Bayesian Networks:

- Probability as a basis for beliefs
  (Foundation)

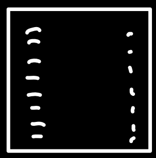
- Bayesian Networks
  (Modeling tool)

  a. Directed Acyclic Graph (DAG) : Causality
  b. Numbers : Probabilities

  $\Big\}$ => Probability Distribution

- Inference

  Compile Bayesian Networks into "Circuits"



- Learning

Notation:

Variable X

Value x

Set of variables $\mathbf{X}$

Instantiation $\mathbf{x}$ → States

$\mathbf{A} = \{X, Y\}$

$\mathbf{a}$: $X = r,\ Y = s$
$X = r,\ Y = m$

$X : \{r, b, g\}$

$Pr(x) : Pr(X = x)$ ← Number probability

$Pr(X)$: Distribution over X

↑ Table

$Pr(X)$

X	
r	.
b	.
g	.

T: binary variable — true / false

T, t : T is true

$\neg T, \bar{t}$: T is false

VARIABLE INDEPENDENCE:

* $Pr(\alpha \mid \beta, \gamma) = Pr(\alpha \mid \gamma)$
  α, β are independent given γ } $\alpha, \beta, \gamma \rightarrow$ Sentences

* Variable sets X, Y, Z
  X and Y are independent given Z. } $I(X, Z, Y)$

$$Pr(x \mid y, z) = Pr(x \mid z)$$

$X = \{A, B\}$ | A, B, C, D, E

$Y = \{C\}$ | binary variable

$Z = \{D, E\}$

$X \rightarrow 4$ states

$Z \rightarrow 2$ states

$Y \rightarrow 4$ states

$I(x, z, y)$

$A \wedge B$ independent of C given $D \wedge E$

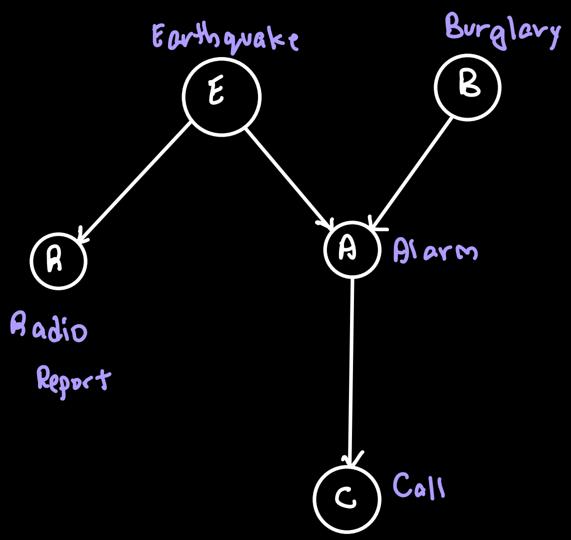
$A \wedge \neg B$ independent of C given $D \wedge E$

32 statements

$\neg A \wedge \neg B$ independent of $\neg C$ given $\neg D \wedge \neg E$

$4 \times 2 \times 4 = 32$

CAPTURING INDEPENDENCE GRAPHICALLY:



Causality!

DAG: Direct Acyclic Graph

$I(R, A, C)$

$I(E, \phi, B)$

- Parents (v):

  A: E, B

  B: E

  C: A

- Descendents (v):

  E: A, C, B

  B: A, C

  C: ϕ

- Non-Descendents (v):

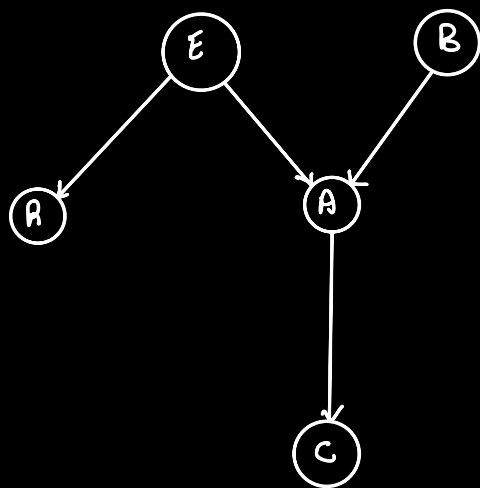
  ↳ exclude v and its parents

  A: B

  E: B

MARKOVIAN ASSUMPTIONS OF A BAYESIAN NETWORK:

$I(V, \text{Parents}(V), \text{Non-Descendents}(V))$



$I(C, A, BER)$

$I(R, E, ABC)$

$I(A, BE, R)$

$I(B, \phi, ER)$

$I(E, \phi, B)$

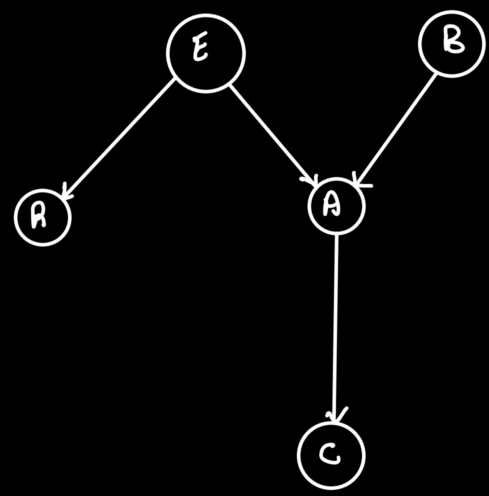
MARKOVIAN ASSUMPTIONS

Markov(G)

$I(C, A, BER)$:

Once there is alarm, whether we get call or not is independent of B, E, R.

PARAMETERIZING THE STRUCTURE:



Variable : C

Parents : A

CONDITIONAL PROBABILITY TABLE (CPT):

CPT For Variable C:

variable (C); state (c)

A	C	Pr(c, a)
t	t	0.8
t	f	0.2
f	t	0.001
f	f	0.999

0.8 → Probability of call give alarm Pr(C = t, A = t)

0.2 → Pr(C = f, A = t)

Pr(C = t, A = t) + Pr(C = f, A = t) = 1

0.001 → Pr(C = t, A = f)

0.999 → Pr(C = f, A = f)

Pr(C = t, A = f) + Pr(C = f, A = f) = 1

4 parameters

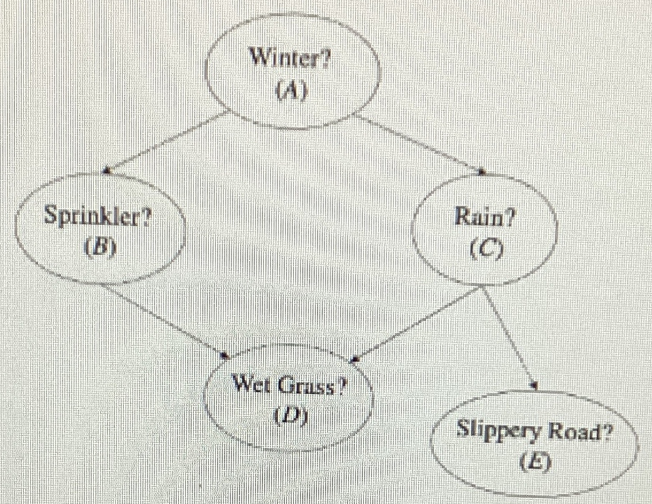
2 independent parameters

E	B	A	$Pr(a \mid e, b)$
t	t	t	x
t	t	f	$1-x$
	$\vdots$		
f	f	t	y
f	f	f	$1-y$

8 parameters

4 independent parameters.

THE DISTRIBUTION OF A BAYESIAN NETWORK:



A	Θ_A
true	.6
false	.4

A	B	$\Theta_{B\|A}$
true	true	.2
true	false	.8
false	true	.75
false	false	.25

A	C	$\Theta_{C\|A}$
true	true	.8
true	false	.2
false	true	.1
false	false	.9

B	C	D	$\Theta_{D\|B,C}$
true	true	true	.95
true	true	false	.05
true	false	true	.9
true	false	false	.1
false	true	true	.8
false	true	false	.2
false	false	true	0
false	false	false	1

C	E	$\Theta_{E\|C}$
true	true	.7
true	false	.3
false	true	0
false	false	1

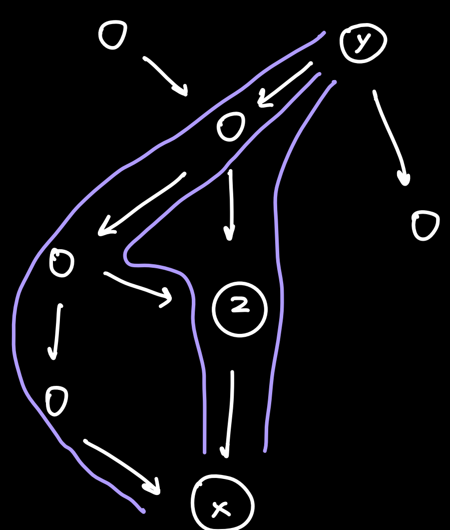
$$\theta_{\bar{d}|b\bar{c}} = Pr(\bar{d} \mid b, \bar{c}) = 0.1$$

