

# Machine Learning in Finance

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From Theory to Practice

 Springer

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*Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.*

—Arthur Conan Doyle

# Introduction

Machine learning in finance sits at the intersection of a number of emergent and established disciplines including pattern recognition, financial econometrics, statistical computing, probabilistic programming, and dynamic programming. With the trend towards increasing computational resources and larger datasets, machine learning has grown into a central computational engineering field, with an emphasis placed on plug-and-play algorithms made available through open-source machine learning toolkits. Algorithm focused areas of finance, such as algorithmic trading have been the primary adopters of this technology. But outside of engineering-based research groups and business activities, much of the field remains a mystery.

A key barrier to understanding machine learning for non-engineering students and practitioners is the absence of the well-established theories and concepts that financial time series analysis equips us with. These serve as the basis for the development of financial modeling intuition and scientific reasoning. Moreover, machine learning is heavily entrenched in engineering ontology, which makes developments in the field somewhat intellectually inaccessible for students, academics, and finance practitioners from the quantitative disciplines such as mathematics, statistics, physics, and economics. Consequently, there is a great deal of misconception and limited understanding of the capacity of this field. While machine learning techniques are often effective, they remain poorly understood and are often mathematically indefensible. How do we place key concepts in the field of machine learning in the context of more foundational theory in time series analysis, econometrics, and mathematical statistics? Under which simplifying conditions are advanced machine learning techniques such as deep neural networks mathematically equivalent to well-known statistical models such as linear regression? How should we reason about the perceived benefits of using advanced machine learning methods over more traditional econometrics methods, for different financial applications? What theory supports the application of machine learning to problems in financial modeling? How does reinforcement learning provide a model-free approach to the Black–Scholes–Merton model for derivative pricing? How does Q-learning generalize discrete-time stochastic control problems in finance?

This book is written for advanced graduate students and academics in financial econometrics, management science, and applied statistics, in addition to quants and data scientists in the field of quantitative finance. We present machine learning as a non-linear extension of various topics in quantitative economics such as financial econometrics and dynamic programming, with an emphasis on novel algorithmic representations of data, regularization, and techniques for controlling the bias-variance tradeoff leading to improved out-of-sample forecasting. The book is presented in three parts, each part covering theory and applications. The first part presents supervised learning for cross-sectional data from both a Bayesian and frequentist perspective. The more advanced material places a firm emphasis on neural networks, including deep learning, as well as Gaussian processes, with examples in investment management and derivatives. The second part covers supervised learning for time series data, arguably the most common data type used in finance with examples in trading, stochastic volatility, and fixed income modeling. Finally, the third part covers reinforcement learning and its applications in trading, investment, and wealth management. We provide Python code examples to support the readers' understanding of the methodologies and applications. As a bridge to research in this emergent field, we present the frontiers of machine learning in finance from a researcher's perspective, highlighting how many well-known concepts in statistical physics are likely to emerge as research topics for machine learning in finance.

## Prerequisites

This book is targeted at graduate students in data science, mathematical finance, financial engineering, and operations research seeking a career in quantitative finance, data science, analytics, and fintech. Students are expected to have completed upper section undergraduate courses in linear algebra, multivariate calculus, advanced probability theory and stochastic processes, statistics for time series (econometrics), and gained some basic introduction to numerical optimization and computational mathematics. Students shall find the later chapters of this book, on reinforcement learning, more accessible with some background in investment science. Students should also have prior experience with Python programming and, ideally, taken a course in computational finance and introductory machine learning. The material in this book is more mathematical and less engineering focused than most courses on machine learning, and for this reason we recommend reviewing the recent book, *Linear Algebra and Learning from Data* by Gilbert Strang as background reading.

## Advantages of the Book

Readers will find this book useful as a bridge from well-established foundational topics in financial econometrics to applications of machine learning in finance. Statistical machine learning is presented as a non-parametric extension of financial econometrics and quantitative finance, with an emphasis on novel algorithmic representations of data, regularization, and model averaging to improve out-of-sample forecasting. The key distinguishing feature from classical financial econometrics and dynamic programming is the absence of an assumption on the data generation process. This has important implications for modeling and performance assessment which are emphasized with examples throughout the book. Some of the main contributions of the book are as follows:

- The textbook market is saturated with excellent books on machine learning. However, few present the topic from the prospective of financial econometrics and cast fundamental concepts in machine learning into canonical modeling and decision frameworks already well established in finance such as financial time series analysis, investment science, and financial risk management. Only through the integration of these disciplines can we develop an intuition into how machine learning theory informs the practice of financial modeling.
- Machine learning is entrenched in engineering ontology, which makes developments in the field somewhat intellectually inaccessible for students, academics, and finance practitioners from quantitative disciplines such as mathematics, statistics, physics, and economics. Moreover, financial econometrics has not kept pace with this transformative field, and there is a need to reconcile various modeling concepts between these disciplines. This textbook is built around powerful mathematical ideas that shall serve as the basis for a graduate course for students with prior training in probability and advanced statistics, linear algebra, times series analysis, and Python programming.
- This book provides financial market motivated and compact theoretical treatment of financial modeling with machine learning for the benefit of regulators, wealth managers, federal research agencies, and professionals in other heavily regulated business functions in finance who seek a more theoretical exposition to allay concerns about the “black-box” nature of machine learning.
- Reinforcement learning is presented as a model-free framework for stochastic control problems in finance, covering portfolio optimization, derivative pricing, and wealth management applications without assuming a data generation process. We also provide a model-free approach to problems in market microstructure, such as optimal execution, with Q-learning. Furthermore, our book is the first to present on methods of inverse reinforcement learning.
- Multiple-choice questions, numerical examples, and more than 80 end-of-chapter exercises are used throughout the book to reinforce key technical concepts.

- This book provides Python codes demonstrating the application of machine learning to algorithmic trading and financial modeling in risk management and equity research. These codes make use of powerful open-source software toolkits such as Google’s TensorFlow and Pandas, a data processing environment for Python.

## **Overview of the Book**

### ***Chapter 1***

Chapter 1 provides the industry context for machine learning in finance, discussing the critical events that have shaped the finance industry’s need for machine learning and the unique barriers to adoption. The finance industry has adopted machine learning to varying degrees of sophistication. How it has been adopted is heavily fragmented by the academic disciplines underpinning the applications. We view some key mathematical examples that demonstrate the nature of machine learning and how it is used in practice, with the focus on building intuition for more technical expositions in later chapters. In particular, we begin to address many finance practitioner’s concerns that neural networks are a “black-box” by showing how they are related to existing well-established techniques such as linear regression, logistic regression, and autoregressive time series models. Such arguments are developed further in later chapters.

### ***Chapter 2***

Chapter 2 introduces probabilistic modeling and reviews foundational concepts in Bayesian econometrics such as Bayesian inference, model selection, online learning, and Bayesian model averaging. We develop more versatile representations of complex data with probabilistic graphical models such as mixture models.

### ***Chapter 3***

Chapter 3 introduces Bayesian regression and shows how it extends many of the concepts in the previous chapter. We develop kernel-based machine learning methods—specifically Gaussian process regression, an important class of Bayesian machine learning methods—and demonstrate their application to “surrogate” models of derivative prices. This chapter also provides a natural point from which to



develop intuition for the role and functional form of regularization in a frequentist setting—the subject of subsequent chapters.

## ***Chapter 4***

Chapter 4 provides a more in-depth description of supervised learning, deep learning, and neural networks—presenting the foundational mathematical and statistical learning concepts and explaining how they relate to real-world examples in trading, risk management, and investment management. These applications present challenges for forecasting and model design and are presented as a reoccurring theme throughout the book. This chapter moves towards a more engineering style exposition of neural networks, applying concepts in the previous chapters to elucidate various model design choices.

## ***Chapter 5***

Chapter 5 presents a method for interpreting neural networks which imposes minimal restrictions on the neural network design. The chapter demonstrates techniques for interpreting a feedforward network, including how to rank the importance of the features. In particular, an example demonstrating how to apply interpretability analysis to deep learning models for factor modeling is also presented.

## ***Chapter 6***

Chapter 6 provides an overview of the most important modeling concepts in financial econometrics. Such methods form the conceptual basis and performance baseline for more advanced neural network architectures presented in the next chapter. In fact, each type of architecture is a generalization of many of the models presented here. This chapter is especially useful for students from an engineering or science background, with little exposure to econometrics and time series analysis.

## ***Chapter 7***

Chapter 7 presents a powerful class of probabilistic models for financial data. Many of these models overcome some of the severe stationarity limitations of the frequentist models in the previous chapters. The fitting procedure demonstrated is also different—the use of Kalman filtering algorithms for state-space models rather

than maximum likelihood estimation or Bayesian inference. Simple examples of hidden Markov models and particle filters in finance and various algorithms are presented.

