1. BASE CASE:

$n = 1$

$$Pr(\alpha_1 | \beta) = Pr(\alpha_1 | \beta)$$

$n = 2$

$$Pr(\alpha_1, \alpha_2 | \beta) = Pr(\alpha_1 | \alpha_2, \beta) \cdot Pr(\alpha_2 | \beta)$$

(BY BAYES CONDITIONING RULE)

INDUCTIVE STEP:

Assume it is true for n

$$Pr(\alpha_1, \dots, \alpha_n | \beta) = Pr(\alpha_1 | \alpha_2, \dots, \alpha_n, \beta) Pr(\alpha_2 | \alpha_3, \dots \alpha_n, \beta) \dots Pr(\alpha_n | \beta)$$

So

$$Pr(\alpha_1, \dots, \alpha_n | \alpha_{n+1}, \beta) = Pr(\alpha_1 | \alpha_2, \dots, \alpha_n, \alpha_{n+1}, \beta) \cdot Pr(\alpha_2 | \alpha_3 \dots \alpha_n \alpha_{n+1}, \beta) \dots Pr(\alpha_n | \alpha_{n+1}, \beta) \quad -(1)$$

Let us prove for $(n+1)$

To Prove:

$$Pr(\alpha_1, \dots, \alpha_{n+1} | \beta) = Pr(\alpha_1 | \alpha_2, \dots, \alpha_{n+1}, \beta) \, Pr(\alpha_2 | \alpha_3, \dots \alpha_{n+1}, \beta) \dots Pr(\alpha_{n+1} | \beta)$$

BY BAYES CONDITIONING :

$$Pr(\alpha_1, \ldots, \alpha_{n+1} | \beta) = Pr(\alpha_1, \ldots, \alpha_n | \alpha_{n+1}, \beta) \cdot Pr(\alpha_{n+1} | \beta)$$

$$= Pr(\alpha_1 | \alpha_2, \ldots, \alpha_{n+1}, \beta) Pr(\alpha_2 | \alpha_3, \ldots \alpha_{n+1}, \beta) \ldots$$

$$Pr(\alpha_n | \alpha_{n+1}, \beta) \cdot Pr(\alpha_{n+1} | \beta)$$

[FROM 1]

HENCE PROVED //

2. Let O denote Oil is present

N denote Natural Gas is present

R denote Positive Result.

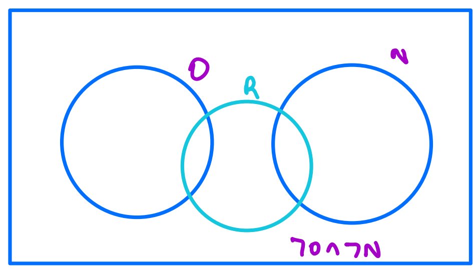
GIVEN:

$Pr(O) = 0.5$

$Pr(N) = 0.2$

$Pr(\neg O \wedge \neg N) = 0.3$

$Pr(O \wedge N) = 0$ [MUTUALLY EXCLUSIVE]



$Pr(R \mid O) = 0.9$

$Pr(R \mid N) = 0.3$

$Pr(R \mid \neg O \wedge \neg N) = 0.1$

So, using CASE ANALYSIS

$Pr(R) = Pr(R, O) + Pr(R, N) + Pr(R, \neg O \wedge \neg N)$

(Only 3 possibilities as O and N are mutually exclusive)

$= Pr(R \mid O) \cdot Pr(O) + Pr(R \mid N) \cdot Pr(N) + Pr(R \mid \neg O \wedge \neg N) \cdot Pr(\neg O \wedge \neg N)$

