FIRST - ORDER LOGIC

INFERENCE

- reducing FOL inference to propositional inference

* simple / restricted method
  - Forward Chaining
  - Backward Chaining

* Resolution

UNIVERSAL INSTANTIATION (UI):

$\forall x \quad King(x) \wedge Greedy(x) \Rightarrow Evil(x)$

$King(John) \wedge Greedy(John) \Rightarrow Evil(John)$

$King(Rick) \wedge Greedy(Rick) \Rightarrow Evil(Rick)$

$King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$

GROUND TERM

$$\frac{\forall x \quad \alpha}{\text{Substitute}(\{x/g\}, \alpha)}$$

g : ground term

TERM

Variable x

Constant

Function (term1, ..., termn)

If finite space, equivalent.

EXISTENTIAL INSTANTIATION (EI):

$\exists x \quad Crown(x) \wedge OnHead(x, John)$

$Crown(C_1) \wedge OnHead(C_1, John)$

$\downarrow$

Skolem constant

$$\frac{\exists x \ \alpha}{Substitute(\{x/k\}, \alpha)}$$

k : constant symbol that does not appear anywhere else in the KB.

FOL KB PL KB

(Satisfiability is preserved)

ENTAILMENT IN FOL IS SEMIDECIDABLE:

Herbrand (1930): If sentence α is entailed by an FOL KB it is entailed by a finite subset of the propositional KB.

Nesting
- John
- Father (John)
- Father (Father (John))

For $n = 0$ to ∞ do

* Create a propositional KB by instantiating with depth-n terms.
* See if α is entailed by this KB.

If $\Delta \models \alpha$ then $\Delta_0 \models \alpha$, $\Delta_1 \models \alpha$, $\Delta_2 \models \alpha$ ...

one of it will satisfy.

DEFINITE CLAUSES:

Horn Clause: at most one positive literal

Definite Clause: exactly one positive literal

$\neg A \vee \neg B \vee C$

$[A \wedge B \Rightarrow C]$

If - then rules with positive literals on each side.

FORWARD - CHAINING:

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

2. $\exists x \; Owns(Nono, x) \wedge Missile(x)$

   $\hookrightarrow Owns(Nono, M_1) \wedge Missile(x)$

3. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

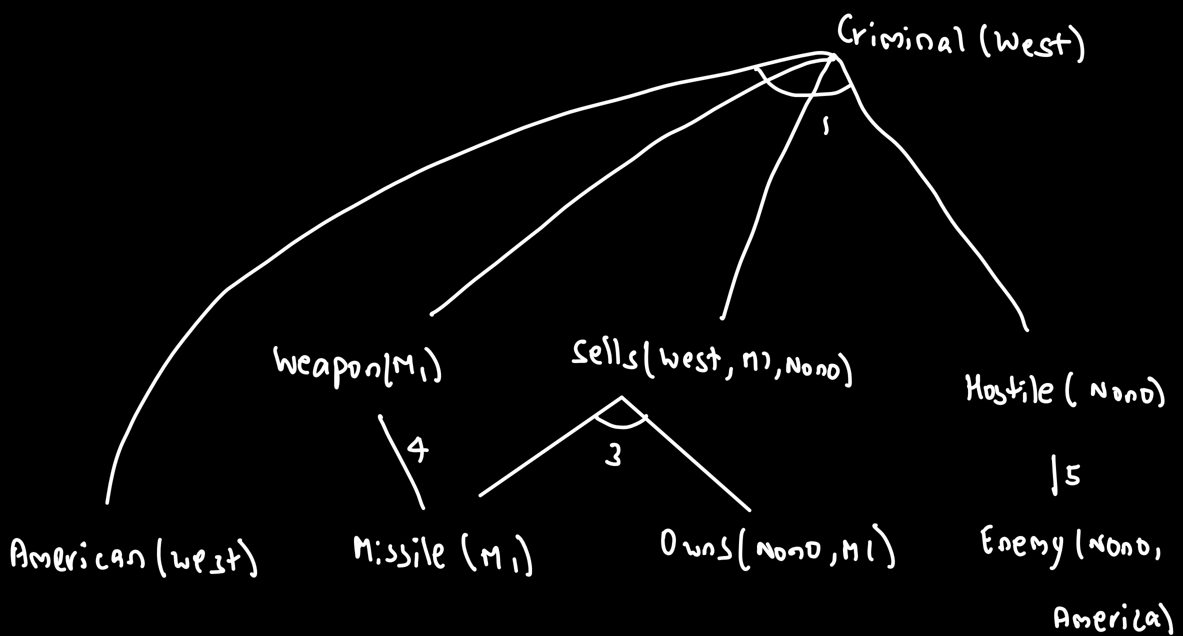
4. $Missile(x) \Rightarrow Weapon(x)$

5. $Enemy(x, America) \Rightarrow Hostile(x)$

6. $American(West)$

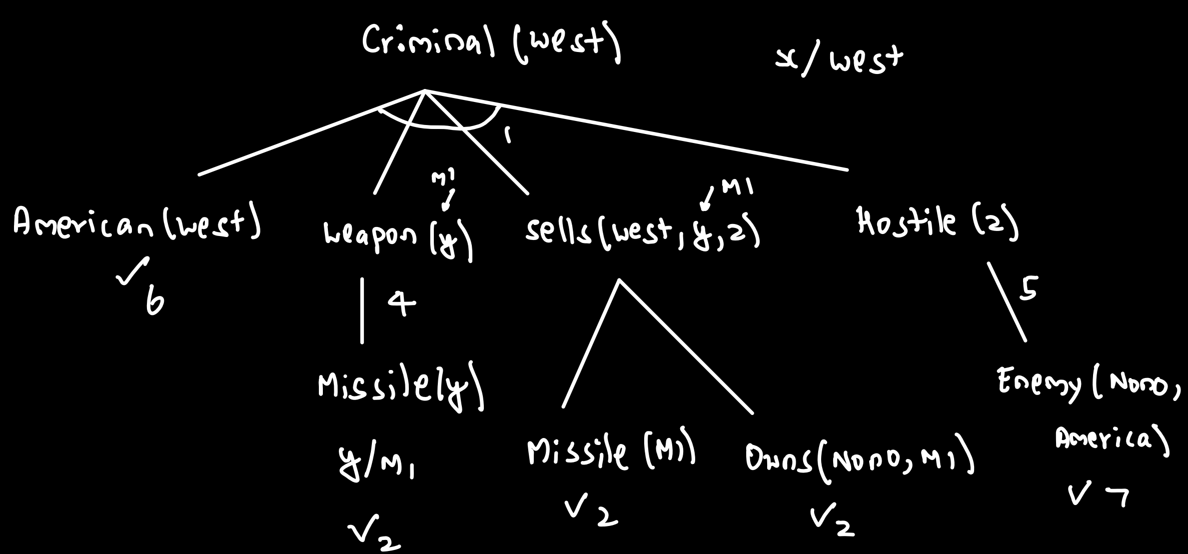
7. $Enemy(Nono, America)$

To show: $Criminal(West)$



"Datalog".

BACKWARD CHAINING:



(with improvements)

(logic programming)

(Prolog)

RESOLUTION IN FOL:

$\Delta \overset{?}{\models} \alpha$ $\underbrace{\Delta \wedge \neg\alpha}_{\text{CNF}}$ check if satisfiable

$\Delta \models \alpha$ iff $\Delta \wedge \neg\alpha$ is unsatisfiable

UNIFICATION:

1. α = Knows(John, x) β = Knows(John, Jane)

Unifier $\rightarrow$ {x / Jane}

α, β

unify$(\alpha, \beta) = \theta$

$\alpha\theta = \beta\theta$

θ = {x / Jane}

2. Knows(John, x)   Knows(y, OJ)

θ = {y / John, x / OJ}

3. Knows(John, x)   Knows(y, Mother(y))

θ = {y / John, x / Mother(John)}

4. $\underbrace{\text{Knows(John, x)} \quad \text{Knows(x, OJ)}}$ Cannot be unified.

