Computer Vision Report: Camera Matrix and Epipolar Geometry

1 Problem 1: Computing a Camera Matrix

1.1 1a: World Coordinate System and Camera Matrix Calculation

Description:
Figure 1 below describes how I chose the world coordinate axes and Table 1 below includes the table of correspondences.

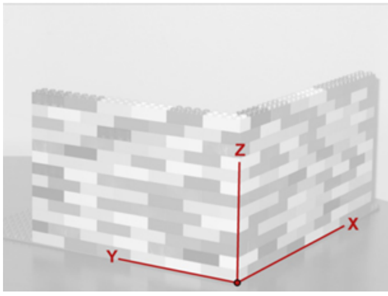

Figure 1: 3D world co-ordinate system

x	y	x'	y'	z'
1547	1840	0	0	0
1557	1267	0	0	67.2
1562	937	0	0	105.6
1073	1749	0	64	0
404	1606	0	160	0
1873	1659	80	0	0
2328	1418	208	0	0
1430	1494	16	0	38.4
732	1372	0	112	38.4
961	1097	0	80	76.8
302	985	0	176	76.8
1320	901	0	32	105.6
849	827	0	96	105.6
409	771	0	160	105.6
1188	1611	0	48	19.2
737	1530	0	112	19.2
305	1443	0	176	19.2
1746	1661	48	0	0
1993	1536	112	0	0
2219	1419	176	0	0
2430	1302	240	0	0
1819	1233	64	0	57.6
2066	1129	128	0	57.6
2283	1020	192	0	57.6
2026	684	112	0	124.8
2357	561	208	0	124.8
852	745	0	96	124.8
198	653	0	192	124.8

Table 1: 2D Image Coordinates in Pixels and Corresponding 3D World Coordinates in mm

Results:
Below is the calculated P matrix:

$$P = \begin{bmatrix} -6.26805785 \times 10^{12} & 2.24941247 \times 10^{8} & 4.52730087 \times 10^{-3} & 1.54665375 \times 10^{3} \\ -9.78591444 \times 10^{8} & 3.66656339 \times 10^{8} & -8.62146328 & -1.83848493 \times 10^{3} \\ 0 & 2.09637702 \times 10^{5} & -8.64770899 \times 10^{-5} & 1 \end{bmatrix}$$

Discussion:
Interpret the results, including any challenges encountered or assumptions made.

1.2 1b: Proof of Camera Matrix Decomposition

Description:
Outline the proof that the camera matrix decomposition algorithm returns K, R, and $\tilde{C}$ such that K is upper-triangular, R is orthogonal, and $KR[I - \tilde{C}]$ equals P.
Discussion:
Comment on the extent of uniqueness in the decomposition and its importance.

1.3 1c: Camera Matrix Decomposition for lego1.jpg

Results:

Intrinsic Matrix K

$$K = \begin{bmatrix} 2.58795903 \times 10^{-1} & 2.98994779 \times 10^{7} & 1.07299996 \times 10^{3} \\ 0 & 4.66801265 \times 10^{3} & 1.74899999 \times 10^{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix R

$$R = \begin{bmatrix} -8.65551697 \times 10^{-9} & 4.12507336 \times 10^{-10} & 1 \\ -1 & 2.22044605 \times 10^{-16} & -8.65551697 \times 10^{-9} \\ 0 & 1 & -4.12507336 \times 10^{-10} \end{bmatrix}$$

Camera Center C

$$C = \begin{bmatrix} 6.89902951 \times 10^{-11} \\ -4.94485147 \times 10^{-6} \\ -4.23549166 \times 10^{2} \end{bmatrix}$$

Discussion:
Discuss the significance of the decomposed components.

2 Problem 2: Computing a Second Camera Matrix

2.1 2a: Repeating the Calculations for lego2.jpg

Description:

Homography Matrix

The homography matrix P is given by:

$$P = \begin{bmatrix} -3.73942007 \times 10^{-3} & 6.00696271 \times 10^{-4} & 2.09880695 \times 10^{-15} & -1.15333684 \times 10^{-1} \\ -7.14513566 \times 10^{-3} & 6.10214747 \times 10^{-4} & 8.90213106 \times 10^{-14} & 1.17161231 \times 10^{-1} \\ 0 & -5.20833333 \times 10^{-3} & 8.68057066 \times 10^{-18} & 1 \end{bmatrix}$$

x	**y**	**x** (3D)	**y** (3D)	**z** (3D)
240	1440	0	192	0
242	1308	0	192	19.2
240	1178	0	192	38.4
242	1045	0	192	57.6
245	905	0	192	76.8
245	786	0	192	96
250	577	0	192	124.8
988	1758	0	0	0
991	1623	0	0	19.2
988	1471	0	0	38.4
991	1320	0	0	57.6
993	1180	0	0	76.8
998	1028	0	0	96
998	799	0	0	124.8
2424	1506	240	0	0
2421	1366	240	0	19.2
2424	1236	240	0	38.4
2434	1109	240	0	57.6
2439	972	240	0	76.8
2439	837	240	0	96
2447	625	240	0	124.8

Table 2: 2D coordinates and their corresponding 3D coordinates

Intrinsic Matrix

The intrinsic matrix K is given by:

$$K = \begin{bmatrix} 8.54211707 \times 10^{-12} & 7.17968654 \times 10^{-1} & -1.15333684 \times 10^{-1} \\ 0 & 1.37186605 & -1.17161231 \times 10^{-1} \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix

The rotation matrix R is given by:

$$R = \begin{bmatrix} -1.24591515 \times 10^{-11} & -1.66666957 \times 10^{-15} & -1.00000000 \\ -1.00000000 & 0 & 1.24591515 \times 10^{-11} \\ 0 & -1.00000000 & 1.66666957 \times 10^{-15} \end{bmatrix}$$

Camera Center

The camera center C is given by:

$$C = \begin{bmatrix} -1.54781397 \\ 1.91995406 \times 10^{2} \\ -2.75640741 \times 10^{12} \end{bmatrix}$$

Results:

Present the new camera matrix and the distance between the two camera centers and the angle between their principal axes.

Discussion:

Analyze the differences between the two matrices and what they imply about the relative camera positions.

2.2 2b: 3D Plot of Camera Positions and World Points

Results:

Include the 3D plot with labeled axes and camera positions. Provide different views to show the camera orientations.

Discussion:

Discuss how the plot clarifies the relative positions and orientations of the cameras.

3 Problem 3: The Image of the World Coordinate System

Description:
Prove that p_4 is a homogeneous representation of the image of the world origin, and p_1, p_2, p_3 are the images of the vanishing points of the world X, Y, and Z axes. Include mathematical derivations.

Results:
Demonstrate these statements using the two computed camera matrices. Include the plots of projected world axes overlaid on the images.

Discussion:
Interpret the results and any observations.

4 Problem 4: Epipolar Geometry

4.1 4a: Calculating Epipoles

Description:
Describe the method used to calculate the epipoles e and e'.

Results:
Provide the de-homogenized coordinates of both epipoles and discuss whether the coordinates make sense.

4.2 4b: Computing the Fundamental Matrix and Epipolar Lines

Description:
Explain how the fundamental matrix F was computed from P and P'.

Results:
Display the epipolar lines on both images. Describe the visualization approach, including the use of color for corresponding lines.

Discussion:
Analyze the correctness of the epipolar lines and any observations on the correspondence between features.

5 Conclusion

Summarize the key findings of the report, including the results of the camera matrix calculations, decomposition, and epipolar geometry analysis.

6 References

List any references, textbooks, or online resources you used.