$= (0.9 \times 0.5) + (0.3 \times 0.2) + (0.1 \times 0.3)$

$= 0.54$

$$Pr(O|R) = \frac{Pr(R|O) \cdot Pr(O)}{Pr(R)}$$

BAYES RULE

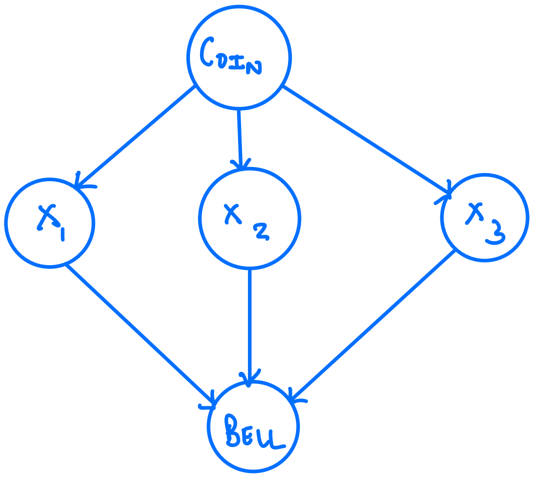
$$= \frac{0.9 \times 0.5}{0.54}$$

$$= \frac{45}{54}$$

$$= 0.833 \,//$$

$$Pr(O|R) = 83.33\% \,//$$

3. BAYESIAN NETWORK:



Coin = {a, b, c}

$X_1 = X_2 = X_3 = \{H, T\}$ where H: Head

T: Tail

Bell : {T, F} where T: On

F: Off

CONDITIONAL PROBABILITY TABLES:

COIN	θ_{Coin}
a	1/3
b	1/3
c	1/3

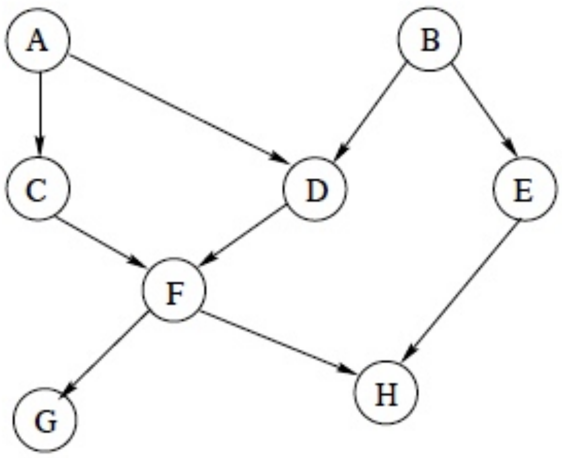
COIN	X_1	$\theta_{x_1 \mid Coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

Coin	X_2	$\theta_{x_2 \mid Coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

Coin	X_3	$\theta_{x_3 \mid Coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

x_1	x_2	x_3	BELL	$\theta_{Bell \mid x_1, x_2, x_3}$
H	H	H	T	1
H	H	H	F	0
H	H	T	T	0
H	H	T	F	1
H	T	H	T	0
H	T	H	F	1
T	H	H	T	0
T	H	H	F	1
H	T	T	T	0
H	T	T	F	1
T	H	T	T	0
T	H	T	F	1
T	T	H	T	0
T	T	H	F	1
T	T	T	T	1
T	T	T	F	0

4.



a. MARKOVIAN ASSUMPTIONS:

$I(A, \phi, BE)$

$I(B, \phi, AC)$

$I(C, A, BDE)$

$I(D, AB, CE)$

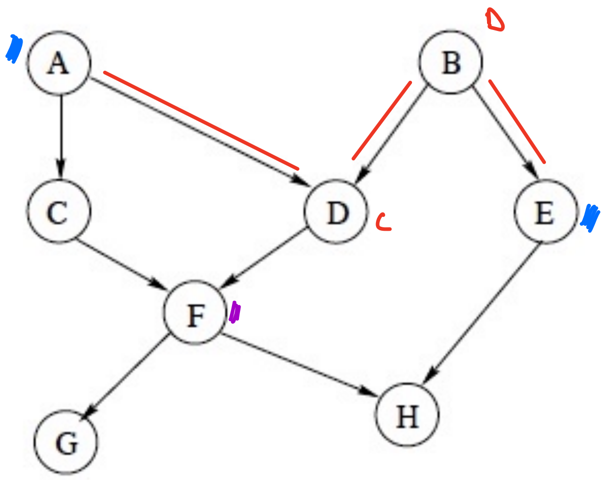
$I(E, B, ACDFG)$

$I(F, CD, ABE)$

$I(G, F, ABCDEH)$

$I(H, EF, ABCDG)$

b. * d-separated (A, F, E)



