PEEPHOLE OPTIMIZATIONS:

Gate Set $\{H, X, RZ, CNOT\}$

1. NOT PROPAGATION:

$$X_q ; H_q = H_q ; Z_q$$

$$X_q ; RZ(k)_q = RZ(1-k)_q ; X_q$$

$$X_{q_1} ; CNOT_{q_1, q_2} = CNOT_{q_1, q_2} ; X_{q_1} ; X_{q_2}$$

$$X_{q_2} ; CNOT_{q_1, q_2} = CNOT_{q_1} ; X_{q_2}$$

Move the X, so that $X_q X_q = I_q$ can be applied wherever possible.

2. HADAMARD REDUCTION:

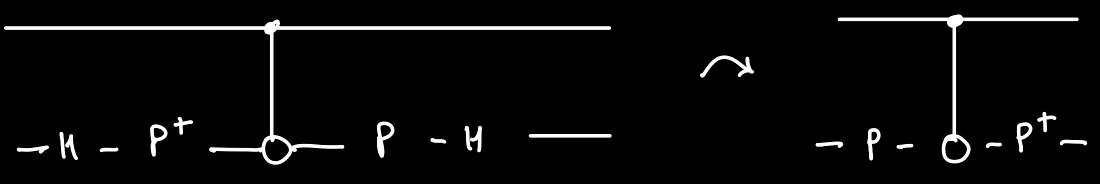
$$P = R_z(1/2)$$

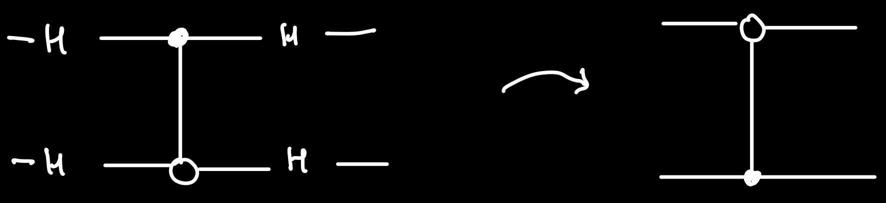
$$P^{\dagger} = R_z(3/2)$$

$$HH = I$$

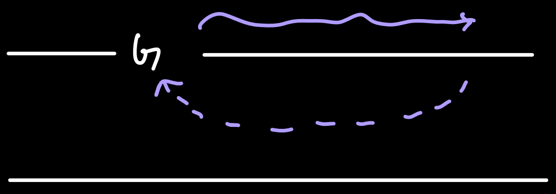
$$HPH \rightarrow P^{\dagger} H P^{\dagger}$$

$$HP^{\dagger}H \rightarrow PHP$$





3. COMMUTATION + CANCELLATION RULES



$HH = I$

$XX = I$

$\text{CNOT CNOT} = I$

$R_z(k)\, R_z(k') = R_z(k+k')$.

```
propagate (
        instruction list,
        a gate to propagate, RULES) {

        if (a cancellation rule applies)
                apply the rule and return the modified
                list
        if (a communication rule applies)
                commute the gate and propagate
```

