VECTOR ALGEBRA

12^{th} Maths - Chapter 10

This is Problem-8 from Exercise 5.8

1. Show that the points A $\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, B $\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and C $\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear, and find the ratio in which B divides AC.

   Solution:
   The input parameters for this problem are available in Table 1

Symbol	Value	Description
A	$\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$	First point
B	$\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$	Second point
C	$\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$	Third point

Table 1

A, **B** and **C** are collinear if,

$$\left\|\overrightarrow{AB}\right\| + \left\|\overrightarrow{BC}\right\| = \left\|\overrightarrow{AC}\right\| \tag{1}$$

Here,

$\overrightarrow{AB}$ can be expressed as follows

$$\overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k} \tag{2}$$

of magnitude

$$\left\|\overrightarrow{AB}\right\| = \sqrt{16+4+36} = 2\sqrt{14} \tag{3}$$

Similarly, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ can be expressed as

$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k} \tag{4}$$

$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k} \tag{5}$$

of magnitudes

$$\left\|\overrightarrow{BC}\right\| = \sqrt{36+9+81} = 3\sqrt{14} \tag{6}$$

$$\left\|\overrightarrow{AC}\right\| = \sqrt{100+25+225} = 5\sqrt{14} \tag{7}$$

where

$$\left\|\overrightarrow{AB}\right\| + \left\|\overrightarrow{BC}\right\| = 2\sqrt{14} + 3\sqrt{14} = 5\sqrt{14} \tag{8}$$

Thus,

$$\left\|\overrightarrow{AB}\right\| + \left\|\overrightarrow{BC}\right\| = \left\|\overrightarrow{AC}\right\| \tag{9}$$

Let **B** divide **AC** in k:1 then,

$$\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \frac{k\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} + 1\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} = \frac{\left(11k+1, 3k-2, 7k-8\right)}{k+1} \tag{10}$$

$\implies k = 2/3 \implies k : 1 \implies 2 : 3$

Hence, **B** divides **AC** in 2:3.