Probability & Statistics

Ans 1) Probability is the measure of uncertainty of events in a random experiment.
The collection of all possible outcomes of random experiment forms a sample space.
The outcomes of random experiment are termed as events.

For eg:- tossing a coin :-
Event is getting head {H} getting tail {T}
Sample space contains both events. {H, T}

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The probability of an event A can be defined as the Number of favorable outcomes of an event $n(A)$ divided by total number of possible outcomes of an event, $n(S)$.

Properties we know:

(i) $0 \leq P(A) \leq 1$ (ii) $\sum_i A_i = 1$

Ans 2 => (a) $f(x) = \frac{x-2}{2}$ for $x = 1, 2, 3, 4$

$f(x)_{x=1} = \frac{1-2}{2} = \frac{-1}{2}$ ∴ can't serve

(b) $g(x) = \frac{x^2}{25}$ for $x = 0, 1, 2, 3, 4$

$g(0) = 0$ $g(1) = \frac{1}{25}$ $g(2) = \frac{4}{25}$, $g(3) = \frac{9}{25}$

$g(4) = \frac{16}{25}$

$\sum g(x) = 1 \Rightarrow 0 + \frac{1}{25} + \frac{4}{25} + \frac{9}{25} + \frac{16}{25} = \frac{6}{5} > 1$

∴ can't serve

(c) $h(x) = \frac{x}{10}$ for $x = 0, 1, 2, 3, 4$

$h(0) = 0$, $h(1) = \frac{1}{10}$, $h(2) = \frac{2}{10}$, $h(3) = \frac{3}{10}$, $h(4) = \frac{4}{10}$

$\sum h(x) = 1 \Rightarrow \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1$

∴ It can serve

(d) $i(x) = \frac{x-3}{5}$ for $x = 0, 1, 2, 3, 4$

$i(0) = -3/5 < 0$ ∴ Can't serve

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Ans 3 ⇒ Two events are said to be mutually exclusive if they cannot occur at the same time or simultaneously.
These are disjoint ie; probability of occurring them at same time is 0.
Eg:- when tossing coin getting head and tail are mutually exclusive events.
Mutually Exhaustive events are a set of events in a sample space such that one of them compulsory occurs while performing the experiment,
i.e; these events together form sample space.
Eg:- getting head & getting tail are mutually exhaustive events.

Ans 4 ⇒ If we have two events from same sample space, like event E and F are two events associated with sample space of a random experiment, the conditional probability of event E given that F has occurred, i.e. $P(E/F)$ is given by

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \quad \text{provided } P(F) \neq 0$$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

E = getting even no greater than 3 $= \{4, 6, 8, 10\}$

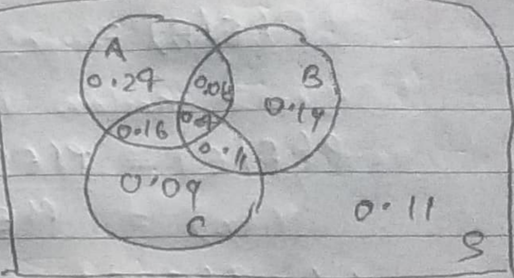
F = number drawn is greater than 3 $= \{4, 5, 6, 7, 8, 9, 10\}$

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$P(F) = \frac{7}{10} \qquad P(E \cap F) = \frac{4}{10}$$

$$P(E/F) = \frac{4/10}{7/10} = \frac{4}{10} \times \frac{10}{7} = \frac{4}{7}$$

Ans 5 ⇒



- $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.1}{0.4} = \dfrac{1}{4}$

- $P(A \cap B | C) = \dfrac{P(A \cap B \cap C)}{P(C)}$

$= \dfrac{0.04}{0.4} = 0.1 = \dfrac{1}{10}$

- $P(A | B \cup C) = \dfrac{P(A \cap (B \cup C))}{P(B \cup C)} = \dfrac{P\{(A \cap B) \cup (A \cap C)\}}{P(B \cup C)}$

$= \dfrac{P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))}{P(B \cup C)}$

$= \dfrac{0.1 + 0.2 - 0.04}{0.4 + 0.4 - 0.15} = \dfrac{0.26}{0.65} = \dfrac{2}{5} = 0.4$

- $P(A | B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = \dfrac{0.04}{0.15} = \dfrac{4}{15}$

- $P(B | A^c) = \dfrac{P(B \cap A^c)}{P(A^c)} = \dfrac{0.3}{1 - 0.5} = \dfrac{0.3}{0.5} = \dfrac{3}{5} = 0.6$

Ans 6 ⇒ If E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of other then they are called independent events.
also, they are said to be independent if

$$P(E \cap F) = P(E) * P(F)$$

E = number appearing multiple of 3 = $\{3, 6\}$
F = number appearing is even = $\{2, 4, 6\}$

$P(E) = \dfrac{2}{6} = \dfrac{1}{3}$ $\qquad P(F) = \dfrac{3}{6} = \dfrac{1}{2}$

$P(E \cap F) = \dfrac{1}{6}$

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$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{6} \times \frac{6}{3} = \frac{1}{3} = P(E)$

$P(F/E) = \frac{P(E \cap F)}{P(E)} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2} = P(F)$

$\because P(E \cap F) = P(E) \times P(F) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$\therefore$ E & F are independent events.

