Computer Vision Report: Camera Matrix and Epipolar Geometry

1 Problem 1: Computing a Camera Matrix

1.1 1a: World Coordinate System and Camera Matrix Calculation

Description:
Figure 1 below describes how I chose the world coordinate axes and Table 1 below includes the table of correspondences which I determined. I calculated the 2D image coordinates by selecting 28 corners of the legos. I then converted it to 3D world coordinates, allowing the intersection on the X-, Y- and Z- coordinate to be (0, 0, 0) and then incrementing the respective amount in mm to the point where I had selected a corner.

x	y	x'	y'	z'
1547	1840	0	0	0
1557	1267	0	0	67.2
1562	937	0	0	105.6
1073	1749	0	64	0
404	1606	0	160	0
1873	1659	80	0	0
2328	1418	208	0	0
1430	1494	16	0	38.4
732	1372	0	112	38.4
961	1097	0	80	76.8
302	985	0	176	76.8
1320	901	0	32	105.6
849	827	0	96	105.6
409	771	0	160	105.6
1188	1611	0	48	19.2
737	1530	0	112	19.2
305	1443	0	176	19.2
1746	1661	48	0	0
1993	1536	112	0	0
2219	1419	176	0	0
2430	1302	240	0	0
1819	1233	64	0	57.6
2066	1129	128	0	57.6
2283	1020	192	0	57.6
2026	684	112	0	124.8
2357	561	208	0	124.8
852	745	0	96	124.8
198	653	0	192	124.8

Table 1: 2D Image Coordinates in Pixels and Corresponding 3D World Coordinates in mm

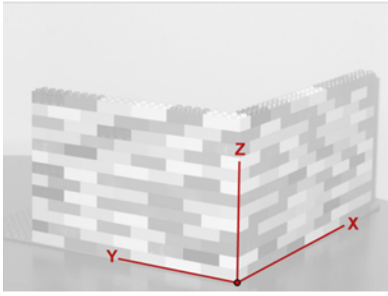

Figure 1: 3D world co-ordinate system

Results:

Below is the calculated P matrix:

Homography Matrix P

$$P = \begin{bmatrix} 2.75416038 \times 10^{-3} & -2.87589499 \times 10^{-3} & 5.85201094 \times 10^{-6} & 6.39998608 \times 10^{-1} \\ -2.34030798 \times 10^{-4} & -3.53935397 \times 10^{-4} & -3.46138168 \times 10^{-3} & 7.68357719 \times 10^{-1} \\ 4.74050139 \times 10^{-7} & 1.15244558 \times 10^{-7} & -5.01027097 \times 10^{-8} & 4.22797122 \times 10^{-4} \end{bmatrix}$$

1.2 1b: Proof of Camera Matrix Decomposition

Description:

The upper triangular matrix has all the elements below the diagonal as zero. As one can observe (see 1.3), this is the case with the intrinsic matrix K.

A square matrix with real numbers or elements is said to be an orthogonal matrix if its transpose is equal to its inverse matrix. The inverse and transpose of the rotation matrix R are:

$$R^{-1} = \begin{bmatrix} 0.24363231 & -0.966366 & 0.08234101 \\ -0.07937672 & -0.10448198 & -0.99135405 \\ 0.966614 & 0.23498992 & -0.10216215 \end{bmatrix}$$

$$R^{T} = \begin{bmatrix} 0.24363231 & -0.966366 & 0.08234101 \\ -0.07937672 & -0.10448198 & -0.99135405 \\ 0.966614 & 0.23498992 & -0.10216215 \end{bmatrix}$$

As one can see, these are identical and hence R is orthogonal.

Discussion:

Comment on the extent of uniqueness in the decomposition and its importance.

1.3 1c: Camera Matrix Decomposition for Lego 1

Results:

Intrinsic Matrix K_1

$$K_1 = \begin{bmatrix} 7.03606595 \times 10^{3} & 1.55093658 \times 10^{2} & 4.04916599 \times 10^{3} \\ 0 & 7.11020543 \times 10^{3} & 9.01944958 \times 10^{1} \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix R_1

$$R_1 = \begin{bmatrix} 0.24363231 & -0.966366 & 0.08234101 \\ -0.07937672 & -0.10448198 & -0.99135405 \\ 0.966614 & 0.23498992 & -0.10216215 \end{bmatrix}$$

Camera Center C_1

$$C_1 = \begin{bmatrix} -739.8902994 \\ -485.37784529 \\ 321.63665888 \end{bmatrix}$$

Discussion:

Discuss the significance of the decomposed components.

