Statistics Exercise 1

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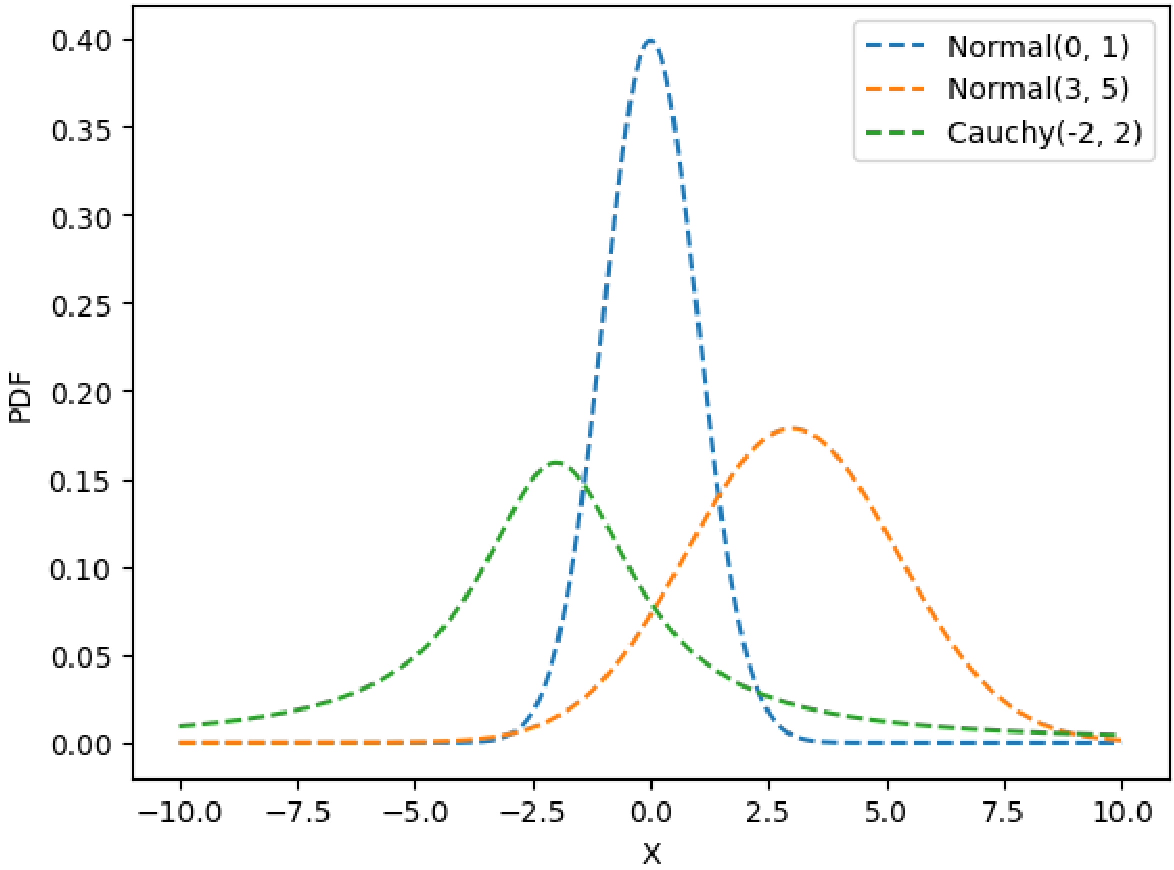
Question 1.a



Key Observation

Normal Distribution - $\mathcal{N}(0,1)$

- Has a narrow curve due to low variance, resulting in a high probability density around the mean (0).
- It has shorter tails - samples drawn from this distribution will be closer to the mean.
- Does not have long tails - chances of drawing extreme values are low.

