Dynamic Modeling of
3-LEGGED JUMPING ROBOT

HOW DO WE JUMP

- **Jumping** or **leaping** is a form of locomotion or movement in which an organism or non-living mechanical system propels itself through the air along a ballistic trajectory.

- Role of muscles : Act like spring?

- Where do energy for jump comes from?

- Why do compression in spring make a body lift off from ground?

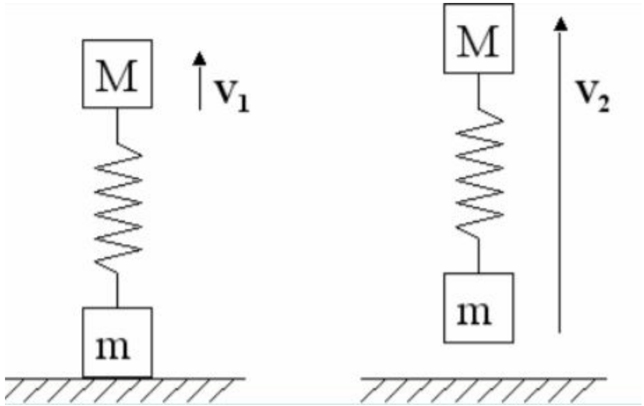
TWO BLOCK PROBLEM

Mechanics:

If the spring is a rigid connection, conservation of momentum equations give:

$$\mathbf{Mv_1 = (M + m)v_2}$$

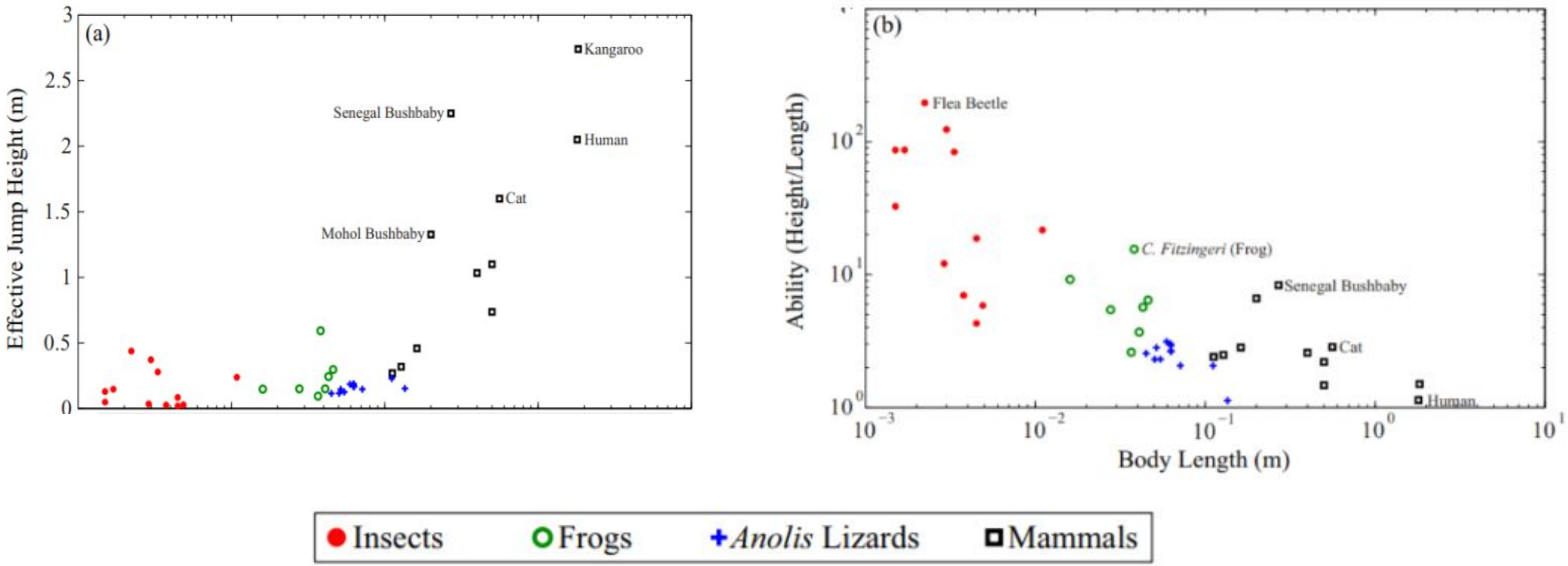
Power amplification in jumps results from energy storage in elastic elements in the appendages responsible for jumping.



