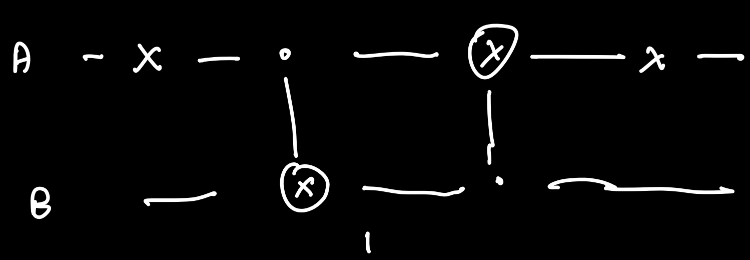
NAME: MADHAV SANKAR KRISHNAKUMAR

UID: 405692669

1) A, B

A	B	A	B
0	0	1	
1	0	1	

If A is 0 we neet to swap



A = 0

becomes 1 after X

changes B to 1

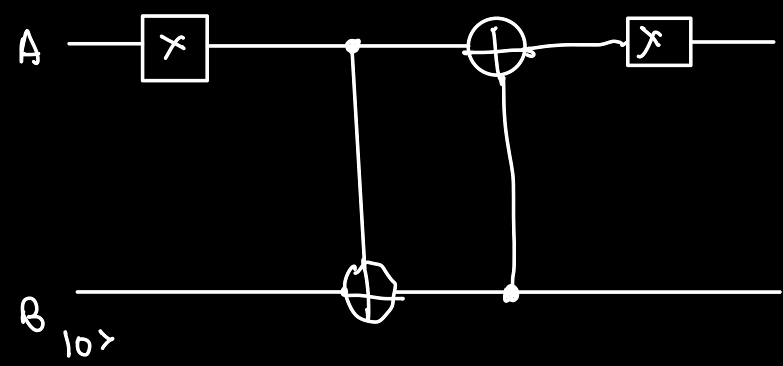
B changes A to 0 $\xrightarrow{X}$ 1

A = 1

A becomes 0

no effect on B

1



If $A = 1$

state $|10\rangle$

$X_A \, CNOT(B, A) \, CNOT(A, B) \, X_A \, |10\rangle$

$|00\rangle$

$CNOT(A, B)$

$|00\rangle$

$CNOT(B, A)$

$|00\rangle$

X_A

$|10\rangle$

A is set to $|1\rangle$

if $A = 0$

state $|00\rangle$

X_A $|10\rangle$

CNOT (A,B) $|11\rangle$

CNOT (B,A) $|01\rangle$

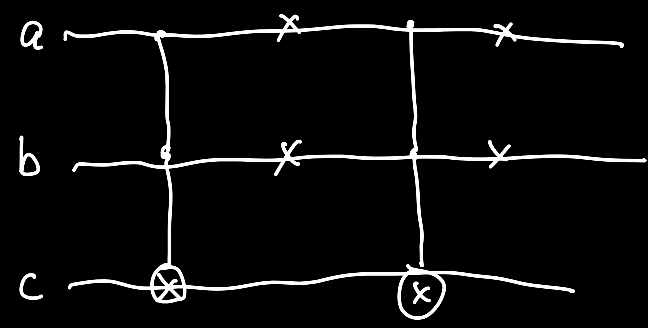
X_A $|11\rangle$

A is set to $|1\rangle$

2. $f(x) = (11 \cdot x) \oplus 1$

x	b	$f(x)$	x	$b \oplus f(x)$
00	0	1	00	1
00	1	1	00	0
01	0	0	01	0
01	1	0	01	1
10	0	0	10	0
10	1	0	10	1
11	0	1	11	1
11	1	1	11	0