Normal Distribution - $\mathcal{N}(3,5)$

In comparison with $\mathcal{N}(0,1)$,

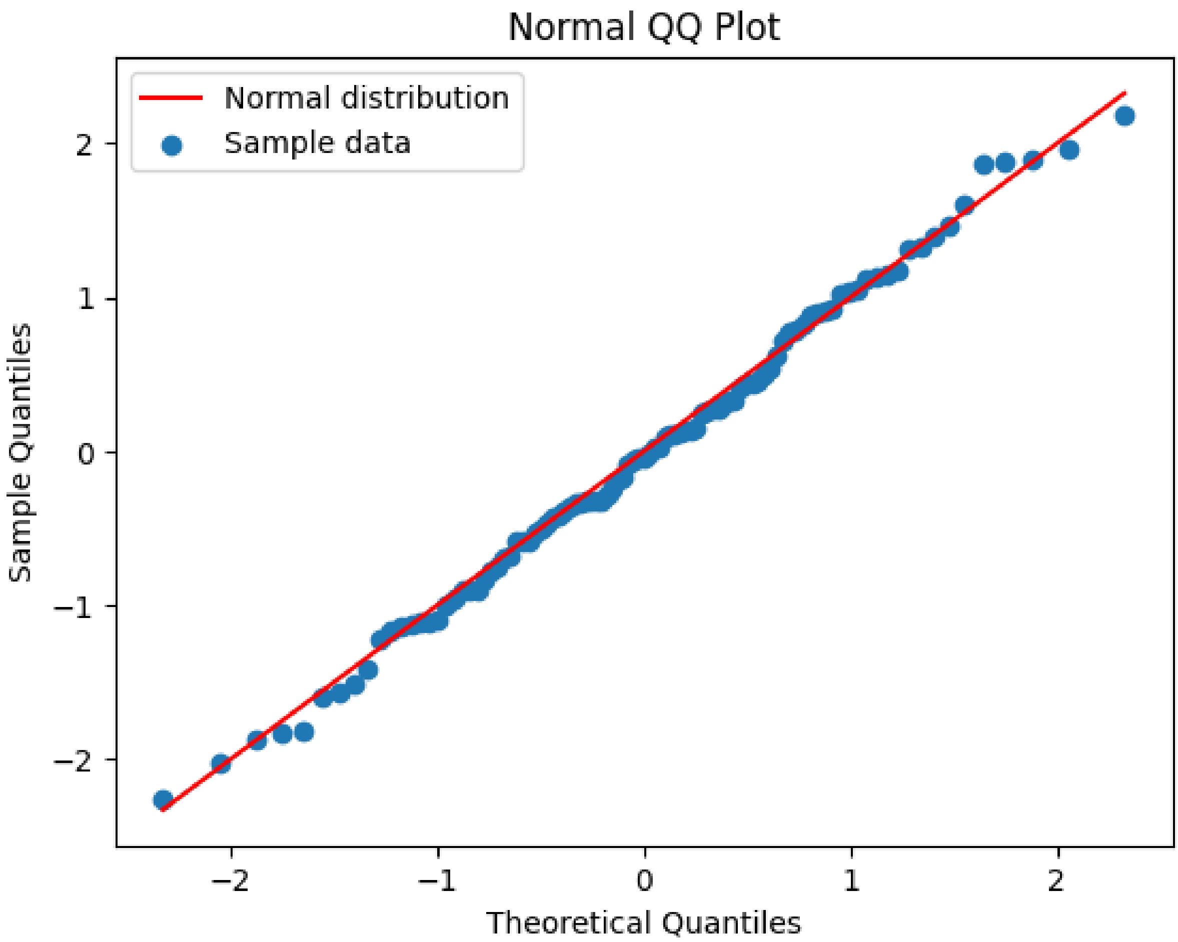
- Has a wider curve due to high variance, resulting in a lower probability density around the mean (3).
- It has longer tails - a significant number of samples drawn from this distribution can be away from the mean (due to high variance).
- Tails are slightly longer than $\mathcal{N}(0,1)$, but chances of drawing extreme values are still low.

Cauchy Distribution - $Cauchy(-2, 2)$

- Has a narrower curve when compared to $\mathcal{N}(3, 5)$ and has longer tails.
- This distribution is not symmetric and does not have a mean or variance.
- Chances of drawing extreme values are higher when compared to Normal distribution.

