

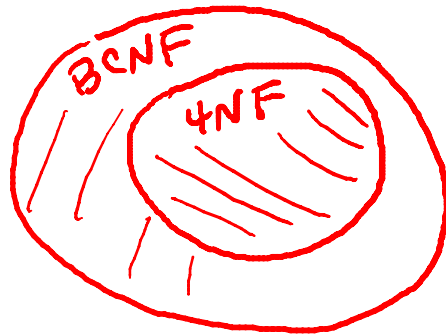


Relational Design Theory

Multivalued Dependencies & 4th Normal Form

Relational design by decomposition

- “Mega” relations + properties of the data
- System decomposes based on properties
- Final set of relations satisfies normal form
 - No anomalies, no lost information
- Functional dependencies \Rightarrow Boyce-Codd Normal Form
- + ■ Multivalued dependences \Rightarrow Fourth Normal Form



Example: College application info.

App1y(SSN, cName, hobby)

FDs? No.

Keys? All attrs.

BCNF? yes.

Good design? No.

5 colleges, 6 hobbies → 30 tuples.

Multivalued Dependency

- Based on knowledge of real world
- All instances of relation must adhere

$$R \quad \underline{\bar{A}} \twoheadrightarrow \bar{B} \quad A_1, \dots, A_n \quad B_1, \dots, B_n$$

$\forall t, u \in R : t[\bar{A}] = u[\bar{A}] \text{ then}$

$\exists v \in R : v[\bar{A}] = t[\bar{A}] \text{ and}$

$v[\bar{B}] = t[\bar{B}] \text{ and}$

$v[\text{rest}] = u[\text{rest}]$

	\bar{A}	\bar{B}	rest
t	\bar{a}	\bar{b}_1	\bar{r}_1
u	\bar{a}	\bar{b}_2	\bar{r}_2
v	\bar{a}	\bar{b}_1	\bar{r}_2
w	\bar{a}	\bar{b}_2	\bar{r}_1

tuple-generating dependencies

Apply(SSN, cName, hobby)

SSN \rightarrow cName SSN \rightarrow hobby

	SSN	cName	hobby
t	123	Stanford.	trumpet
u	123	Berkeley	tennis.
v	123	Stanford	tennis
w	123	Berkeley	trumpet
	⋮	⋮	⋮

Modified example

Apply(SSN, cName, hobby) ★

Reveal hobbies to colleges selectively ★

MVDs? *None*

Good design? *Yes.*

Expanded example

Apply(SSN, cName, date, major, hobby)

Reveal hobbies to colleges selectively ✓

Apply once to each college *one day*

May apply to multiple majors ✓

SSN, cName → date

SSN, cName, date → major "rest" hobby

Trivial Multivalued Dependency

$$\bar{A} \twoheadrightarrow \bar{B} \quad \bar{B} \subseteq \bar{A} \text{ or } \bar{A} \vee \bar{B} = \text{all attributes}$$

Nontrivial MVD

otherwise.

no "rest"

\bar{A}	\bar{B}	
a	b	—

Rules for Multivalued Dependencies

FD-is-an-MVD rule

$\bar{A} \rightarrow \bar{B}$ then $\bar{A} \twoheadrightarrow \bar{B}$

$$\bar{b}_1 = \bar{b}_2$$

	\bar{A}	\bar{B}	rest
t	$\bar{a} \bullet$	\bar{b}_1	\bar{r}_1
u	$\bar{a} \bullet$	\bar{b}_2	\bar{r}_2
$\rightarrow v$	\bar{a}	$\bar{b}_1 = \bar{b}_2$	\bar{r}_2
		\vdots	

