MODELING, REASONING AND LEARNING UNDER UNCERTAINTY:

Classical logic is "monotonic"

MONOTONICITY:

If $\Delta \models \alpha$

then $\Delta \wedge \beta \models \alpha$

new information

i.e.

$M(\Delta) \subseteq M(\alpha)$

$\Rightarrow$ $M(\Delta \wedge \beta) \subseteq M(\alpha)$

$M(\Delta) \cap M(\beta)$

Δ: Tweety is a bird

α: Does it fly? Yes

β: Tweety is a penguin

α: Does it fly? No!

NOT MONOTONIC

$\forall x \; bird(x) \Rightarrow flies(x) \quad]\, \Delta$

good $\quad \Delta \wedge bird(tweety) \models flies(Tweety)$

bad $\quad \Delta \wedge bird(Tweety)$ contradiction when penguin

$\forall x \; bird(x) \wedge \neg ab(x) \Rightarrow flies(x)$

good $\quad \Delta \wedge bird(Tweety) \wedge \neg flies(Tweety)$ ✓

bad $\quad \Delta \wedge bird(Tweety) \overset{?}{\models} flies(Tweety)$

cannot say now!

So we want it to have a generic truth like birds fly but when given more information, we might have to change to no does not fly. Not Classical logic.

$$\Delta \wedge \underbrace{\;\;}_{\text{Assumptions } \neg ab} \models flies(Tweety)$$

non-monotonic logic / non-classical logic.

Δ : $Quaker(x) \wedge \neg ab_q(x) \Rightarrow Pacifist(x)$

$Republican(x) \wedge \neg ab_r(x) \Rightarrow \neg Pacifist(x)$

↳ abnormal

$\left. \begin{array}{l} Quaker(Nixon) \\ Republican(Nixon) \end{array} \right\} Pacifist(Nixon) \wedge \neg Pacifist(Nixon)$

BELIEF REVISION:

"degree of belief" α follow or not

$\downarrow$

belief(α) [0,1]

Probability.

PROBABILITY AS A BASIS FOR REPRESENTING BELIEFS:

$$Pr(\alpha) = \sum_{\omega \models \alpha} Pr(\omega)$$

	E	B	A	
world	Earthquake	Burglary	Alarm	Pr(.)
ω_1	true	true	true	.0190
ω_2	true	true	false	.0010
ω_3	true	false	true	.0560
ω_4	true	false	false	.0240
ω_5	false	true	true	.1620
ω_6	false	true	false	.0180
ω_7	false	false	true	.0072
ω_8	false	false	false	.7128

$Pr(\text{Earthquake}) = 0.1$

$Pr(B) = 0.2$

$Pr(\neg B) = 0.8$

$Pr(A) = 0.2442$

- Sentence α

  $0 \leq Pr(\alpha) \leq 1$

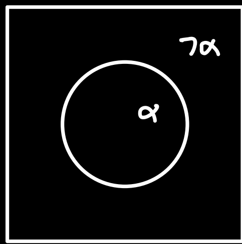
- α is unconsistent (unsatisfiable)

  $Pr(\alpha) = 0$

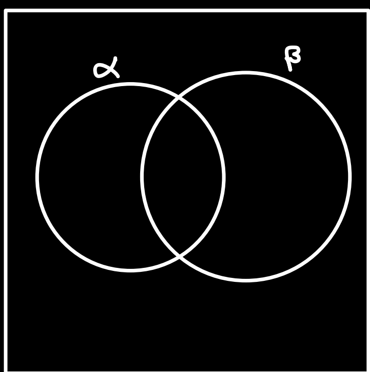
- α is valid

  $Pr(\alpha) = 1$

- $Pr(\alpha) + Pr(\neg\alpha) = 1$



- $Pr(\alpha \vee \beta) = Pr(\alpha) + Pr(\beta) - Pr(\alpha \wedge \beta)$



- If α, β are mutually exclusive

  then $Pr(\alpha \vee \beta) = Pr(\alpha) + Pr(\beta)$

  $M(\alpha) \wedge M(\beta) = \phi$

BELIEF CHANGE :

- Δ $\qquad M(\Delta)$

- $\Delta \wedge \beta$ $\qquad M(\Delta \wedge \beta) = M(\Delta) \cap M(\beta)$

Initial : $Pr(\cdot)$

Updated : $Pr(\cdot \mid \beta)$

β ↳ new information

world	Earthquake (E)	Burglary (B)	Alarm (A)	Pr(.)
ω_1	true	true	true	.0190/0.2442
ω_2	true	true	false	.0010 ×
ω_3	true	false	true	.0560/0.2442
ω_4	true	false	false	.0240 ×
ω_5	false	true	true	.1620/0.2442
ω_6	false	true	false	.0180 ×
ω_7	false	false	true	.0072/0.2442
ω_8	false	false	false	.7128 ×

New Information : A (alarm triggered)

"evidence"

$$Pr(A) = 0.2442$$

$$Pr(\omega \mid \beta) \overset{def}{=} \begin{cases} 0 & \text{if} \quad \omega \models \neg\beta \\ \dfrac{Pr(\omega)}{Pr(\beta)} & \text{if} \quad \omega \models \beta \end{cases}$$

evidence

BAYES CONDITIONING:

$$\boxed{Pr(\alpha \mid \beta) = \frac{Pr(\alpha \wedge \beta)}{Pr(\beta)}}$$

conditional probability.