Valve D → open

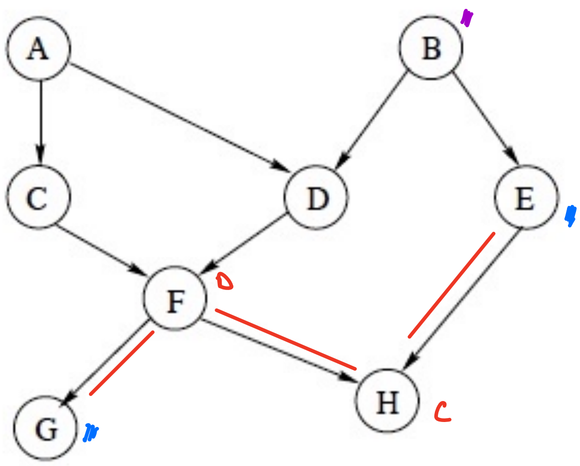
Valve B → open

So Path A-D-B-E is not blocked.

A and E are not d-separated by F.

FALSE //

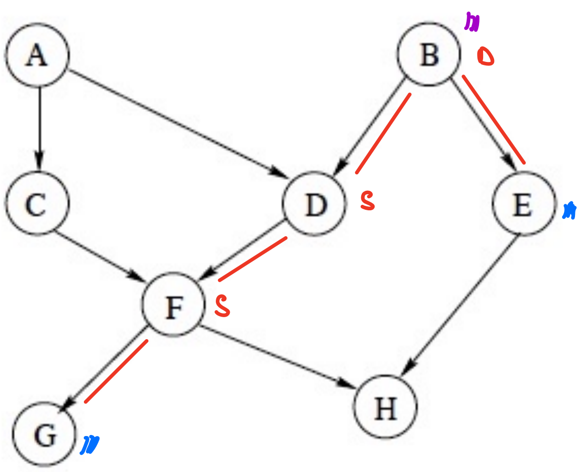
* d-separated (G,B,E)



Valve F - open

Valve H - closed

So G-F-H-E-B is closed.

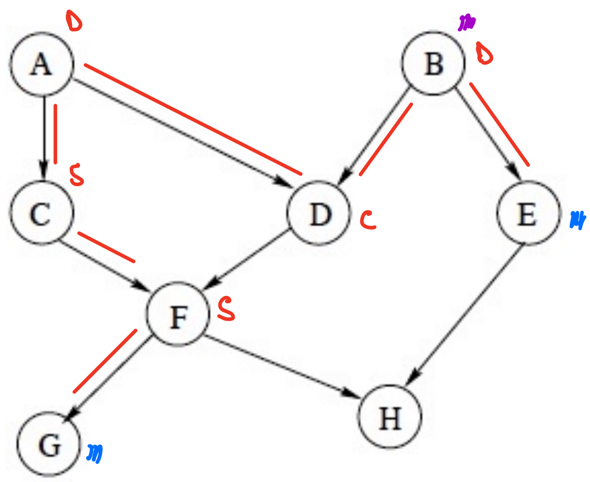


Valve F - open

Valve D - open

Valve B - closed

Path G-F-D-B-E is blocked



Valve F - open

Valve C - open

Valve A - open

Valve D - closed

Valve B - closed

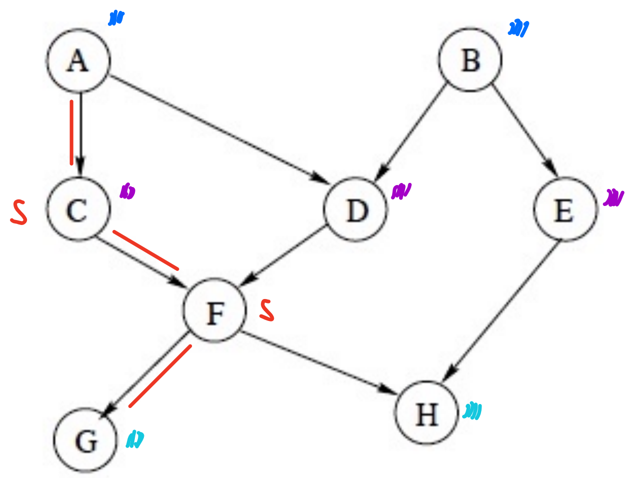
So, path G-F-C-A-D-B-E is blocked.

