Option Pricing

Financial Engineering

What is an option?

- An option provides the holder with the **right** to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- Since it is a right and **not an obligation**, the holder can choose not to exercise the right and allow the option to expire.
- There are two types of options - **call** options (right to buy) and **put** options (right to sell).

Option Pricing Models

The amount that an investor must pay for an option contract. This price is based upon **factors** such as the **underlying security** as well as the left until the options **expiration date(T)**.

The two popular models are:

1. The **Binomial** Model

2. **Black - Scholes** Model

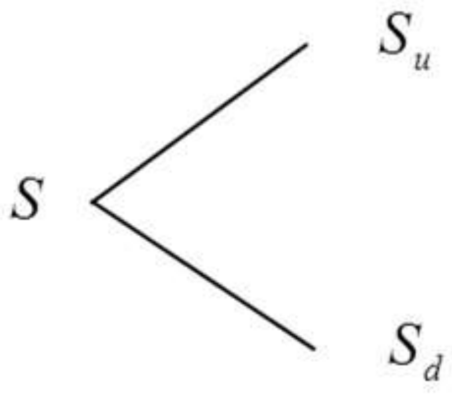
The Binomial Model

The model assumes,

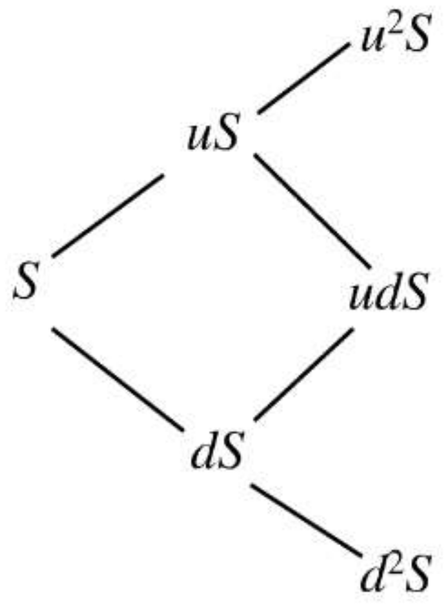
- The price of asset can only go up or go down in fixed amounts in discrete time.
- There is no arbitrage between the option and the replicating portfolio composed of underlying asset and risk-less asset.

The Binomial Model Parameters

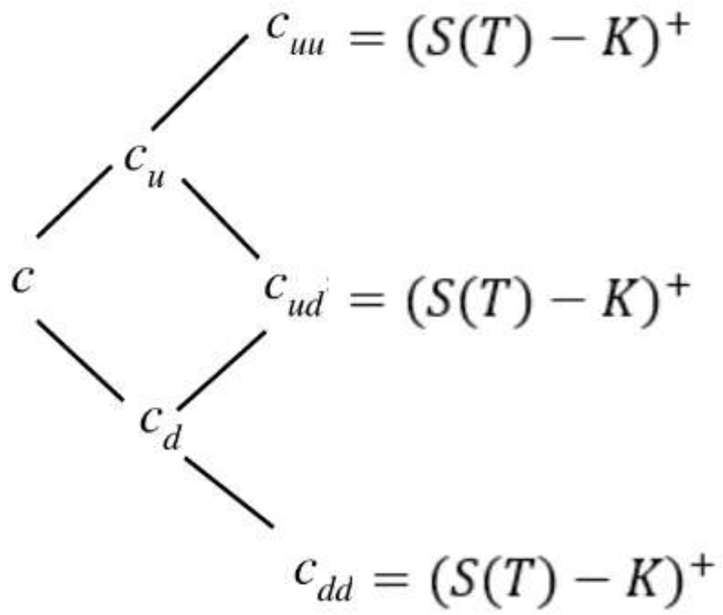
- Current stock price = S
- d <R <u (no risk free arbitrage possible)
- Time period for T
- r is risk free interest rate
- K is the Strike price or exercise price
- σ as volatility



Two period Binomial model



Stock Price Diagram



Payoff Diagram

Stock we chose: Amazon(AMZN)

Historical Data

	Open	High	Low	Close	Adj Close	Volume
Date						
2023-12-22	153.770004	154.350006	152.710007	153.419998	153.419998	29480100
2023-12-26	153.559998	153.979996	153.029999	153.410004	153.410004	25067200
2023-12-27	153.559998	154.779999	153.119995	153.339996	153.339996	31434700
2023-12-28	153.720001	154.080002	152.949997	153.380005	153.380005	27057000
2023-12-29	153.100006	153.889999	151.029999	151.940002	151.940002	39789000