## ***Chapter 8***

Chapter 8 presents various neural network models for financial time series analysis, providing examples of how they relate to well-known techniques in financial econometrics. Recurrent neural networks (RNNs) are presented as non-linear time series models and generalize classical linear time series models such as  $AR(p)$ . They provide a powerful approach for prediction in financial time series and generalize to non-stationary data. The chapter also presents convolution neural networks for filtering time series data and exploiting different scales in the data. Finally, this chapter demonstrates how autoencoders are used to compress information and generalize principal component analysis.

## ***Chapter 9***

Chapter 9 introduces Markov decision processes and the classical methods of dynamic programming, before building familiarity with the ideas of reinforcement learning and other approximate methods for solving MDPs. After describing Bellman optimality and iterative value and policy updates before moving to Q-learning, the chapter quickly advances towards a more engineering style exposition of the topic, covering key computational concepts such as greediness, batch learning, and Q-learning. Through a number of mini-case studies, the chapter provides insight into how RL is applied to optimization problems in asset management and trading. These examples are each supported with Python notebooks.

## ***Chapter 10***

Chapter 10 considers real-world applications of reinforcement learning in finance, as well as further advances the theory presented in the previous chapter. We start with one of the most common problems of quantitative finance, which is the problem of optimal portfolio trading in discrete time. Many practical problems of trading or risk management amount to different forms of dynamic portfolio optimization, with different optimization criteria, portfolio composition, and constraints. The chapter introduces a reinforcement learning approach to option pricing that generalizes the classical Black–Scholes model to a data-driven approach using Q-learning. It then presents a probabilistic extension of Q-learning called G-learning and shows how it

can be used for dynamic portfolio optimization. For certain specifications of reward functions, G-learning is semi-analytically tractable and amounts to a probabilistic version of linear quadratic regulators (LQRs). Detailed analyses of such cases are presented and we show their solutions with examples from problems of dynamic portfolio optimization and wealth management.

## ***Chapter 11***

Chapter 11 provides an overview of the most popular methods of inverse reinforcement learning (IRL) and imitation learning (IL). These methods solve the problem of optimal control in a data-driven way, similarly to reinforcement learning, however with the critical difference that now rewards are *not* observed. The problem is rather to learn the reward function from the observed behavior of an agent. As behavioral data without rewards are widely available, the problem of learning from such data is certainly very interesting. The chapter provides a moderate-level description of the most promising IRL methods, equips the reader with sufficient knowledge to understand and follow the current literature on IRL, and presents examples that use simple simulated environments to see how these methods perform when we know the “ground truth” rewards. We then present use cases for IRL in quantitative finance that include applications to trading strategy identification, sentiment-based trading, option pricing, inference of portfolio investors, and market modeling.

## ***Chapter 12***

Chapter 12 takes us forward to emerging research topics in quantitative finance and machine learning. Among many interesting emerging topics, we focus here on two broad themes. The first one deals with unification of supervised learning and reinforcement learning as two tasks of perception-action cycles of agents. We outline some recent research ideas in the literature including in particular information theory-based versions of reinforcement learning and discuss their relevance for financial applications. We explain why these ideas might have interesting practical implications for RL financial models, where feature selection could be done within the general task of optimization of a long-term objective, rather than outside of it, as is usually performed in “alpha-research.”

The second topic presented in this chapter deals with using methods of reinforcement learning to construct models of market dynamics. We also introduce some advanced physics-based approaches for computations for such RL-inspired market models.

## *Source Code*

Many of the chapters are accompanied by Python notebooks to illustrate some of the main concepts and demonstrate application of machine learning methods. Each notebook is lightly annotated. Many of these notebooks use TensorFlow. We recommend loading these notebooks, together with any accompanying Python source files and data, in Google Colab. Please see the appendices of each chapter accompanied by notebooks, and the `README.md` in the subfolder of each chapter, for further instructions and details.