Parameter

A	B	C	D	E	
t	t	t	t	t	$0.6 \times 0.2 \times 0.8 \times 0.95 \times 0.7$
t	t	f	t	f	$0.6 \times 0.2 \times 0.2 \times 0.9 \times 0.1$
		⋮			
f	f	f	f	f	$0.4 \times 0.25 \times 0.9 \times 1 \times 1 = 0.09$

d-SEPARATION:

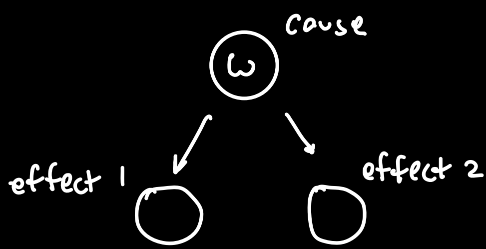
We want to check if $I(x, z, y)$



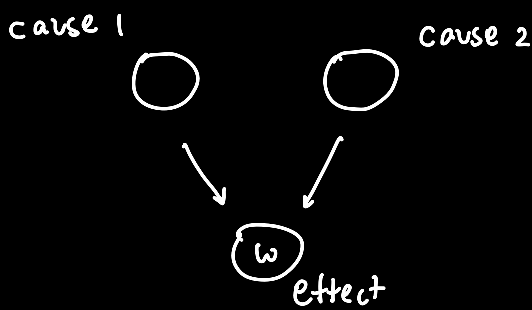
Paths between x and y.

If any of the path is not blocked by z, not independent.

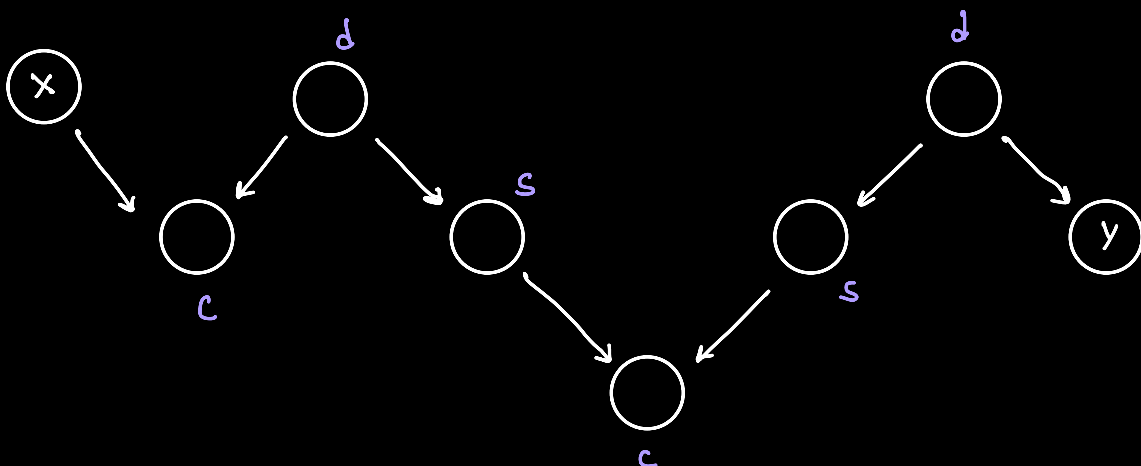
Sequential
(s)



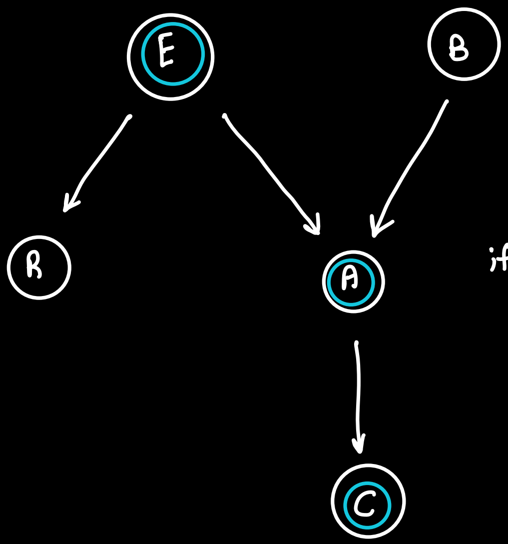
Divergent
(d)