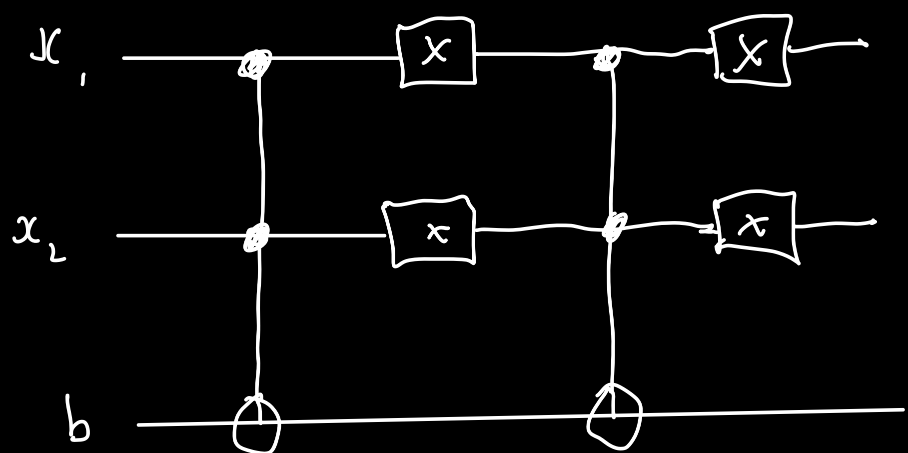
	000	001	010	011	100	101	110	111
000	0	1	0	0	0	0	0	0
001	1	0	0	0				
010	0	0	1					
011	0	0	0	1				
100					1			
101						1		
110							0	1
111							1	0



if $a=1, b=1$, reverse c

$a=0, b=0$

reverse c



$$((x_1 \wedge x_2) \oplus b) \vee ((\neg x_1 \wedge \neg x_2) \oplus b)$$

if $x = 00/11 \rightarrow f(x) = 0 \oplus 1 = 1$

$x = 01/10 \rightarrow f(x) = 0$

So $f(x)=1$ when

$x=00$ / 11

i.e. $(x_1 \wedge x_2)$ or $(\neg x_1 \wedge \neg x_2)$

which gives

$$f(x)=((x_1 \wedge x_2) \oplus b) \vee ((\neg x_1 \wedge \neg x_2) \oplus b)$$

3. $p = 95\%$

8 length

$f(x) = ax + b$

Bernstein Vazirani finds a with $p = 1$ //

Run it only once after

finding b classically //

4) 95%

32 → 32

P(failing after 4m iterations) $< e^{-m}$

$$e^{-m} < \frac{5}{100}$$

$$-m < \ln\left(\frac{5}{100}\right)$$

$$-m < -\ln\left(100/5\right)$$

$$-m < -\ln(20)$$

$$m > \ln(20)$$

$$m > 2.995$$

Choose $m = 3$

Run 4m iterations

= 12 times // for 1 independent equation

$n = 32$

$(31) \times 12 = 372$ times //

5. 32 → 1bit

$N = 2^{32}$

x_1

$$k \approx \frac{\pi}{4}\sqrt{N} - \frac{1}{2}$$

$$\approx \frac{\pi}{4}\sqrt{2^{32}} - \frac{1}{2}$$

$$= \frac{\pi}{4} \times 2^{16} - \frac{1}{2}$$

$$= \pi \times 2^{14} - \frac{1}{2}$$

$$\approx 51471.35$$

Ans 51471 times//

6) $$\left[\, 0\begin{pmatrix} 0 & 1/\sqrt{2} \\ 1 & 1/\sqrt{2} \end{pmatrix} \quad 1/\sqrt{2}\begin{pmatrix} 0 & 1/\sqrt{2} \\ 1 & 1/\sqrt{2} \end{pmatrix} \quad 1/\sqrt{2}\begin{pmatrix} 0 & 1/\sqrt{2} \\ 1 & 1/\sqrt{2} \end{pmatrix} \right]$$

$$\begin{bmatrix} 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1/\sqrt{2} & 1/2 & 1/\sqrt{2} & 1/2 \end{bmatrix}$$

7) $\frac{3}{5\sqrt{2}}|00\rangle - \frac{4}{5\sqrt{2}}|01\rangle - \frac{3}{5\sqrt{2}}|10\rangle + \frac{4}{5\sqrt{2}}|11\rangle$

8. $$(Z \otimes H)\, \text{CNOT} \left(\frac{3}{5}|01\rangle - \frac{4}{5}|10\rangle \right)$$

$$(Z \otimes H)\left(\frac{3}{5}|01\rangle - \frac{4}{5}|11\rangle \right)$$

$$\frac{3}{5}|0\rangle \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \right) - \frac{4}{5}\left(-|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \right)$$

$$\frac{3}{5\sqrt{2}}|00\rangle - \frac{3}{5\sqrt{2}}|01\rangle + \frac{4}{5\sqrt{2}}|10\rangle - \frac{4}{5\sqrt{2}}|11\rangle$$

