1. a) $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

	P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
w_1	T	T	F	F
w_2	T	F	T	T
w_3	F	T	T	T
w_4	F	F	T	T

$M(P \Rightarrow \neg Q) = \{w_2, w_3, w_4\}$

$M(Q \Rightarrow \neg P) = \{w_2, w_3, w_4\}$

$M(P \Rightarrow \neg Q) = M(Q \Rightarrow \neg P)$

So $(P \Rightarrow \neg Q) \Leftrightarrow (Q \Rightarrow \neg P)$

b. $\alpha = P \Leftrightarrow \neg Q$ $\beta = ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

	P	Q	$P \Rightarrow \neg Q$	$\neg Q \Rightarrow P$	$P \Leftrightarrow \neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	β
ω_1	T	T	F	T	F	F	F	F
ω_2	T	F	T	T	T	T	F	T
ω_3	F	T	T	T	T	F	T	T
ω_4	F	F	T	F	F	F	F	F

$M(\alpha) = \{\omega_2, \omega_3\}$

$M(\beta) = \{\omega_2, \omega_3\}$

$M(\alpha) = M(\beta)$

So $\alpha \Leftrightarrow \beta$

$(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

2. Let Smoke be S

Fire be F

Heat be H.

a. $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$

$\alpha = S \Rightarrow F$

$\beta = \neg S \Rightarrow \neg F$

S	F	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$\alpha \Rightarrow \beta$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

$M(\alpha \Rightarrow \beta)$ is not all worlds. Neither is it ϕ.

So, $\alpha \Rightarrow \beta$ is satisfiable, neither valid nor unsatisfiable//

b. $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

$\alpha = S \Rightarrow F \qquad \beta = ((S \vee H) \Rightarrow F)$

S	F	H	α	$S \vee H$	β	$\alpha \Rightarrow \beta$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

$M(\alpha \Rightarrow \beta)$ is not all worlds. Neither is it ϕ.

So, $\alpha \Rightarrow \beta$ is satisfiable, neither valid nor unsatisfiable//

c. $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$\alpha = (S \wedge H) \Rightarrow F$

$\beta = (S \Rightarrow F) \vee (H \Rightarrow F)$

S	F	H	$S \wedge H$	α	$S \Rightarrow F$	$H \Rightarrow F$	β	$\alpha \Leftrightarrow \beta$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

$M(\alpha \Leftrightarrow \beta)$ is all worlds.

Therefore $\alpha \Leftrightarrow \beta$ is Valid //

3. Let mythical be A

immortal be B

mortal be $\neg B$

mammal be C

horned be D

magical be E

Q. $A \Rightarrow B$

$\neg A \Rightarrow (\neg B \wedge C)$

$(B \vee C) \Rightarrow D$

$D \Rightarrow E$

Δ : $(A \Rightarrow B) \wedge ((\neg A \Rightarrow (\neg B \wedge C)) \wedge ((B \vee C) \Rightarrow D) \wedge (D \Rightarrow E)$

b. 1. $A \Rightarrow B$

2. $\neg A \Rightarrow (\neg B \wedge C)$

3. $(B \vee C) \Rightarrow D$

4. $D \Rightarrow E$

STEP 1 : Remove $\Rightarrow$

1. $\neg A \vee B$

2. $A \vee (\neg B \wedge C)$

3. $\neg(B \vee C) \vee D$

4. $\neg D \vee E$

STEP 2 : DE - MORGAN'S LAWS

3. $\neg(B \vee C) \vee D$

$(\neg B \wedge \neg C) \vee D$

STEP 3 : Distribution of $\wedge$

2. $A \vee (\neg B \wedge C)$

$(A \vee \neg B) \wedge (A \vee C)$

3. $(\neg B \vee D) \wedge (\neg C \vee D)$

Δ in CNF:

