Lecture 12.3

Mechanism design with money: There is an allocation rule which we implement with the help of suitable payment rules.

Mechanism design without money: No money is allowed.

Ex: Stable matching, stable roommate, House allocation etc.

Stable Matching

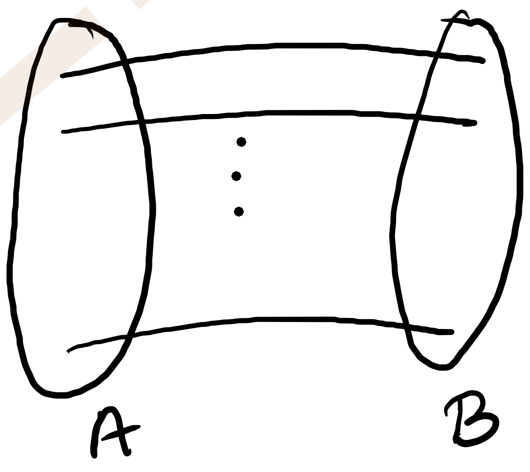
Setup: Set A of n men

set B of n women.

Each man has a preference which is a complete order over B.

Each woman has a preference which is a complete order over A.

Goal: Match each man with an woman so that the matching is "stable".



Blocking Pair: In a matching M of a stable matching instance, a pair $(a, b) \in A \times B$ is called a blocking pair if any of the following conditions hold.

(i) Both a and b are unmatched in M.

(ii) a is unmatched and $a >_b M(b)$, that is woman b prefers man a over her current partner in M.

(iii) b is unmatched and $b >_a M(a)$, that is man a prefers woman b over her current partner in M.

(iv) Both a and b are matched, $b >_a M(a)$ and $a >_b M(b)$, that is both a and b prefer each other over their current partners in M.

Stable matching: A matching having <u>no</u> blocking pair.

Gale-Shapley Theorem: Every stable matching instance has a stable matching. Moreover, one such stable matchings can be computed in polynomial time.

Proof: We present the Gale-Shapley algorithm.

Men-proposing deferred acceptance algorithm.

$m_1 : w_1 > w_2 > w_3$

$m_2 : w_2 > w_3 > w_1$

$m_3 : \underline{w_2} > \underline{w_1} > w_3$

$w_1 : m_2 > \underline{\underline{m_1}} > \underline{\underline{m_3}}$

$w_2 : m_1 > \underline{\underline{m_2}} > \underline{\underline{m_3}}$

$w_3 : m_3 > m_2 > m_1$

At the beginning all men and women are unmatched.

<u>Iteration 1:</u> m_1 proposes w_1.

current solution : $\{(m_1, w_1)\}$

<u>Iteration 2:</u> m_2 proposes w_2.

current solution : $\{(m_1, w_1), (m_2, w_2)\}$

Iteration 3: m_3 proposes w_2

w_2 rejects m_3 proposal since $m_2 >_{w_2} m_3$

current solution: $\{(m_1, w_1), (m_2, w_2)\}$

Iteration 4: m_3 proposes w_1

w_1 rejects m_3 since $m_1 >_{w_1} m_3$

current solution: $\{(m_1, w_1), (m_2, w_2)\}$

Iteration 5: m_3 proposes w_3

current solution: $\{(m_1, w_1), (m_2, w_2), (m_3, w_3)\}$