QUANTUM ERROR CORRECTION

CLASSICAL ERROR

$0 \rightarrow 1$

$1 \rightarrow 0$

QUANTUM ERROR

BIT FLIP:

$(\alpha|0\rangle + \beta|1\rangle) \xrightarrow{X} (\alpha|1\rangle + \beta|0\rangle)$

PHASE FLIP:

$(\alpha|0\rangle + \beta|1\rangle) \xrightarrow{Z} (\alpha|0\rangle - \beta|1\rangle)$

ARBITRARY ERROR:

$(\alpha|0\rangle + \beta|1\rangle) \rightarrow (\alpha'|0\rangle + \beta'|1\rangle)$

CLASSICAL COPYING

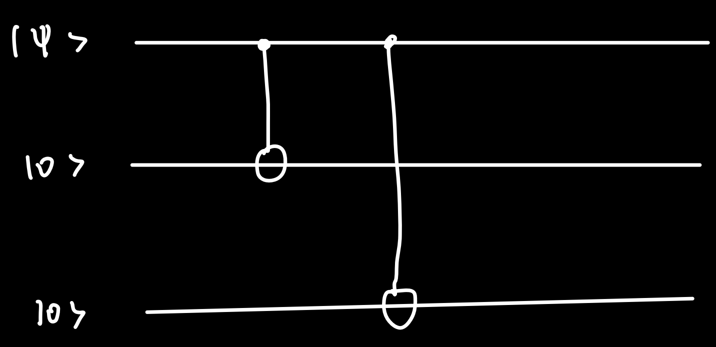
$0 \rightarrow 000$

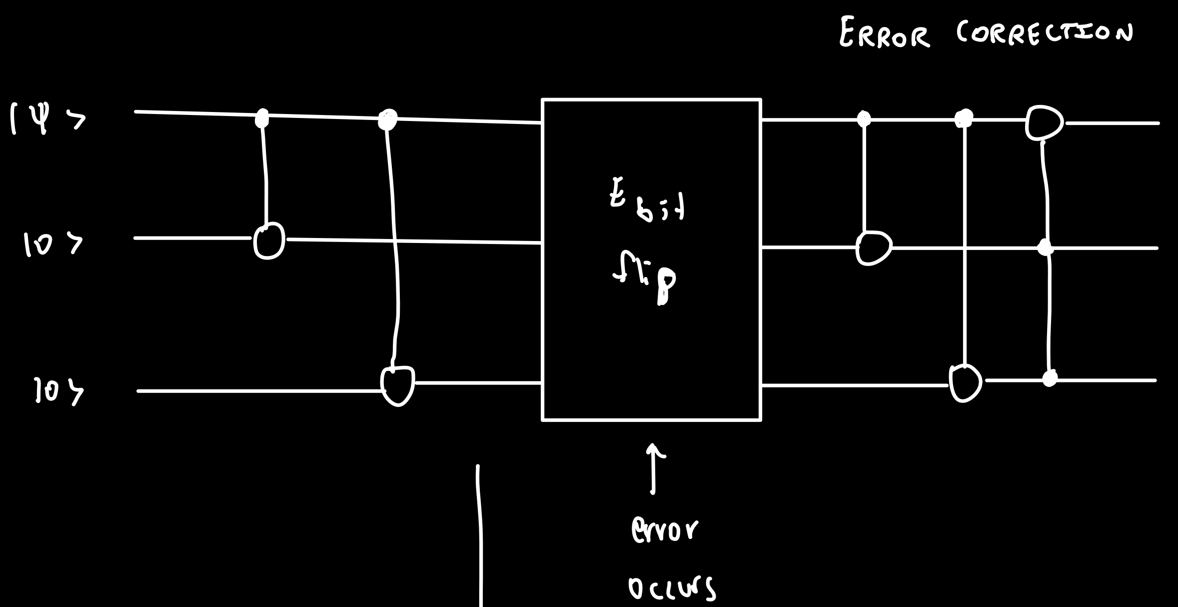
$1 \rightarrow 111$

NO CLONING THEOREM

AIM:

$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$





$$a|000\rangle + b|111\rangle \xrightarrow{\text{error}_2} a|010\rangle + b|101\rangle$$