1. $\neg A \vee B$

2. $A \vee \neg B$

   $A \vee C$

3. $\neg B \vee D$

   $\neg C \vee D$

4. $\neg D \vee E$

Δ: $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

c. i. α : A

Δ : $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

To check $\Delta \models \alpha$, check $\Delta \wedge \neg\alpha$

RESOLUTION:

1. $\neg A \vee B$
2. $A \vee \neg B$
3. $A \vee C$
4. $\neg B \vee D$
5. $\neg C \vee D$
6. $\neg D \vee E$
7. $\neg A$

---

8. C (3, 7)
9. D (5, 8)
10. $\neg B$ (2, 7)
11. $\neg A$ (1, 10)
12. E (6, 9)
13. True (1, 2)
14. $B \vee C$ (1, 3)
15. $(\neg A \vee D)$ (1, 4)

We can no longer apply resolution and no contradiction found. So, $\Delta \wedge \neg\alpha$ is satisfiable. Therefore $\Delta \not\models \alpha$, We cannot prove that unicorn is mythical.

ii) $\alpha = E$

Δ : $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

To CHECK $\Delta \models \alpha$, check $\Delta \wedge \neg\alpha$

RESOLUTION:

1. $\neg A \vee B$
2. $A \vee \neg B$
3. $A \vee C$
4. $\neg B \vee D$
5. $\neg C \vee D$
6. $\neg D \vee E$
7. $\neg E$

---

8. $\neg D$ (6,7)
9. $\neg C$ (5,8)
10. $\neg B$ (4,8)
11. $\neg A$ (2,10)
12. C (3,11)
13. FALSE (9,12)

CONTRADICTION.

There is a contradiction. $\Delta \wedge \neg\alpha$: unsatisfiable. Therefore, $\Delta \models \alpha$.

Hence, using Δ, we can prove that unicorn is magical.

iii. $\alpha = D$

Δ : $(\neg A \vee B) \wedge (A \vee \neg B) \wedge (A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) \wedge (\neg D \vee E)$

To check $\Delta \models \alpha$, check $\Delta \wedge \neg\alpha$

RESOLUTION:

1. $\neg A \vee B$
2. $A \vee \neg B$
3. $A \vee C$
4. $\neg B \vee D$
5. $\neg C \vee D$
6. $\neg D \vee E$
7. $\neg D$

---

8. $\neg B$ (4,7)
9. $\neg C$ (5,7)
10. A (3,9)
11. $\neg A$ (1,8)

CONTRADICTION.

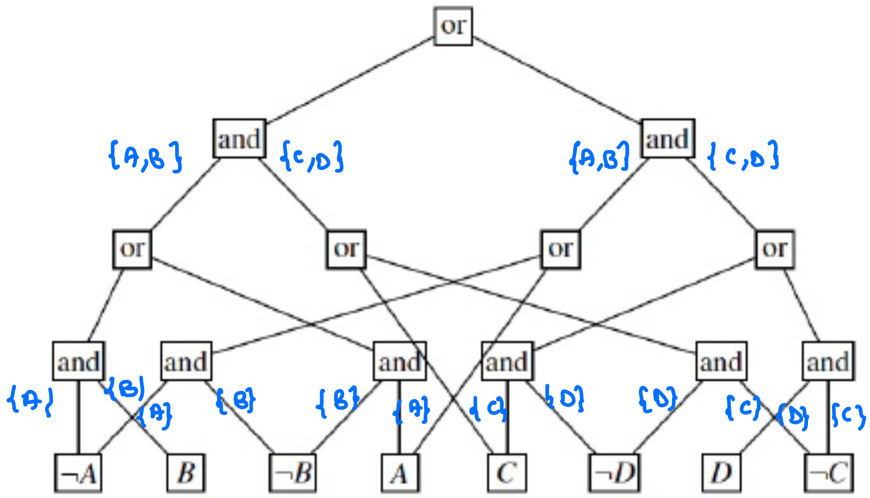
12. FALSE (10,11)

There is a contradiction. $\Delta \wedge \neg\alpha$: unsatisfiable. Therefore, $\Delta \models \alpha$.