Parameters

- Stock price S(0) = 173.85
- 10 Year Treasury Rate (r) : 4.62%
- Strike Price (K) = 170
- Up and Down factor

$$u = \frac{1}{d} = e^{\sigma\sqrt{\frac{T}{n}}}$$

Annual volatility.

```
historical_prices = stock_data['Adj Close']
returns = historical_prices.pct_change().dropna()
volatility = np.sqrt(252) * returns.std()  # Assuming 252 trading days in a year
print("\nEstimated Annual Volatility:", volatility)
```

- Estimated Annual Volatility (σ): 33.026%

- **Probability**

$$p = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

- **Call Option Price**

$$C(0) = e^{-r\Delta T} E((S(T) - K)^{+})$$

- **Put Option Price**

$$P(0) = C(0) - S(T) + Ke^{-r\Delta T}$$

Black Scholes Model :

The Black-Scholes formula calculates the theoretical prices of European call and put options based on the specified inputs.

The model assumes,

- Constant Risk-Free Rate
- Constant Volatility
- Options can only be exercised at expiration, unlike American options that can be exercised at any time before expiration
- Underlying asset's price follows a log-normal distribution

Black-Scholes Model: Option Pricing Formula

$$C = SN(d_1) - Ke^{-rT} N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

where

- S = Underlying Asset Price
- K = Strike Price
- T = Time to Expiration
- r = Risk-Free Rate
- σ = Volatility of the Underlying Asset

Why Black scholes model is better than binomial model?

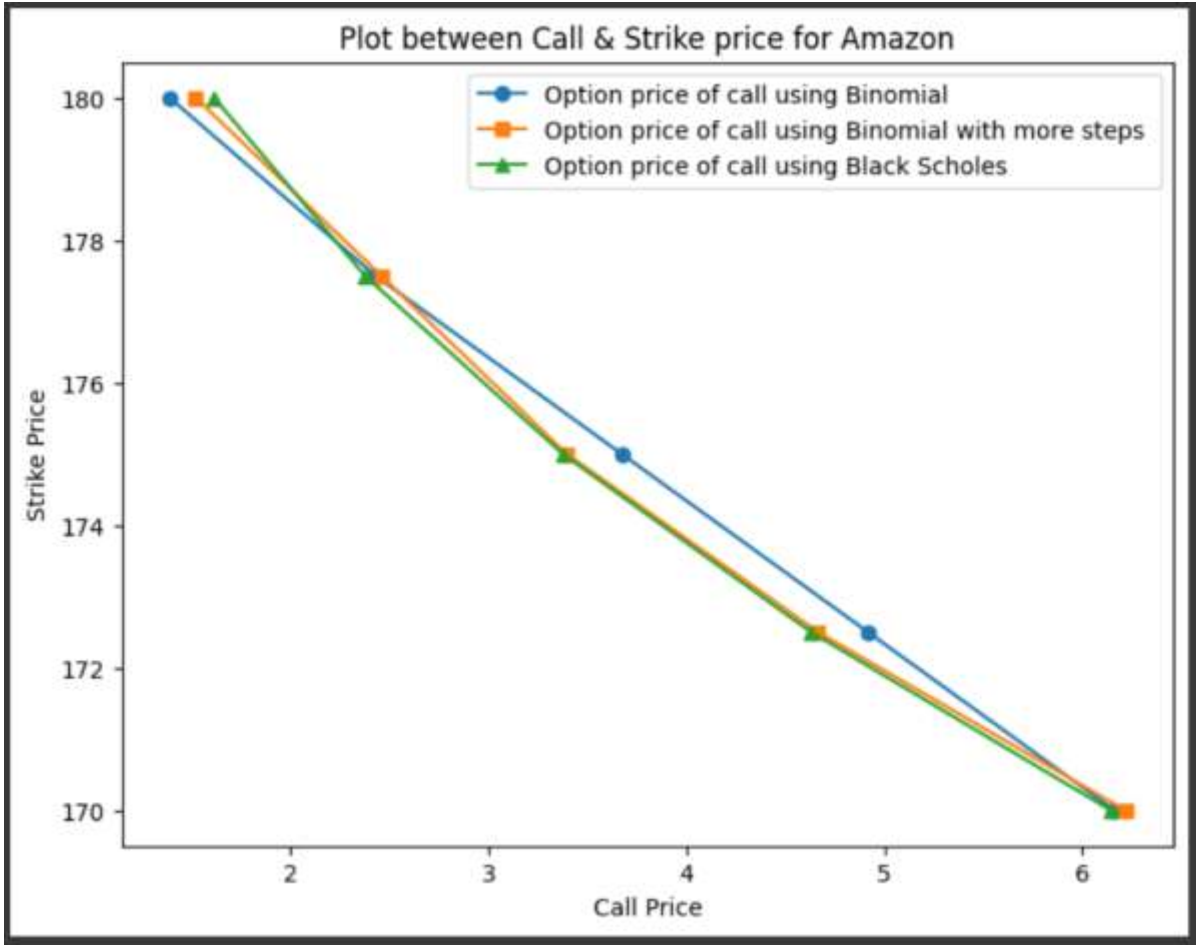
1. **Continuous Model:** The Black-Scholes model uses continuous time, which is more aligned with the real-world trading environment. The binomial model, on the other hand, uses discrete time steps.
2. **Arbitrage-Free:** Both models can be arbitrage-free but achieving this for binomial model can be more complex.
3. **Simplicity:** The Black-Scholes model has a closed-form solution, making it easier and faster to calculate option prices whereas the binomial model requires iterative calculations at each time step.
4. **Flexibility:** The Black-Scholes model can be extended to price a wide range of derivative securities, including options on dividend-paying stocks, currencies, and futures contracts. While the binomial model is versatile, it may require more adjustments and modifications to accommodate certain market features.

