INFERENCE:

Knowledge base $\Delta = \{\beta_1, \beta_2, \dots \beta_n\}$

Set of sentences

$\beta_1 \wedge \beta_2 \dots \wedge \beta_n$

QUERY α

Does Δ imply α ?

$\Delta \models \alpha$ reads : Δ implies α, Δ entails α

α follows from Δ.

INFERENCE METHODS:

1. TRUTH TABLE (MODEL ENUMERATION):

Δ imply α : $M(\Delta) \subseteq M(\alpha)$

(Brute Force)

2. INFERENCE RULES:

* Keep applying inference rule until you get what you want.

RESOLUTION

$$\frac{\alpha, \ \alpha \Rightarrow \beta}{\beta}$$

3. By REDUCTION SAT (Satisfiability) - SEARCH (SAT)

4. CONVERTING KB (Knowledge Base) into "tractable forms"
(advanced queries)

state of the art.

METHOD 1: TRUTH TABLES, ENUMERATING MODELS

KB Δ : A , $A \lor B \Rightarrow C$

Query α : C

Does $\Delta \models \alpha$? $\Delta \models \alpha$ iff $M(\Delta) \subseteq M(\alpha)$

	A	B	C	A	$A \lor B \Rightarrow C$	Δ	α
1	T	T	T	✓	✓	✓	✓
2	T	T	F	✓	✗	✗	✗
3	T	F	T	✓	✓	✓	✓
4	T	F	F	✓	✗	✗	✗
5	F	T	T	✗	✓	✗	✓
6	F	T	F	✗	✗	✗	✗
7	F	F	T	✗	✓	✗	✓
8	F	F	F	✗	✓	✗	✗
						$\{1, 3\}$	$\{1, 3, 5, 7\}$

$M(\Delta) = \{1, 3\}$

$M(\alpha) : \{1, 3, 5, 7\}$ $\qquad M(\Delta) \subseteq M(\alpha) \Rightarrow \Delta \models \alpha$

DISADVANTAGE:

Exponential time in number of variables.

ADVANTAGES:

- Easy, fallback
- Can test equivalence etc.

REFUTATION THEOREM:

$\Delta \models \alpha$: Δ implies α

$\Delta \wedge \neg\alpha$ contractory

not satisfiable

Assume α is false and prove that Δ is false.

i.e. $M(\Delta \wedge \neg\alpha)$ is null set.

Reducing logical implication to SAT solving//

$\Delta \Rightarrow \alpha$

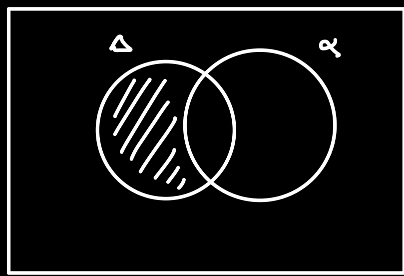
$\neg\alpha \Rightarrow \neg\Delta$

$\alpha \vee \neg\Delta$ is always true

$\neg(\alpha \vee \neg\Delta)$ is not satisfiable

$\neg\alpha \wedge \Delta$

We want $M(\Delta) \subseteq M(\alpha)$



if subset, shaded portion $= \phi$

$$M(\Delta \wedge \neg\alpha) = \phi$$

METHOD 2: INFERENCE RULES:

$$\frac{\text{Pattern 1}, \text{Pattern 2}}{\text{Pattern 3}}$$

MODUS PONENS:

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta}$$

OR - INTRODUCTION:

$$\frac{\alpha, \beta}{\alpha \vee \beta}$$

AND - INTRODUCTION:

$$\frac{\alpha, \beta}{\alpha \wedge \beta}$$

INFERENCE RULES R

$\Delta \models \alpha$

$\Delta \vdash_R \alpha$: α derived from Δ using inference rules R.

Are the rules complete?

Should be able to prove any $\Delta \models \alpha$

INFERENCE RULES R ARE COMPLETE

if $\Delta \models \alpha$ then $\Delta \vdash_R \alpha$.

INFERENCE RULES R ARE SOUND

if $\Delta \vdash_R \alpha$ then $\Delta \models \alpha$.

