Discrete Mathematics- Quiz 1

Question 1: Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Solution: Let P be the total number of possible passwords, and let P_6, P_7, and P_8 denote the number of possible passwords of length 6, 7, and 8, respectively. By the sum rule,

$$P = P_6 + P_7 + P_8.$$

We will now find P_6, P_7, and P_8. Finding P_6 directly is difficult. To find P_6 it is easier to find the number of strings of uppercase letters and digits that are six characters long, including those with no digits, and subtract from this the number of strings with no digits. By the product rule, the number of strings of six characters is 36^6, and the number of strings with no digits is 26^6. Hence,

$$P_6 = 36^6 - 26^6 = 2,176,782,336 - 308,915,776 = 1,867,866,560.$$

Similarly, we have

$$P_7 = 36^7 - 26^7 = 78,364,164,096 - 8,031,810,176 = 70,332,353,920$$

and

$$P_8 = 36^8 - 26^8 = 2,821,109,907,456 - 208,827,064,576 = 2,612,282,842,880.$$

Consequently,

$$P = P_6 + P_7 + P_8 = 2,684,483,063,360.$$

Question 2: Discuss ways in which the current telephone numbering plan can be extended to accommodate the rapid demand for more telephone numbers. For each new numbering plan you discuss, show how to find the number of different telephone numbers it supports.

Solution:

To accommodate the rapid increase in demand for telephone numbers, various extensions to the current telephone numbering plans can be considered. In the existing systems, telephone numbers typically consist of a fixed number of digits, with the format often being something like `(XXX) XXX-XXXX`, where `X` represents a digit.

We will explore a few options to extend the numbering plan.

1. Expanding the Length of Numbers

One way to accommodate more telephone numbers is to increase the length of the phone number. For example, if the current plan uses 10 digits (such as `(XXX) XXX-XXXX`), it could be extended to 11 or 12 digits.

Example:

If we extend the number to 11 digits, the total number of possible telephone numbers is calculated by:

$$\text{Total number of telephone numbers} = 10^{11} = 100,000,000,000.$$

Thus, 100 billion different telephone numbers could be supported in this plan.

2. Increasing the Range of Digits

Another option is to use a larger range of digits in the telephone number. Currently, telephone numbers are usually composed of digits 0 to 9, which provides a base-10 numbering system. By expanding to a base-16 system, for example, we could increase the number of available numbers per digit.

Example:

If we change each digit to be one of 16 possible characters (0-9 and A-F), and if the telephone number length remains 10 digits, the total number of different telephone numbers would be:

$$\text{Total number of telephone numbers} = 16^{10} = 1,099,511,627,776.$$

So, using base-16 digits, we could support over 1 trillion telephone numbers.

3. Area Code and Number Prefix Modification

A further modification involves changing the area code or the prefix used in the telephone number to create additional combinations. Currently, area codes and prefixes are often restricted to certain values, but relaxing these restrictions could allow for more combinations.

Example:

Suppose we have 3 digits for the area code and 7 digits for the local number. If we allow the area code to consist of any combination of 3 digits (using 10 choices per digit), and the local number to consist of 7 digits, the total number of possible telephone numbers would be:

$$\text{Total number of telephone numbers} = 10^3 \times 10^7 = 10^{10} = 10,000,000,000.$$

This approach offers the same number of possibilities as the previous 10-digit numbering plan but makes the area code flexible.

4. Using Alphanumeric Numbers

A more radical change could involve using alphanumeric telephone numbers, where both letters and numbers are part of the phone number. This is already in use in some systems, such as vanity phone numbers, where letters can be mapped to digits.

Example:

If we allow each character in the phone number to be a letter (A-Z) or a digit (0-9), this gives us a total of 36 choices for each character. Assuming a 10-digit phone number, the total number of possible combinations is:

$$\text{Total number of telephone numbers} = 36^{10} = 3,656,158,440,062,976.$$

This dramatically increases the number of possible telephone numbers, offering trillions of combinations.

Conclusion

Several methods can be employed to increase the number of available telephone numbers. By expanding the length of the number, increasing the base of the digits, modifying area codes and prefixes, or using alphanumeric combinations, we can greatly extend the available pool of telephone numbers. The choice of which strategy to use will depend on the specific needs and the feasibility of implementation.