CS & IT ENGINEERING

Algorithm

Miscellaneous Topics

Lecture No.- 08

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Recap of Previous Lecture

Topic Introduction to Dynamic Programming

Topic The General Method

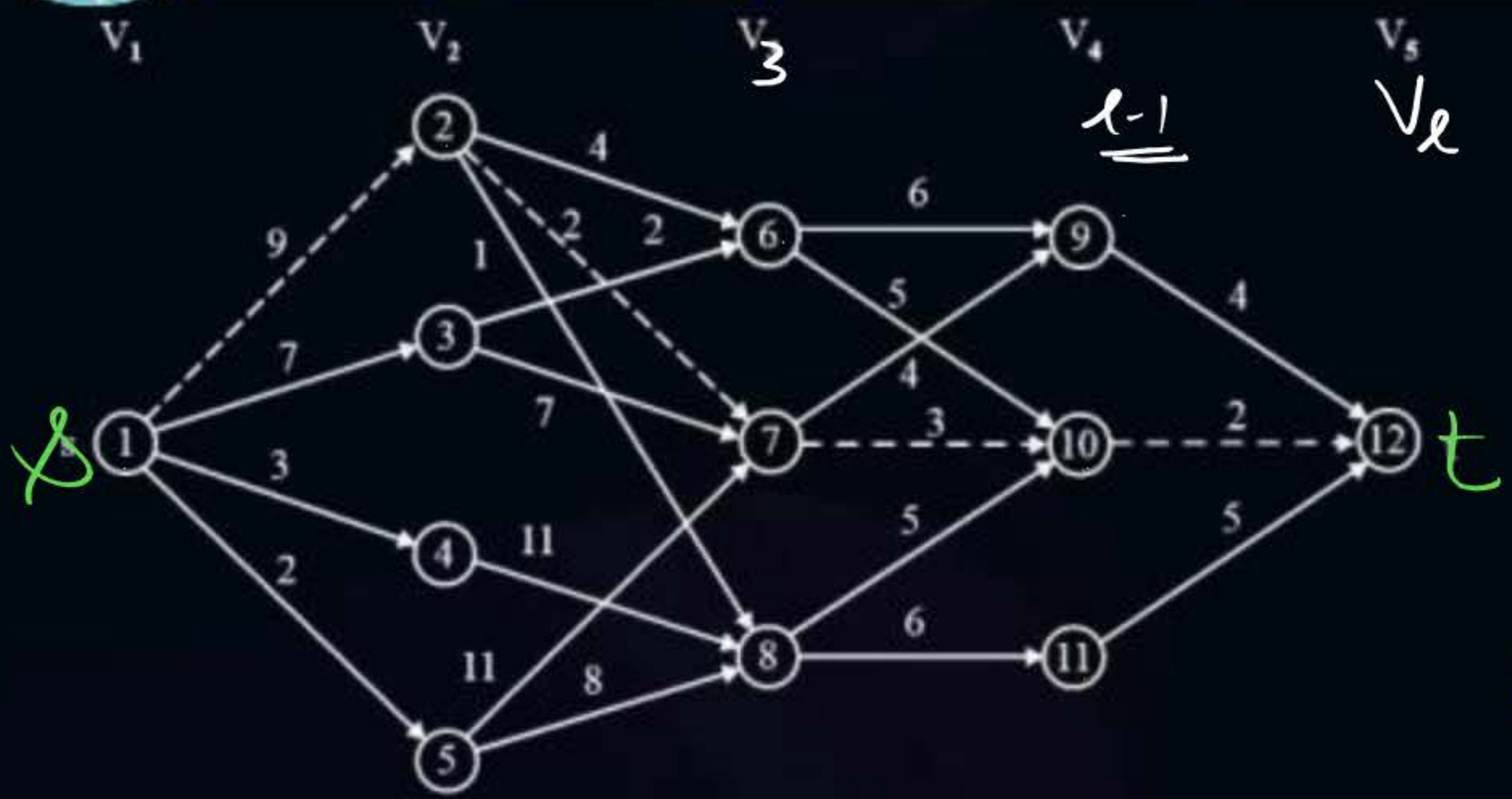
DP Vs DandC

Fibonacci Implementation

Topics to be Covered

Topic Multistage Graphs

Topic Travelling Salesperson Problem



$COST(i, j)$ = Cost of Path from vertex 'j' in Stage 'i' to reach dest. vertex 't';

$$\overset{i\ j}{COST(1,1)} = \min_{K \in V_2} \{ c(\overset{j}{1},K) + COST(2,K) \} \quad -\ (1)$$