Convergent
(c)



SEQUENTIAL VALVE:

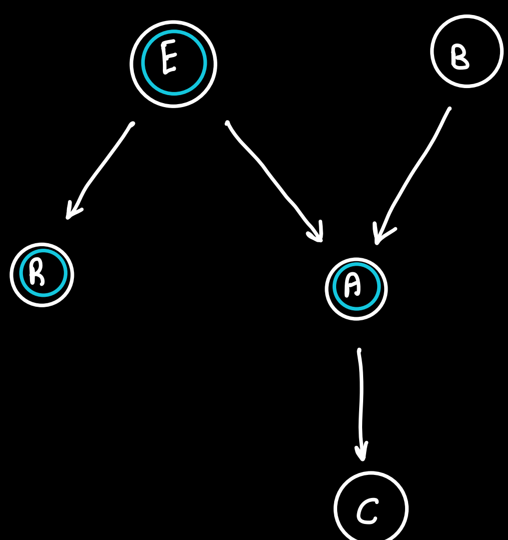


if we know A, then the valve is closed, E and C are independent along this path.

Given that we know Z

A sequential valve $\rightarrow w \rightarrow$ is closed iff variable $w \in Z$

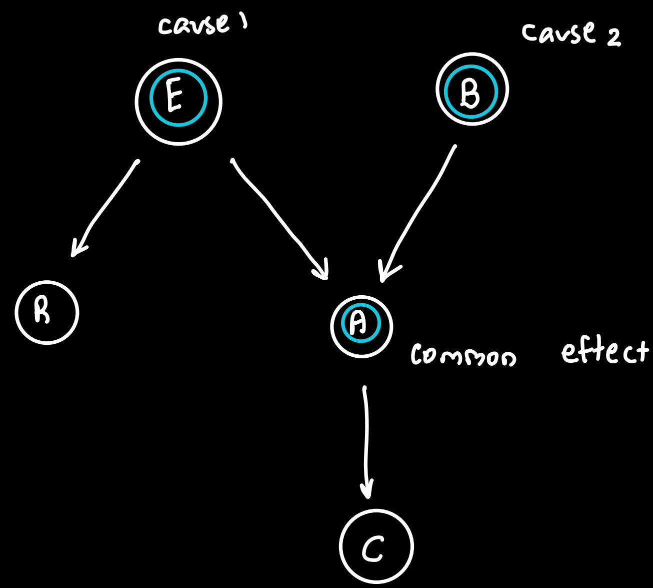
DIVERSION VALVE:



If we know E, R and A are independent.

A divergent valve $\leftarrow W \rightarrow$ is closed iff variable $W \in Z$.

Convergent Valve:



A convergent valve $\rightarrow W \leftarrow$ is closed iff neither variable W nor any of its descendents are in Z.

d-Separation:

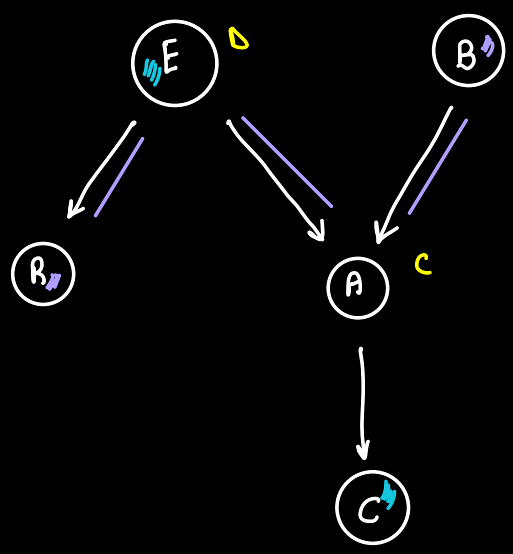
- X and Y are d-separated given Z

$$dsep(X, Z, Y)$$

iff every path between a node in X and a node in Y is blocked by Z.