Every path between G and E are blocked by B.

G and E are d-separated by B.

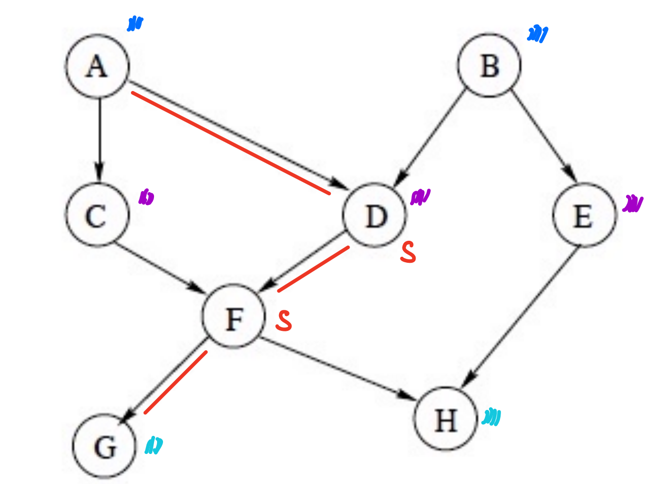
TRUE//

- d- separated (AB, CDE, GH)



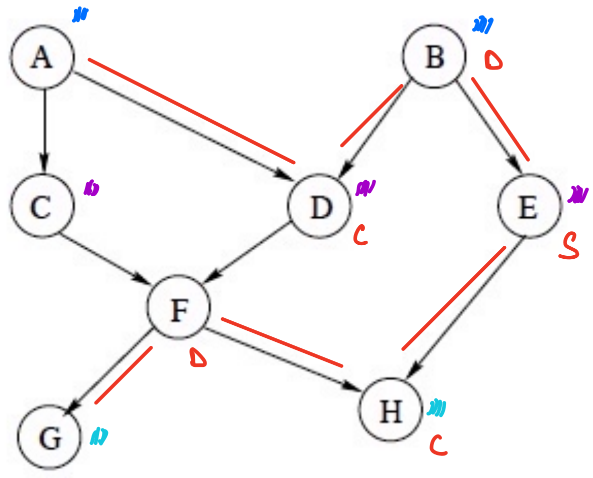
Closed at C

Path A-C-F-G is blocked.



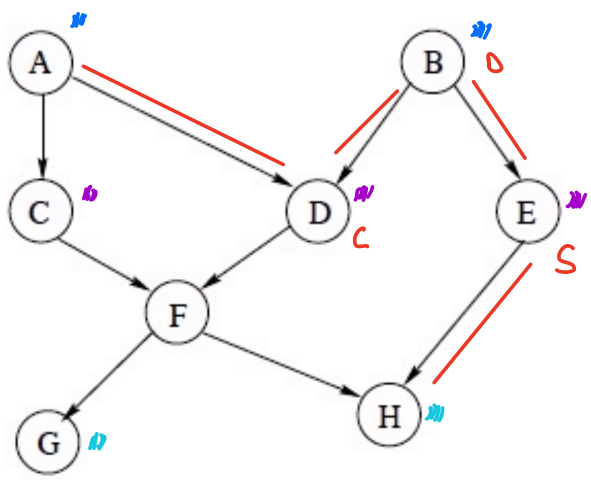
Closed at D

Path A-D-F-G is blocked.