2 Problem 2: Computing a Second Camera Matrix

2.1 2a: Repeating the Calculations for Lego 2

Description:

x	**y**	**x** (3D)	**y** (3D)	**z** (3D)
240	1440	0	192	0
242	1308	0	192	19.2
240	1178	0	192	38.4
242	1045	0	192	57.6
245	905	0	192	76.8
245	786	0	192	96
250	577	0	192	124.8
988	1758	0	0	0
991	1623	0	0	19.2
988	1471	0	0	38.4
991	1320	0	0	57.6
993	1180	0	0	76.8
998	1028	0	0	96
998	799	0	0	124.8
2424	1506	240	0	0
2421	1366	240	0	19.2
2424	1236	240	0	38.4
2434	1109	240	0	57.6
2439	972	240	0	76.8
2439	837	240	0	96
2447	625	240	0	124.8

Table 2: 2D Image Coordinates in Pixels and Corresponding 3D World Coordinates in mm

Results:

Homography Matrix P

$$P = \begin{bmatrix} -3.45737581 \times 10^{-3} & 1.84680728 \times 10^{-3} & -1.34003533 \times 10^{-5} & -4.87876478 \times 10^{-1} \\ 2.22192346 \times 10^{-4} & 3.96427196 \times 10^{-4} & 3.84370894 \times 10^{-3} & -8.72895156 \times 10^{-1} \\ -2.06695621 \times 10^{-7} & -3.04302600 \times 10^{-7} & 4.29616113 \times 10^{-8} & -4.94925813 \times 10^{-4} \end{bmatrix}$$

Intrinsic Matrix K_2

$$K_2 = \begin{bmatrix} 1.05251026 \times 10^{4} & -5.99949679 \times 10^{1} & 1.10856424 \times 10^{3} \\ 0 & 1.04504974 \times 10^{4} & -1.04111217 \times 10^{1} \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix R_2

$$R_2 = \begin{bmatrix} -0.82783123 & 0.5608882 & -0.00999388 \\ 0.05685095 & 0.10160475 & 0.9931991 \\ -0.55808908 & -0.82163307 & 0.11599862 \end{bmatrix}$$

Camera Center C_2

$$C_2 = \begin{bmatrix} -721.05444208 \\ -1082.93989766 \\ 380.46969249 \end{bmatrix}$$

Discussion:

Analyze the differences between the two matrices and what they imply about the relative camera positions.

2.2 2b: 3D Plot of Camera Positions and World Points

Results:
Include the 3D plot with labeled axes and camera positions. Provide different views to show the camera orientations.

Discussion:
Discuss how the plot clarifies the relative positions and orientations of the cameras.

3 Problem 3: The Image of the World Coordinate System

Description:

Let's express the 3×4 matrix $\mathbf{P}$ as

$$\mathbf{P} = [\mathbf{p}_1\ \mathbf{p}_2\ \mathbf{p}_3\ \mathbf{p}_4].$$

Here, $\mathbf{p}_4$ is a homogeneous representation of the image of the world origin, and $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$ are the images of the vanishing points of the world X-, Y-, and Z-axes, respectively. Note that the ideal point along the world x-axis is given by

$$[1\ 0\ 0\ 0]^T.$$

Since $\mathbf{P} \cdot [1\ 0\ 0\ 0]^T = \mathbf{p}_1$, we have that the image of the ideal point is formed at the pixel whose homogeneous representation is $\mathbf{p}_1$. By the same reasoning, $\mathbf{p}_2$ is the image pixel for the ideal point along the y-axis, and $\mathbf{p}_3$ is the image pixel for the ideal point along the z-axis. The last column, $\mathbf{p}_4$, is the image of the world origin. The homogeneous representation of the world origin is

$$[0\ 0\ 0\ 1]^T.$$

Its image is $\mathbf{P} \cdot [0\ 0\ 0\ 1]^T = \mathbf{p}_4$.

Results:

Demonstrate these statements using the two computed camera matrices. Include the plots of projected world axes overlaid on the images.

Discussion:

Interpret the results and any observations.

4 Problem 4: Epipolar Geometry

4.1 4a: Calculating Epipoles

Description:

Describe the method used to calculate the epipoles e and e'.

Results:

Provide the de-homogenized coordinates of both epipoles and discuss whether the coordinates make sense.

4.2 4b: Computing the Fundamental Matrix and Epipolar Lines

Description:

Explain how the fundamental matrix F was computed from P and P'.

Results:

Display the epipolar lines on both images. Describe the visualization approach, including the use of color for corresponding lines.

Discussion:

Analyze the correctness of the epipolar lines and any observations on the correspondence between features.

5 Conclusion

Summarize the key findings of the report, including the results of the camera matrix calculations, decomposition, and epipolar geometry analysis.

6 References

List any references, textbooks, or online resources you used.