PS-1

Q1. a)

optimisation problem of the hh:

$$\max_{\{C_t, N_t, B_t, M_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U(X_t, N_t)\right]$$

$U(X_t, N_t) = \dfrac{X_t^{1-\gamma} - 1}{1-\gamma} - \chi \dfrac{N_t^{1+\varphi}}{1+\varphi}$

$X_t = \left[(1-\theta)C_t^{1-\nu} + \theta\left(\dfrac{M_t}{P_t}\right)^{1-\nu}\right]^{1/1-\nu}$

s.t. $P_t C_t + B_t M_t \leq \omega_t N_t + Q_{t-1}B_{t-1} + M_{t-1} + P_t(TR_t + PR_t)$

$$\mathcal{L} = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \left\{\frac{X_t^{1-\gamma}-1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}\right\} + \lambda_t\left(\omega_t N_t + Q_{t-1}B_{t-1} + M_{t-1} + P_t(TR_t + PR_t) - P_t C_t - B_t - M_t\right)\right]$$

FOCs

C_t

$$\beta^t X_t^{-\gamma} \frac{1}{1-\nu}\left[(1-\theta)C_t^{1-\nu} + \theta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{\nu}{1-\nu}} \cdot (1-\theta)(1-\nu)C_t^{-\nu} - \beta_t \lambda_t P_t = 0$$

Simplifying:

$$X_t^{-\gamma+\nu}(1-\theta)(C_t)^{-\gamma} = \lambda_t P_t \quad — (1)$$

M_t $\quad \beta^t X_t^{-\gamma} \frac{1}{1-v} \left[(1-\theta)c_t^{1-v} + \theta\left(\frac{M_t}{P_t}\right)^{1-v}\right]^{\frac{v}{1-v}}.$

$$\theta(1-v) M_t^{-v} P_t^{v-1} - \beta^t \lambda_t + \beta^{t+1}\mathbb{E}_t(\lambda_{t+1}) = 0$$

Simplifying:

$$X_t^{-\gamma+v}\, \theta\, (M_t)^{-v} (P_t)^{v-1} + \beta \mathbb{E}_t(\lambda_{t+1}) = \lambda_t \quad - ②$$

B_t $\quad \beta \mathbb{E}_t(\lambda_{t+1} Q_t) = \lambda_t \quad - ③$

N_t $\quad \beta^t \cdot \chi N_t^{\varphi} + \beta^t \lambda_t w_t = 0$

$$\chi N_t^{\varphi} = \lambda_t w_t \quad - ④ \qquad \# N_t^s$$

using ① & ③, we get the EE

$$X_t^{v-\gamma} \frac{(1-\theta) c_t^{-v}}{P_t} = \beta . \mathbb{E}_t\left[X_{t+1}^{v-\gamma} \frac{(1-\theta) C_{t+1}^{-\gamma}}{P_{t+1}} . Q_t\right]$$

$$\beta . \mathbb{E}_t\left[Q_t . \frac{P_t}{P_{t+1}} \left(\frac{X_{t+1}}{X_t}\right)^{v-\gamma} . \left(\frac{c_{t+1}}{c_t}\right)^{-v}\right] = 1 \quad - ⑤$$

using ① & ②, we get the labor leisure tradeoff:

$$\chi N_t^{\varphi} = X_t^{v-\gamma} \frac{(1-\theta)}{P_t} . C_t^{-v} . w_t$$

$$\Rightarrow \frac{w_t}{P_t} = \frac{\chi N_t^{\varphi}}{(1-\theta) c_t^{-v} X_t^{v-\gamma}} \quad - ⑥$$