Hence, using Δ, we can prove that unicorn is horned.

4. a.

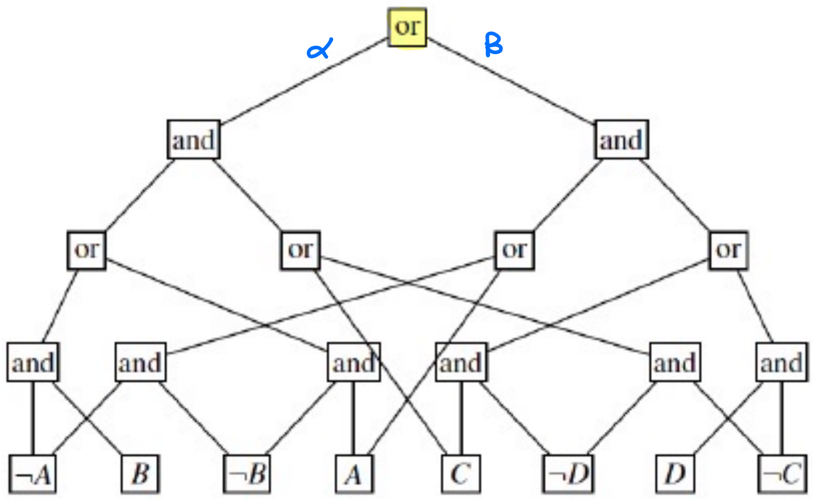
DECOMPOSABILITY :



YES

It is decomposable since the inputs to the AND gate do not share any variable

DETERMINISTIC:



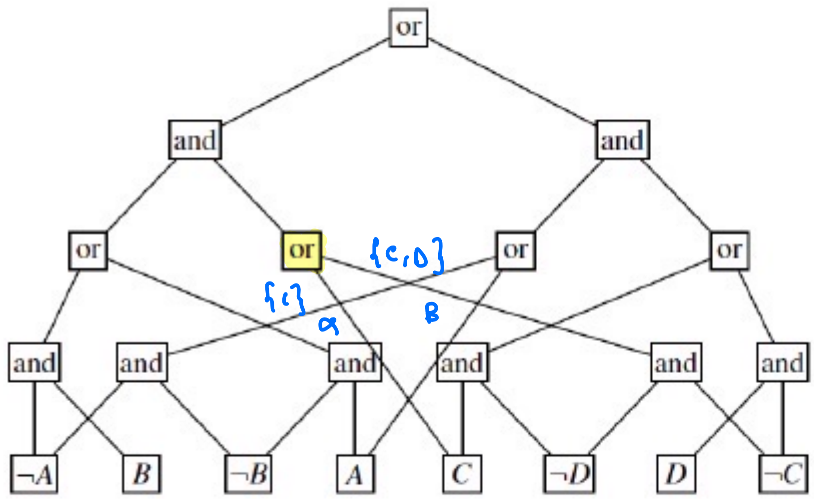
$\boxed{\text{No}}$

$$\alpha = \underbrace{((\neg A \wedge B) \vee (\neg B \wedge A))}_{\text{True if } A \text{ is T, } B \text{ is F}} \wedge \underbrace{(C \vee (\neg D \wedge \neg C))}_{\text{True if } C \text{ is true}}$$

$$\beta = \underbrace{((\neg A \wedge \neg B) \vee A)}_{\text{True if } A \text{ is true}} \wedge \underbrace{((C \wedge \neg D) \vee (D \wedge \neg C))}_{\text{True if } C \text{ is T, } D \text{ is F}}$$

So, for input A = True, B = False, C = True, D = False, both α and β are true. But to be deterministic, only one input to an OR gate should be true for a give input. So, it is NOT DETERMINISTIC.