$(1 - (K \sim\sim\sim t))$: $COST(1,1)$

$COST(2,K)$

$K \in V_2$

$\langle 1,K \rangle \in E$

$$COST(i,j) = \min_{\substack{K \in V_{i+1} \\ \langle j,K \rangle \in E}} \{ c(j,K) + COST(i+1,K) \} \quad -\ (2)$$

$COST(l-1, j) = c(j, t)$

C	1	2	3	. .	12
1					
2					
3					
⋮					
12					

I/P

$c(i,j)$ = edge cost

$$COST(i,j) = \min_{\substack{k \in V_{i+1} \\ \langle j,k\rangle \in E}} \{ c(j,k) + COST(i+1,k) \} \quad \text{— ①}$$

$$COST(k-1,j) = c(j,t) \quad \text{— ②}$$

$$D(i,j) = \text{'}\underline{k}\text{' that minimizes Eq ①}$$

$$COST(1,1) = \min\{ \underbrace{c(1,2)+COST(2,2)}_{9+7}^{k=2},\ \underbrace{c(1,3)+COST(2,3)}_{7+9}^{k=3},\ \underbrace{c(1,4)+COST(2,4)}_{3+18}^{k=4},\ \underbrace{c(1,5)+COST(2,5)}_{2+15}^{k=5} \}$$

$$= \boxed{16}$$

$$D(1,1) = 2$$

Path Construction:

$$\langle 1 - 2 - 7 - 10 - 12 \rangle$$

$$D(1,1)$$

$$D(2, D(1,1)) = D(2,2) \qquad D(3,7)$$

$COST(4,9) = C(9,t) = 4$

$COST(4,10) = C(10,t) = 2$

$COST(4,11) = C(11,t) = 5$

$$COST(3,6) = \min\left\{ \underbrace{C(6,9) + COST(4,9)}_{k=9:\ 6+4},\ \underbrace{C(6,10) + COST(4,10)}_{k=10:\ 5+2} \right\} = 7$$