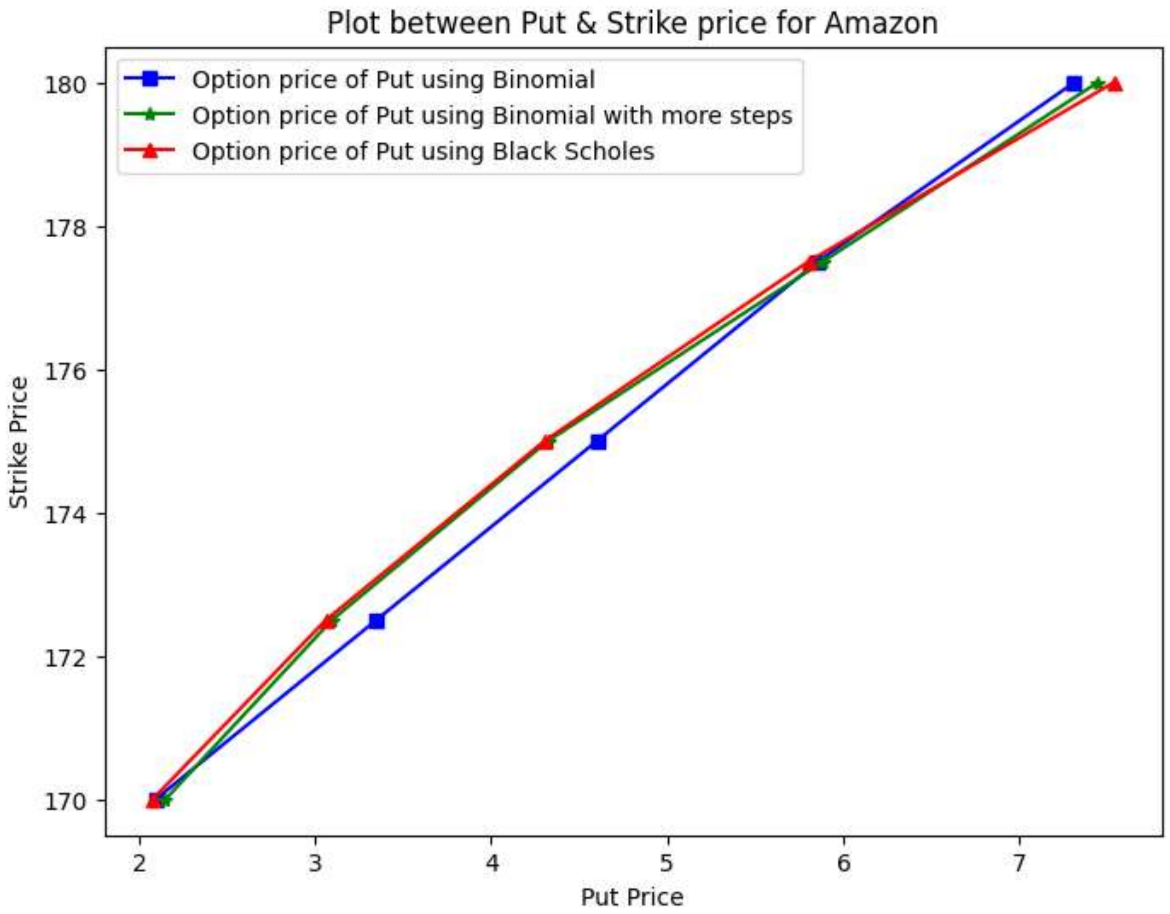
```python
def black_scholes_call(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    call_price = S * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)
    return call_price

stock_price = 173.85 # Current price of the underlying asset
strike_prices = [170, 172.5, 175, 177.5, 180]# Different strike prices
expiration_dates = [0.0277, 0.0555]  # Expiration dates in days
risk_free_rate = r  # Risk-free interest rate
volatility = sigma  # Volatility of the underlying asset

option_prices_CB = []

for expiration_date in expiration_dates:
    print(f"\nCall Option prices for expiration date {expiration_date} days:")
    for strike_price in strike_prices:
        option_price_CB = black_scholes_call(stock_price, strike_price, expiration_date, risk_free_rate, volatility)
        option_prices_CB.append(option_price_CB)  # Store the option price in the list

        print(f"Strike Price: {strike_price}, Option Price: {option_price_CB:.2f}")
```

```
Call Option prices for expiration date 0.0277 days:
Strike Price: 170, Option Price: 6.14
Strike Price: 172.5, Option Price: 4.63
Strike Price: 175, Option Price: 3.38
Strike Price: 177.5, Option Price: 2.38
Strike Price: 180, Option Price: 1.61

Call Option prices for expiration date 0.0555 days:
Strike Price: 170, Option Price: 7.74
Strike Price: 172.5, Option Price: 6.31
Strike Price: 175, Option Price: 5.06
Strike Price: 177.5, Option Price: 3.99
Strike Price: 180, Option Price: 3.10
```

```python
def black_scholes_put(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    put_price = K * np.exp(-r * T) * norm.cdf(-d2) - S * norm.cdf(-d1)
    return put_price

stock_price = 173.85 # Current price of the underlying asset
strike_prices = [170, 172.5, 175, 177.5, 180] # Different strike prices
expiration_dates = [0.0277, 0.0555]  # Expiration dates in days
risk_free_rate = r  # Risk-free interest rate
volatility = sigma  # Volatility of the underlying asset

option_prices_PB = []

for expiration_date in expiration_dates:
    print(f"\nPut Option prices for expiration date {expiration_date} days:")
    for strike_price in strike_prices:
        option_price_PB = black_scholes_put(stock_price, strike_price, expiration_date, risk_free_rate, volatility)
        option_prices_PB.append(option_price_PB)  # Store the option price in the list

        print(f"Strike Price: {strike_price}, Option Price: {option_price_PB:.2f}")
```