SMOOTHNESS:

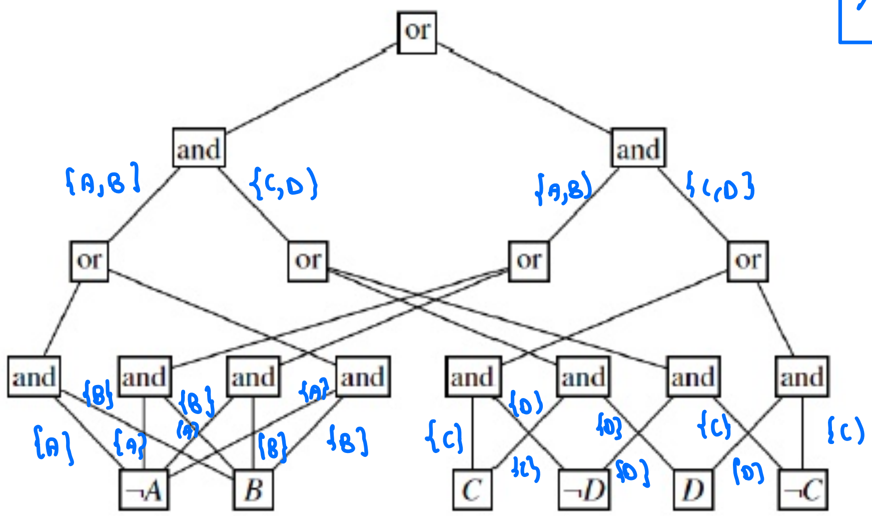


$\boxed{\text{No}}$

For smoothness, both the inputs of OR gate should have same variables. But as seen α has variables from {C} and β from {C,D}. They are not same. Therefore, it is NOT SMOOTH.

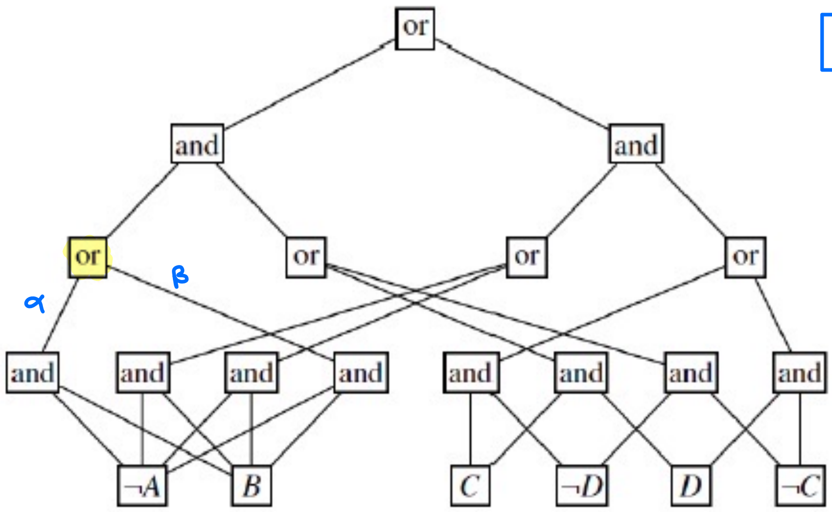
b. DECOMPOSABILITY:

YES



It is decomposable since the inputs to the AND gate do not share any variable

DETERMINISTIC:



$\boxed{\text{No}}$

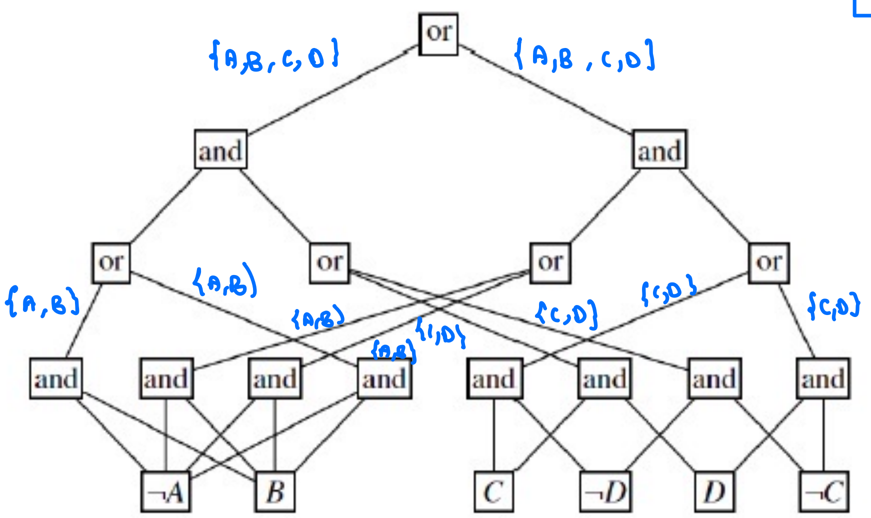
$\alpha = \neg A \wedge B$

$\beta = \neg A \wedge B$

So, for input A = False and B = True both α and β are true. But to be deterministic, only one input to an OR gate should be true for a give input. So, it is NOT DETERMINISTIC.

SMOOTHNESS:

YES



The circuit is SMOOTH as all OR gate have the same set of variables.

5. a.

A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	F	F	F

Models are $w(A, \neg B)$ and $w(\neg A, B)$

$$WMC = w(A, \neg B) + w(\neg A, B)$$

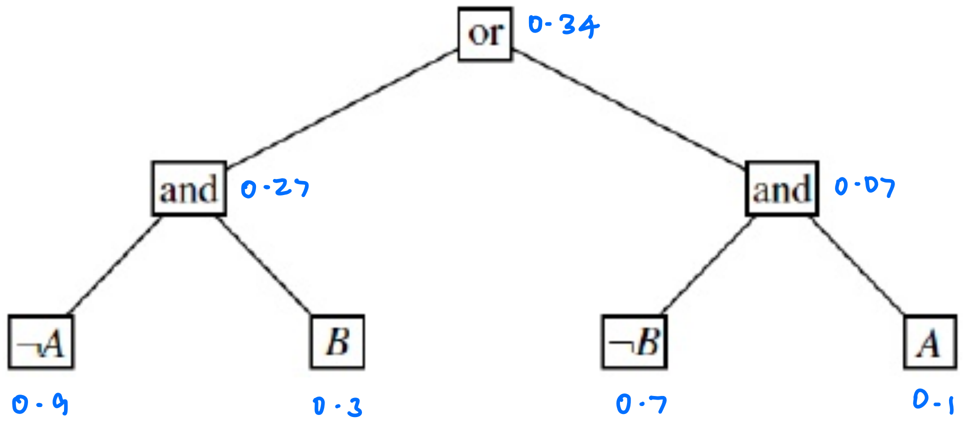
$$= w(A) \cdot w(\neg B) + w(\neg A) \cdot w(B)$$

$$= 0.1 \times 0.7 + 0.9 \times 0.3$$

$$= 0.07 + 0.27$$

$$= 0.34 \,//$$

b.



Count on the root : 0.34

Count on the root and the value calculated as per WMC are the same.

