Lecture 6.2

Correlated Equilibrium

Examle: (Traffic light) Suppose there are two cars at a road junction. If both cars go, then there will be an accident.

	stop	go
stop	0, 0	0, 1
go	1, 0	−100, −100

(Stop, go) and (go, stop) are two PSNEs of this game.

$\sigma : \left((\text{stop}, \text{go}) : \frac{1}{2} \; , (\text{go}, \text{stop}) : \frac{1}{2} \right) \leftarrow$

Let $\sigma^* \in \Delta\left(\bigtimes_{i=1}^{n} S_i \right)$ be a probability distribution over strategy profiles. A trusted third party samples a strategy profile from σ^*, and it conveys each player its strategy only. Then, σ^* will be called a correlated equilibrium if no player has any incentive to deviate from its "advised" strategy, assuming every other player follow the expert's advice.

$\sigma = \left((\text{stop}, \text{go}) : \frac{1}{2}, (\text{go}, \text{stop}) : \frac{1}{2} \right)$ a correlated equilibrium of the traffic light game.

Definition (CE): Given a normal form game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, a probability distribution $\sigma \in \Delta\left(\bigtimes_{i=1}^{N} S_i \right)$ is called a CE if

$\forall i \in N,\ \forall s_i, s_i' \in S_i,$

$$\mathop{\mathbb{E}}_{(s_i, s_{-i}) \sim \sigma}\left[u_i(s_i, s_{-i}) \mid s_i \right] \geq \mathop{\mathbb{E}}_{(s_i, s_{-i}) \sim \sigma}\left[u_i(s_i', s_{-i}) \mid s_i \right]$$

Equivalently, for every (switching) function $\delta : S_i \to S_i$

$$\mathop{\mathbb{E}}_{(s_i, s_{-i}) \sim \sigma}\left[u_i(s_i, s_{-i}) \right] \geq \mathop{\mathbb{E}}_{(s_i, s_{-i}) \sim \sigma}\left[u_i(\delta(s_i), s_{-i}) \right]$$

Theorem: Finding a CE is polynomial time solvable.

Proof: Write a linear program for finding a CE.

Variables: $x(s)$, $s \in \bigtimes_{i=1}^{n} S_i$

$x(s) \geq 0 \quad \forall s \in \bigtimes_{i=1}^{n} S_i$, $\sum_{s \in \bigtimes_{i=1}^{n} S_i} x(s) = 1$.

$\forall s_i, s_i' \in S_i$

$$\underbrace{\sum_{s_{-i} \in S_{-i}} x(s_i, s_{-i})\, u_i(s_i, s_{-i})}_{\mathbb{E}_{(s_i, s_{-i}) \sim \sigma}\left[u_i(s_i, s_{-i}) \mid s_i\right]} \geq \sum_{s_{-i} \in S_{-i}} x(s_i', s_{-i})\, u_i(s_i, s_{-i})$$

Any feasible solution to the above linear program is a CE. $\blacksquare$

Q. Does a CE always exist in a finite strategic form game?

Ans. YES! Because, every mixed strategy Nash equilibrium can be equivalently viewed as a CE. $\blacksquare$