* A path is blocked by z iff at least one value on the path is closed given z.

EXAMPLE 1:



2 values : E, A

dsep (B, EC, R)

z = {E, C}

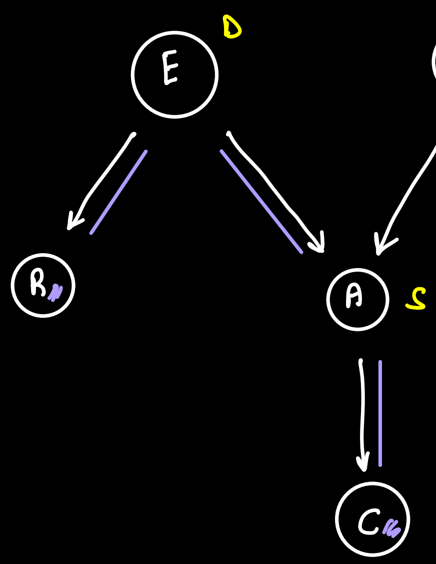
↑

known / observed.

E ⟶ closed A → open

So dsep (B, EC, R) //

EXAMPLE 2:



2 values → E, A

E → open

A → open

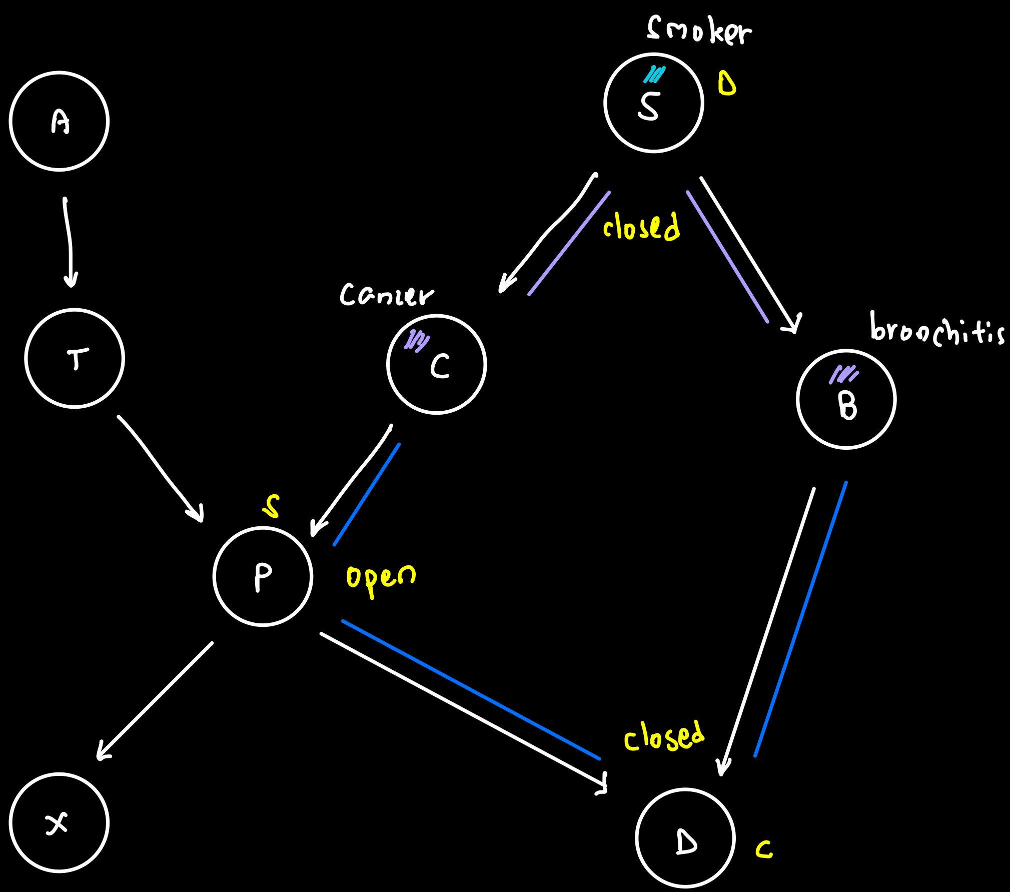
dsep (R, ϕ, C)

Z : empty no variables are known or observed.

No d-separation.

Cannot infer independence.

EXAMPLE 3:



dsep (C, S, B)

↓

Yes!

Z: {S}

So independence inferred.