Lecture 9.3

Main take-away from revelation principle:

"the search for indirect mechanisms to implement a social choice function is trivial"

Next big question of mechanism design: characterize the set of all social choice functions which are implementable in DSE / BNE.

Useful Properties of Social Choice Functions

(1) Ex-post efficiency / Pareto optimality / Efficiency :-

A social choice function $f: \underset{i\in[n]}{X} \Theta_i \longrightarrow \mathcal{X}$ is called ex-post efficient if, for every type profile, the out-come chosen is pareto-optimal. That is, there does not exist any outcome $x \in \mathcal{X}$ such that

$\forall (\theta_1, \ldots, \theta_n) \in \underset{i\in[n]}{X} \Theta_i$:

$u_i(x, (\theta_1, \ldots, \theta_n)) \geq u_i(f(\theta_1, \ldots, \theta_n), (\theta_1, \ldots, \theta_n)) \quad \forall i \in [n]$

and there exists a player $j \in [n]$ such that

$u_j(x, (\theta_1, \ldots, \theta_n)) > u_j(f(\theta_1, \ldots, \theta_n), (\theta_1, \ldots, \theta_n))$.

(2) **Non-Dictatorship:** A player $d \in [n]$ is called a dictator if $\forall (\theta_1, \ldots, \theta_n) \in \bigtimes_{i=1}^{n} \Theta_i$

$$u_d(f(\theta_1, \ldots, \theta_n), (\theta_1, \ldots, \theta_n)) \geq u_d(x, (\theta_1, \ldots, \theta_n)) \quad \forall x \in X$$

A social choice function is called a dictatorship

if there exists a dictator. Otherwise the social choice function is called non-dictatorship.

(3) **Individual Rationality**: Let $\bar{u}_i(\theta_i)$ be the utility of player i when it does not parcipate in the mechanism and its type is $\theta_i \in \Theta_i$.

(a) **Ex-post Individual Rationality**: A social choice function f is called ex-post IR if

$\forall\, i \in [n]$,

$$u_i\Big(f(\theta_i, \underline{\theta}_i), (\theta_1, \dots, \theta_n)\Big) \geq \bar{u}_i(\theta_i) \quad \forall\, (\theta_i, \underline{\theta}_i) \in \Theta$$

(b) **Interim Individual Rationality:** A social choice function f is called interim IR if

$\forall\, i \in [n]$, $\forall\, \theta_i \in \Theta_i$

$$\mathbb{E}\Big[\, u_i\big(f(\theta_i, \underline{\theta}_i), (\theta_1, \dots, \theta_n)\big) \,\Big|\, \theta_i \Big] \geq \bar{u}_i(\theta_i) \quad \forall\, \theta_i \in \Theta_i$$

Prior $\underline{\underline{p}} \in \Delta\Big(\underset{i=1}{\overset{n}{\times}} \Theta_i\Big)$

(c) Ex-Ante Individual Rationality: A social choice function f is ex-ante IR if

$$\forall i \in [n], \quad \mathbb{E}\left[u_i\left(f(\theta_i, \underline{\theta}_{-i}), (\theta_i, \underline{\theta}_{-i})\right)\right] \geq \mathbb{E}\left[\bar{u}_i(\theta_i)\right]$$

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