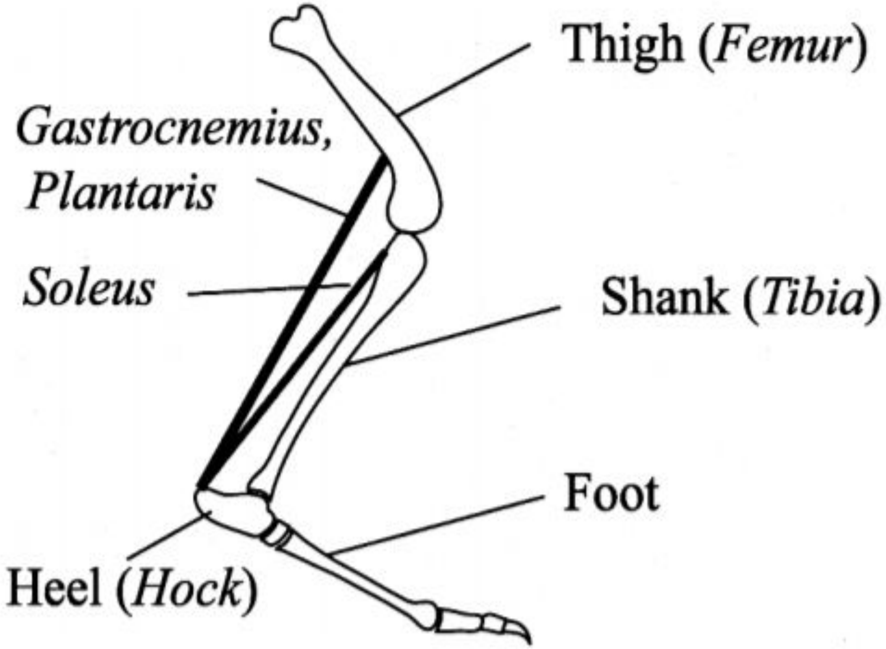
WHY JUMPING IS BENEFICIAL FOR ROBOTS

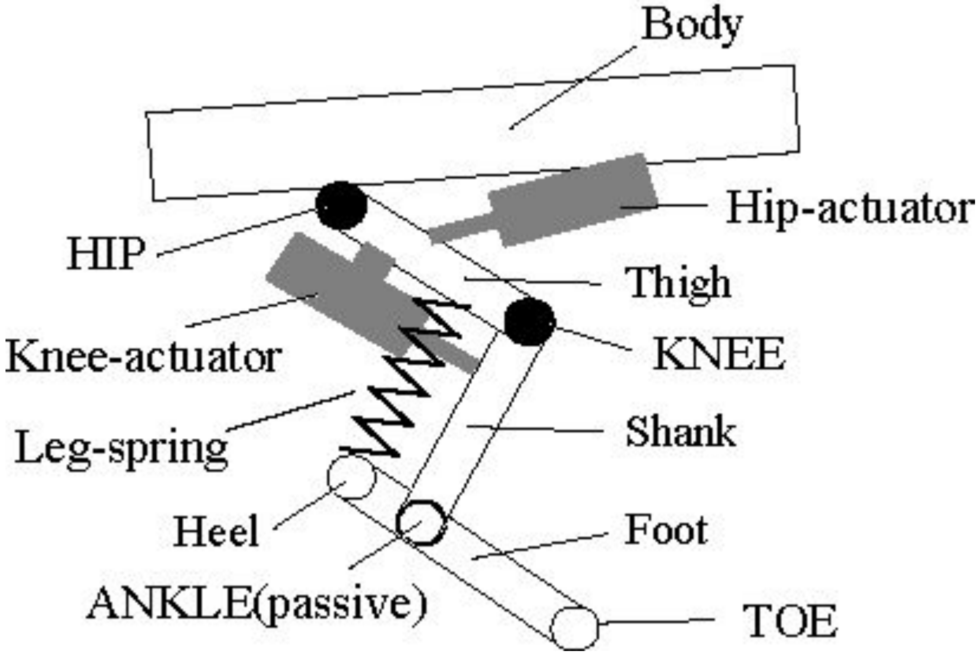
- Robots Skipping Obstacles.
- Efficient Way To Travel Over Rough Surface
- Able To Store Energy Recovered From Environment By Compressing From Elastic- Mechanism And Quickly Release Energy In One Jump.
- Replication Of Hopping

Is JUMPING ability related to body height?