9. Left qubit = 1

$\frac{1}{2}|10\rangle \quad \frac{-1}{2}|11\rangle$

State: $\dfrac{\frac{1}{2}|10\rangle}{\sqrt{\frac{1}{4}+\frac{1}{4}}} - \dfrac{\frac{1}{2}}{\sqrt{\frac{1}{4}+\frac{1}{4}}}|11\rangle$

$= \frac{1}{\sqrt{2}}|10\rangle - \frac{1}{\sqrt{2}}|11\rangle$

Both equally likely

Other qubit

$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$

10. P(getting 0 on left) $= \frac{16}{(5\sqrt{2})^2} + \frac{9}{(5\sqrt{2})^2}$

$= \frac{25}{50} = 1/2$

50%

State of other qubit

$\sqrt{\alpha_1^2 + \alpha_2} = \sqrt{\left(\frac{4}{5\sqrt{2}}\right)^2 + \left(\frac{3}{5\sqrt{2}}\right)^2}$

$= 1/\sqrt{2}$

$\frac{\frac{4}{5\sqrt{2}}}{1/\sqrt{2}} |0\rangle + \frac{\frac{3}{5\sqrt{2}}}{\frac{1}{\sqrt{2}}} |01\rangle$

$\frac{4}{5} |0\rangle + \frac{3}{5} |1\rangle$

$P(1) = 9/50$

Other qubit $\rightarrow |0\rangle$

11. $(H \otimes H \otimes X)\,|000\rangle$

$\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)|1\rangle$

$\frac{1}{2}\left(|001\rangle + |010\rangle + |101\rangle + |111\rangle\right)$

CNOT $(2,3)$

$\frac{1}{2}\left(|001\rangle + |011\rangle + |101\rangle + |110\rangle\right)$

CNOT $(1,3)$

$\frac{1}{2}\left(|001\rangle + |011\rangle + |100\rangle + |111\rangle\right)$

$\times\ (H^{\otimes 2})$

$\frac{1}{2}\Big[|1\rangle\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$

$+\ |1\rangle\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$

$$+ |0\rangle \left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle \right) \left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle \right)$$

$$+ |0\rangle \left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle \right) \left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle \right)$$

$$\frac{1}{4} \Big(|100\rangle - \underline{|101\rangle} + |110\rangle - |111\rangle$$

$$+ |100\rangle + \underline{|101\rangle} - |110\rangle - |111\rangle$$

$$+ |000\rangle - |001\rangle + |010\rangle - |011\rangle$$

$$+ |000\rangle + |001\rangle - |010\rangle - |011\rangle \Big)$$

101 cancels out//

$$P(101) = 0$$

12. $H(q_2)$ $P^+(q_2)$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$P^+ = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

00

01

10

11

CASE 1:

00

LEFT

$|00\rangle$

$H \underbrace{P^+ P}_{I} H$

$HH = I$

CNOT is not applied as qubit 1 = 0

RIGHT

$PP^+ = I$

Same //

Case 2:

01

$HP^{+}PH = I$

$PP^{+} = I$

same //

Case 3 and 4

when $|1\rangle$ in first qubit

$(HP\,CNOT\,P^{+}H)|0\rangle$

$HP\,CNOT\,P^{+}|+\rangle$

$\frac{1}{\sqrt{2}} HP\,CNOT\,(|0\rangle - i|1\rangle)$

$\frac{1}{\sqrt{2}} HP(|1\rangle - i|0\rangle)$

$\frac{1}{\sqrt{2}}\, i|1\rangle - i|0\rangle$

$-iH|-\rangle = -i|1\rangle$

$$P^{\dagger}\ (CNOT)\ P|0\rangle$$

$$P^{\dagger}\ (CNOT)\ |0\rangle$$

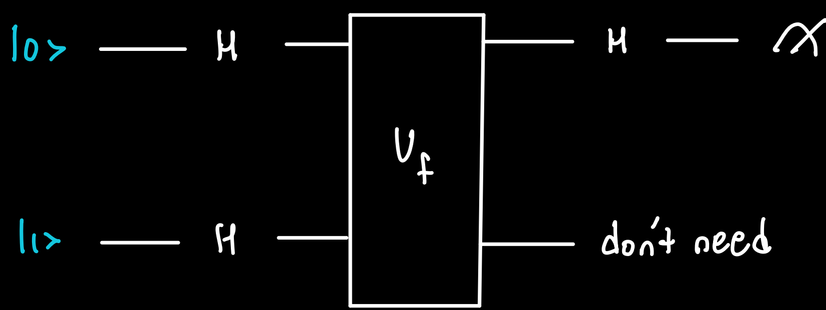
$$P^{\dagger}\,|1\rangle$$

$$-i\,|1\rangle$$

LHS = RHS

13. $f(0) = 0$

$f(1) = 0$



$(H \otimes I)\, U_f\, (H \otimes H)\, |01\rangle$ $\qquad (0+1)(0-1)$

$= (H \otimes I)\, U_f\, \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle \right)$

$U_f |x\rangle |b\rangle = |x\rangle |b \oplus f(x)\rangle$

$= (H \otimes I)\, \frac{1}{2}\big(|0\rangle |0 \oplus f(0)\rangle - |0\rangle |1 \oplus f(0)\rangle + |1\rangle |0 \oplus f(1)\rangle$

$- |1\rangle |1 \oplus f(1)\rangle \big)$

$= (H \otimes I)\, \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle \right)$

$|0+1)(0-1)$

$= (H \otimes I)\, |+\rangle \underbrace{|-\rangle}_{\text{Discard}}$

$= |0\rangle$

$\downarrow$

measure , we will get 0 => CONSTANT

14.