## *Scope*

We recognize that the field of machine learning is developing rapidly and to keep abreast of the research in this field is a challenging pursuit. Machine learning is an umbrella term for a number of methodology classes, including supervised learning, unsupervised learning, and reinforcement learning. This book focuses on supervised learning and reinforcement learning because these are the areas with the most overlap with econometrics, predictive modeling, and optimal control in finance. Supervised machine learning can be categorized as generative and discriminative. Our focus is on discriminative learners which attempt to partition the input space, either directly, through affine transformations or through projections onto a manifold. Neural networks have been shown to provide a universal approximation to a wide class of functions. Moreover, they can be shown to reduce to other well-known statistical techniques and are adaptable to time series data.

Extending time series models, a number of chapters in this book are devoted to an introduction to reinforcement learning (RL) and inverse reinforcement learning (IRL) that deal with problems of optimal control of such time series and show how many classical financial problems such as portfolio optimization, option pricing, and wealth management can naturally be posed as problems for RL and IRL. We present simple RL methods that can be applied for these problems, as well as explain how neural networks can be used in these applications.

There are already several excellent textbooks covering other classical machine learning methods, and we instead choose to focus on how to cast machine learning into various financial modeling and decision frameworks. We emphasize that much of this material is not unique to neural networks, but comparisons of alternative supervised learning approaches, such as random forests, are beyond the scope of this book.

## ***Multiple-Choice Questions***

Multiple-choice questions are included after introducing a key concept. The correct answers to all questions are provided at the end of each chapter with selected, partial, explanations to some of the more challenging material.

## ***Exercises***

The exercises that appear at the end of every chapter form an important component of the book. Each exercise has been chosen to reinforce concepts explained in the text, to stimulate the application of machine learning in finance, and to gently bridge material in other chapters. It is graded according to difficulty ranging from (\*), which denotes a simple exercise which might take a few minutes to complete, through to (\*\*\*), which denotes a significantly more complex exercise. Unless specified otherwise, all equations referenced in each exercise correspond to those in the corresponding chapter.

## **Instructor Materials**

The book is supplemented by a separate Instructor's Manual which provides worked solutions to the end of chapter questions. Full explanations for the solutions to the multiple-choice questions are also provided. The manual provides additional notes and example code solutions for some of the programming exercises in the later chapters.

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# Contents

## Part I Machine Learning with Cross-Sectional Data

<b>1</b>	<b>Introduction</b>	3
1	Background	3
1.1	Big Data—Big Compute in Finance	4
1.2	Fintech	6
2	Machine Learning and Prediction	8
2.1	Entropy	11
2.2	Neural Networks	14
3	Statistical Modeling vs. Machine Learning	16
3.1	Modeling Paradigms	16
3.2	Financial Econometrics and Machine Learning	18
3.3	Over-fitting	21
4	Reinforcement Learning	22
5	Examples of Supervised Machine Learning in Practice	28
5.1	Algorithmic Trading	29
5.2	High-Frequency Trade Execution	32
5.3	Mortgage Modeling	34
6	Summary	40
7	Exercises	41
	References	44
<b>2</b>	<b>Probabilistic Modeling</b>	47
1	Introduction	47
2	Bayesian vs. Frequentist Estimation	48
3	Frequentist Inference from Data	51
4	Assessing the Quality of Our Estimator: Bias and Variance	53
5	The Bias–Variance Tradeoff (Dilemma) for Estimators	55
6	Bayesian Inference from Data	56
6.1	A More Informative Prior: The Beta Distribution	60
6.2	Sequential Bayesian updates	61

6.3	Practical Implications of Choosing a Classical or Bayesian Estimation Framework .....	63
7	Model Selection .....	63
7.1	Bayesian Inference .....	64
7.2	Model Selection .....	65
7.3	Model Selection When There Are Many Models .....	66
7.4	Occam's Razor .....	69
7.5	Model Averaging .....	69
8	Probabilistic Graphical Models .....	70
8.1	Mixture Models .....	72
9	Summary .....	76
10	Exercises .....	76
	References .....	80
<b>3</b>	<b>Bayesian Regression and Gaussian Processes .....</b>	<b>81</b>
1	Introduction .....	81
2	Bayesian Inference with Linear Regression .....	82
2.1	Maximum Likelihood Estimation .....	86
2.2	Bayesian Prediction .....	88
2.3	Schur Identity .....	89
3	Gaussian Process Regression .....	91
3.1	Gaussian Processes in Finance .....	92
3.2	Gaussian Processes Regression and Prediction .....	93
3.3	Hyperparameter Tuning .....	94
3.4	Computational Properties .....	96
4	Massively Scalable Gaussian Processes .....	96
4.1	Structured Kernel Interpolation (SKI) .....	97
4.2	Kernel Approximations .....	97
5	Example: Pricing and Greeking with Single-GPs .....	98
5.1	Greeking .....	101
5.2	Mesh-Free GPs .....	101
5.3	Massively Scalable GPs .....	103
6	Multi-response Gaussian Processes .....	103
6.1	Multi-Output Gaussian Process Regression and Prediction .....	104
7	Summary .....	105
8	Exercises .....	106
8.1	Programming Related Questions* .....	107
	References .....	108
<b>4</b>	<b>Feedforward Neural Networks .....</b>	<b>111</b>
1	Introduction .....	111
2	Feedforward Architectures .....	112
2.1	Preliminaries .....	112
2.2	Geometric Interpretation of Feedforward Networks .....	114
2.3	Probabilistic Reasoning .....	117