Ans 7 ⇒ $P(A) = 0.60$ $P(B) = 0.40$ $P(A \cap B) = 0.24$

- $P(A|B) = P(A)$ To verify.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.40} = \frac{3}{5} = 0.60$

$P(A) = 0.60 = P(A/B)$ $\therefore$ verified

- To verify $P(A|B^c) = P(A)$

$P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$= \frac{0.60 - 0.24}{0.60} = \frac{0.36}{0.60} = \frac{3}{5} = 0.60$

$= P(A)$ $\therefore$ verified

- $P(B/A) = P(B)$ To verify

$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.24}{0.60} = \frac{2}{5} = 0.40 = P(B)$

$\therefore$ verified

- $P(B|A^c) = P(B)$ To verify

$P(B/A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{0.40 - 0.24}{1 - 0.60} = \frac{0.16}{0.40}$

$= \frac{2}{5} = 0.40 = P(B)$ $\therefore$ verified

ns 8 ⇒ If $E_1, E_2, \ldots E_n$ are n non empty events which

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constitute a partition of sample space S, and A is any event of nonzero probability, then

$$P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A|E_j)}$$

Now, Three boxes - I, II, III each contains 2 coins.

Box I — 2 Gold coins Box II - 2 Silver coins

Box III - 1 Gold & 1 Silver coins

E_i = other coin is gold, i.e. it is from Box I.

A = getting a coin gold.

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$$

$$= \frac{1}{3}\times 1 + \frac{1}{3}\times 0 + \frac{1}{3}\times\frac{1}{2} = \frac{1}{3}\times\frac{3}{2} = \frac{1}{2}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{\sum P(E_i)\,P(A|E_i)} = \frac{P(E_1)\,P(A|E_1)}{P(A)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{2}} = \frac{1}{3}\times 2 = \frac{2}{3}$$

Ans 9 ⇒ A random variable is a real valued function whose domain is the sample space of a random experiment.

Domain of random variable is sample space of random experiment. and

Codomain is set of real numbers.

Eg:- Consider tossing a coin sample space = {Head, Tail}

$$X(s) = \begin{cases} 1 & \text{if } s = H \\ -1 & \text{if } s = T \end{cases}$$

X(S) is a random variable whose domain is {Head, Tail} and range is {-1, 1}.

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There two types of random variable:-
• Discrete random variable • continuous random variable

Now;
$X(x)$ = Random variable for numbers of kings.

$x = 0, 1, 2$

$P\{X(x=0)\} = \frac{48}{52} \times \frac{48}{52} = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

$P\{X(x=1)\} = \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = 2 \times \frac{1}{13} \times \frac{12}{13}$

$= \frac{24}{169}$

$P\{X(x=2)\} = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Ans 10 ⇒ R = getting red ball from urn B = getting black ball from urn
$n(R) = 5$, $n(B) = 2$
X = no. of black balls = 0, 1, 2 (possible values of X)
yes X is a random variable

$P(X=0) = \frac{5}{7} \times \frac{4}{6} = \frac{10}{21}$

$P(X=1) = \frac{2}{7} \times \frac{5}{6} + \frac{5}{7} \times \frac{2}{6} = \frac{5}{21} + \frac{5}{21} = \frac{10}{21}$

$P(X=2) = \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

Ans 11 ⇒ Let X be a random variable whose possible values $x_1, x_2, x_3, \dots x_n$ occur with probabilities $p_1, p_2, \dots p_n$ respectively. The mean of X_i denoted by μ is the number

$$\mu = \sum_{i=1}^{n} x_i p_i$$

i.e, mean is weighted average of possible values of X,

μ is also called as expectation of x denoted by $E(x)$. It is a measure of central tendency that it roughly allocates average value of random variable, i.e; around which value x is supposed to circulate.

X = sum of number appear on two dice.

$X = 2, 3, 4, \ldots 12$

$P(X=2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ $\quad X=2 = \{(1,1)\}$

$P(X=3) = P\{(1,2),(2,1)\} = \frac{2}{36} = \frac{1}{18}$

$P(X=4) = P\{(1,3),(2,2),(3,1)\} = \frac{3}{36} = \frac{1}{12}$

$P(X=5) = P\{(1,4),(2,3),(3,2),(4,1)\} = \frac{4}{36} = \frac{1}{9}$

$P(X=6) = P\{(1,5),(2,4),(3,3),(4,2),(5,1)\} = \frac{5}{36}$

$P(X=7) = P\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} = \frac{6}{36} = \frac{1}{6}$