$D(3,6) = 10$

$COST(3,7) = 5$

$D(3,7) = 10$

$COST(3,8) = 7$

$D(3,8) = 10$

$COST(2,2) = \min\{4+7;\ 2+5;\ 1+7\} = 7$

$D(2,2) = 7$

$COST(2,3) = \min\{2+7;\ 7+5\} = 9$

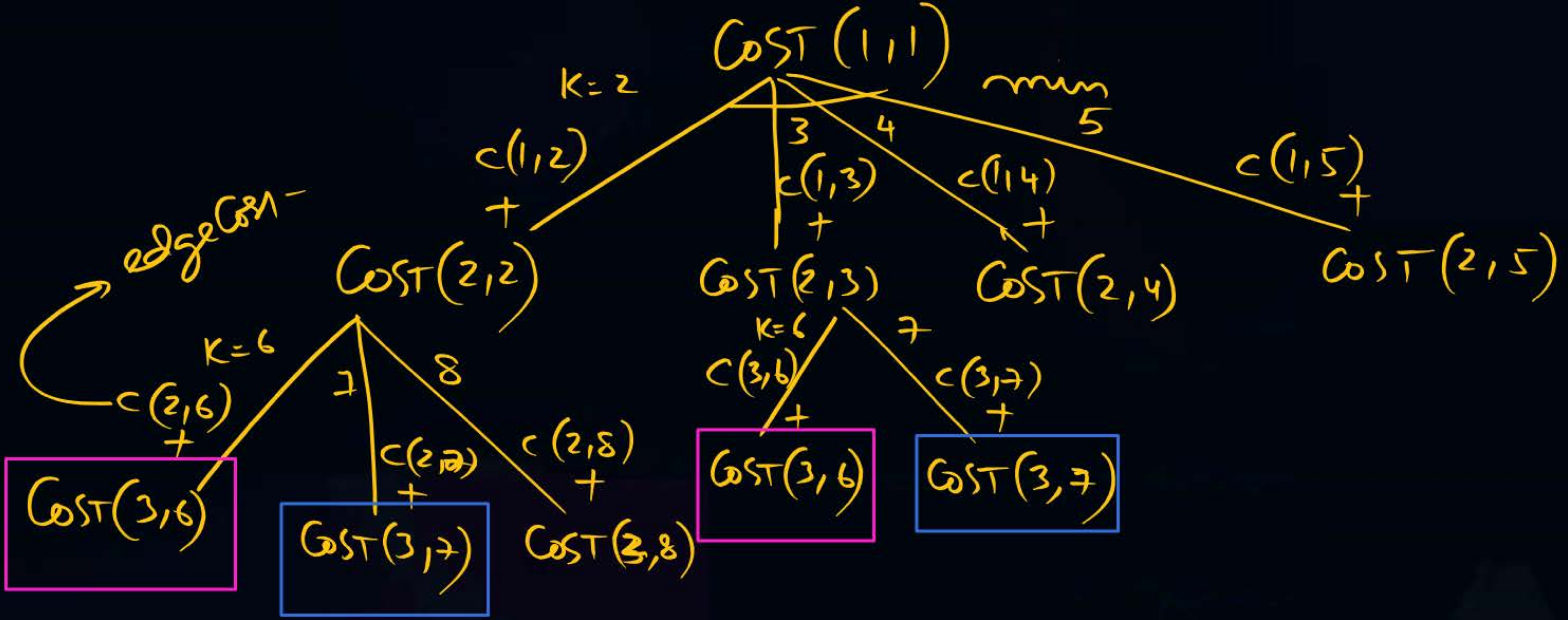
$D(2,3) = 6$

$COST(2,4) = 18$

$D(2,4) = 8$

$COST(2,5) = 15$

$D(2,5) = 8$


COST (1,1)
K=2
min
3
4
5
c(1,2)
+
c(1,3)
+
c(1,4)
+
c(1,5)
+
edgeCost
COST(2,2)
COST(2,3)
COST(2,4)
COST(2,5)
K=6
7
8
c(2,6)
+
c(2,7)
+
c(2,8)
+
K=6
7
c(3,6)
+
c(3,7)
+
COST(3,6)
COST(3,7)
COST(3,8)
COST(3,6)
COST(3,7)

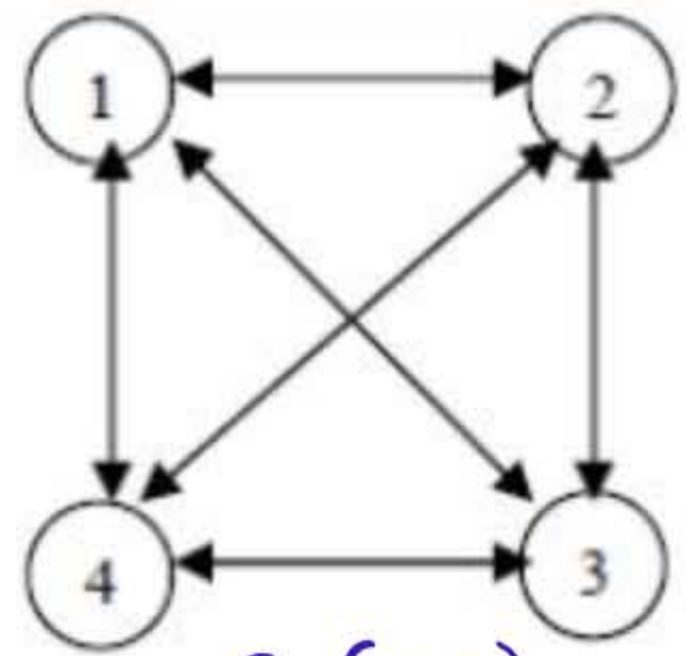
Topic : Dynamic Programming: (DP)

2) Travelling Salesperson Problem (TSP)

"Tour"

v_0 = home city

⇒ The tour of TSP should start from home city v_0 & visit remaining $(n-1)$ cities exactly once & come back to home city (v_0), S.T. the cost of the tour is minimum