2.4	Function Approximation with Deep Learning*	119
2.5	VC Dimension	120
2.6	When Is a Neural Network a Spline?*	124
2.7	Why Deep Networks?	127
3	Convexity and Inequality Constraints	132
3.1	Similarity of MLPs with Other Supervised Learners	138
4	Training, Validation, and Testing	140
5	Stochastic Gradient Descent (SGD)	142
5.1	Back-Propagation	143
5.2	Momentum	146
6	Bayesian Neural Networks*	149
7	Summary	153
8	Exercises	153
8.1	Programming Related Questions*	156
	References	164
<b>5</b>	<b>Interpretability</b>	167
1	Introduction	167
2	Background on Interpretability	168
2.1	Sensitivities	168
3	Explanatory Power of Neural Networks	169
3.1	Multiple Hidden Layers	170
3.2	Example: Step Test	170
4	Interaction Effects	170
4.1	Example: Friedman Data	171
5	Bounds on the Variance of the Jacobian	172
5.1	Chernoff Bounds	174
5.2	Simulated Example	174
6	Factor Modeling	177
6.1	Non-linear Factor Models	177
6.2	Fundamental Factor Modeling	178
7	Summary	183
8	Exercises	184
8.1	Programming Related Questions*	184
	References	188

## Part II Sequential Learning

<b>6</b>	<b>Sequence Modeling</b>	191
1	Introduction	191
2	Autoregressive Modeling	192
2.1	Preliminaries	192
2.2	Autoregressive Processes	194
2.3	Stability	195
2.4	Stationarity	195
2.5	Partial Autocorrelations	197

2.6	Maximum Likelihood Estimation .....	199
2.7	Heteroscedasticity .....	200
2.8	Moving Average Processes .....	201
2.9	GARCH .....	202
2.10	Exponential Smoothing.....	204
3	Fitting Time Series Models: The Box–Jenkins Approach .....	205
3.1	Stationarity .....	205
3.2	Transformation to Ensure Stationarity .....	206
3.3	Identification .....	206
3.4	Model Diagnostics .....	208
4	Prediction .....	210
4.1	Predicting Events .....	210
4.2	Time Series Cross-Validation .....	213
5	Principal Component Analysis .....	213
5.1	Principal Component Projection .....	215
5.2	Dimensionality Reduction.....	216
6	Summary.....	217
7	Exercises .....	218
	Reference.....	220
<b>7</b>	<b>Probabilistic Sequence Modeling .....</b>	<b>221</b>
1	Introduction .....	221
2	Hidden Markov Modeling .....	222
2.1	The Viterbi Algorithm .....	224
2.2	State-Space Models.....	227
3	Particle Filtering .....	227
3.1	Sequential Importance Resampling (SIR) .....	228
3.2	Multinomial Resampling .....	229
3.3	Application: Stochastic Volatility Models .....	230
4	Point Calibration of Stochastic Filters.....	231
5	Bayesian Calibration of Stochastic Filters .....	233
6	Summary.....	235
7	Exercises .....	235
	References .....	237
<b>8</b>	<b>Advanced Neural Networks .....</b>	<b>239</b>
1	Introduction .....	239
2	Recurrent Neural Networks .....	240
2.1	RNN Memory: Partial Autocovariance .....	244
2.2	Stability .....	245
2.3	Stationarity .....	246
2.4	Generalized Recurrent Neural Networks (GRNNs).....	248
3	Gated Recurrent Units.....	249
3.1	$\alpha$ -RNNs .....	249
3.2	Neural Network Exponential Smoothing .....	251
3.3	Long Short-Term Memory (LSTM) .....	254

