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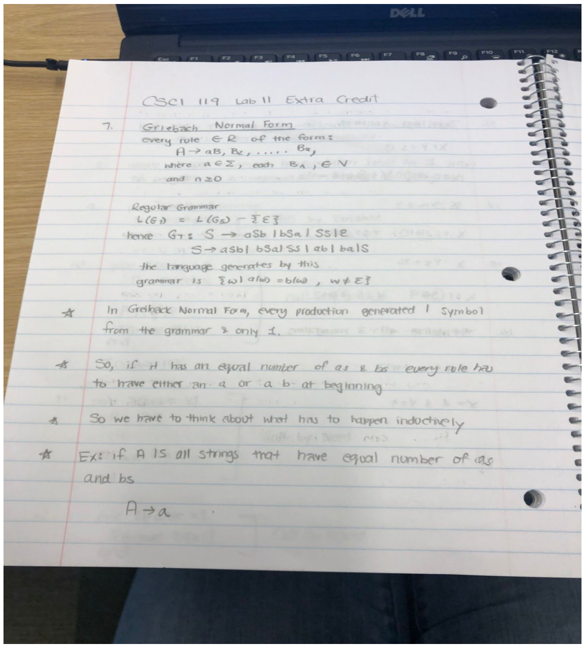
CSCI 119 Lab 11 Extra Credit

7.



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7. Griebach Normal Form
every rule $\in R$ of the form:
$A \to aB_1 B_2, \ldots B_n,$
where $a \in \Sigma$, each $B_i \in V$
and $n \geq 0$

Regular Grammar
$L(G_7) = L(G_5) - \{\varepsilon\}$
hence G_7: $S \to aSb \mid bSa \mid SS \mid \varepsilon$
$S \to aSb \mid bSa \mid SS \mid ab \mid ba \mid S$
the language generates by this grammar is $\{w \mid a(w) = b(w),\ w \neq \varepsilon\}$

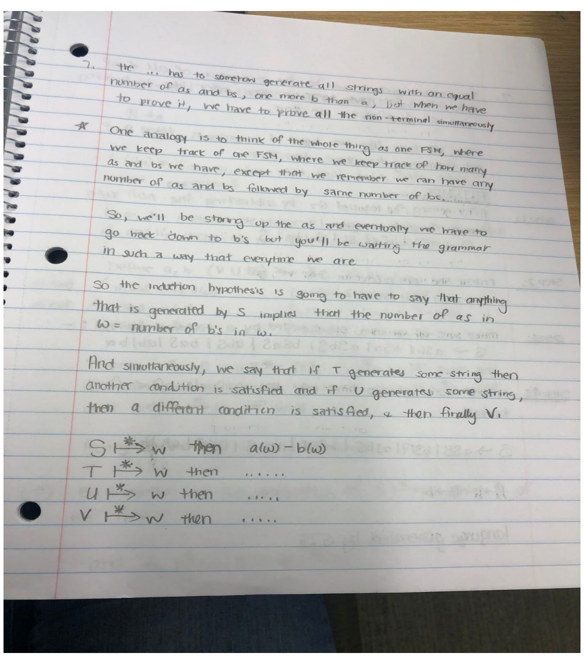
☆ In Greiback Normal Form, every production generated 1 symbol from the grammar & only 1.

☆ So, if it has an equal number of as & bs every rule has to have either an a or a b at beginning

☆ So we have to think about what has to happen inductively

☆ Ex: if A is all strings that have equal number of as and bs

$A \to a$



7. the .. has to somehow generate all strings with an equal number of as and bs, one more b than a, but when we have to prove it, we have to prove all the non-terminal simultaneously

☆ One analogy is to think of the whole thing as one FSM, where we keep track of one FSM, where we keep track of how many as and bs we have, except that we remember we can have any number of as and bs followed by same number of bs.

So, we'll be storing up the as and eventually we have to go back down to b's but you'll be waiting the grammar in such a way that everytime we are

So the induction hypothesis is going to have to say that anything that is generated by S implies that the number of as in w = number of b's in w.

And simultaneously, we say that if T generates some string then another condition is satisfied and if U generates some string, then a different condition is satisfied, & then finally V.

$S \overset{*}{\vdash\!\!\to} w$ then $a(w) - b(w)$
$T \overset{*}{\vdash\!\!\to} w$ then ......
$U \overset{*}{\vdash\!\!\to} w$ then .....
$V \overset{*}{\vdash\!\!\to} w$ then .....

7. The induction hypothesis will be the conjugation of all 4 of these and we'll have to prove that after the induction step all 4 of these are true.

So, ~~it~~ looks like we have 4 invariants that we are maintaining throughout derivation.

ATTEMPT

Step 1: After getting the required G_7 by subtracting the null rule from G_5 we get

$S \to aSb \mid bSa \mid SS \mid ab \mid ba \mid S$

Step 2: Remove the unit production $S \to S$ we get

$S \to aSb \mid bSa \mid SS \mid ab \mid ba$

Step 3: make sure all variables are preceded by a terminal symbol : –

$S \to aSb \mid bSa \mid aSbS \mid bSaS \mid abS \mid baS \mid ab \mid ba$

Step 4: make sure all rules have only non-terminals after first terminals

$S \to aSB \mid bSA \mid aSBS \mid bSAS \mid aBS \mid bAS \mid aB \mid bA$

$A \to a \quad B \to b$

language generated by G7:

7. $S \to aSB \mid bSA \mid aSBS \mid bSAS \mid aBS \mid bAS \mid aB \mid bA$

$A \to a$
$B \to b$

$L(G_7) = \{ab, ba, abab, abba, baab, baba, aabb, bbaa, aaabbb\}$

Here, the grammar describes all strings with equal number of a's and b's except the one with no a's and b's.