$P(X=8) = P\{(2,6),(3,5),(4,4),(5,3),(6,2)\} = \frac{5}{36}$

$P(X=9) = P\{(3,6),(4,5),(5,4),(6,3)\} = \frac{4}{36} = \frac{1}{9}$

$P(X=10) = P\{(4,6),(5,5),(6,4)\} = \frac{3}{36} = \frac{1}{12}$

$P(X=11) = P\{(5,6),(6,5)\} = \frac{2}{36} = \frac{1}{18}$

$P(X=12) = P\{(6,6)\} = \frac{1}{36}$

$$\mu = E(x) = \sum_{i=2}^{12} x_i p_i$$

$$= \frac{2}{36} + \frac{3\times2}{36} + \frac{4\times3}{36} + \frac{5\times4}{36} + \frac{6\times5}{36} + \frac{7\times6}{36}$$

$$+ \frac{8\times5}{36} + \frac{9\times4}{36} + \frac{10\times3}{36} + \frac{11\times2}{36} + \frac{12\times1}{36}$$

$$= \frac{1}{36}\{2+6+12+20\ 30+42+40+36+30+22+12\}$$

$$= \frac{1}{36}\times 252 = \underline{7}$$

Ans12 ⇒ The mean of a random variable does not give as information about the variability in the values of random variable. In fact, if the variance is small, then the values of random variable are close to the mean. The variability or spread in the values of a random variable may be measured by variance.

Ans 12 ⇒ Let x be a random variable whose possible values $x_1, x_2, \ldots x_n$ occur with probabilities $p(x_1), p(x_2), \ldots p(x_n)$ respectively. Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by $Var(X)$ or σ_x^2 is defined as

$$\sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)$$

The non negative number $\sigma_x = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$

is the Standard deviation of X.

also, $Var(X) = E(X^2) - [E(X)]^2$

Significance is to measure the variability or spread in values of random variable.

X = sum of no. appear on two dice

$X = 2, 3, --- 12$

$P(X=x_i) = \frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36}$

$\mu = E(X) = 7$

$E(X^2) = \sum_{i=2}^{12} x_i^2 \, p(x_i)$

$= \frac{1}{36}\{4 + 9\times2 + 16\times3 + 25\times4 + 36\times5 + 49\times6 + 64\times5 + 81\times4 + 100\times3 + 121\times2 + 144\times1\}$

$= \frac{1}{36}\{4 + 18 + 48 + 100 + 180 + 294 + \cancel{420}\ 320 + 324 + 300 + 242 + 144\}$

$= \frac{1}{36}\{1974\} = 54.8\bar{3}$

$\sigma_x^2 = Var(X) = 54.8\bar{3} - (7)^2 = 54.8\bar{3} - 49$

$= 5.8\bar{3}$

Standard deviation $= \sqrt{Var(X)} = \sqrt{5.8\bar{3}}$ $(= 1.2341)$

Ans 13 ⇒ $f(x) = x, \quad 0 \le x \le 2$

$\left[f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty \right]$

$E(X) = \int_0^2 x\, f(x)\, dx$

$= \int_0^2 x \cdot x\, dx = \int_0^2 x^2 dx = \left[\frac{x^3}{3}\right]_0^2$

$= 8/3$

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X = continuous random variable

$$Var(X) = \sigma_X^2 = E|(X-\mu)^2| = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$\therefore \sigma^2 x = \int_0^2 (x - 8/3)^2 x\,dx$$

/*

wrong

$$= \int_0^2 \left\{x^2 - \frac{8x}{3}\right\} dx = \left[\frac{x^3}{3}\right]_0^2 - \frac{8}{3\times 2}\left[x^2\right]_0^2$$

$$\frac{8}{3} - \frac{8}{3}\times 2 = -\frac{8}{3}$$

*/

$$\sigma^2_x = \int_0^2 \left(x^2 + \frac{64}{9} - \frac{16x}{3}\right) x\,dx$$

$$= \int_0^2 \left\{x^3 + \frac{64x}{9} - \frac{16x^2}{3}\right\} dx$$

$$= \frac{[x^4]_0^2}{4} + \frac{64[x^2]_0^2}{9\times 2} - \frac{16}{3\times 3}[x^3]_0^2$$

$$= 4 + \frac{128}{9} - \frac{128}{9}$$

$$= 4$$

Ans 14 ⇒ X = number appear on fair dice rolled = {1, 2, 3, 4, 5, 6

$P(X=1) = \frac{1}{6} = P(X=2) = P(X=3) = P(X=4) = P(X=5) = P(X$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$\sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot p(x_i)$$

$$E(X) = \sum_{i=1}^{n} x_i p_i = \frac{1}{6}\{1+2+3+4+5+6\} = 2$$

$\frac{91}{6}$ $\frac{49}{4}$ $\frac{182-147}{12}$ 35

$$\mu = \frac{7}{2}$$

$$\sigma_x^2 = \left(1-\frac{7}{2}\right)^2 \cdot \frac{1}{6} + \left(2-\frac{7}{2}\right)^2 \cdot \frac{1}{6} + \cdots$$

$$\left(6-\frac{7}{2}\right)^2 \cdot \frac{1}{6}$$

$$\sigma_x^2 = \frac{1}{6}\left\{\frac{25}{4} + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{25}{4}\right\}$$

$$= \frac{1}{6 \times 4} \times 70 = \frac{35}{12}$$

Unit-II Probability & Statistics

Ans 1 ⇒ Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

- There should be finite