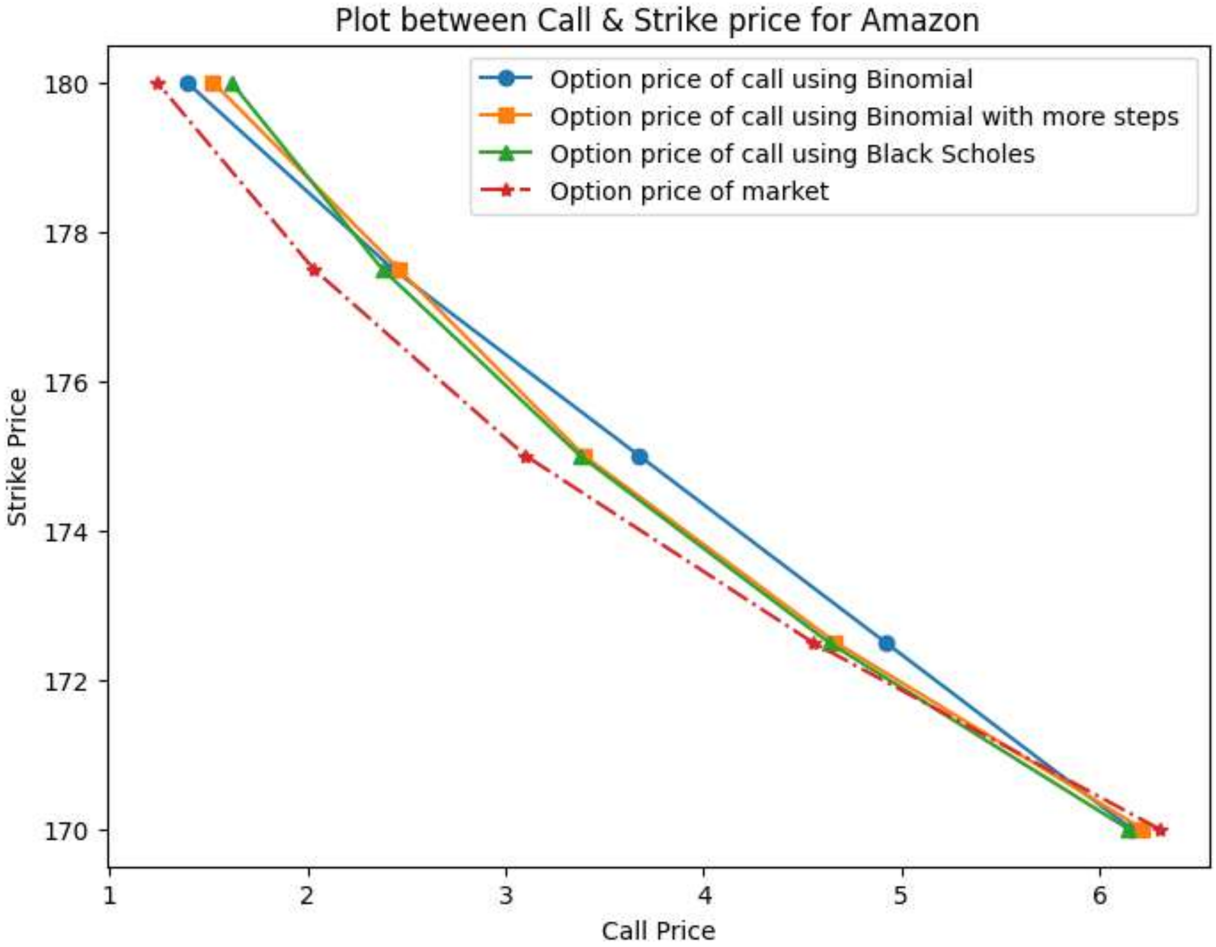
```
Put Option prices for expiration date 0.0277 days:
Strike Price: 170, Option Price: 2.08
Strike Price: 172.5, Option Price: 3.06
Strike Price: 175, Option Price: 4.30
Strike Price: 177.5, Option Price: 5.80
Strike Price: 180, Option Price: 7.53

Put Option prices for expiration date 0.0555 days:
Strike Price: 170, Option Price: 3.46
Strike Price: 172.5, Option Price: 4.52
Strike Price: 175, Option Price: 5.77
Strike Price: 177.5, Option Price: 7.19
Strike Price: 180, Option Price: 8.79
```


