a stoplight, the amount of torque passed through the torque converter is very small, so keeping the car still requires only a light pressure on the brake pedal.

In addition to allowing the car to come to a complete stop without stalling the engine, the torque converter gives the car more torque when it accelerates out of a stop. Modern torque converters can multiply the torque of the engine by two to three times. This effect only happens when the engine is turning much faster than the transmission. At higher speeds, the transmission catches up to the engine, eventually moving at *almost* the same speed. Ideally, though, the transmission should move at exactly the same speed as the engine, because the difference in speed wastes power. To counter this effect, many cars have a torque converter with a lockup clutch. When the two halves of the torque converter get up to speed, this clutch locks them together, eliminating the slippage and improving efficiency.

The torque converter is typically unlocked as soon as the driver removes his/her foot from the accelerator pedal and steps on the brakes. This allows the engine to keep running even if the driver brakes to slow the wheels down.

The major components of the torque converter are a pump, a turbine and the transmission fluid. The fins that make up the pump of the torque converter are attached to the flywheel of the engine. The pump therefore turns at the same speed as the engine. The turbine is connected to the transmission and causes the transmission to spin at the same speed as the turbine, this basically moves the car. The coupling between the turbine and the pump is through the transmission fluid. Torque is transmitted from the pump to the turbine of the torque converter.

Transmission dynamics

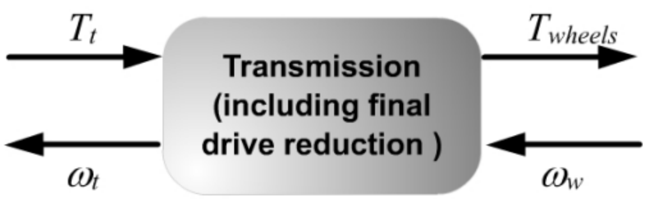
Let R be the gear ratio of the transmission. The value of R depends on the operating gear and includes the final gear reduction in the differential. In general, R1 and increases as the gear shifts upwards.

The turbine torque T_t is the input torque to the transmission. Let the torque transmitted to the wheels be T_{wheels} . At steady state operation under the first, second or higher gears of the transmission, the torque transmitted to the wheels is

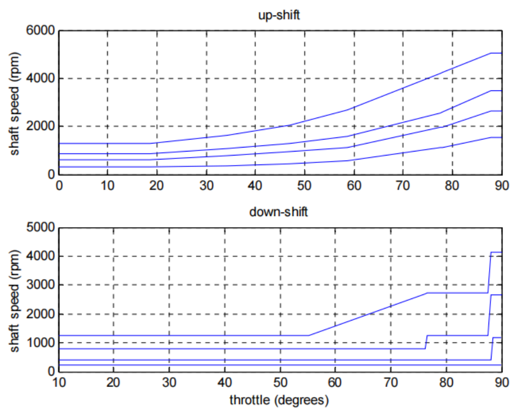
$$T_{wheels} = \frac{1}{R} T_t$$

The relation between the transmission and wheel speeds is

$$\omega_t = \frac{1}{R} \omega_w$$



The steady state gear ratio R depends on the operating gear. The operating gear is determined by a gear shift schedule that depends on both the transmission shaft speed and the throttle opening (with fully open throttle angle being counted as 90 degrees)



The gear change is assumed to be complete when T_{wheel} and w_t converge to 1/R*T_t and w_w/R within a threshold value.

Engine dynamics

The engine rotational speed dynamics can be described by the equation

$$I_e\dot{\omega}_e = T_i - T_f - T_a - T_p \tag{4.35}$$

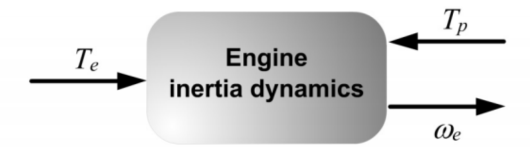
where T_i is the engine combustion torque, T_f are the torque frictional losses, T_a is the accessory torque and T_p is the pump torque and represents the load on the engine from the torque converter.