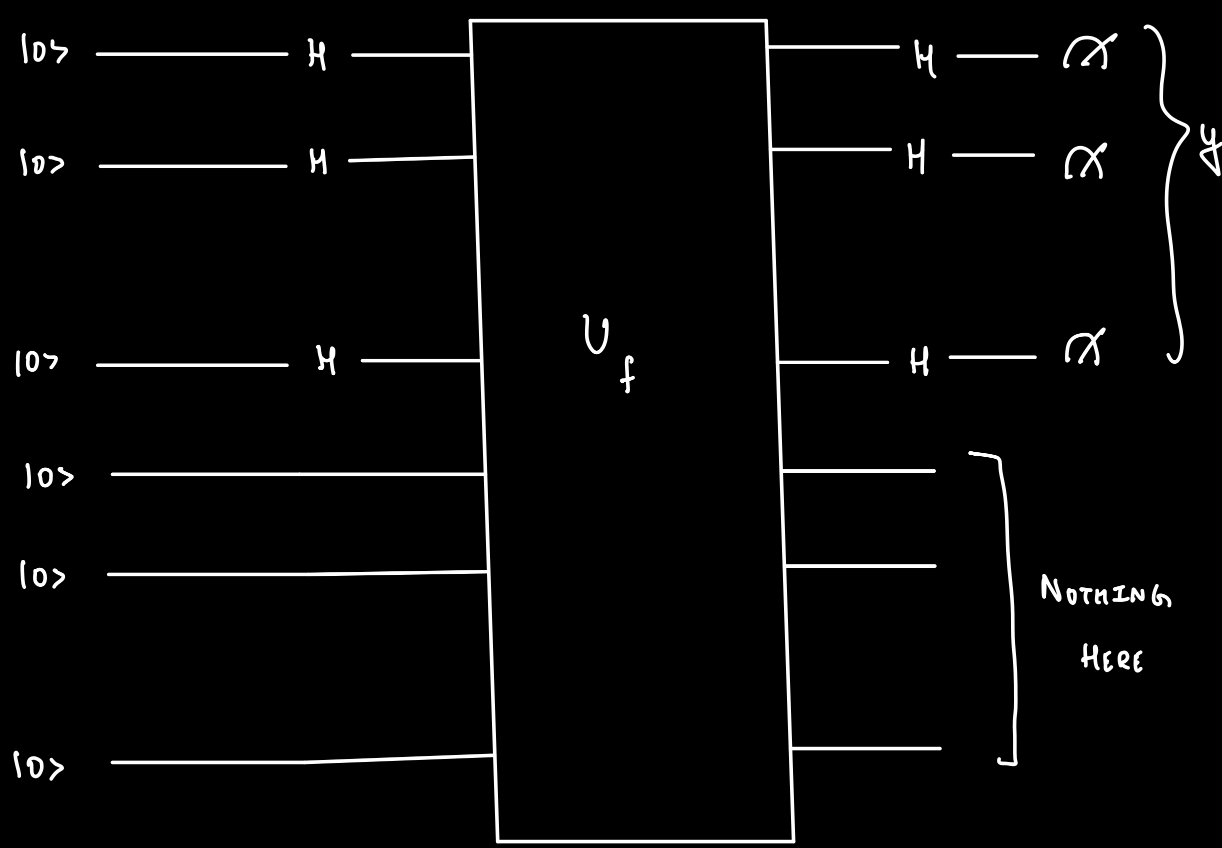
$n = 3$

$f(000) = f(101) = 001$

$f(001) = f(100) = 010$

$f(010) = f(111) = 100$

$f(011) = f(110) = 111$



STEP 1:

$$(H \otimes H \otimes H \otimes I \otimes I \times I)\,|000000\rangle$$

$$= \left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right) \otimes |000\rangle$$

$$= \tfrac{1}{2\sqrt{2}}\left(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle\right) \otimes |000\rangle$$

STEP 2:

APPLY U_f

$$U_f(x, b) = (x, \overset{|000\rangle}{b} \oplus f(x)) = (x, f(x))$$

$$\tfrac{1}{2\sqrt{2}}\left(|000\,\underline{001}\rangle + |001\,\underline{010}\rangle + |010\,\underline{100}\rangle + |011\,\underline{111}\rangle + |100\,\underline{010}\rangle + |101\,\underline{001}\rangle + |110\,\underline{111}\rangle + |111\,\underline{100}\rangle\right)$$

STEP 3:

APPLY $H^{\otimes 3}$

$$H^{\otimes 3}|000\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + \cancel{|001\rangle} + |010\rangle + \cancel{|011\rangle} + \cancel{|100\rangle} + |101\rangle + \cancel{|110\rangle} + |111\rangle\right)$$

$$H^{\otimes 3}|001\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - \cancel{|001\rangle} + |010\rangle - \cancel{|011\rangle} + \cancel{|100\rangle} - |101\rangle + \cancel{|110\rangle} - |111\rangle\right)$$

$$H^{\otimes 3}|010\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + \cancel{|001\rangle} - |010\rangle - \cancel{|011\rangle} + \cancel{|100\rangle} + |101\rangle - \cancel{|110\rangle} - |111\rangle\right)$$

$$H^{\otimes}|011\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - \cancel{|001\rangle} - |010\rangle + \cancel{|011\rangle} + \cancel{|100\rangle} - |101\rangle - \cancel{|110\rangle} + |111\rangle\right)$$

$$H^{\otimes 3}|100\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + \cancel{|001\rangle} + |010\rangle + \cancel{|011\rangle} - \cancel{|100\rangle} - |101\rangle - \cancel{|110\rangle} - |111\rangle\right)$$

$$H^{\otimes 3}|101\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - \cancel{|001\rangle} + |010\rangle - \cancel{|011\rangle} - \cancel{|100\rangle} + |101\rangle - \cancel{|110\rangle} + |111\rangle\right)$$

$$H^{\otimes 3}|110\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + \cancel{|001\rangle} - |010\rangle - \cancel{|011\rangle} - \cancel{|100\rangle} - |101\rangle + \cancel{|110\rangle} + |111\rangle\right)$$

$$H^{\otimes 3}|111\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - \cancel{|001\rangle} - |010\rangle + \cancel{|011\rangle} - \cancel{|100\rangle} + |101\rangle + \cancel{|110\rangle} - |111\rangle\right)$$

$$\frac{1}{4}\big(|000\rangle + |010\rangle + |101\rangle + |111\rangle\big) \otimes |001\rangle$$