Plot between Call & Strike price for Amazon
Option price of call using Binomial
Option price of call using Binomial with more steps
Option price of call using Black Scholes
Strike Price
180
178
176
174
172
170
Call Price
2
3
4
5
6

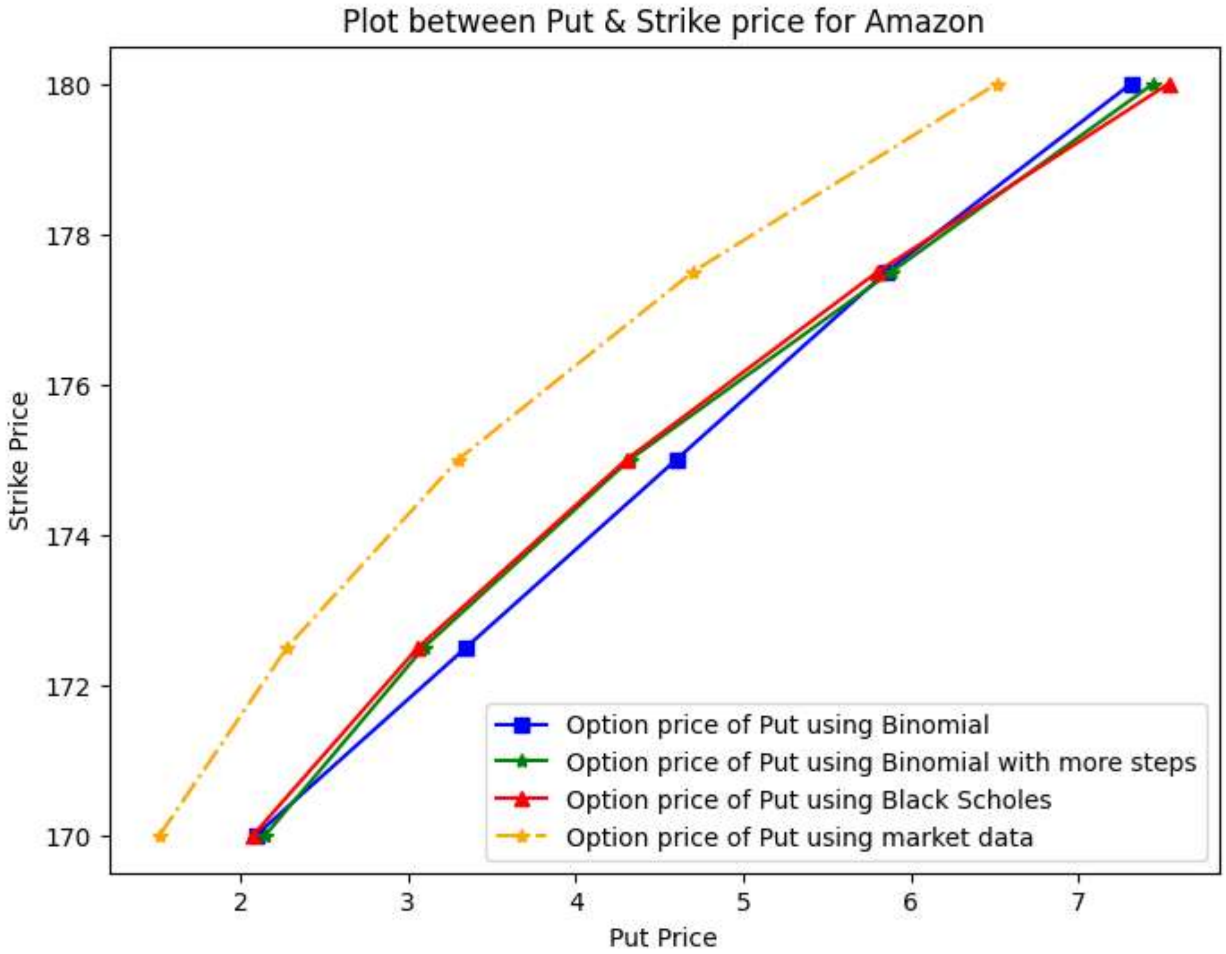


Plot between Put & Strike price for Amazon
Option price of Put using Binomial
Option price of Put using Binomial with more steps
Option price of Put using Black Scholes
Strike Price
180
178
176
174
172
170
Put Price
2
3
4
5
6
7

Comparison with market



Comparison with market



Delta Neutral Portfolio

A delta-neutral portfolio is a strategy used by options traders to hedge against directional risk in their positions.

- Delta measures the sensitivity of the option’s price to changes in the price of the underlying asset.
- A delta neutral portfolio aims to have a total delta of zero.
    - If a trader holds a long position (owns) in call options, which have a positive delta, they may sell (short) shares of the underlying stock to offset the positive delta.
    - Conversely, if a trader holds a short position (sold) in call options, they may buy shares of the underlying stock to offset the negative delta.

Implied volatility

Implied volatility(IV) is derived from option prices, indicating the market's expectation of future volatility for a financial instrument. It reflects traders' perceptions of potential price movements in the underlying asset. Higher IV suggests greater anticipated price swings, influencing option prices. IV helps forecast market volatility, is crucial in options pricing models, and can vary across different options, providing insights into market sentiment and risk.

```
Implied Volatility for Call Option: 32.9849057417159 %
Implied Volatility for Put Option: 33.0472184748532 %
```

Applications of Black Scholes Model :

- Option Pricing: Provides a theoretical framework for pricing European options.
- Hedging: Helps investors and institutions hedge against price fluctuations in the underlying asset.
- Risk Management

Limitations:

- Assumption Sensitivity: The model's results can be sensitive to changes in its assumptions.
- Market Dynamics: May not accurately capture real-world market dynamics, especially during extreme market conditions.
- Other Models: Mention models like the Binomial Model that can address some limitations of the Black-Scholes model.