4	Python Notebook Examples .....	255
4.1	Bitcoin Prediction.....	256
4.2	Predicting from the Limit Order Book.....	256
5	Convolutional Neural Networks .....	257
5.1	Weighted Moving Average Smoothers .....	258
5.2	2D Convolution .....	261
5.3	Pooling .....	263
5.4	Dilated Convolution .....	264
5.5	Python Notebooks .....	265
6	Autoencoders .....	266
6.1	Linear Autoencoders.....	267
6.2	Equivalence of Linear Autoencoders and PCA .....	268
6.3	Deep Autoencoders .....	270
7	Summary.....	271
8	Exercises .....	272
8.1	Programming Related Questions* .....	273
	References.....	275

### Part III Sequential Data with Decision-Making

9	Introduction to Reinforcement Learning .....	279
1	Introduction .....	279
2	Elements of Reinforcement Learning .....	284
2.1	Rewards .....	284
2.2	Value and Policy Functions .....	286
2.3	Observable Versus Partially Observable Environments .....	286
3	Markov Decision Processes .....	289
3.1	Decision Policies.....	291
3.2	Value Functions and Bellman Equations .....	293
3.3	Optimal Policy and Bellman Optimality.....	296
4	Dynamic Programming Methods .....	299
4.1	Policy Evaluation .....	300
4.2	Policy Iteration.....	302
4.3	Value Iteration .....	303
5	Reinforcement Learning Methods .....	306
5.1	Monte Carlo Methods .....	307
5.2	Policy-Based Learning .....	309
5.3	Temporal Difference Learning .....	311
5.4	SARSA and Q-Learning.....	313
5.5	Stochastic Approximations and Batch-Mode Q-learning .....	316
5.6	Q-learning in a Continuous Space: Function Approximation .....	323
5.7	Batch-Mode Q-Learning .....	327
5.8	Least Squares Policy Iteration.....	331
5.9	Deep Reinforcement Learning .....	335

6	Summary .....	337
7	Exercises .....	337
	References .....	345
<b>10</b>	<b>Applications of Reinforcement Learning .....</b>	<b>347</b>
1	Introduction .....	347
2	The QLBS Model for Option Pricing .....	349
3	Discrete-Time Black–Scholes–Merton Model .....	352
3.1	Hedge Portfolio Evaluation .....	352
3.2	Optimal Hedging Strategy .....	354
3.3	Option Pricing in Discrete Time .....	356
3.4	Hedging and Pricing in the BS Limit .....	359
4	The QLBS Model .....	360
4.1	State Variables .....	361
4.2	Bellman Equations .....	362
4.3	Optimal Policy .....	365
4.4	DP Solution: Monte Carlo Implementation .....	368
4.5	RL Solution for QLBS: Fitted Q Iteration .....	370
4.6	Examples .....	373
4.7	Option Portfolios .....	375
4.8	Possible Extensions .....	379
5	G-Learning for Stock Portfolios .....	380
5.1	Introduction .....	380
5.2	Investment Portfolio .....	381
5.3	Terminal Condition .....	382
5.4	Asset Returns Model .....	383
5.5	Signal Dynamics and State Space .....	383
5.6	One-Period Rewards .....	384
5.7	Multi-period Portfolio Optimization .....	386
5.8	Stochastic Policy .....	386
5.9	Reference Policy .....	388
5.10	Bellman Optimality Equation .....	388
5.11	Entropy-Regularized Bellman Optimality Equation .....	389
5.12	G-Function: An Entropy-Regularized Q-Function .....	391
5.13	G-Learning and F-Learning .....	393
5.14	Portfolio Dynamics with Market Impact .....	395
5.15	Zero Friction Limit: LQR with Entropy Regularization .....	396
5.16	Non-zero Market Impact: Non-linear Dynamics .....	400
6	RL for Wealth Management .....	401
6.1	The Merton Consumption Problem .....	401
6.2	Portfolio Optimization for a Defined Contribution Retirement Plan .....	405
6.3	G-Learning for Retirement Plan Optimization .....	408
6.4	Discussion .....	413
7	Summary .....	413