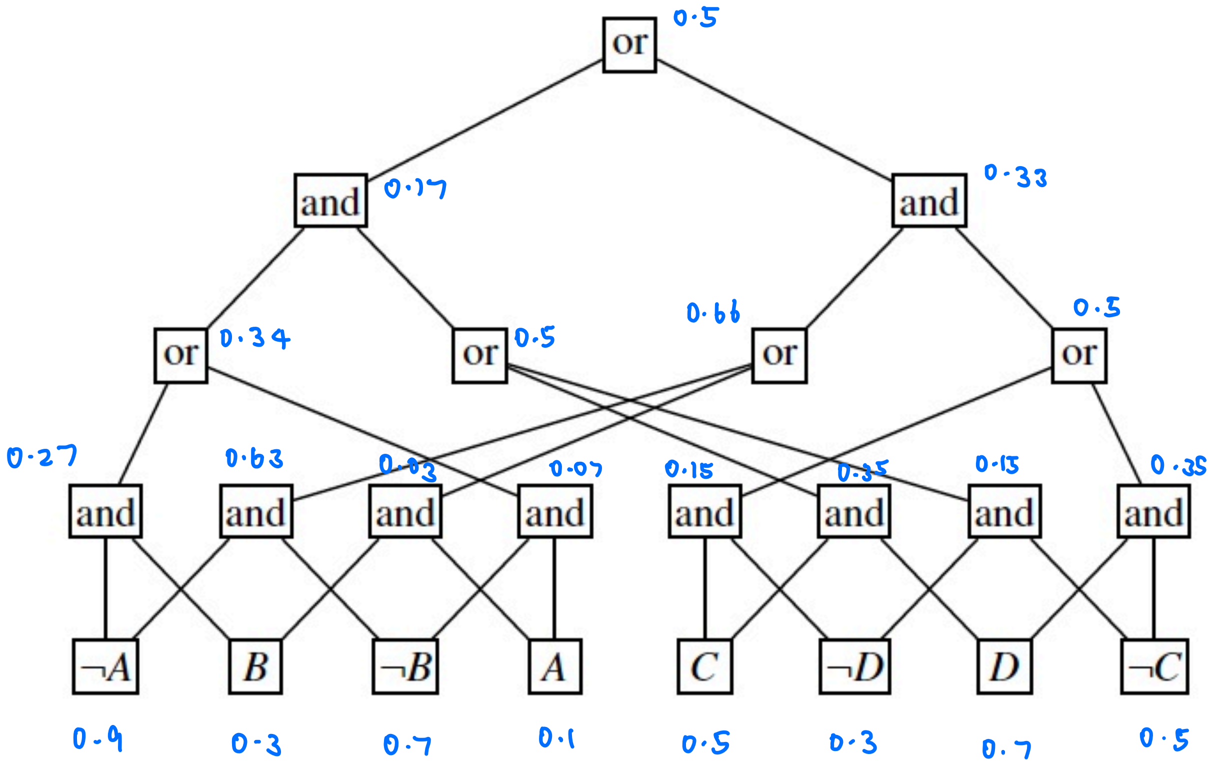
As we can see, the formula of the NNF circuit is

$(\neg A \wedge B) \vee (\neg B \wedge A)$,

which is basically the sum of the satisfying assignments or worlds in WMC. For each satisfying assignment, $\omega(\alpha\beta) = \omega(\alpha) \cdot \omega(\beta)$, just like how we multiply in d-NNF.

Therefore, we multiply the literal values in an assignment and add the assignments together. So, we get the same value in NNF cirwit root and WMC.

c.



WMC $= 0.5$ //

We can also apply WMC formula to

$[((\neg A \wedge B) \vee (\neg B \wedge A)) \wedge ((C \wedge D) \vee (\neg D \wedge \neg C))] \vee$

$[((\neg A \wedge \neg B) \vee (A \wedge B)) \wedge ((C \wedge \neg D) \vee (D \wedge \neg C))]$

$$\text{WMC} = [(w(\neg A, B) + w(\neg B, A)) \times (w(C,D) + w(\neg C, \neg D))] +$$

$$[(w(\neg A, \neg B) + w(A,B)) \times (w(C, \neg D) + w(D, \neg C))]$$

$$= [(0.9 \times 0.3 + 0.7 \times 0.1) \times (0.5 \times 0.7 + 0.3 \times 0.5)] +$$

$$[(0.9 \times 0.7 + 0.3 \times 0.1) \times (0.5 \times 0.3 + 0.7 \times 0.5)]$$

$$WMC = (0.34) \times (0.5) + (0.66) \times (0.5) = 0.5 \,//$$