Question 1.c

QQ plot for samples from $\mathcal{N}(0, 1)$

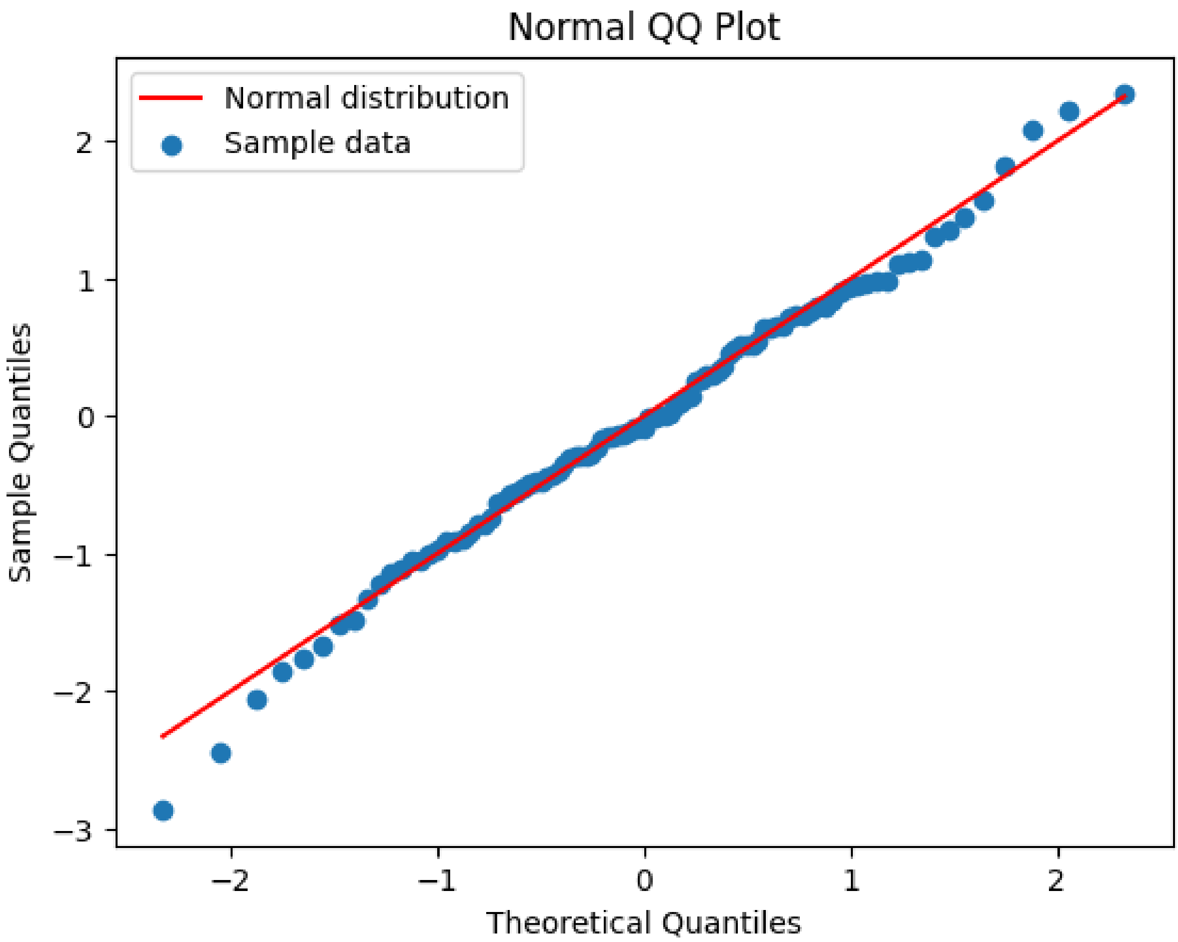


From the QQ plot,

- We can observe that the samples drawn from $\mathcal{N}(0, 1)$ closely align with the theoretical quantiles of normal distribution.
- They form a straight line, indicating that the samples are normally distributed.