JUMPING MODELS

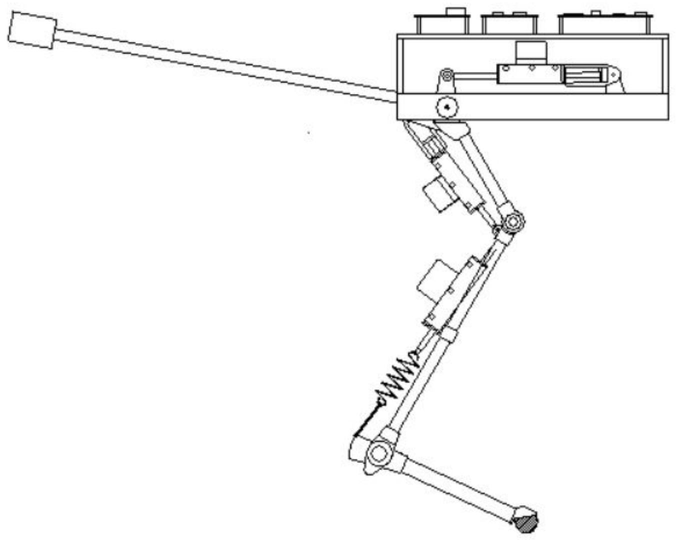




UNIROO

The Uniroo was a 3-leg-link hopping robot based on kangaroo locomotion, with a soleus spring arrangement .

In robotics, biologically inspired designs have resulted in machines that are better able to traverse diverse terrain than wheeled vehicles.



SLIP MODEL

- Hopping has generally been characterized by the spring loaded inverted pendulum (SLIP) model, which is the basis for most hopping robot designs and is a simpler model than those of maximal height jumping.

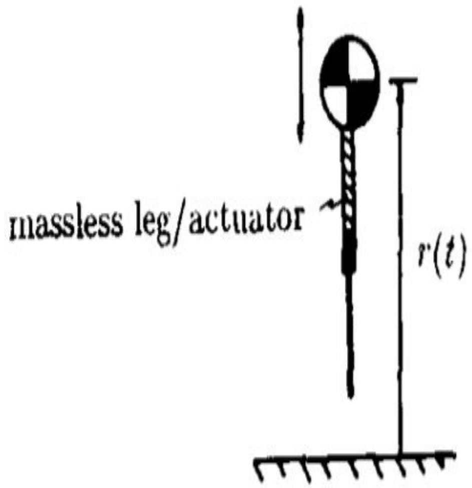
- The basis for our robot is a 1D variant of the SLIP model, which has a mass-pneumatic arrangement with an actuated mass, which we actuate sinusoidally.

1D-SLIP MODEL

- The actuator is a pneumatic cylinder with restoring force, $F \sim \frac{1}{r}$ where r is the length of the cylinder or leg.

- The stiffness of the nonlinear spring may be adjusted by changing the supply pressure to the cylinder.

- The vertical hopping cycle may be decomposed into four phases.



1D-SLIP MODEL

1. Thrust Phase:

The j^{th} hopping cycle begins at time t_j when the leg reaches its maximum compression: **$r'(t_j) = 0$.** A constant supply pressure is applied to the leg cylinder for a fixed time.

This results in a constant thrust force, T, which is the product of the supply pressure and the cross section of the pneumatic cylinder. The robot equation of motion during this phase is:

$$r'' - r + g = 0 \quad \text{for } t_j <= t <= t_j + \delta_t$$

where g is the gravitational constant.

1D-SLIP MODEL

2.Decompression Phase:

At the end of the thrust phase, the valves are closed, defining an effective spring constant, $n_2 = Tr_{et}$, where r_{et} is the body position at the end of thrust phase. The equation of motion of the system during this phase is:

$$\ddot{r} - \frac{Tr_{et}^{j}}{r} + g = 0 \quad r_{et} \leq r \leq r_{\max}$$

where r_{max} is the uncompressed pneumatic cylinder reference length. The robot loses contact with the ground when it reaches the height r_{max}.

