VECTOR ALGEBRA

12^{th} Maths - Chapter 10

This is Problem-8 from Exercise 5.8

1. Show that the points A $\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, B $\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and C $\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear, and find the ratio in which B divides AC.

   Solution:
   The input parameters for this problem are available in Table **??**

Symbol	Value	Description
A	$\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$	First point
B	$\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$	Second point
C	$\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$	Third point

Table 1

Points **A**, **B** and **C** are on a line if

$$\text{rank}\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} < 3 \tag{1}$$

Substituting, we must find the rank of

$$\mathbf{M} = \begin{pmatrix} 1 & 5 & 11 \\ -2 & 0 & 3 \\ -8 & -2 & 7 \end{pmatrix} \tag{2}$$

Using row reduction methods to bring $\mathbf{M}$ into row-reduced echelon form,

$$\begin{pmatrix} 1 & 5 & 11 \\ -2 & 0 & 3 \\ -8 & -2 & 7 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + 2R_1} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ -8 & -2 & 7 \end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_3 \to R_3 + 8R_1} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 38 & 95 \end{pmatrix} \tag{4}$$

$$\xleftrightarrow{R_3 \to R_3 - \frac{19}{5} R_2} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

Clearly, the rank of $\mathbf{M}$ is 2, and hence the given points are collinear. Fig. **??** verifies that the three points are indeed collinear as claimed.
Let $\mathbf{B}$ divide $\mathbf{AC}$ in k:1 then,

$$\frac{k\mathbf{C} + \mathbf{A}}{k+1} = \mathbf{B} \tag{6}$$

$$\implies k\mathbf{C} + \mathbf{A} = \mathbf{B}(k+1) \implies k(\mathbf{C} - \mathbf{B}) = (\mathbf{B} - \mathbf{A}) \tag{7}$$

Multiplying with $(\mathbf{C} - \mathbf{B})^\top$ on both sides,

$$k(\mathbf{C} - \mathbf{B})(\mathbf{C} - \mathbf{B})^\top = (\mathbf{B} - \mathbf{A})\,\mathbf{C} - \mathbf{B}^\top$$

The value of k is as follows,

$$k = \frac{(\mathbf{B} - \mathbf{A})(\mathbf{C} - \mathbf{B})^\top}{\|\mathbf{C} - \mathbf{B}\|^2} \tag{8}$$

where,

$$(\mathbf{B} - \mathbf{A}) = \left(\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \tag{9}$$

$$(\mathbf{C} - \mathbf{B}) = \left(\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}\right) = \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} \tag{10}$$

$$(\mathbf{C} - \mathbf{B})^{\top} = \begin{pmatrix} 6 & 3 & 9 \end{pmatrix}$$

Substituting the values in equation (**??**) the value of k is $2/3$.
Hence, **B** divides **AC** in the ratio 2 : 3.

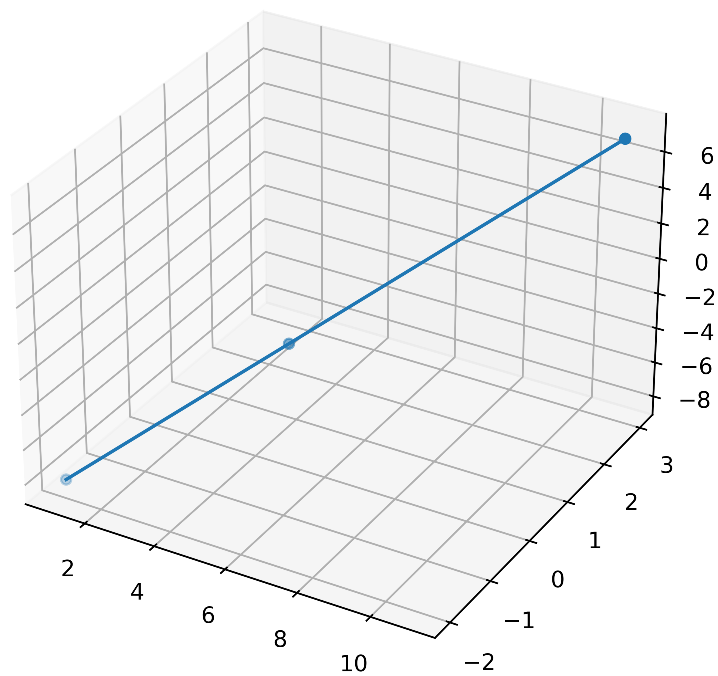

Figure 1