KNOWLEDGE REPRESENTATION AND REASONING LOGIC:

Statements in Logic → Logical Deduction → Conclusion

↑

Observation

Expert Systems " Common Sense Reasoning "

CLASSICAL LOGICS:

→ Propositional Logic

or

→ First-order Logic

* Symbolic Representation

or

* Numeric Representation

↳ Using probability theory

(Bayesian Networks)

* Model or Learn

Integrate / Fuse these approaches ← Knowledge - Symbolic

- numeric

↑ Data.

LOGIC (PROPOSITIONAL LOGIC) BOOLEAN LOGIC:

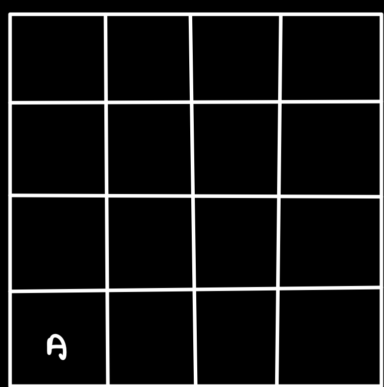
1. Syntax (Grammar)

2. Semantics (Meaning)

   ↳ α is equivalent to β

   → α implies β → Deduction.

WUMPUS WORLD



- Agent
- Pits : Cells adjacent to a pit are Breezy
- Wumpus : Cells adjacent to wumpus are smelly.

SYNTAX:

1. Basic Syntax

2. Normal Forms

a. Variables / Boolean Variables / Atoms / Atomic Variables / Propositional

Symbols:

$x_1 \quad x_2 \quad \dots \quad x_n$

$\downarrow$

True / False

1 / 0

b. Logical connectors

$\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\Leftrightarrow$

AND OR NOT imply equivalent

c. - Variable is a sentence

- if S is a sentence

then $\neg S$ is a sentence

- if S_1 and S_2 are sentences

then
$$\left.\begin{array}{l} S_1 \wedge S_2 \\ S_1 \vee S_2 \\ S_1 \Rightarrow S_2 \\ S_1 \Leftrightarrow S_2 \end{array}\right\} \text{are sentences}$$

eg: $\left[\neg((x_1 \wedge \neg x_2) \Rightarrow x_7) \right] \wedge x_4$

$S_1 \wedge S_2$ CONJUNCTION

CONJUNCT

$S_1 \vee S_2$ DISJUNCTION

DISJUNCT

$S_1 \Rightarrow S_2$

Premise / Antecedent

Consequent.

	B_1	
B_4	P	B_2
	B_3	

Cells adjacent to pit are breezy

$P \Rightarrow (B_1 \wedge B_2 \wedge B_3 \wedge B_4)$

This is stronger than $P \Rightarrow (B_1 \vee B_2 \vee B_3 \vee B_4)$

1. $\alpha \Rightarrow \beta$

$\neg\beta \Rightarrow \neg\alpha$ CONTRAPOSITIVE

2. $\alpha \Rightarrow \beta$

$\neg\alpha \vee \beta$ DE MORGAN'S LAW

3. $\neg(\alpha \wedge \beta)$

$\neg\alpha \vee \neg\beta$

4. $\neg(\alpha \vee \beta)$

$\neg\alpha \wedge \neg\beta$

$$P \Rightarrow (B_1 \wedge B_2 \wedge B_3 \wedge B_4)$$

$$\neg(B_1 \wedge B_2 \wedge B_3 \wedge B_4) \Rightarrow \neg P$$

$$\neg B_1 \vee \neg B_2 \vee \neg B_3 \vee \neg B_4 \Rightarrow \neg P$$

NORMAL FORMS:

* CONJUNCTIVE NORMAL FORM (CNF):

Literal :

x positive literal

$\neg x$ Negative literal

Clause :

Disjunction of literals

$(A \vee \neg B)$

CNF :

Conjuction of clauses.

$(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D) \wedge \dots$

* DISJUNCTIVE NORMAL FORM (DNF):

Term :

Conjunction of literals

DNF :

Disjunction of terms

$(A \wedge B) \vee (A \wedge \neg C) \vee (A \wedge \neg D \wedge E) \vee \dots$

IS A FORM UNIVERSAL?