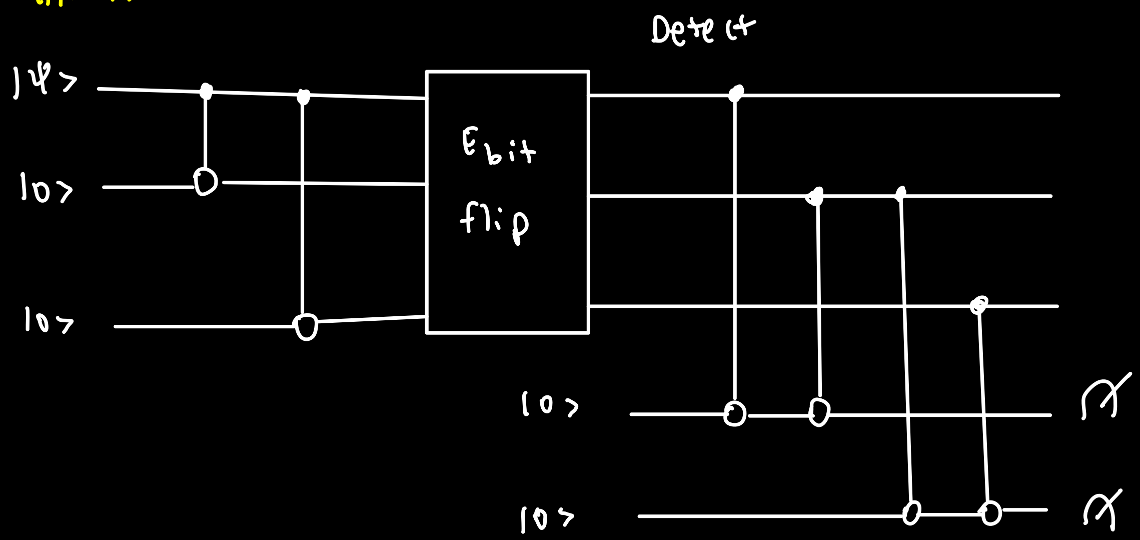
$$\xrightarrow{cx(1,2)} a|010\rangle + b|111\rangle$$

$$\xrightarrow{cx(1,3)} a|010\rangle + b|110\rangle$$

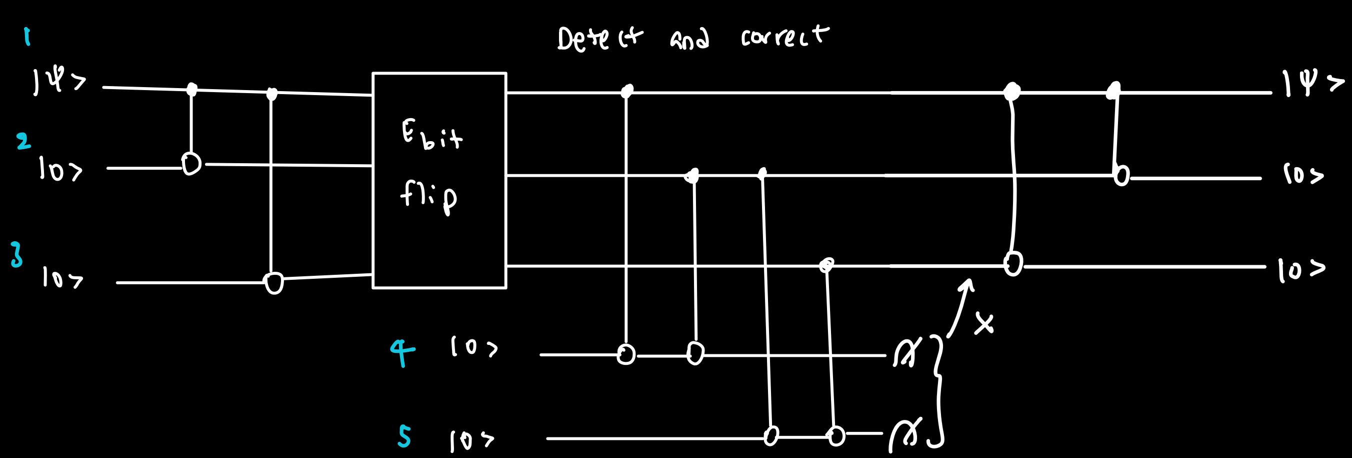
$$\xrightarrow{ccx(3,2,1)} a|010\rangle + b|110\rangle$$

$$a|0\rangle + b|1\rangle \; //$$

PARITY CHECKING :



00	do nothing	
10	flip qubit 1	
11	flip qubit 2	
01	flip qubit 3.	