Define $a, b\ (V \cup \Sigma)^* - \{\varepsilon\} \to \mathbb{N}$ by

$a(Aw)$ / $a(aw) = 1 + a(w)$	$b(Aw)$ / $b(aw) = b(w)$
$a(Bw)$ / $a(bw) = a(w)$	$b(Bw)$ / $b(bw) = 1 + b(w)$
$a(Sw) = a(w)$	$b(Sw) = b(w)$

Theorem: for all $w \in \Sigma^* - \{\varepsilon\}$, $S \overset{*}{\mapsto} w$ iff there are an equal number of a's and b's in w and $A \overset{*}{\mapsto} w$ iff $w = a$ and $B \overset{*}{\mapsto} w$ iff $w = b$

Proving Biconditional in both directions: –

$(\to)$ By induction on the derivation of $S \overset{*}{\mapsto} w$ which is of the form;

and $A \overset{*}{\mapsto} w$ and $B \overset{*}{\mapsto} w$

7. $A, B\ S \mapsto \omega_1 \mapsto \omega_2 \mapsto \ldots\ldots \mapsto \omega$

But the induction will fail because $A, B, S, \omega_1, \omega_2, \ldots \in (V \cup \Sigma)^* - \{\varepsilon\}$

Hence, we have to expand the domain of ω to $(V \cup \Sigma)^* - \{\varepsilon\}$ so we can use the IH on all strings intermediate to the production of ω.

Hence, the relations a, b have been defined from $(V \cup \Sigma)^* - \{\varepsilon\} \to \mathbb{N}$ and they satisfy the identities

$$(A+) \Rightarrow a(u \circ v) = a(u) + a(v)$$

$$(B+) \Rightarrow b(u \circ v) = b(u) + b(v)$$

<u>Lemma 1:</u>

Take all $\omega \in (V \cup \Sigma)^* - \{\varepsilon\}$ if $S \overset{*}{\mapsto} \omega$ and $A \overset{*}{\mapsto} \omega$ and $B \overset{*}{\mapsto} \omega$ then $a(\omega) = b(\omega)$ if $S \overset{*}{\mapsto} \omega$ and $\omega = a$ if $A \overset{*}{\mapsto} \omega$ and $\omega = b$ if $B \overset{*}{\mapsto} \omega$

Define multi-step $\overset{*}{\mapsto}$ relation

$$\frac{\alpha \overset{*}{\mapsto} \beta \qquad \beta \mapsto \gamma}{\alpha \overset{*}{\mapsto} \gamma}$$

Proof: By induction on the derivations of $S \overset{*}{\mapsto} \omega$, $A \overset{*}{\mapsto} \omega$ and $B \overset{*}{\mapsto} \omega$

<u>Case S1:</u> $\omega = S \vee \omega = A \vee \omega = B$

if $\omega = S$ then

$a(\omega) = a(S) = 0 = b(S) = b(\omega)$

hence $\omega =$ satisfies the consequent

<u>Case S2:</u> $S \overset{*}{\mapsto} \omega'$ and $\omega' \mapsto \omega$

$\vee\ A \mapsto \omega' \ \wedge\ \omega' \mapsto \omega$

$\vee\ B \mapsto \omega' \ \wedge\ \omega' \mapsto \omega$

Applying the IH on the smaller derivations $S \overset{*}{\mapsto} \omega'$, $A \overset{*}{\mapsto} \omega'$ $B \overset{*}{\mapsto} \omega'$ we get that they satisfy corresponding consequents.

<u>Goal:</u> ω satisfies the consequent as well in all 3 cases i.e. the consequent in all 3 cases

Consequent is a variant of the step relation in all 3 cases.

By the rule for the single-step relation $\mapsto$, there must exist a equation $V \to \gamma$ in G_7 and 2 strings $\alpha, \beta, \in (V \cup \Sigma)^* - \{\varepsilon\}$ such that $\omega' = \alpha V \beta$,

$\omega = \alpha V \beta$

We have 3 non-terminals in G_5 and it follows from IH on all 3 cases that respectively

if $S \overset{*}{\mapsto} \omega$ then $a(\omega') = b(\omega')$

$\Rightarrow a(\alpha V \beta) = b(\alpha V \beta)$

$\Rightarrow a(\alpha S \beta) = b(\alpha S \beta)$

7. But (A+) and (B+) state: –

$$a(\alpha \cdot S\beta) = a(\alpha) + a(S\beta)$$
$$= a(\alpha) + a(\beta)$$

and $b(\alpha \cdot S\beta) = b(\alpha) + b(S\beta)$
$$= b(\alpha) + b(\beta)$$

by associativity of concatenation

$(*) \rightarrow a(\alpha) + a(\beta) = b(\alpha) + b(\beta)$

and if $A \overset{*}{\mapsto} \omega'$, then $\omega' = a$

and if $B \overset{*}{\mapsto} \omega'$, then $\omega' = b$