Any sentence in boolean form can be expressed.

CNF, DNF => Universal.

* HORN:

HORN CLAUSE:

Clause with at most one positive literal

$(A \lor \neg B \lor \neg C)$ ✓

$(A \lor B \lor \neg C)$ ×

HORN FORM:

Conjunction of Horn Clause

TRACTABLE:

Something hard becomes easy to do in Horn.

TRACTABLE FOR SAT:

Horn, DNF, DNNF.

It is not universal.

* NNF (NEGATION NORMAL FORM):

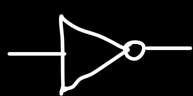
$\neg, \wedge, \vee$ allowed like CNF, DNF.

$\neg$ can only appear next to variables

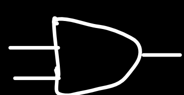
$(\neg x \vee y) \vee (x \wedge z) \wedge \dots$ allowed

$\neg((\neg x \vee y) \vee (x \wedge z)$

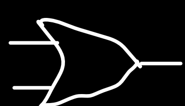
↳ not allowed



NOT



AND



OR

=> UNIVERSAL

* DECOMPOSABLE NNF CIRCUIT (DNNF CIRCUIT):

→ Tractable:

SAT can be done in linear.

For all AND Gates:

The variables involved in the 2 inputs should not have an overlap.

$$\Big(\big(\underbrace{(A \vee B)}_{\{A,B\}} \wedge \underbrace{C}_{\{C\}} \big) \vee D \Big) \wedge \big(\underbrace{(E \vee F)}_{\{E,F\}} \wedge \underbrace{G}_{\{G\}} \big)$$

$\{A, B, C, D\}$ $\{E, F, G\}$

SEMANTICS:

(Meaning)

Earthquake (E)

Burglary (B)

Alarm (A)

→ Sentence α holds in some world ω

$$\omega \models \alpha \quad (\alpha \text{ is true at } \omega)$$

WORLD	E	B	A
ω_1	T	T	T
ω_2	T	T	F

if $\alpha : (B \vee E) \Rightarrow A$

$$\neg(B \vee E) \vee A$$

At ω_1 $\quad \neg(T \vee T) \vee T = T$

So $\omega_1 \models \alpha$

At ω_2 $\quad \neg(T \vee T) \vee F = F$

$$\omega_2 \not\models \alpha$$

KNOWLEDGE:

Say $(E \vee B) \Rightarrow A$ is a knowledge

$\neg(E \vee B) \vee A$

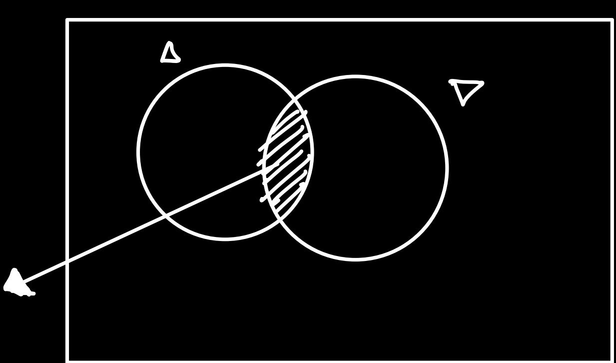
$(\neg E \wedge \neg B) \vee A$

So worlds where

E	B	A
T	T	F
T	F	F
F	T	F

are not possible anymore.

possible worlds



New possible worlds.

Δ : worlds that satisfy knowledge K_1

$\nabla \equiv K_2$

$M(\alpha)$ = set of worlds where α holds at

$\downarrow$

Meaning of α
Models of α $\Big\}$ $\{ \omega : \omega \models \alpha \}$

* α equivalent to β

$$M(\alpha) = M(\beta)$$

* α is contradictory / inconsistent

$$M(\alpha) = \phi$$

* α is valid / tautology

$$M(\alpha) = \text{all worlds}$$

* α and β are mutually exclusive

$$M(\alpha) \cap M(\beta) = \phi$$

* α implies β

$$M(\alpha) \subseteq M(\beta)$$