Money dd:

$$1 = 1/Q_t + \theta/1-\theta \left[\frac{(M_t/P_t)}{c_t}\right]^{-v}$$

$$M_t/P_t = (1 - 1/Q_t)^{-1/v} . (\theta/1-\theta)^{1/v} C_t \quad - ⑦$$

b1. Govt's budget constraint

$$B_t + M_t = P_t(TR_t) + Q_{t-1} \cdot B_{t-1} + M_{t-1} \quad -(8)$$

firm's problem

$$\max_{N_t} \quad Y_t - \frac{W_t}{P_t} \cdot N_t$$

$$\# \quad Y_t = A_t N_t$$

FOC

$$N_t : \quad A_t = W_t / P_t \quad -(9) \qquad \# N_t^d$$

mkt. clearing

$$Y_t = C_t \quad -(10)$$
$$N_t^d = N_t^s$$
$$B_t^d = B_t^s$$
$$M_t^d = M_t^s$$

$$\text{TFP} : \quad A_t = A_{t-1} \exp(\varepsilon_{at}) \quad -(11)$$

$$\text{Money ss} : \quad M_t^s = M_{t-1}^s \exp(\varepsilon_{mt}) \quad -(12)$$

neutrality $\Rightarrow$ real outcomes are independent of price levels & nominal var (money)

say eq^n 4 & 9 (which must be equivalent in the equil., $\because$ mkt clearing)

$$\frac{X N_t^{\varphi}}{(1-\theta)} C_t^{\nu} \cdot X_t^{\gamma-\nu} = A_t = \frac{W_t}{P_t} \qquad \# N_t^d = N_t^s$$

$X_t = X_t(M_t)$ # by def^n

Say $v = \gamma \Rightarrow X_t^{\gamma - v} = 1$

$\therefore A_t = \chi . \dfrac{N_t^{\varphi}}{(1-\theta)} . C_t^{-v} \Rightarrow$ independent of M_t

(same arg. for EE & cons^n if $\gamma = v$)

c) $A = 1$

$\therefore Y = N$

& $W/P = 1$

$\because Y = C \Leftrightarrow N = C.$

$N_t^d = N_t^s \Rightarrow \chi \dfrac{N^{\varphi}}{(1-\theta)} C^v . X^{\gamma - v - 1}$

$$\Rightarrow (1-\theta) X^{v-\gamma} = \chi \, C^{\varphi + v} \quad - (13)$$

@ SS $C_t = C_{t+1}$

$\phi_t . \dfrac{P_t}{P_{t+1}} = R_t$ is constant (ass^n)

$\therefore$ By EE

$\beta R = 1 \Rightarrow R = 1/\beta$

ass^n constant ϕ, $P_{t+1}/P_t = \pi$

$$\Rightarrow \pi = \phi \beta \quad - (14)$$

$\therefore$ money dd eq^n:

$$\frac{M_t}{P_t} = \left(1-\beta/\Pi\right)^{-1/\nu}\left(\frac{\theta}{1-\theta}\right)^{1/\nu}\cdot c \qquad -(15)$$

$$\Rightarrow \left(\frac{M_t}{P_t}\right)^{1-\nu} = \left(1-\frac{\beta}{\Pi}\right)^{-\left(\frac{1-\nu}{\nu}\right)}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-\nu}{\nu}} \cdot c^{1-\nu}$$

$$X = \left\{(1-\theta)c^{1-\nu} + \theta\left[\left(1-\frac{\beta}{\Pi}\right)^{-\left(\frac{1-\nu}{\nu}\right)}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-\nu}{\nu}} c^{1-\nu}\right]\right\}^{\frac{1}{1-\nu}}$$

$$= \left\{(1+\theta) + \theta\left[\left(1-\frac{\beta}{\Pi}\right)^{-\frac{(1-\nu)}{\nu}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-\nu}{\nu}}\right]\right\}^{\frac{1}{1-\nu}}\cdot c \qquad -(16)$$

sub. for X from (16) into (13)

$$\chi c^{\varphi+\nu} = (1-\theta)\left[(1-\theta)+\theta\left[\left(1-\frac{\beta}{\Pi}\right)^{-\frac{(1-\nu)}{\nu}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-\nu}{\nu}}\right]\right]^{\frac{\nu-\gamma}{1-\nu}} c^{\nu-\gamma}$$

$$\therefore\ c = \left\{\frac{(1-\theta)}{\chi}\left[(1-\theta)+\theta\left[\left(1-\frac{\beta}{\Pi}\right)^{-\frac{1-\nu}{\nu}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-\nu}{\nu}}\right]\right]^{\frac{\nu-\gamma}{1-\nu}}\right\}^{\frac{1}{\varphi+\gamma}} \qquad -(17)$$