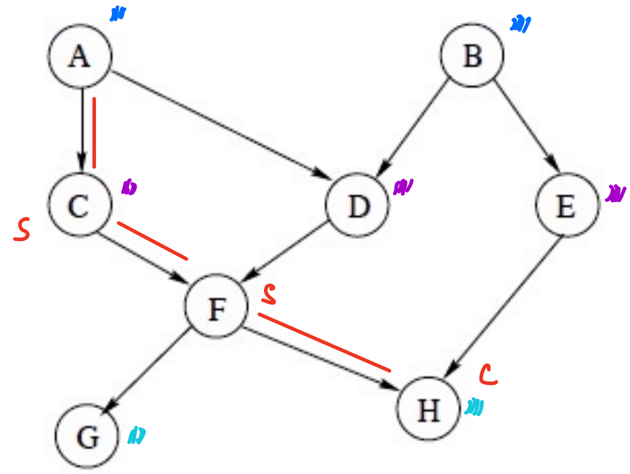
Closed at E

Path A-D-B-E-H-F-G is blocked.



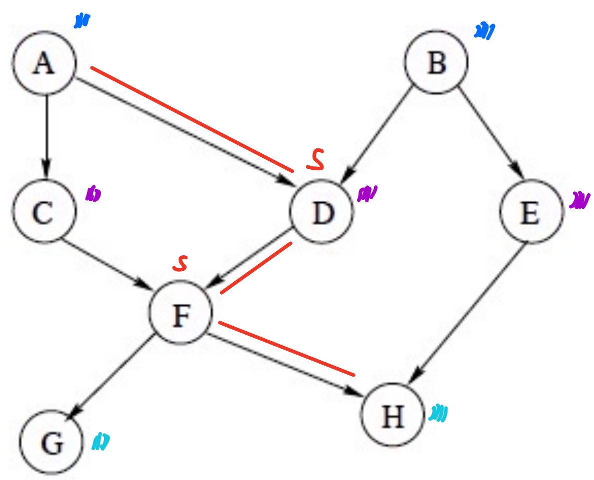
Closed at E

Path A-D-B-E-H is blocked.



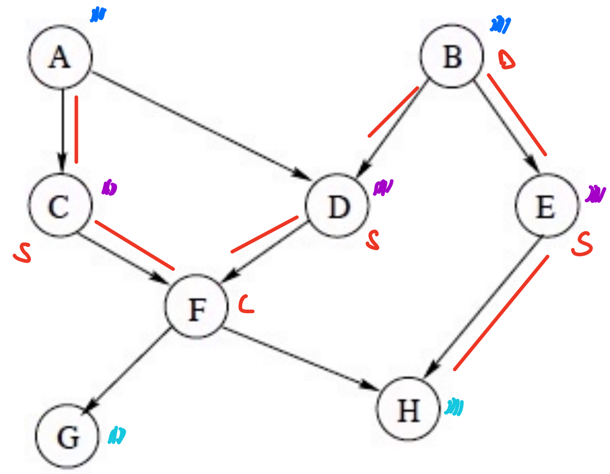
Closed at C

Path A~C~F~H is blocked.



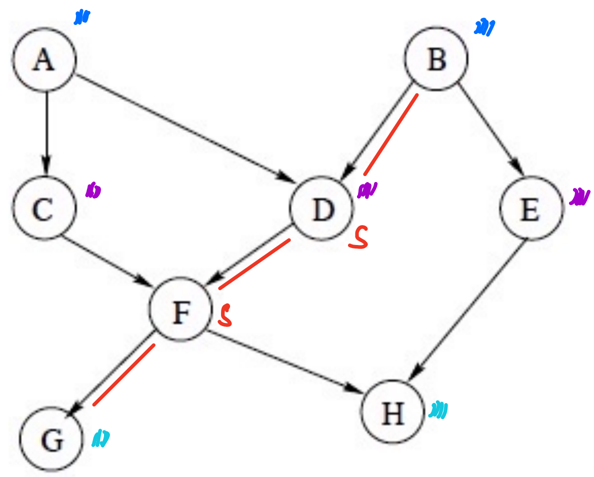
Closed at D

Path A~D~F~H is blocked.



