# Problem Statement:

To run empirical studies to analyze the length of the longest edge in the MST and the number of graph edges that are not longer than that one.

# Procedure:

To understand the ratio of the number edges shorter than longest edge in MST to that of total number of edges, the following steps were followed -

* A graph generator was used to generate random edge weighted graphs. The weights of edges have uniform distribution.
* To calculate the MST of the generated graph, we use Lazy Prim’s algorithm.
* The number of edges smaller than the largest edge of the MST is calculated.
* For each case of vertices and edges, we obtain the above mentioned ratio on an average of 100 random experiments.

# Analysis:

The results from the different random experiments are tabulated as below -

|  |  |  |  |
| --- | --- | --- | --- |
| **# of Vertices** | **Max # of Edges** | **# of Edges** | **Ratio** |
| 10 | 45 | 10 | 0.807 |
| 20 | 0.605 |
| 30 | 0.478 |
| 40 | 0.327 |
| 45 | 0.314 |
| 100 | 4950 | 50 | 0.980 |
| 100 | 0.973 |
| 200 | 0.907 |
| 300 | 0.766 |
| 400 | 0.636 |
| 450 | 0.580 |
| 1000 | 0.258 |
| 2000 | 0.128 |
| 3000 | 0.086 |
| 4000 | 0.064 |
| 4500 | 0.057 |
| 1000 | 499500 | 1000 | 0.997 |
| 5000 | 0.721 |
| 10000 | 0.377 |
| 50000 | 0.075 |
| 200000 | 0.019 |
| 10000 | 49995000 | 50000 | 0.904 |
| 500000 | 0.098 |

The results of above recorded random experiments were examined. The pattern in the behavior of required ratio can be generalized [for simple graphs with uniform weight distribution]. We observe a steep decline of the number of edges shorter than the largest of MST. As the graph grows bigger and denser, the decline can be seen rapidly.

# Conclusion:

The number of edges smaller than the largest of an MST declines steeply as the grows bigger and denser. This is however, only applicable to uniformly distributed weights in the graph.

# References:

[Robert Sedgewick - Algorithms Website](https://algs4.cs.princeton.edu/code/)