1D-SLIP MODEL

3.Flight Phase: We assume that the lift-off and touchdown heights are identical to r_{max}, and that air drag during flight is negligible. The equation of motion in this phase is:

$$r'' + g = 0 \quad r > r_{max}$$

1D-SLIP MODEL

4.Compression Phase: At touchdown , (t^j_{td} , r_{max}), an initial pressure exists in the leg, fixing the spring constant during compression at . The equation of motion during this phase is:

$$\ddot{r} - \frac{\eta}{r} + g = 0$$

At the end of this phase, a new minimum height r_{j+1} is reached at time t_{j+1}, and a new hopping cycle starts.

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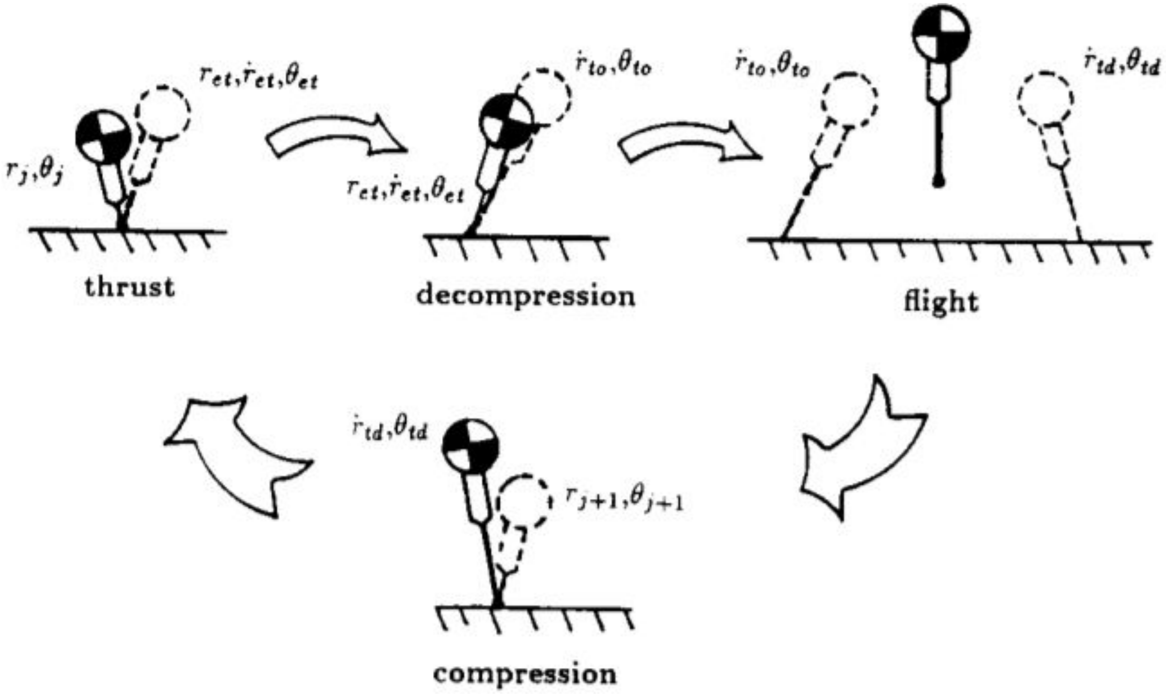
2D-SLIP MODEL

Now comes the same model for 2D plane, where it will jump along with motion, this type of model is very common and useful in our project.



2D-SLIP MODEL

1. Thrust Phase:

At time t_j the leg is at its minimum radial length, $r_j = r(t_j)$ (i.e.,$r'(t_j) = 0$,the control valves are opened and a constant supply pressure is connected to the leg cylinder for a fixed time, δ_t . This applies a constant radial force T

$$m\left(\vec{r} - r\dot{\theta}^2\right) - \tilde{\tau} + mg\cos(\theta) = 0 \quad \text{for} \quad t_j \leq t \leq t_j + \delta_t$$

$$m(\vec{\theta} + 2\dot{r}\dot{\theta}) - mg\sin(\theta) = 0$$

2D-SLIP
MODEL

2. Decompression Phase: At the end of the thrust phase the effective spring constant is r'($r-r_{zero}$), where r_{zero} is the complementary piston length.
The actual piston length is r_{max} - r_{zero}. The spring force acts radially until the cylinder reaches length r_{max}, which denotes the beginning of the flight.
The equations of motion are:

$$m\left(\ddot{r}-r\dot{\theta}^2\right)-\frac{\tilde{\tau}\left(r_{et}-r_{zero}\right)}{r-r_{zero}}+mg\cos(\theta)=0$$

$$m(r\ddot{\theta}+2\dot{r}\dot{\theta})-mg\sin(\theta)=0 \quad r_{et}\leq r\leq r_{\max}$$