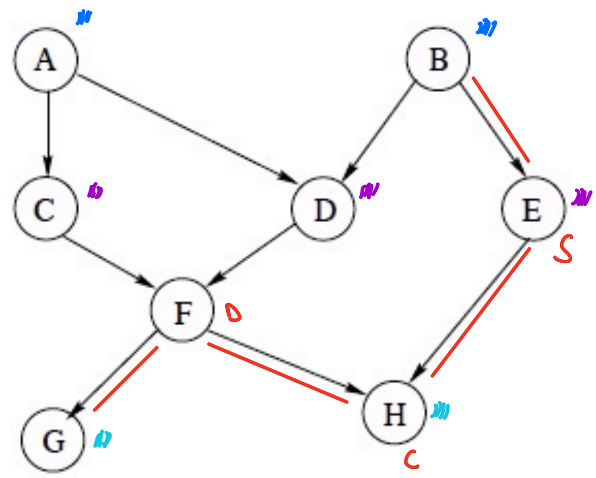
Closed at E

Path A-C-F-D-B-E-H is blocked.



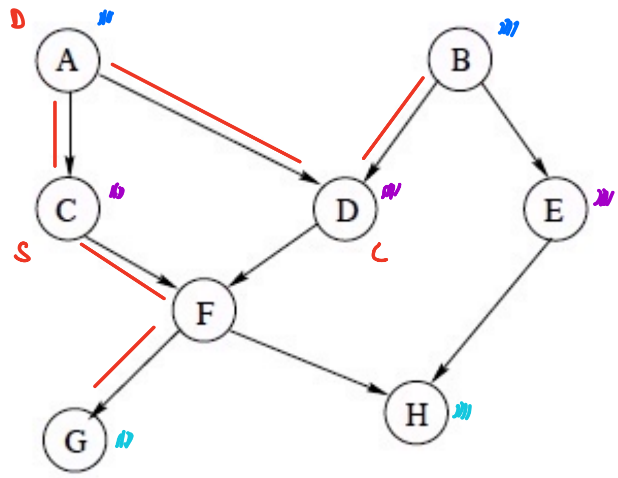
Closed at D

Path B-D-F-G is blocked.



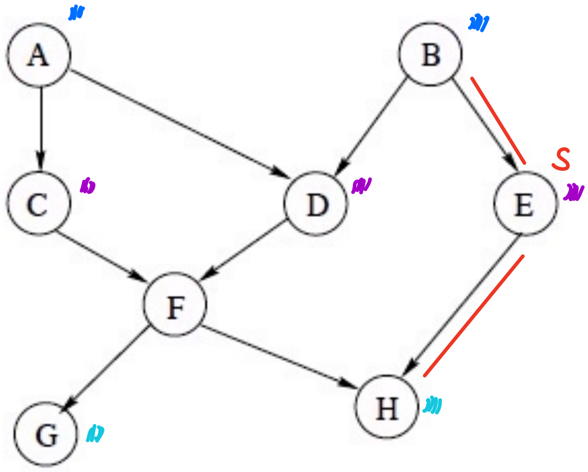
Closed at E

Path B-E-H-F-G is blocked.



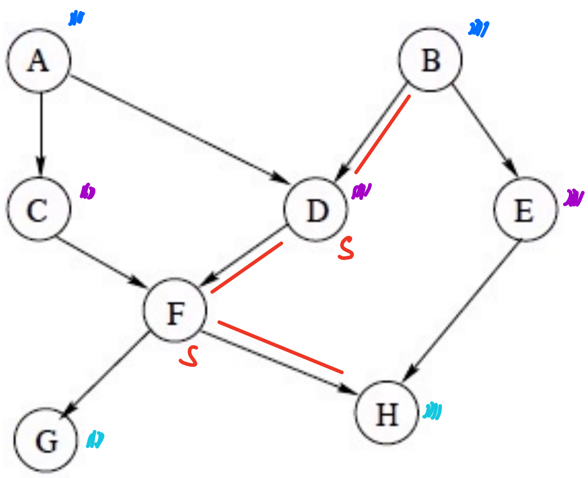
Closed at C

Path B-D-A-C-F-G is blocked.