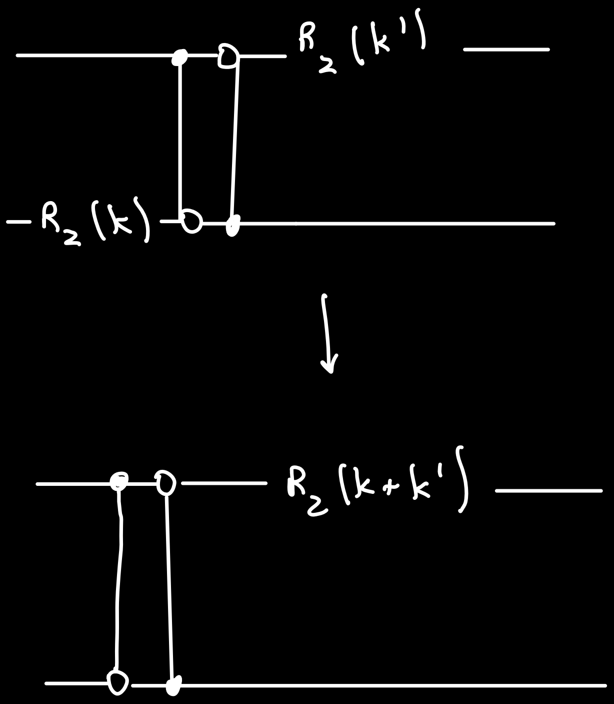
recursively

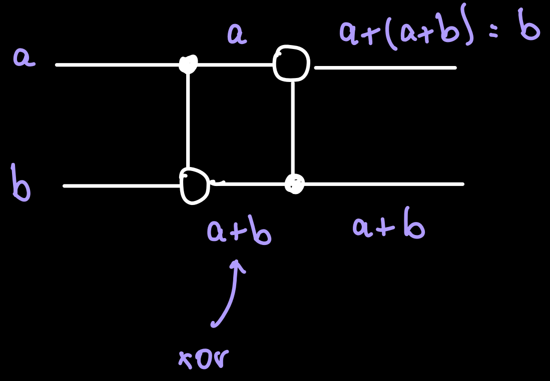
else return the original list.

}

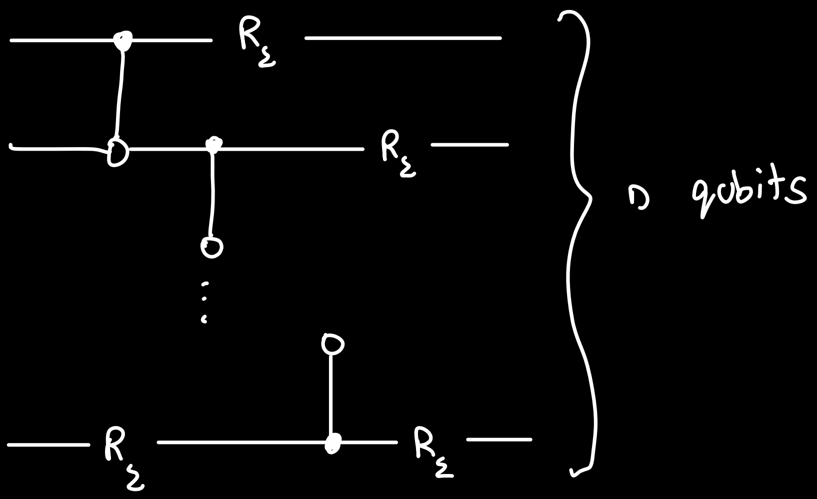
4. SAME AS 3 WITH 2-QUBIT GATES.

5. ROTATION MERGING:





Assume all CNOT, R_z:

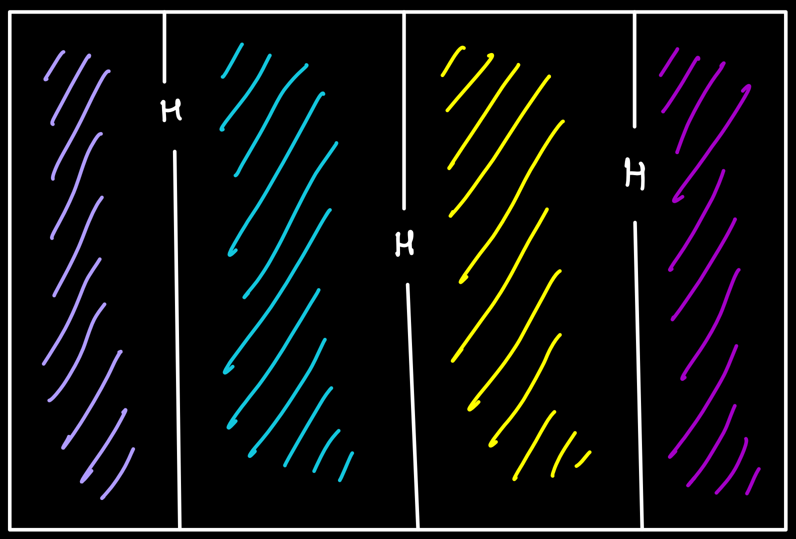


1. (Forget all R_z): Work out the symbolic value "everywhere".

2. For places in circuit with same symbolic value: Merge R_z.

This works in the sections between H.

Quantum Circuit:



PHASE ORDERING:

Sequence of optimization:

1 2 4 3 4 2 3 5 4 3

5 ↳ Rotation merging.

Single qubit cancellation

These 2 are the most helpful.

	ORIGINAL	QISKIT	t|ket>	NAM	THIS (ABOVE)
SPEED (RANK)				1	2
LOC					
Adder	900	805	775	606	682
Mult	883	804	806	712	705
gcla	884	793	780	624	723
Reduction		10.1%	10.6%	24.8%	17.8%
CORRECTNESS (RANK)					1 (THEOREM PROVER)