The piston length in the 1-DOF model does not matter because a "massless" extension may be added and the dimensions rescaled. The behavior of the 2-DOF model is affected because the change in theta is dictated by an angular momentum effect which becomes more pronounced as the mass approaches the foot pivot point.

2D-SLIP MODEL

3. Flight Phase:

The final conditions of the decompression phase are converted from polar to Cartesian representation. Neglecting air drag during flight, the equations of motion are:

$$x''=0\ ;$$

$$y''+g=0$$

The touchdown conditions at the end of flight are determined by the foot placement algorithm

2D-SLIP MODEL

4. Compression Phase:

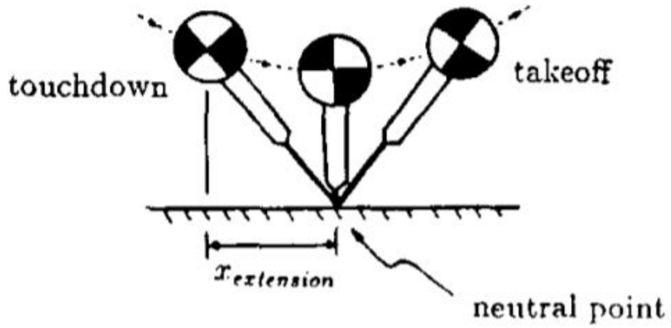
At touchdown, the Cartesian variables are mapped to polar coordinates. The spring constant during this phase, n , is a parameter chosen by the designer. The end of the compression phase is signaled by the condition r' = 0. The equations of motion are:

$$m\left(\vec{r}-r\dot{\theta}^2\right)-\frac{\tilde{\eta}}{r-r_{zero}}+mg\cos(\theta)=0$$

$$m(r\vec{\theta}+2\dot{r}\dot{\theta})-mg\sin(\theta)=0$$

At the end of this phase a new minimum height is reached and the cycle repeats.

FOOT PLACEMENT ALGORITHM



- In contrast to the 1-DOF model, the 2-DOF model requires a foot placement algorithm to actively control and balance its forward motion.
- The FPA determines where the robot foot should be located at touchdown. This determines the initial conditions for the ensuing phases.
- Given a desired forward velocity, $x'_{desired}$, the algorithm places the foot a distance $x'_{extension}$ in front of the body to regulate the actual forward velocity. $y'_{extension}$ is computed as:

$$x_{\text{extension}} \approx \frac{\dot{x}T_{\text{stance}}}{2} + \kappa_{\dot{x}}\left(\dot{x} - \dot{x}_{desired}\right)$$

APPROACH FOR ANY MODEL

- Torque and force equations can be calculated the same way.
- Solving them will provide us trajectory, and hence angles/current of actuators.
- When working with higher DOF, these equations become complicated.
- Euler-Lagrangian or Newton Euler equations hence comes to field.

EULER-LAGRANGIAN APPROACH

Lagrangian energy(L) : **L = K.E. - P.E**

In our system of equations, energy may depend on many control variables, for eg in two DOF link hopping robot it depends on Two Angles(that of actuator) and coordinates of COM.

So this may be written in the form of

$$\boldsymbol{\varkappa = [x_a \ y_a \ \Theta \ \phi]^T.}$$

and Euler-lagrangian equation is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

RESULT

- Results of such equation is required trajectory.

We get our control variables by solving these equations.

- Any sort of model can be solved through this kind of dynamic modeling.

- After getting control parameters we need to link these parameters with robot’s source of action.

HOPPING TO
THREE LEGGED JUMP

- Model including shifting of COM laterally.
- Division of total mass in three parts.
- Lateral distance for three legs.
- Respective trajectory for three legs.
- Controls and learning afterwards make everything perfect.

THANK YOU

DO PARTICIPATE, DO LEARN!!

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