$G=(V,E)$

$$C = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 9 & 10 \\ 3 & 6 & 13 & 0 & 12 \\ 4 & 8 & 8 & 9 & 0 \end{array}$$

Greedy method $(v_0 = 1)$: $(1-2-3-4-1)$: 39

$(1-2-4-3-1)$: 35

$10+10+9+6$

Brute-force : $(n-1)!$

Let $g(i, S)$ repr. Cost of the Tour of T.S.P from vertex 'i', & visiting all vertices in the set 'S' exactly once and terminating the tour at V_0;

$V_0 = 1$

$$g\Big(1, \underbrace{\{2,3,4\}}_{}^{S}\Big) = \min_{\substack{K \in S \\ \langle 1,K\rangle \in E}} \Big\{ c(1,K) + g\big(K, S - \{K\}\big) \Big\}$$

$$\Big\langle \overset{c(1,K)}{\underline{1 - K}} \sim\sim\sim\sim\sim V_0 \Big\rangle$$

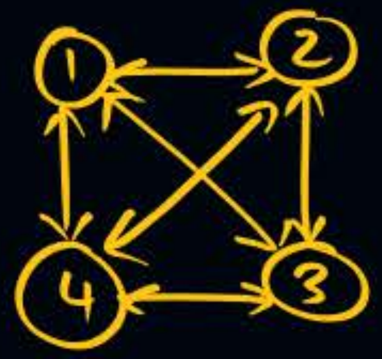
$$K \in S$$
$$\langle 1,K \rangle \in E$$

$$\underline{g(i, S)} = \min_{\substack{K \in S \\ \langle i,K\rangle \in E}} \Big\{ c(i,K) + \underline{g\big(K, S - \{K\}\big)} \Big\} \quad \text{— (1)}$$

$$g(i, \phi) = c(i, V_0)$$

$$i - V_0$$

$J(i, S) \equiv$ Value of 'K' that minimizes Eq (1)



c	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

$v_0 = 1$

$|S| = 3$

$$g(1, \{2,3,4\}) = \min\Big\{ \underbrace{c(1,2)}_{10} + \underbrace{g(2,\{3,4\})}_{25},\ \underbrace{c(1,3)}_{15} + \underbrace{g(3,\{2,4\})}_{25},\ \underbrace{c(1,4)}_{20} + \underbrace{g(4,\{2,3\})}_{23} \Big\} = 35$$

$k=2$, $k=3$, $k=4$

$$J(1, \{2,3,4\}) = 2$$

$|S| = 0$

$g(2, \emptyset) = c(2,1) = 5$

$g(3, \emptyset) = c(3,1) = 6$

$g(4, \emptyset) = c(4,1) = 8$

$|S| = 1$

$g(2, \{3\}) = c(2,3) + g(3, \emptyset) = 9 + 6 = 15$

$g(2, \{4\}) = 10 + 8 = 18$

$g(3, \{2\}) = 13 + 5 = 18$

$g(3, \{4\}) = 12 + 8 = 20$

$g(4, \{2\}) = 8 + 5 = 13$

$g(4, \{3\}) = 9 + 6 = 15$

$$g(2, \{3,4\}) = \min\Big\{ \underbrace{c(2,3) + g(3,\{4\})}_{9 + 20},\ \underbrace{c(2,4) + g(4,\{3\})}_{10 + 15} \Big\} = 25$$

$k=3$, $k=4$

$$J(2, \{3,4\}) = 4$$

$|S| = 2$

$g(2, \{3,4\}) = 25$

$J(2, \{3,4\}) = 4$

$g(3, \{2,4\}) = 25$

$J(3, \{2,4\}) = 4$

$g(4, \{2,3\}) = 23$

$J(4, \{2,3\}) = 2$

Tour-Construction :