$a|\underline{01}000\rangle + b|\underline{10}100\rangle$ 2nd Qubit error

$\xrightarrow{cx(1,4)}$ $a|01000\rangle + b|10110\rangle$

$\xrightarrow{cx(2,4)}$ $a|01010\rangle + b|10110\rangle$

$\xrightarrow{cx(2,5)}$ $a|01011\rangle + b|10110\rangle$

$\xrightarrow{cx(3,5)}$ $a|010\underline{11}\rangle + b|101\underline{11}\rangle$

$\xrightarrow{flip(2)}$ $a|00011\rangle + b|11111\rangle$

$\xrightarrow{cx(1,3)+cx(1,2)}$ $a|00011\rangle + b|10011\rangle$

PHASE FLIP ERROR:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

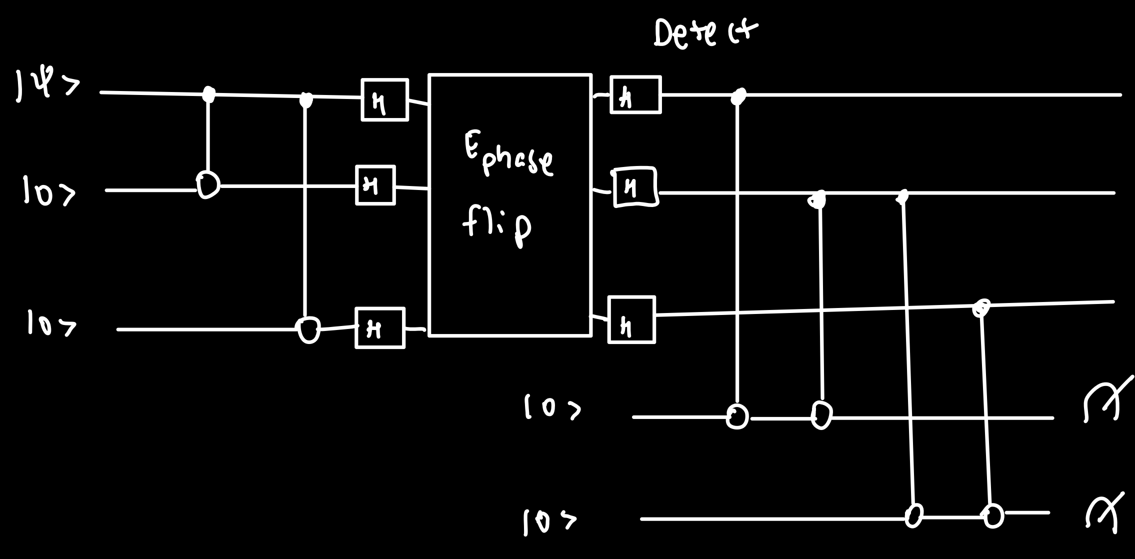
$Z|+\rangle = |-\rangle$

$Z|-\rangle = |+\rangle$

So z to $|+/-\rangle$ is same as x to $|0/1\rangle$

$H|0\rangle = |+\rangle$

$H|1\rangle = |-\rangle$



$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