(15) : ss value of M_t/P_t
(13) : ss value of $N_t (= C_t)$

d| to solve for ss, take Φ & nominal money supply M_t^s as given:
specify values for the model parameters:
$\langle \beta, \gamma, \nu, \theta, \varphi, \chi, \dots \rangle$

using (14) (i.e. $\Phi = \Pi/\beta$) get c from (17)
using $N = c$ & (13) get N
Using (16) get M_t/P_t

Given M_t^s, get P_t

e) Given the knowledge of parameters (like v), we can caliberate θ from (17) where c can be written as a fn of θ & then using (16) (given M_t^s & P_t @ SS) we can solve for θ using different values for v.

f) (15) : $\frac{M_t}{1} = (1-\beta/\pi)^{-1/v} \left(\frac{\theta}{1-\theta}\right)^{1/v} \cdot c$ $\rightarrow \approx N$

from (13)

$$\Rightarrow \left[\frac{(1-\theta)x^{v-\gamma}}{\chi}\right]^{1/\varphi+v} = c$$

$$\therefore M_t = (1-\beta/\pi)^{-1/v} \left(\frac{\theta}{1-\theta}\right)^{1/v} \cdot \left[\frac{(1-\theta)x^{v-\gamma}}{\chi}\right]^{1/\varphi+v}$$

g) $$\frac{w_t}{P_t} = \frac{\chi N_t^{\varphi}}{(1-\theta) C_t^{-v} \cdot X_t^{v-\gamma}} \qquad (\text{eq}^n \ (6))$$

$$\ln(w_t) - \ln(P_t) = \ln(\chi) + \varphi \ln(N_t) - \ln(1-\theta) + v\ln(C_t) + (\gamma - v)\ln(X_t).$$

$$\ln(w) + \frac{1}{w}(w_t - w) - \left[\ln(P) + \frac{1}{P}(P_t - P)\right]$$
$$= \ln(\chi) + \varphi\left[\ln(N) + \frac{1}{N}(N_t - N)\right]$$
$$- \ln(1-\theta) + v\left[\ln(C) + \frac{1}{C} \cdot (C_t - C)\right]$$
$$+ (\gamma - v)\left[\ln(X) - \frac{1}{X}(X_t - X)\right]$$

denote $(z_t - z) = \hat{z}$

$$\ln(\omega) + \hat{\omega}_t - \ln(P) - \hat{P}_t = \ln(\chi) + \varphi[\ln(N) + \hat{N}_t]$$
$$- \ln(1-\Phi) + v(\ln(C) + \hat{C}_t) + (\gamma - v)(\ln(x) + \hat{x}_t)$$

rearranging

$$\hat{\omega}_t - \hat{P}_t = \varphi \hat{N}_t + v\hat{C}_t + (\gamma - v)\hat{x}_t.$$

Similar for EE (eq^n ⑤).

$$(v-\gamma)\hat{x}_t - v\hat{C}_t - \hat{P}_t = (v-\gamma)\hat{x}_{t+1} - v\hat{C}_{t+1} - \hat{P}_{t+1}$$
$$+ (v-\gamma)\hat{x}_t - v\hat{M}_t + v\hat{P}_t - \hat{P}_t$$

$$-v\hat{C}_t = (v-\gamma)\hat{x}_{t+1} - v\hat{C}_{t+1} - \hat{P}_{t+1} - v\hat{M}_t + v\hat{P}_t$$

from eq^n ⑨.

$$\hat{A}_t = \hat{\omega}_t - \hat{P}_t.$$

h) code attached.

It is the best attempt I could do.

j) $\uparrow$ in $M \Rightarrow P \downarrow$ & interest rate $\downarrow$

$\therefore$ value of money $\downarrow$ w.r.t consn & real bonds

v is the inverse elasticity of substitution b/w C_t & (M_t/P_t). when v is small (<1)

$M_t^S \uparrow \Rightarrow MU_c \downarrow \rightarrow N_t^S \downarrow \rightarrow Y_t \downarrow$ & $C \downarrow$

$M_t^S \uparrow \Rightarrow P \uparrow$ to sustain $M_t^d \uparrow$, interest on bonds $\downarrow$

$\Rightarrow$ opp hold when ($v > 1$) large.