Using the notation

$$T_e = T_i - T_f - T_a \tag{4.36}$$

to represent the net engine torque after losses, we have

$$I_e\dot{\omega}_e = T_e - T_p \tag{4.37}$$



Summary of longitudinal vehicle dynamic equations		
		Primary Vehicle Dynamic Equation $m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin(\theta)$
1	Front longitudinal tire force	$F_{xf} = C_{\sigma f}\sigma_{xf}$ where $\sigma_{xf} = \frac{r_{eff}\omega_{wf} - \dot{x}}{\dot{x}}$ during braking $\sigma_{xf} = \frac{r_{eff}\omega_{wf} - \dot{x}}{r_{eff}\omega_{wf}}$ during acceleration
2	Rear longitudinal tire force	$F_{xr} = C_{\sigma r}\sigma_{xr}$ where $\sigma_{xr} = \frac{r_{eff}\omega_{wr} - \dot{x}}{\dot{x}}$ during braking $\sigma_{xr} = \frac{r_{eff}\omega_{wr} - \dot{x}}{r_{eff}\omega_{wr}}$ during acceleration
3	Rolling resistance	$R_{xf} + R_{xr} = f(F_{zf} + F_{zr})$ where the front normal tire force is $F_{zf} = \frac{-F_{aero}h_{aero} - m\ddot{x}h - mgh\sin(\theta) + mg\ell_r\cos(\theta)}{\ell_f + \ell_r}$ and the rear normal tire force is $F_{zr} = \frac{F_{aero}h_{aero} + m\ddot{x}h + mgh\sin(\theta) + mg\ell_f\cos(\theta)}{}$
4	Aerodynamic drag force	$F_{aero} = \frac{1}{2}\rho C_d A_F(\dot{x} + V_{wind})^2$

PID Controller

A PID controller is a feedback controller that utilizes a combination of three control actions to manipulate the output of a system. These actions are based on the discrepancy between a desired setpoint and the actual output measured by a sensor. The controller continuously adjusts the system's input to minimize this error.

The three control actions in a PID controller are:

- **Proportional (P) Control:** This action delivers an output adjustment directly proportional to the current error. A larger error magnitude translates to a larger output correction.
- **Integral (I) Control:** This action addresses long-term system behavior by accumulating the error over time and incorporating this summation into the output adjustment. This integral action eliminates steady-state errors, where a constant error persists even after the initial transient response.
- **Derivative (D) Control:** This action anticipates future errors by considering the rate of change of the error signal. The controller adjusts the output based on how quickly the error is increasing or decreasing. This anticipatory action allows for faster system response and reduces overshoot (when the output transiently exceeds the setpoint).

Each gain term plays a specific role in influencing the controller's behavior:

- **Kp** influences the response to the current error. A higher Kp results in a faster response, but may also introduce oscillations.
- **Ki** eliminates steady-state errors. While a higher Ki value reduces steady-state error, it can slow down the system's response.
- **Kd** improves the transient response by reducing overshoot. However, a higher Kd can make the system more sensitive to noise.

Advantages of PID Controllers

PID controllers offer several advantages that make them a popular choice in various control applications:

- **Simplicity:** The underlying concept of PID control is relatively straightforward, making it accessible to a broad range of engineers.
- **Effectiveness:** PID controllers can effectively regulate a wide variety of systems, encompassing simple temperature control loops to complex motion control systems.
- **Robustness:** They exhibit relative insensitivity to variations in plant dynamics (the behavior of the system being controlled).
- **Tunability:** The gains (Kp, Ki, Kd) can be adjusted to optimize the controller's performance for a specific system, ensuring the desired response characteristics.

Stanley Controller

Unlike traditional feedback controllers that rely on error signals, geometric path tracking controllers utilize the vehicle's position and the path geometry to determine the steering angle. This approach leverages the vehicle's kinematic model, which describes the relationship between steering input and its motion.

The Stanley controller specifically focuses on two key aspects of path following:

Heading Alignment: Ensuring the vehicle's heading aligns with the direction of the path at a designated point.

Cross Track Error (CTE) Reduction: Minimizing the lateral distance between the vehicle and the closest point on the path.

The Stanley Controller Algorithm

The Stanley controller operates by iteratively calculating the appropriate steering angle based on the following:

1. **Path Representation:** The path is defined as a sequence of waypoints or a mathematical function.
2. **Lookahead Distance:** A predetermined distance ahead on the path is chosen as a reference point.
3. **Heading Error:** The difference between the vehicle's heading and the tangent of the path at the lookahead point is calculated.
4. **Cross Track Error (CTE):** The lateral distance between the vehicle's front axle and the closest point on the path at the lookahead point is determined.

The steering angle is then computed as a weighted sum of:

- A term proportional to the heading error, driving the vehicle to align with the path direction.
- A term proportional to the CTE, steering the vehicle towards the path.

The weights applied to these terms are crucial for controller tuning and depend on factors like vehicle dynamics and desired response characteristics.

Benefits of the Stanley Controller

The Stanley controller offers several advantages for path tracking in autonomous vehicles:

- **Simplicity:** The underlying geometric approach is intuitive and relatively easy to implement.
- **Computational Efficiency:** The controller calculations involve basic geometric relationships, making it suitable for real-time applications.
- **Smooth Path Following:** By considering both heading and CTE, the Stanley controller promotes smooth and stable path tracking.