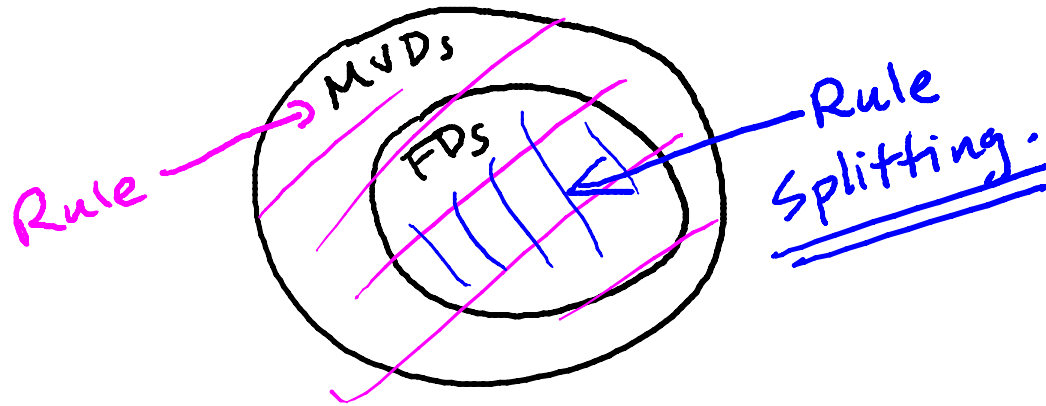
Rules for Multivalued Dependencies

Intersection rule

$$\bar{A} \twoheadrightarrow \bar{B} \quad \bar{A} \twoheadrightarrow \bar{C} \quad \text{then} \quad \bar{A} \twoheadrightarrow \bar{B} \cap \bar{C}$$

Transitive rule

$$\bar{A} \twoheadrightarrow \bar{B} \quad \bar{B} \twoheadrightarrow \bar{C} \quad \text{then} \quad \underline{\bar{A} \twoheadrightarrow \bar{C} - \bar{B}}$$



Fourth Normal Form

Relation R with MVDs is in 4NF if:

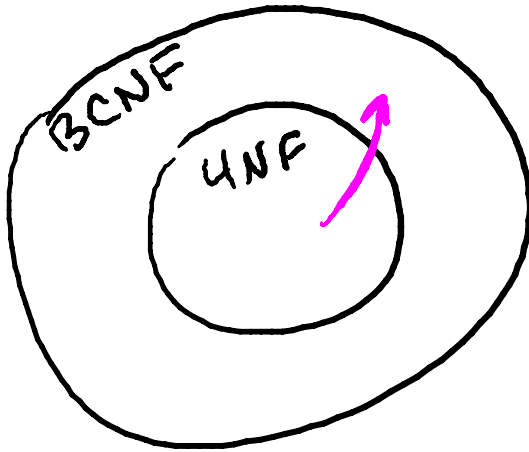
For each nontrivial $A \twoheadrightarrow B$, A is a key

	\overline{A}	\overline{B}	rest
t	a	b ₁	r ₁
u	a	b ₂	r ₂

Fourth Normal Form \Rightarrow BCNF

Relation R with MVDs is in 4NF if:

For each nontrivial $A \twoheadrightarrow B$, A is a key



$A \rightarrow B$ A is a key
 $\hookrightarrow A \twoheadrightarrow B$ ↗

4NF decomposition algorithm

Input: relation R + FDs for R + MVDs for R

Output: decomposition of R into 4NF relations with "lossless join"

Compute keys for R ✓

Repeat until all relations are in 4NF: ✓

Pick any R' with nontrivial $A \twoheadrightarrow B$ that violates 4NF

Decompose R' into $R_1(A, B)$ and $R_2(A, \text{rest})$

Compute FDs and MVDs for R_1 and R_2

Compute keys for R_1 and R_2 ✓

4NF Decomposition Example #1

Apply(SSN, cName, hobby)

SSN \Rightarrow cName No keys.

A1(SSN, cName)

No FDs
No MVDs

A2(SSN, hobby)



4NF Decomposition Example #2

Apply(SSN, cName, date, major, hobby)

① ✓ SSN, cName \rightarrow date ☆ No Keys -

✓ SSN, cName, date \rightarrow major X

~~A1 (SSN, cName, date, major)~~

~~A2 (SSN, cName, date, hobby)~~

\rightarrow A3 (SSN, cName, date)

A4 (SSN, cName, major)

\rightarrow A5 (SSN, cName, hobby)



Relational design

- Functional dependencies & Boyce-Codd Normal Form

$R(A, B, C) \quad \underline{A} \rightarrow \underline{B}$

- Multivalued dependencies & Fourth Normal Form

$R(\underline{A}, \underline{B}, \underline{C}, \underline{D}) \quad \underline{\bar{A}} \twoheadrightarrow \underline{\bar{B}} \leftarrow$