Question 1.d

QQ plot for samples from $\mathcal{N}(3, 5)$

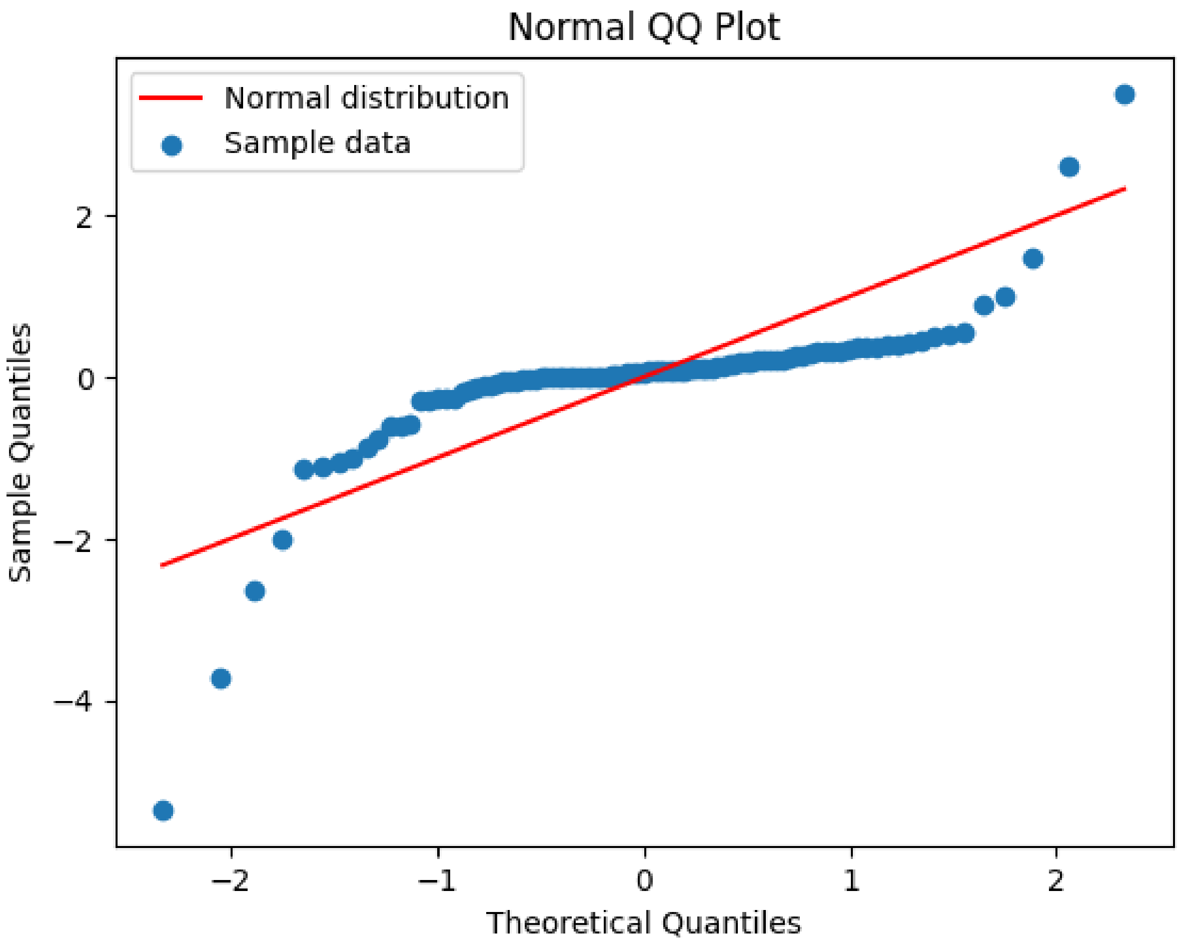


From the QQ plot,

- We can observe that the samples drawn from $\mathcal{N}(3, 5)$ closely align with the theoretical quantiles of normal distribution.
- We can see a few points away from the straight line in the plot. This is because of the high variance in the distribution.
- Ignoring a few outliers, the samples are closer to a straight line, indicating that the samples are normally distributed.

Question 1.e

QQ plot for samples from $Cauchy(-2, 2)$



From the QQ plot,

- We can observe that the samples drawn from $Cauchy(-2, 2)$ do not form a straight line - indicating that the samples are not normally distributed.
- In addition, we can see multiple outliers in the plot - indicating a longer tail.

Question 2.a

Given data: $x_1 - \bar{x}, x_2 - \bar{x}, x_3 - \bar{x} \ldots x_9 - \bar{x}$ and a mean $\bar{x}$ of 10 data points.

Mean is defined as:

$$\bar{x} = (x_1 + x_2 + x_3 \ldots + x_9 + x_{10})/10$$

Multiply both sides by 10

$$10.\bar{x} = (x_1 + x_2 + x_3 \ldots + x_9 + x_{10})$$

Subtract $9.\bar{x}$ from both sides

$$10.\bar{x} - 9.\bar{x} = (x_1 + x_2 + x_3 \ldots + x_9 + x_{10}) - 9.\bar{x}$$

Rearrange terms

$$\bar{x} = (x_1 - \bar{x} + x_2 - \bar{x} + x_3 - \bar{x} \ldots + x_9 - \bar{x}) + x_{10}$$

$$x_{10} = (x_1 - \bar{x} + x_2 - \bar{x} + x_3 - \bar{x} \ldots + x_9 - \bar{x}) - \bar{x}$$

By substituting the given data points and mean, we get the value of x_{10}.

Question 2.b

Sample data points: $x_1, x_2, x_3 \ldots x_9, x_{10}$.

$$s^2 = \frac{\sum_{i=1}^{i=n} (x_i - \bar{x})^2}{n - 1}$$

$\bar{x}$ is the mean of the sample data points and not the population mean.

In this case of sample variance, degrees of freedom is n-1 because we use the sample mean which is a calculated measure from the sample data. For n = 10, degrees of freedom is 9.

Given $n - 1$ data points and a mean of n data points, we can easily calculate the missing data point. Which means only $n - 1$ data points can vary, hence we lose one degree of freedom.

4.a