RESOLUTION:

$$\frac{\alpha \vee \beta, \ \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

$\alpha \vee \beta$ is $\neg\alpha \Rightarrow \beta$

$\neg\beta \vee \gamma$ is $\beta \Rightarrow \gamma$

$$\frac{\neg\alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

$\hookrightarrow \alpha \vee \gamma$

Resolution is typically applied to CNF.

$$\frac{A \vee B, \quad \neg B \vee C \vee D}{A \vee C \vee D}$$

$$\frac{\neg A \vee C, \quad B \vee \neg C \vee X}{\neg A \vee B \vee X}$$

We resolved $\neg A \vee C$ with $B \vee \neg C \vee X$ over variable C and we got the resolvent $\neg A \vee B \vee X$.

$\Rightarrow$ Is it complete?

No!

$\Delta = A \vee B$

$\alpha = A \vee B \vee C$

$\Delta \nvdash_{\text{Resolution}} \alpha$ but $\Delta \models \alpha$

But resolution is "refutation complete" if applied to a CNF.

REFUTATION COMPLETE:

It will derive a contradiction if CNF is unsatisfiable.

So convert $\Delta \models \alpha$ to $\Delta \wedge \neg\alpha$ and then convert into CNF $\rightarrow$ Apply resolution.

UNIT RESOLUTION:

$A \vee \neg B \vee C$ $\leftarrow$ length 3 (literals)

$A \vee \neg B$ $\leftarrow$ length 2 (binary clause)

$\neg B$ $\leftarrow$ length 1 (unit clause)

When you resolve 2 clauses, one of them is unit.

1. $\dfrac{A \vee B, \; \neg B \vee C \vee D}{A \vee C \vee D}$

2. $\underbrace{\dfrac{A, \; \neg A \vee B \vee C}{B \vee C}}_{\text{unit resolution.}}$

Resolution $\longrightarrow$ exponential time

Unit Resolution $\rightarrow$ Linear time! But it is not refutation complete.

RESOLUTION EXAMPLE:

Δ: $A \vee \neg B \Rightarrow C$

$C \Rightarrow D \vee \neg E$

$E \vee D$

α: $A \Rightarrow D$

Is $\Delta \models \alpha$

$\Delta \wedge \neg \alpha$ prove unsat

contradiction $\Rightarrow$ $\Delta \models \alpha$

no contradiction $\Rightarrow$ $\Delta \not\models \alpha$

Assume Δ is in CNF

Δ: $\neg A \vee C$ $\qquad$ $\neg \alpha$: A

$B \vee C$ $\qquad$ $\neg D$

$\neg C \vee D \vee \neg E$

$E \vee D$

0. $\neg A \vee C$

1. $B \vee C$

2. $\neg C \vee D \vee \neg E$

3. $E \vee D$

4. A

5. $\neg D$

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6. C	0, 4	Unit	
7. $D \vee \neg E$	0, 6	Unit	RESOLUTION PROOF.
8. $\neg E$	5, 7	Unit	
9. E	3, 5	Unit	
10. Contradiction	8, 9		

$\Delta \wedge \neg \alpha$ is unsat

$\Delta \models \alpha$

CONVERTING SENTENCES INTO CNF:

Any sentence can be converted into CNF.

1. Get rid of all connections but for $\wedge, \vee, \neg$

$$\alpha \Rightarrow \beta \rightarrow \neg\alpha \vee \beta \quad - \text{①}$$

$$\alpha \Leftrightarrow \beta \rightarrow (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$$

use ①

2. Use de Morgan's law to push negations inside

$$\neg(\alpha \wedge \beta) \rightarrow \neg\alpha \vee \neg\beta$$

$$\neg(\alpha \vee \beta) \rightarrow \neg\alpha \wedge \neg\beta$$

3. Distribute $\vee$ over $\wedge$

$$(\alpha \wedge \beta) \vee \gamma \rightarrow (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$$

EXAMPLE:

$$A \Leftrightarrow (B \vee C)$$

1. $(A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$

$(\neg A \vee (B \vee C)) \wedge (\neg(B \vee C) \vee A)$

2. $\underbrace{(\neg A \vee (B \vee C))} \wedge ((\neg B \wedge \neg C) \vee A)$

Clause 1: $\neg A \vee B \vee C$

3. $(\neg B \vee A) \wedge (\neg C \vee A)$

Clauses:

$\neg A \vee B \vee C$

$\neg B \vee A$

$\neg C \vee A$

$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$