$v_0 = 1$

$$\boxed{1 - 2 - 4 - 3 - 1} = 35$$

$$J(1, \{2,3,4\}) = 2$$

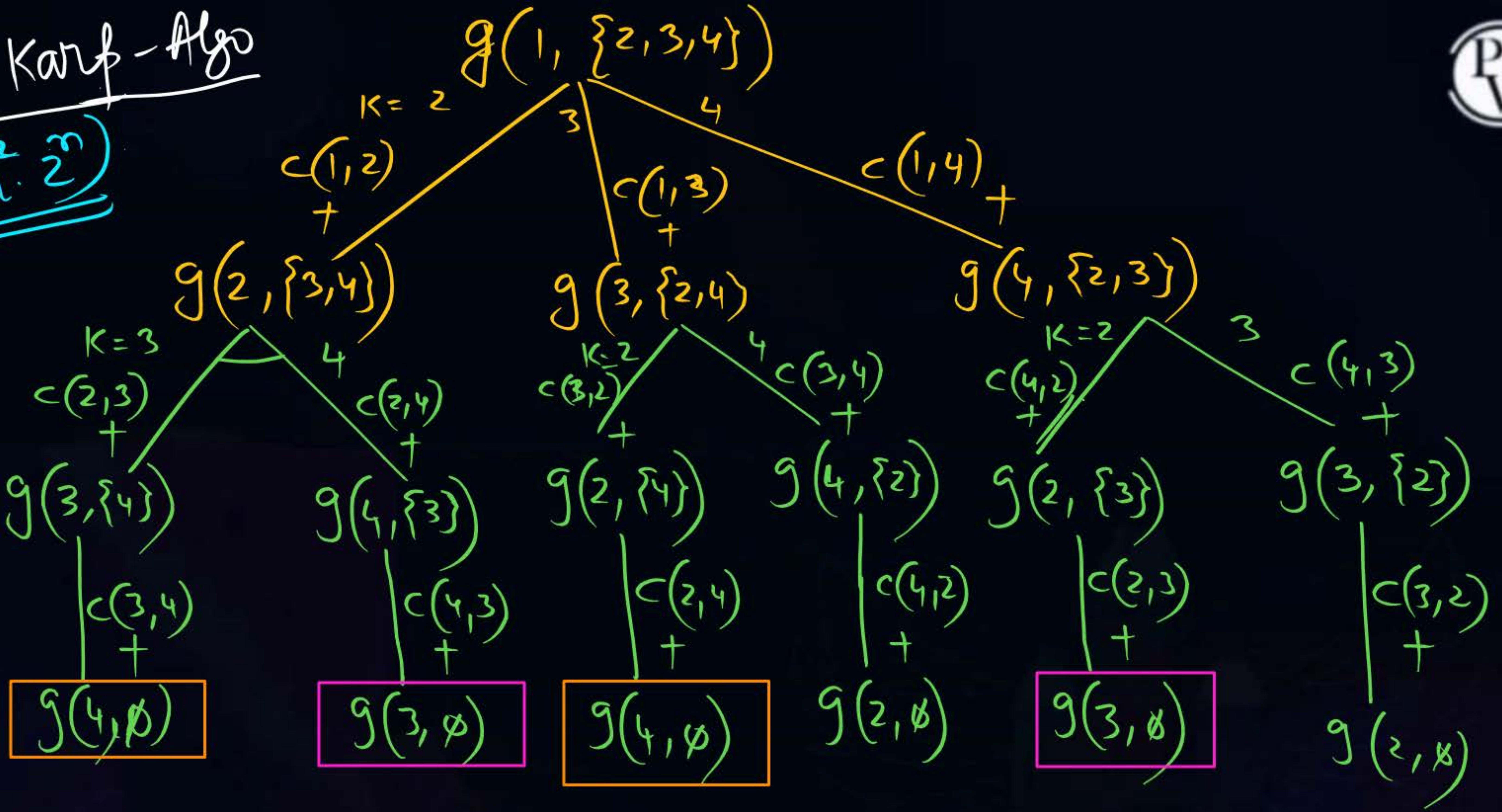
$$J(2, \{3,4\}) = 4$$

$v_0 = 4$ ✓

H/w

Held-Karp-Algo

$O(n^2 \cdot 2^n)$



(T.S.P is one of the Problem, for which there is Polynomial Time Algo. in the Literature)

3) All-Pairs Shortest Paths (Flyod-warshall's Algorithms)

1. (Solvable by Greedy Method also) → Single Src. Sht. Paths (Dijkstra's Algo)

(Apply it n-times, changing Src vertex each Time)

2. Flyod-warshall's Algo (D.P)

i K j

THANK - YOU