$\forall x \quad Rich(x) \Rightarrow Unhappy(x)$

$Rich(ken)$

CNF:

$\neg Rich(x) \lor Unhappy(x)$

$Rich(ken)$

$\theta = \{x/ken\}$

$Unhappy(ken)$

RESOLUTION EXAMPLE:

Jack owns a dog

Every dog owner is an animal lover

No animal lover kills an animal

Either Jack or curiosity killed the cat

Cats are animals

Did curiosity kill the cat?

Δ: $\exists x \; Dog(x) \wedge Own(Jack, x)$

$\forall x \; (\exists y \; Owns(x, y) \wedge Dog(y)) \Rightarrow Alover(x)$

$\forall x \; Alover(x) \Rightarrow (\forall y \; Animal(y) \Rightarrow \neg Kills(x, y))$

$Kills(Jack, Tuna) \vee Kills(Cur, Tuna)$

$Cat(Tuna)$

$\forall x \; Cat(x) \Rightarrow Animal(x)$

α: $Kills(Cur, Tuna)$

CNF:

Δ : 1. $Dog(D)$

2. $Owns(Jack, D)$

3. $\neg Owns(x, y) \vee \neg Dog(y) \vee Alover(x)$

4. $\neg Alover(x) \vee \neg Animal(y) \vee \neg Kills(x, y)$

5. $Kills(Jack, Tuna) \vee Kills(Cur, Tuna)$

6. $Cat(Tuna)$

7. $\neg Cat(x) \vee Animal(x)$

$\neg\alpha$ 0. $\neg Kills(Cur, Tuna)$

8. $\neg Owns(x, D) \lor Alover(x)$ — $(1, 3 \quad \theta = \{y/D\})$

9. $Alover(Jack)$ — $(2, 8 \quad \theta = \{x/Jack\})$

10. $Animal(x)$ — $(6, 7 \quad \theta = \{x/Tuna\})$

11. $\neg Alover(x) \lor \neg Kills(x, Tuna)$ — $(4, 10 \quad \theta = \{y/Tuna\})$

12. $\neg Kills(Jack, Tuna)$ — $(9, 11 \quad \theta = \{x/Jack\})$

13. $Kills(Jack, Tuna)$ — $(0, 5)$

14. Empty clause — $(12, 13)$

Contradiction

$\Delta \land \neg\alpha$ is unsatisfiable

$\Delta \models \alpha$

CONVERSION TO CNF:

$\forall x \, [\forall y \; Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y \; Loves(y,x)]$

1. Eliminate $\Rightarrow$, $\Leftrightarrow$

$\forall x \, [\neg \forall y \; \neg Animal(y) \vee Loves(x,y)] \vee [\exists y \; Loves(y,x)]$

2. Move $\neg$ inwards

$$\neg(\alpha \wedge \beta) = \neg\alpha \vee \neg\beta$$

$$\neg(\alpha \vee \beta) = \neg\alpha \wedge \neg\beta$$

$$\neg\forall x \, \alpha = \exists x \, \neg\alpha$$

$$\neg\exists x \alpha = \forall x \, \neg\alpha$$

$\forall x \, [\exists y \, \neg(\neg Animal(y) \vee Loves(x,y))] \vee [\exists y \; Loves(y,x)]$

$\forall x \, [\exists y \; Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y \; Loves(y,x)]$

3. Standardize variables: each quantifier should use a different one.

$\forall x \, [\exists y \; Animal(y) \wedge \neg Loves(x,y)] \vee [\exists z \; Loves(z,x)]$

4. Skolemize

$$\forall x\ [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

Skolem functions

5. Drop Universal Quantifiers

$$[Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

6. Distribute

$$[Animal(F(x)) \vee Loves(G(x), x)] \wedge [\neg Loves(x, F(x)) \vee Loves(G(x), x)] \;//$$

$\vee \longrightarrow |$

$\wedge \longrightarrow \&$

$\sim \longrightarrow \neg$

$=>$

$<=>$

$\exists$ E

$\forall$ A