$$\xrightarrow{H^{\otimes 3}} a|{+}{+}{+}\rangle + b|{-}{-}{-}\rangle$$

$$\xrightarrow{\text{error } 3} a|{+}{+}{-}\rangle + b|{-}{-}{+}\rangle$$

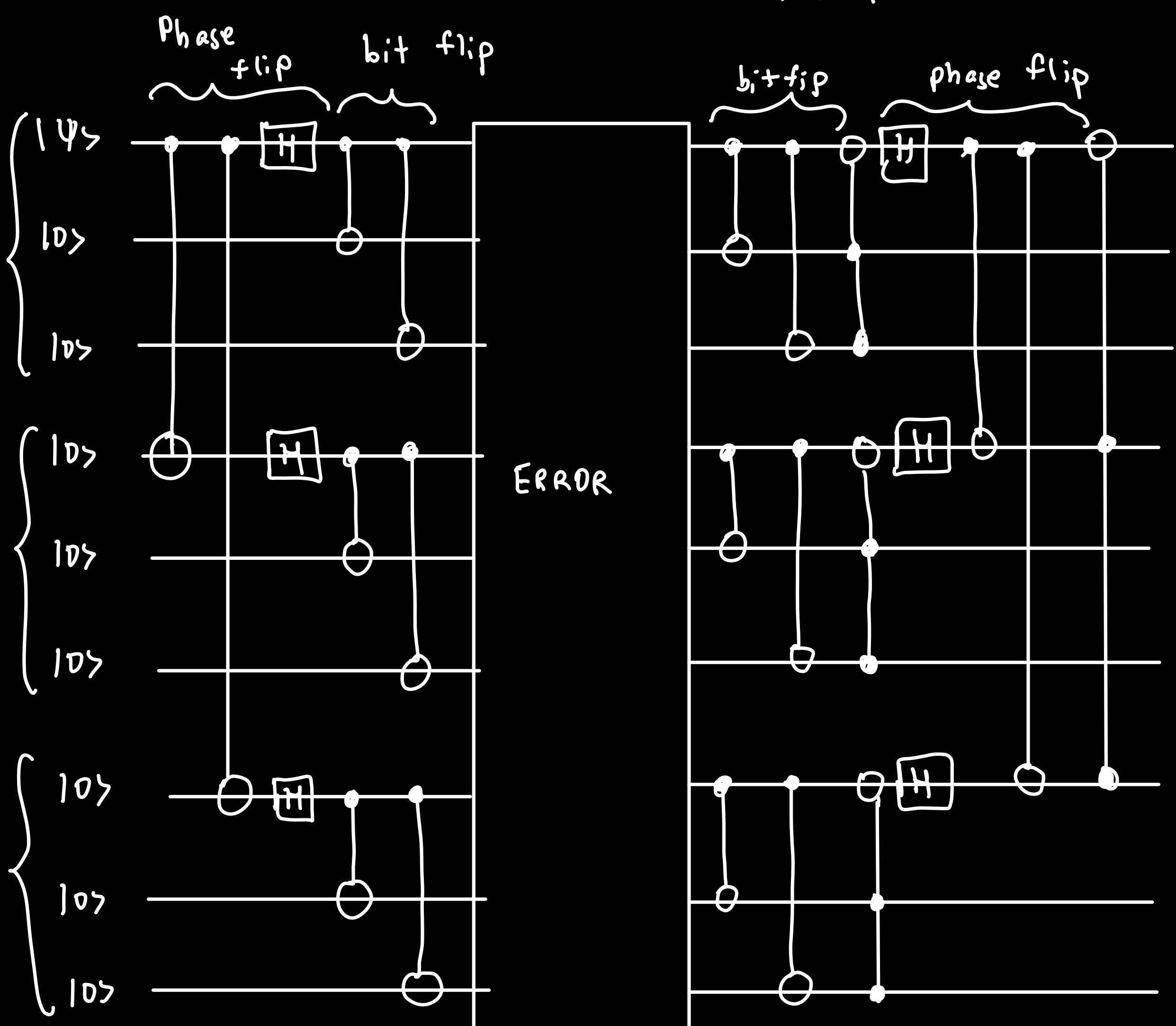
$$\xrightarrow{H^{\otimes 3}} a|001\rangle + b|110\rangle$$

Same as before.

SHOR CODE:

Shor Code = (Phase flip code) • (Bit flip code)

concatenation



THE THRESHOLD THEOREM:

Upper bound on error rate.

Simulation 10^{-3}

Theory 10^{-5}.

Eric Hudson, Wes Campbell

SPAM Reliability $\longrightarrow$ 99.97%.

10^{-4} error rate.

$$P(\text{2 errors in the shor code}) = \binom{9}{2} \cdot p^2 (1-p)^7$$

$$\approx 36 \cdot p^2$$

$$36p^2 < p$$

$$\boxed{p < \frac{1}{36}}$$

2019: Gidney, Ekeria

Shor's Algorithm 2048 bit RSA integers.

10^-3 : 8 hours

20 million qubits.