8	Exercises .....	414
	References .....	416
<b>11</b>	<b>Inverse Reinforcement Learning and Imitation Learning .....</b>	<b>419</b>
1	Introduction .....	419
2	Inverse Reinforcement Learning .....	423
2.1	RL Versus IRL .....	425
2.2	What Are the Criteria for Success in IRL? .....	426
2.3	Can a Truly Portable Reward Function Be Learned with IRL? .....	427
3	Maximum Entropy Inverse Reinforcement Learning .....	428
3.1	Maximum Entropy Principle .....	430
3.2	Maximum Causal Entropy .....	433
3.3	G-Learning and Soft Q-Learning .....	436
3.4	Maximum Entropy IRL .....	438
3.5	Estimating the Partition Function .....	442
4	Example: MaxEnt IRL for Inference of Customer Preferences .....	443
4.1	IRL and the Problem of Customer Choice .....	444
4.2	Customer Utility Function .....	445
4.3	Maximum Entropy IRL for Customer Utility .....	446
4.4	How Much Data Is Needed? IRL and Observational Noise .....	450
4.5	Counterfactual Simulations .....	452
4.6	Finite-Sample Properties of MLE Estimators .....	454
4.7	Discussion .....	455
5	Adversarial Imitation Learning and IRL .....	457
5.1	Imitation Learning .....	457
5.2	GAIL: Generative Adversarial Imitation Learning .....	459
5.3	GAIL as an Art of Bypassing RL in IRL .....	461
5.4	Practical Regularization in GAIL .....	464
5.5	Adversarial Training in GAIL .....	466
5.6	Other Adversarial Approaches* .....	468
5.7	f-Divergence Training* .....	468
5.8	Wasserstein GAN* .....	469
5.9	Least Squares GAN* .....	471
6	Beyond GAIL: AIRL, f-MAX, FAIRL, RS-GAIL, etc.* .....	471
6.1	AIRL: Adversarial Inverse Reinforcement Learning .....	472
6.2	Forward KL or Backward KL? .....	474
6.3	f-MAX .....	476
6.4	Forward KL: FAIRL .....	477
6.5	Risk-Sensitive GAIL (RS-GAIL) .....	479
6.6	Summary .....	481
7	Gaussian Process Inverse Reinforcement Learning .....	481
7.1	Bayesian IRL .....	482
7.2	Gaussian Process IRL .....	483

8	Can IRL Surpass the Teacher? .....	484
8.1	IRL from Failure .....	485
8.2	Learning Preferences .....	487
8.3	T-REX: Trajectory-Ranked Reward EXtrapolation .....	488
8.4	D-REX: Disturbance-Based Reward EXtrapolation .....	490
9	Let Us Try It Out: IRL for Financial Cliff Walking .....	490
9.1	Max-Causal Entropy IRL.....	491
9.2	IRL from Failure .....	492
9.3	T-REX .....	493
9.4	Summary .....	494
10	Financial Applications of IRL .....	495
10.1	Algorithmic Trading Strategy Identification.....	495
10.2	Inverse Reinforcement Learning for Option Pricing .....	497
10.3	IRL of a Portfolio Investor with G-Learning .....	499
10.4	IRL and Reward Learning for Sentiment-Based Trading Strategies.....	504
10.5	IRL and the “Invisible Hand” Inference .....	505
11	Summary.....	512
12	Exercises .....	513
	References.....	515
<b>12</b>	<b>Frontiers of Machine Learning and Finance .....</b>	<b>519</b>
1	Introduction .....	519
2	Market Dynamics, IRL, and Physics .....	521
2.1	“Quantum Equilibrium–Disequilibrium” (QED) Model.....	522
2.2	The Langevin Equation .....	523
2.3	The GBM Model as the Langevin Equation .....	524
2.4	The QED Model as the Langevin Equation .....	525
2.5	Insights for Financial Modeling .....	527
2.6	Insights for Machine Learning .....	528
3	Physics and Machine Learning .....	529
3.1	Hierarchical Representations in Deep Learning and Physics .....	529
3.2	Tensor Networks .....	530
3.3	Bounded-Rational Agents in a Non-equilibrium Environment .....	534
4	A “Grand Unification” of Machine Learning? .....	535
4.1	Perception-Action Cycles .....	537
4.2	Information Theory Meets Reinforcement Learning.....	538
4.3	Reinforcement Learning Meets Supervised Learning: Predictron, MuZero, and Other New Ideas .....	539
	References.....	540
	<b>Index.....</b>	<b>543</b>

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