Examples of VCG Mechanism

Lecture 10.3

Example 1: (Vickrey auction for selling Multiple Identical Objects)

- 3 identical items with one seller
- 5 buyers each wanting to buy one item.
- We use any allocatively efficient allocation rule.

Set of allocations $(\mathcal{X}) = \{(1,1,1,0,0), (1,0,1,1,0), \ldots\}$

allocation rule: choose three buyers having highest valuation

We will use Clarke's payment rule. Since the allocation rule is allocatively efficient, we know due to <u>Grove's Theorem</u> that the resulting mechanism is dominant strategy incentive compatible. This allows us to assume wlog that the buyers report their true valuation of the item to the mechanism designer.

- The valuations of the item to the players are 20, 15, 12, 10, 8.

The allocation chosen : (1,1,1,0,0)

The valuation of (1,1,1,0,0) to player 1 is 20

〃 2 is 15

〃 3 is 12

〃 4 is 0

〃 5 is 0

Sum of valuations of (1,1,1,0,0) is $20+15+12 = 47$.

Payments:

Payment received by player 1 $= (15+12) - (15+12+10) = -10$

$\text{———〃———}\ 2 = (20+12) - (20+12+10) = -10$

$\text{———〃———}\ 3 = (20+15) - (20+15+10) = -10$

$\text{———〃———}\ 4 = (20+15+10) - (20+15+10) = 0$

$\text{———〃———}\ 5 = (20+15+10) - (20+15+10) = 0$

Vickrey discount to player 1 $= 20 - 10 = 10$

———"——— 2 $= 15 - 10 = 5$

———"——— 3 $= 12 - 10 = 2$

———"——— 4 $= 0 - 0 = 0$

———"——— 5 $= 0 - 0 = 0$.

Example 2: (Combinatorial Auction)

- One seller having two items, say A and B.
- Three buyers having the following valuations:

	{A}	{B}	{A, B}
Player 1	*	*	12
Player 2	5	*	*
Player 3	*	4	*

Allocatively efficient allocation : {A, B} to player 1.

Payment received by player 1 $= (0+0) - (5+4) = -9$

———————— 2 $= (12+0) - (12+0) = 0$

———————— 3 $= (12+0) - (12+0) = 0.$

Vickrey discount to player 1 $= 12 - 9 = 3.$

—————〃————— $2 = 0 - 0 = 0$

—————〃————— $3 = 0 - 0 = 0.$