

VECTOR ALGEBRA

12th Maths - Chapter 10

This is Problem-8 from Exercise 5.8

1. Show that the points $A \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, $B \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and $C \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear, and find the ratio in which B divides AC.

Solution:

The input parameters for this problem are available in Table 1

Symbol	Value	Description
A	$\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$	First point
B	$\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$	Second point
C	$\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$	Third point

Table 1

Points **A**, **B** and **C** are on a line if

$$\text{rank}(\mathbf{A} \ \mathbf{B} \ \mathbf{C}) < 3 \quad (1)$$

Substituting, we must find the rank of

$$\mathbf{M} = \begin{pmatrix} 1 & 5 & 11 \\ -2 & 0 & 3 \\ -8 & -2 & 7 \end{pmatrix} \quad (2)$$

Using row reduction methods to bring \mathbf{M} into row-reduced echelon form,

$$\begin{pmatrix} 1 & 5 & 11 \\ -2 & 0 & 3 \\ -8 & -2 & 7 \end{pmatrix} \xleftrightarrow{R_2 \rightarrow R_2 + 2R_1} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ -8 & -2 & 7 \end{pmatrix} \quad (3)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 + 8R_1} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 38 & 95 \end{pmatrix} \quad (4)$$

$$\xleftrightarrow{R_3 \rightarrow R_3 - \frac{19}{5}R_2} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

Clearly, the rank of \mathbf{M} is 2, and hence the given points are collinear. Fig. 1 verifies that the three points are indeed collinear as claimed.

Let \mathbf{B} divide \mathbf{AC} in $k:1$ then,

$$\frac{k\mathbf{C} + \mathbf{A}}{k + 1} = \mathbf{B} \quad (6)$$

$$\implies k\mathbf{C} + \mathbf{A} = \mathbf{B}(k + 1) \implies k(\mathbf{C} - \mathbf{B}) = (\mathbf{B} - \mathbf{A}) \quad (7)$$

Multiplying with $(\mathbf{C} - \mathbf{B})^\top$ on both sides,

$$k(\mathbf{C} - \mathbf{B})(\mathbf{C} - \mathbf{B})^\top = (\mathbf{B} - \mathbf{A})\mathbf{C} - \mathbf{B}^\top$$

The value of k is as follows,

$$k = \frac{(\mathbf{B} - \mathbf{A})(\mathbf{C} - \mathbf{B})^\top}{\|\mathbf{C} - \mathbf{B}\|^2} \quad (8)$$

where,

$$(\mathbf{B} - \mathbf{A}) = \left(\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix} \right) = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \quad (9)$$

$$(\mathbf{C} - \mathbf{B}) = \left(\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \right) = \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} \quad (10)$$

$$(\mathbf{C} - \mathbf{B})^{\top} = (6 \quad 3 \quad 9)$$

Substituting the values in equation (8) the value of k is $2/3$.
Hence, \mathbf{B} divides \mathbf{AC} in the ratio $2 : 3$.

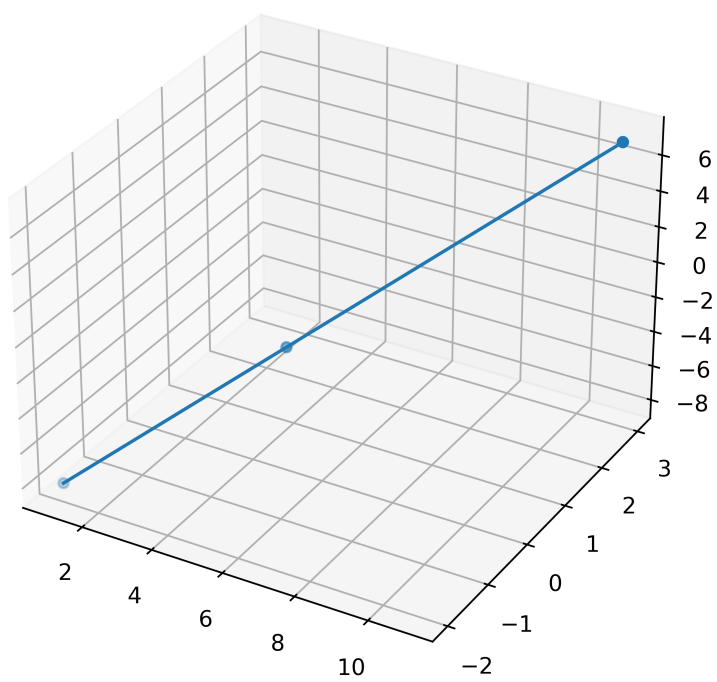


Figure 1