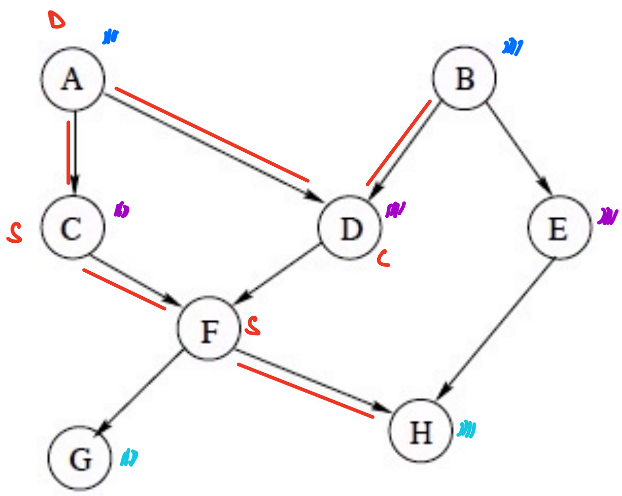
Closed at E

Path B-E-H is blocked.



Closed at D

Path B-D-F-H is blocked.



Closed at C

Path B-D-A-C-F-H is blocked.

Every path between {A,B} and {G,H} are blocked by {C,D,E}

{A,B} and {G,H} are d-separated by {C,D,E}

TRUE //

C.

$$Pr(a,b,c,d,e,f,g,h) = Pr(a) \cdot Pr(b) \cdot Pr(c|a) \cdot Pr(d|a,b) \cdot$$

$$Pr(e|b) \cdot Pr(f|c,d) \cdot Pr(g|f) \cdot Pr(h|e,f) .$$

d. $Pr(A=1, B=1)$

A and B are independent from Markovian Assumption

$I(A, \phi, BE)$

So, $Pr(A=1, B=1) = Pr(A=1) \cdot Pr(B=1)$

$= 0.2 \times 0.7$

$= 0.14$ //

$Pr(E=0 \mid A=0)$

By BAYES CONDITIONING

$$Pr(E=0 \mid A=0) = \frac{Pr(E=0, A=0)}{Pr(A=0)}$$

A and E are independent from Markovian Assumption

$I(A, \phi, BE)$

So, $Pr(E=0, A=0) = Pr(E=0) \cdot Pr(A=0)$

$$\text{So} \quad Pr(E=0 \mid A=0) = \frac{Pr(E=0) \cdot Pr(A=0)}{Pr(A=0)} = Pr(E=0)$$

Using Case Analysis

$Pr(E=0) = Pr(E=0 \mid B=0) \cdot Pr(B=0) + Pr(E=0 \mid B=1) \cdot Pr(B=1)$

$= 0.1 \times 0.3 + 0.9 \times 0.7$

$= 0.03 + 0.63 = 0.66$

$Pr(E=0 \mid A=0) = 0.66$ //

5. a. $M(\alpha) = \{w_0, w_2, w_3\}$

b. $Pr(\alpha) = Pr(\omega_0) + Pr(\omega_2) + Pr(\omega_3)$

$= 0.3 + 0.1 + 0.4$

$= 0.8$ //

c. $Pr(A, B \mid \alpha)$

$$Pr(w \mid \alpha) = \begin{cases} 0 & \text{if} \quad w \models \neg\alpha \\ Pr(w)/Pr(\alpha) & \text{if} \quad w \models \alpha \end{cases}$$

	A	B	$Pr(A, B \mid \alpha)$
w_0	T	T	$0.3/0.8 = 0.375$
w_1	T	F	0
w_2	F	T	$0.1/0.8 = 0.125$
w_3	F	F	$0.4/0.8 = 0.5$

d. $Pr(A \Rightarrow \neg B \mid \alpha)$

$\Delta = A \Rightarrow \neg B$

A	B	$A \Rightarrow \neg B$
T	T	F
T	F	T
F	T	T
F	F	T

$M(\Delta) = \{w_1, w_2, w_3\}$

$Pr(\Delta \mid \alpha) = Pr(w_1 \mid \alpha) + Pr(w_2 \mid \alpha) + Pr(w_3 \mid \alpha)$

$= 0 + 0.125 + 0.5$

$= 0.625$ //