$$|000\rangle + |010\rangle - |101\rangle - |111\rangle \otimes |010\rangle$$

$$|000\rangle - |010\rangle + |101\rangle - |111\rangle \otimes |100\rangle$$

$$|000\rangle - |010\rangle - |101\rangle + |111\rangle \otimes |111\rangle$$

When we measure we get

$y_1 : 000$

$y_2 : 010$

with equal probability of $1/4$

$y_3 = 101$

$y_4 = 111$

$(000) \cdot s = 0$

$(010) \cdot s = 0$

$(101) \cdot s = 0$

$(111) \cdot s = 0$

these imply if $s = s1 . s2 s3$

$$\begin{cases} s_2 = 0 \\ s_1 \oplus s_3 = 0 \end{cases}$$

$$s_1 \oplus s_2 \oplus s_3 = 0$$

2 equations are enough

$$s_2 = 0$$

$$s_1 \oplus s_3 = 0$$

So either $s_1 = s_3 = 0$ } all 0 case. ignored

or $s_1 = s_3 = 1$ ✓

$s = 101$ //

15. $f(00) = f(01) = 0$

$f(10) = f(11) = 1$

GROVER'S ALGORITHM:

$$G = -H^{\otimes n} Z_0 H^{\otimes n} Z_f$$

$x = |0^n\rangle$ (n $|0\rangle$ qubits)

$H^{\otimes n} x$

repeat { apply G to x } $O(\sqrt{2^n})$ times

measure x and output the result

$x = |00\rangle$

1. $H^{\otimes n} x$

$$\tfrac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

2. Apply G

$$-H^{\otimes n} Z_0 H^{\otimes n} Z_f \left[\tfrac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \right]$$

$$= -H^{\otimes n} Z_0 H^{\otimes n} \left[\tfrac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle) \right]$$

$\cancel{(2\,1)(0-1)}$

$$= -H^{\otimes n} Z_0 \Big[\tfrac{1}{4} \big[(\underline{|00\rangle} + \underline{|01\rangle} + |10\rangle + \underline{|11\rangle})$$

$$+ (\underline{|00\rangle} - \underline{|01\rangle} + |10\rangle - \underline{|11\rangle})$$

$(0-1)(0+1)$

$$- (\underline{|00\rangle} + \underline{|01\rangle} - |10\rangle - \underline{|11\rangle})$$

$(0+1)(0-1)$

$$- (\underline{|00\rangle} - \underline{|01\rangle} - |10\rangle + \underline{|11\rangle}) \big] \Big]$$

$$= -H^{\otimes} Z_0 \big[|10\rangle \big]$$

$$= -H^{\otimes} \big[|10\rangle \big]$$

$$= \left(\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$$

$$= -\frac{1}{2} \{ |00\rangle + |01\rangle - |10\rangle - |11\rangle \}$$

2. Apply to

$$-H^{\otimes n} Z_0 H^{\otimes n} Z_f \left[-\tfrac{1}{2} (|00\rangle + |01\rangle - |10\rangle - |11\rangle) \right]$$

$$= +H^{\otimes n} Z_0 H^{\otimes n} \left[\tfrac{1}{2} \left(|00\rangle + |01\rangle + |10\rangle + |11\rangle \right) \right]$$

$$= +H^{\otimes n} Z_0 \left\{ \tfrac{1}{4} \left[\left(|00\rangle + \underline{|01\rangle} + \underline{|10\rangle} + \underline{|11\rangle} \right) \right. \right.$$

$$+ \left(|00\rangle - \underline{|01\rangle} + \underline{|10\rangle} - \underline{|11\rangle} \right)$$

$$+ \left(|00\rangle + \underline{|01\rangle} - \underline{|10\rangle} - \underline{|11\rangle} \right)$$

$$\left. \left. + \left(|00\rangle - \underline{|01\rangle} - \underline{|10\rangle} + \underline{|11\rangle} \right) \right] \right.$$

$$= +H^{\otimes} Z_0 \left[|00\rangle \right]$$

$$= +H^{\otimes} \left[|00\rangle \right]$$

$P = 1/2$ only

Both cases probability of 10 or 11 is $1/2$. It is same//

$N = 4$

$$k = \frac{\pi}{4\theta} - \frac{1}{2} = \frac{\pi\sqrt{N}}{4} - \frac{1}{2}$$

$$k = \frac{\pi \times 2}{4} - \frac{1}{2} = \frac{1}{2}(\pi - 1) = \frac{2.14}{2} \approx 1$$

Apply $\theta \not\approx \sin\theta$ as $\sin\theta = 1/2$

So this equation may not apply