$Pr(B) = 0.2$

$Pr(B \mid A) \simeq 0.741 \uparrow$

$Pr(E) = 0.1$

$Pr(E \mid A) \simeq 0.307 \uparrow$ $\qquad Pr(E \mid B) = 0.1$

$Pr(B) = 0.2$

$Pr(B \mid E) = 0.2$

BAYESIAN NETWORKS
- Causal graph
- numbers.

$$\begin{tikzcd} B \arrow[dr] & & E \arrow[dl] \\ & A & \end{tikzcd}$$

$Pr(B \mid A, E) \simeq 0.253$

$Pr(B \mid A, \neg E) \simeq 0.957$

INDEPENDENCE :

$Pr(E) = 0.1$

$Pr(B) = 0.2$

$Pr(E \mid B) = 0.1$ $\qquad$ $Pr(B \mid E) = 0.2$

α is independent of β

$Pr(\alpha \mid \beta) = Pr(\alpha)$ $\qquad$ Also $Pr(\alpha \mid \beta) = Pr(\alpha)$

$$\frac{Pr(\alpha \cap \beta)}{Pr(B)} = Pr(\alpha) \qquad Pr(\beta \mid \alpha) = Pr(\beta)$$

$$Pr(\alpha \cap \beta) = Pr(\alpha) \cdot Pr(\beta)$$

$\left.\begin{array}{l} Pr(B) = 0.2 \\ Pr(B \mid E) = 0.2 \end{array}\right\}$ B, E independent

$\left.\begin{array}{l} Pr(B \mid A) = 0.741 \\ Pr(B \mid A, E) = 0.253 \end{array}\right\}$ not anymore

So, independence is dynamic.

Pr finds α conditionally independent of β given γ

$$Pr(\alpha \mid \beta \wedge \gamma) = Pr(\alpha \mid \gamma)$$

$$Pr(\alpha \wedge \beta \mid \gamma) = Pr(\alpha \mid \gamma)\, Pr(\beta \mid \gamma)$$

PROPERTIES OF BELIEFS:

$$Pr(\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) = Pr(\alpha_1 \mid \alpha_2 \wedge \dots \alpha_n)\, Pr(\alpha_2 \mid \alpha_3 \wedge \dots \wedge \alpha_n) \dots Pr(\alpha_n)$$

Chain Rule

CASE ANALYSIS (LAW OF TOTAL PROBABILITY):

$$Pr(\alpha) = \sum_{i=1}^{n} Pr(\alpha \wedge \beta_i)$$

where $\beta_1, \beta_2, \dots \beta_n$ are mutually exclusive and exhaustive.

$$Pr(\alpha) = Pr(\alpha \wedge \beta) + Pr(\alpha \wedge \neg\beta)$$

$$\sum_{i=1}^{n} Pr(\alpha \cap \beta_i) = \sum_{i=1}^{n} Pr(\alpha | \beta_i) Pr(\beta_i)$$

BAYES RULE:

$$Pr(\alpha | \beta) = \frac{Pr(\alpha \cap \beta)}{Pr(\beta)} \qquad Pr(\beta | \alpha) = \frac{Pr(\alpha \cap \beta)}{Pr(\alpha)}$$

$$Pr(\alpha \cap \beta) = Pr(\alpha | \beta) Pr(\beta) \qquad Pr(\alpha \cap \beta) = Pr(\beta | \alpha) \cdot Pr(\alpha)$$

$$Pr(\alpha | \beta) Pr(\beta) = Pr(\beta | \alpha) Pr(\alpha)$$

$$\boxed{Pr(\alpha | \beta) = \frac{Pr(\beta | \alpha) Pr(\alpha)}{Pr(\beta)}}$$

eg: α : cause like disease

β : effect like symptoms

① $Pr(\alpha | \beta) \rightarrow$ Pr of cause given effect
(disease) (symptoms)

② $Pr(\beta | \alpha) \rightarrow$ Pr of effect given cause.
(symptoms) (disease)

② is easier to calculate.

D: D has disease, $\neg D$ does not

T: T Test came out positive, $\neg T$ test came out negative.

Find: $Pr(D|T)$

We know: $Pr(D) = 1/1000$

False positive ratio 2%

$Pr(T | \neg D) = 2/100$

$\hookrightarrow Pr(\neg T | \neg D) = 98/100$

False negative ratio of 5%

$Pr(\neg T | D) = 5/100$

$\hookrightarrow Pr(T | D) = 95/100$

$$Pr(D|T) = \frac{Pr(T|D)\,Pr(D)}{Pr(T)} = \frac{95/100 \times 1/1000}{2093/100{,}000} = \frac{95}{2093} \approx 4.5\%$$

$$Pr(T) = Pr(T|D) \cdot Pr(D) + Pr(T|\neg D) \cdot Pr(\neg D)$$

$$= 95/100 \cdot 1/1000 + 2/100 \times 999/1000$$

$$= \frac{2093}{100{,}000}$$

PROPOSITIONAL LOGIC:

$X, \neg X$

$T, \neg T$

$D, \neg D$

What if X : red, green, blue.

$(x = red) \vee (x = blue) \Rightarrow (y = large)$

$X \to$ Short hand for $(x = True)$

$\neg X \to (x = False)$

$$Pr(T) = \underbrace{Pr(T|D)\, Pr(D)}_{D = true} + \underbrace{Pr(T|\neg D)\, Pr(\neg D)}_{D = false}$$

$$Pr(T) = Pr(T \mid x = red)\, P(x = red) + \ldots$$